BUDAPEST UNIVERSITY OF TECHNOLOGY AND ECONOMICS
FACULTY OF MECHANICAL ENGINEERING

Optimisation of compressor-driven heat pumps for building services

PhD theses booklet

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September, 2011, Budapest

1. STATE OF THE ART AND OBJECTIVES OF RESEARCH

My dissertation focuses on the energetic analysis and revision of sizing of compressor driven heat pumps. In Hungary 80% of the working heat pump systems are systems with U-tube installations. Having revised the problems of designing, operating and establishing heat pump systems I discovered that sufficient knowledge of united, integrated system's theory methods of design and operation of heat pump systems is missing both from the education and from the skills of designing and project engineers. Furthermore, there are not such methods, by which we could reliably calculate the amount of extractable heat capacity from U-tubes, the COP value of heat pumps in the function of both high and low temperature limits, the mass flow of the refrigerant, and the mass flow of the heating system. A method is missing, that could be used for energetic analysis of heating systems with heat pumps. Furthermore, we lack energetic and economic input-output model. Besides, such input-output model is missing, which would describe the operation of heating systems with heat pumps together with energy and material flows with the system of balance equations. Finally, we also lack a model, by which we could precisely calculate the necessary ground heat extraction and the COP value of the heat pump to a given heat demand. Or the other way around, an important question arises: by a given ground heat extraction what kind of heat demand can be fulfilled, considering of course the thermodynamic characteristics of consumers' appliances.

The objective of my dissertation was to construct the input – output energetic analysis method of heat pump systems installed with U-tubes, to describe the connections of each part with the help of a system of balance equations and their resolution, aiming to achieve in case of newly designed systems that to given designed heat demand we design a system, which provides minimal investment and operational costs. In case of working systems the aim was to operate the heat pump system at desired heat demand with minimal energy consumption, with maximum COP value and with minimal upkeep. My objective was to develop two

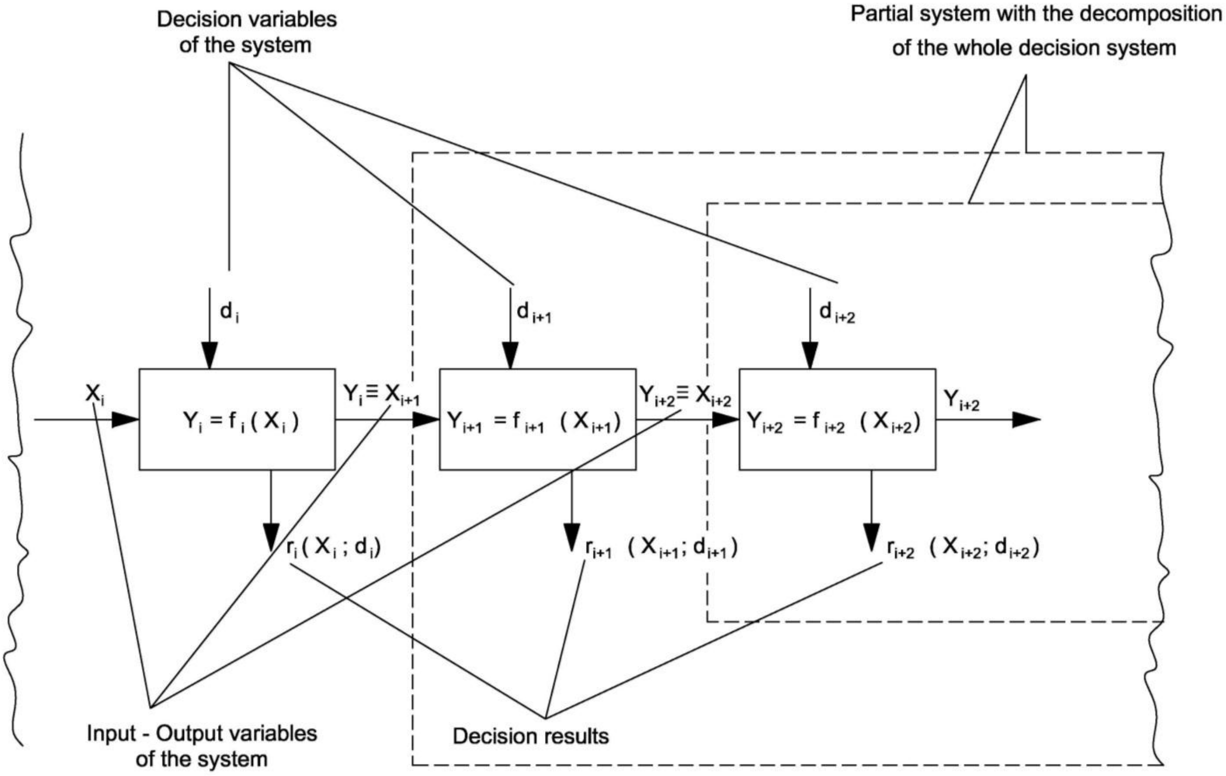
– in fact connected –methods: firstly that of the optimal designing and establishing parameters, and secondly that of the optimal method of the system's upkeep.

Having thoroughly studied the literature I found that in the input - output analysis of the system elements the U-tube is not yet a revealed element, whose input – output analysis, primarily the simulation of extractable heat capacity and its comparison with measured data in the function of the depth and technical development of the U-tube, ground characteristics and the circulated mass flow is under-researched. Therefore, I conducted analysis of the amount of extractable heat capacity from vertical U-tubes, for the mass flow of the circulated fluid in the function of time, outer months and spacing of U-tube legs. Even though more research has been conducted in this field, the obtained results are neither reliable, nor can they be generalised.

2. UTILISED METHODOLOGY

I utilise the so called white box model [Garbai, 2006], [Zadeh - Polak, 1972] for the input – output analysis and the description of the operation of heat pump systems with U-tube installation. In the white box model I divide the system into elements, and I depict the stage relationship of system elements with a graph. I make the system elements to be the junctions of the graph. I "enlarge" the junctions, which therefore become boxes. I write into the boxes the balance equations linking the inputs and the outputs, together with the so called decision. The graphical mode of the white box models is presented in Graph 1. I state the input and output variables of the elements, the decision variables and the decision results. By mathematical connection of balance equations of the system elements it is possible to conduct the uniform input – output analysis of the entire system and the optimisation, together with the minimising of the upkeep and set-up objective function.

In optimisation I utilise the method of dynamic programming and I refer to the work by Nemhauser [Nemhauser, 1966] and Bellmann [Bellmann, 1957]. Until today these authors provide the best summary of mathematical system theory applicable for mechanical systems. From the system theoretical model I expect a more comprehensible study of operation and investment costs of heat pump systems.



1. graph: Graphical mode of white box models [Garbai, 2006]

3. NEW SCIENTIFIC RESULTS

Thesis 1: [1], [2], [3], [4], [5], [6], [7].

I developed a calculation model for the modelling of temperature streaming in vertical U-tubes. I constructed that system of linked differential equations, by which I could calculate the warming of the circulating water and the total extractable heat capacity. I provide exact result for the system of linked differential equation.

- For the pipe leading underground from 0 m to 15 m:

$$\frac{dT_{pv}(H)}{dH}=\frac{s}{\dot{m}_p\cdot c_p}+\frac{1}{R_{overall}\cdot\dot{m}_p\cdot c_p}\cdot\left(G\cdot H^3+I\cdot H^2+E\cdot H+F-T_{pv}(H)\right)+ \\ +\frac{\dot{q}'}{\dot{m}_p\cdot c_p}, \tag{1}$$

where G, I, E, F are constants for the given months.

- For the upcoming pipe from 15 m to 0 m:

$$\frac{dT_{pe}(H)}{dH}=\frac{s}{\dot{m}_p\cdot c_p}+\frac{1}{R_{overall}\cdot\dot{m}_p\cdot c_p}\cdot \\ \left(G_1\cdot(15-H)^3+I_1\cdot(15-H)^2+E_1\cdot(15-H)+F_1-T_{pe}(H)\right)+\frac{\dot{q}'}{\dot{m}_p\cdot c_p}, \tag{2}$$

where G_1, I_1, E_1, F_1 are constants for the given months.

- For the pipe leading underground from 15 m to 100 m, presupposing linear heat change:

$$\frac{dT_{pv}(H)}{dH}=\frac{s}{\dot{m}_p\cdot c_p}+\frac{1}{R_{overall}\cdot\dot{m}_p\cdot c_p}\cdot\left(0{,}0706\cdot H+8{,}9412-T_{pv}(H)\right)+\frac{\dot{q}'}{\dot{m}_p\cdot c_p}. \tag{3}$$

- For the upcoming pipe from 100 m to 15 m, presupposing linear heat change:

$$\frac{dT_{pe}(H)}{dH}=\frac{s}{\dot{m}_p\cdot c_p}+\frac{1}{R_{overall}\cdot\dot{m}_p\cdot c_p}\cdot\left(-0{,}0706\cdot(100-H)+16-T_{pe}(H)\right)+\frac{\dot{q}'}{\dot{m}_p\cdot c_p}. \tag{4}$$

I introduced the concept of quasi-stationary heat resistance coefficient, which plays significant role in the differential equations, and I provided a formula for its calculation.

$$R_{overall}=R_{ground}+R_{grout}+R_{pipe}+R_{fluid}. \tag{5}$$

We can calculate the heat resistance of the ground by the following formula:

For small values of the Fourier (Fo) number we can state:

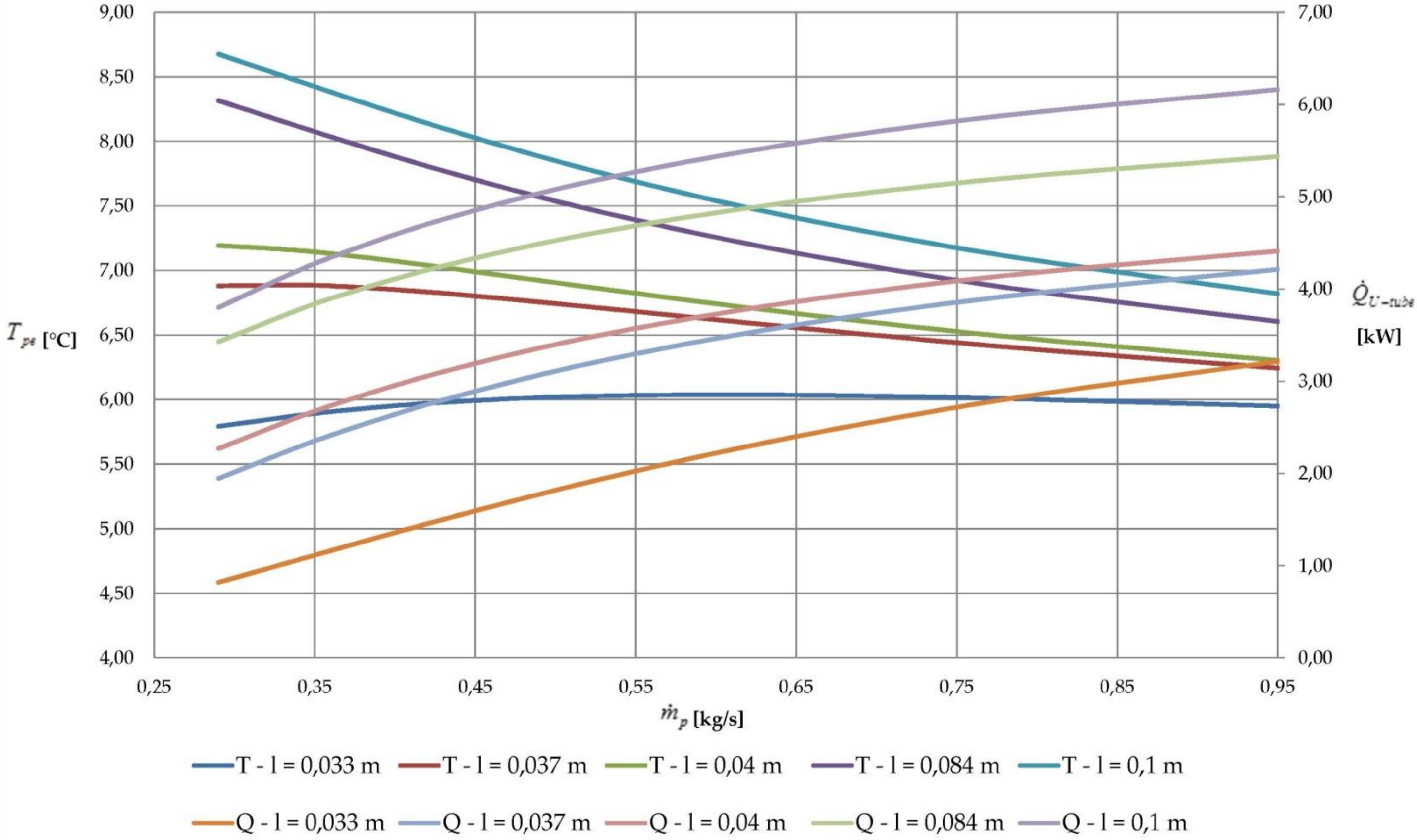
$$R_{ground}=\frac{r_0}{2\cdot\lambda_{ground}\cdot\left\{(\pi\cdot Fo)^{-\frac{1}{2}}+\frac{1}{2}-\frac{1}{4}\left(\frac{Fo}{\pi}\right)^{\frac{1}{2}}+\frac{1}{8}\cdot Fo...\right\}}, \tag{6}$$

and for the big values of the Fourier (Fo) number:

$$R_{ground} = \frac{r_0}{2 \cdot \lambda_{ground} \cdot \left\{ \frac{1}{\ln(4Fo) - 2\gamma} - \frac{\gamma}{[\ln(4Fo) - 2\gamma]^2} - .... \right\}} . \qquad (7)$$

Thesis 2: [1], [3], [4], [7].

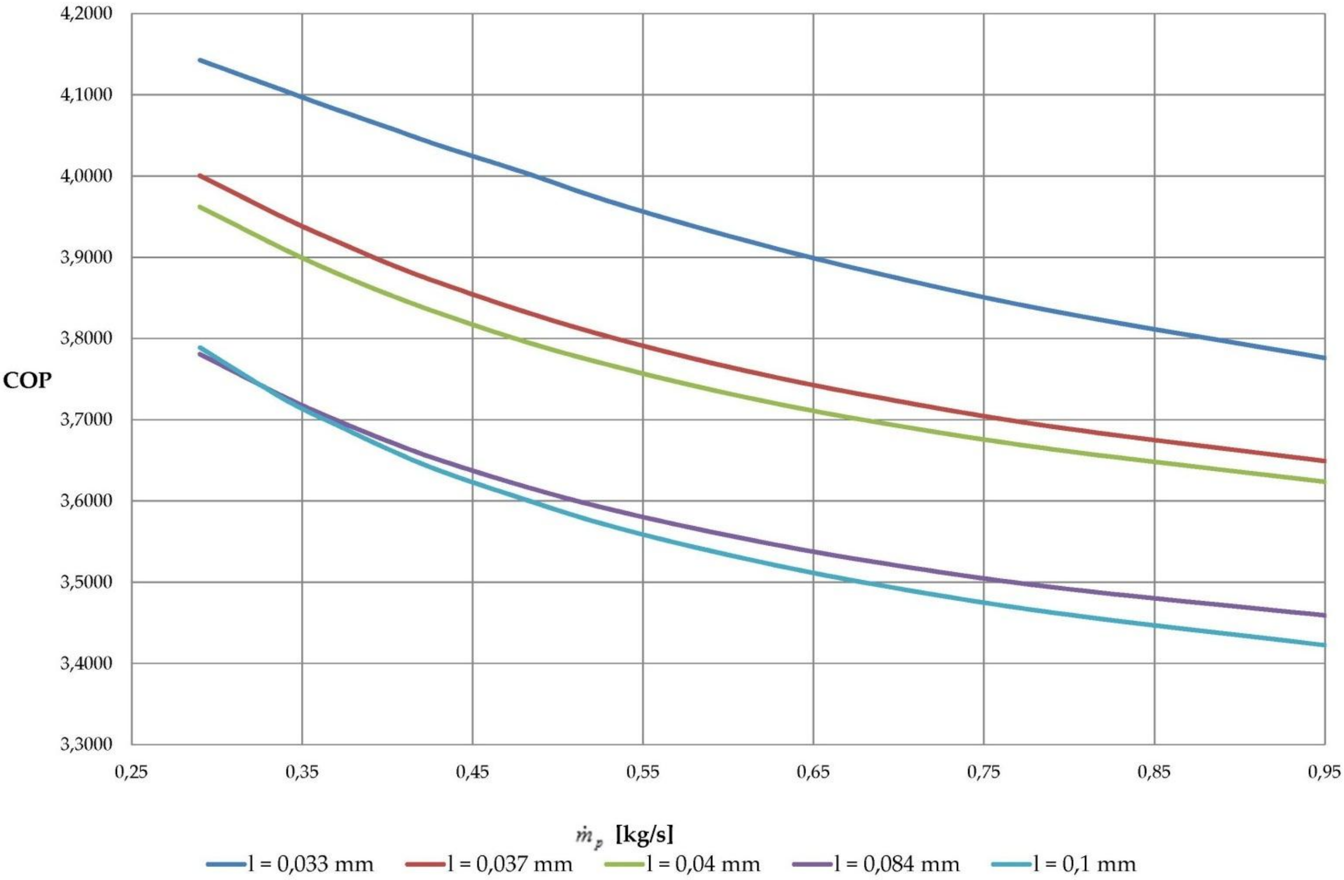
I calculated and depicted in diagrams the extractable heat capacity for the standardly used pipes with Ø32 mm and Ø40 mm diameter, and typical ground characteristics, together with the amount of the temperature of the upcoming fluid from the ground in the function of depth, time (months), mass flow and the position of U-tube legs.



2. graph: Extractable heat capacity ($\dot{Q}_{U-tube}$) and the upcoming temperature (T_{pe}) change in the function of mass flow ($\dot{m}_p$) in the month of **February** (after 1 year of operation, in case of U-tube pipe with a diameter of Ø32 mm)

Thesis 3: [1], [2], [3], [4], [5], [6], [7].

I have obtained results for time following 1 year of operation, besides I computed for a 10-year-cycle the change of COP values of the heating cycle in R407c refrigerant by fixing the typical 50 °C condensation temperature in the function of evaporating temperature, which is the function of circulated primer mass flow, upcoming fluid temperature and the extractable heat capacity. These latter are the functions of borehole depth and distance of the U-tube legs, which are depicted in the calculations and diagrams.

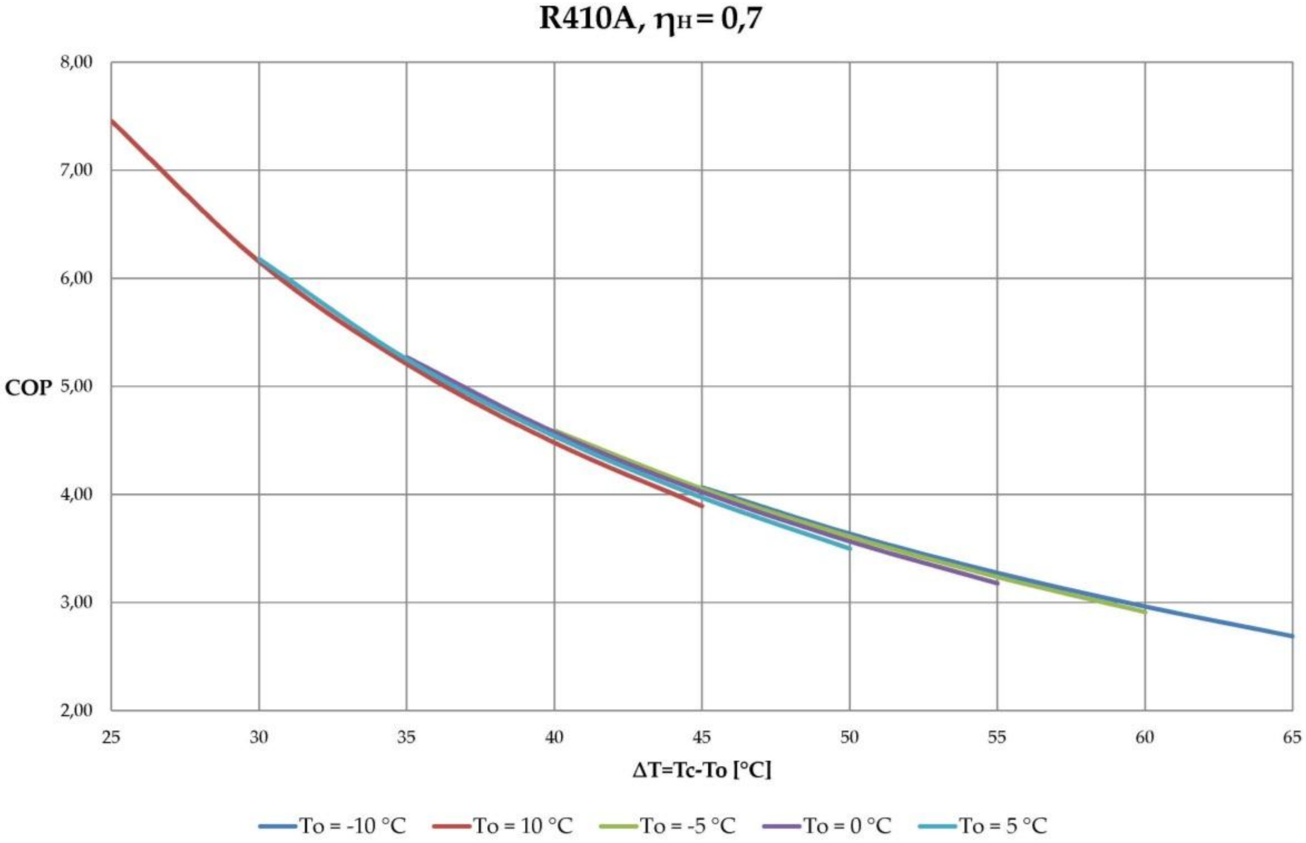


3. graph: Change of the COP value in the function of mass flow ($\dot{m}_p$) and the distance of U-tube legs (l) in the month of **February** after 1 year of operation, in case of U-tube pipe with Ø32 mm diameter, T_c = 50 °C

(values of T_0 are listed in my dissertation, M3.1 table of Appendix nr. 3.)

Thesis 3.1:

I computed the COP value functions of R407c and R410A refrigerants, standardly used in heat pumps working with U-tubes, by utilising the hydraulic efficiency of 0,7 (which is mostly considered typical) for different ΔT=Tc – To values and by different To parameter values.



4. graph: Change of COP value in the function of $\Delta T = T_c - T_0$ value and evaporating temperature (T_0)

Thesis 4: [8], [9],[10], [11], [12].

I conducted the systematic analysis of electronic compressor driven heat pumps with U-tube installation. I stated the white box models of system parts and of the input-output of the linked system, the transformation equations linking input – output variables and balance equations. I defined the so called basic and inverse task, by which operational analysis of heat pump systems can be conducted and its working points can be stated.

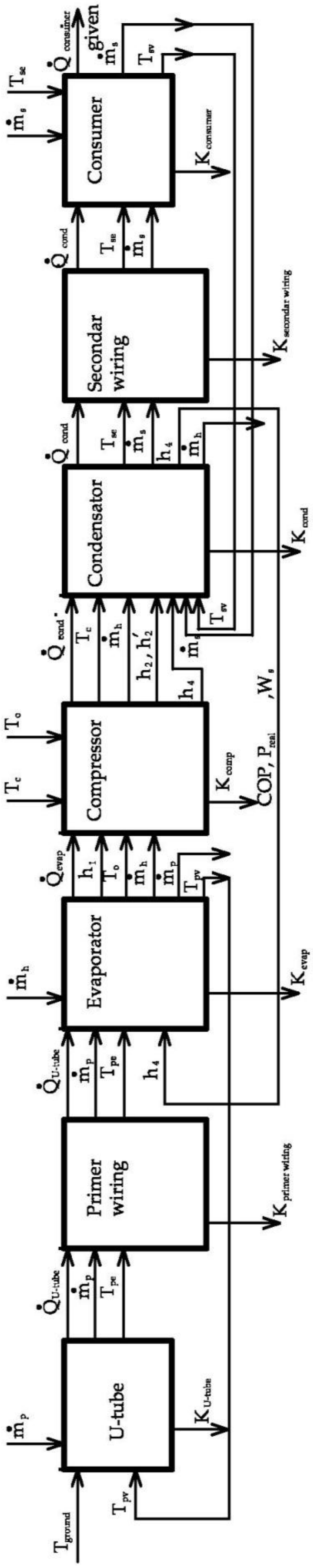
In the basic task from the inputs of the system elements, we define output values of system elements primarily from the value of temperature extracted by U-tube and its characteristics, together with deliverable heat capacity. In the meanwhile, we can freely decide about operation of inner elements.

In the inverse task regarding the given known heat demand we define the operational mode of inner system parts and within this primarily the state variables of the heat pump heating circle and the heat capacity extracted from the ground. For the invers task we put forward an optimisation and objective function determining the optimisation.

Thesis 5: [8], [9],[10], [11], [12].

I constructed the system theoretic decision scheme, aiming for optimisation of the operation of heating system of the existing electronic compressor-driven heat pump with U-tube installation. The objective function is the total operation cost of the system, of which three main elements are the costs of the flow of the fluid in the U-tubes, costs of the electricity used by the compressor of the heat pump, and the costs of electricity, used for flowing of the heating fluid in the heating system. I compiled the input, output and decision variables of the decision model. I utilised the method of dynamic programing for optimisation, for which I stated the so called recursive function equations, which express the optimisation of certain decision phases.

When we refer to the optimisation of an existing system, we mean the following: we search for those operation parameters of an already installed and operating system – in the function of consumers' demand - , by which the operation costs of the system are minimal. For such we have to know exactly the type of certain system elements, their size and capacity, together with the exact demands of consumers.



5. graph: Decision system theory scheme of an operating heat pump system

Objective function of the heat pump decision system, depicted in Graph 5 is as follows:

$$K(\dot{Q}_{consumer}) = \min \Sigma K_{\ddot{u}} = \min(K_{consumer} + K_{condensator} + K_{compressor} + K_{evaporator} + +K_{U-tube})_{\ddot{u}} \,. \qquad (8)$$

The objective function stated in equation (8) by the already existing systems considers only operation costs. In such cases we neglect the costs of installation, since the system elements are given.

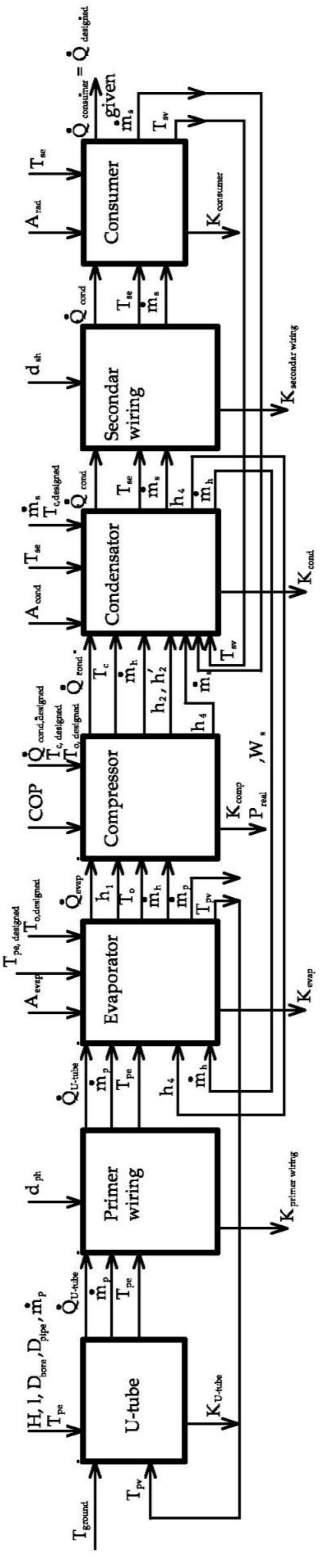
The inputs and the decision variables, together with the transformation equations linking the outputs I describe in the 4th chapter of my dissertation. Furthermore, I elaborate on the process of optimisation and expose the optimisation equations in the 6.1.2 chapter of my dissertation.

Thesis 6: [8], [9],[10], [11], [12].

I developed the system theory decision scheme, aiming for optimisation of design and installation of new electronic compressor-driven heat pump heating systems with U-tubes. The objective function expresses the minimum costs of establishment and operation of the entire system as requirement of design and installation. I combine the annual description of establishment costs with annual operation costs. I put forward the input, output and decision variables of the decision model. I utilised the method of dynamic programing for the purposes of optimisation. For this I stated the so called recursive function equations, which express the optimisation of certain decision phases. It is important to note that these recursive function equation in their structure, content and regarding their transformation coherence significantly differ from the decision models, aiming for optimum of operation.

Graph 6 depicts the system theory decision white box models of design and installation. The system is linear with mass and energy linking.

Objective function, expressing the minimum of establishment and operation costs:



6. graph: System theory, decision white box model of design and establishment

$$K(\dot{Q}_{consumer}) = \min\left[\Sigma K_{ü} + \Sigma K_{B}\right] = \min\left\{\begin{array}{l}\left[\begin{array}{l}(K_{consumer} + K_{secondarywiring} + K_{condensator} + K_{compressor} + \\ + K_{evaporator} + K_{primarywiring} + K_{U-tube})_{ü}\end{array}\right] + \\ + \left[\begin{array}{l}K_{consumer} + K_{secondarywiring} + K_{condensator} + K_{compressor} + \\ + K_{evaporator} + K_{primarywiring} + K_{U-tube})_{b}\end{array}\right]\end{array}\right\}, \quad (9)$$

which expresses, that in the function of consumers' demand we minimise the initial and operation costs by all phases of the system.

I describe the input and decision variables, together with transformation equations linking the outputs in the 4th chapter of my dissertation. Besides, I elaborate on the process of optimisation and describe optimisation equations in chapter 6.2 of my dissertation.

4. APPLICABILITY OF THE RESULTS

Based on the system theory decision white box models developed any optimal operational work phase of heat pump heating system with U-tubes can be calculated. Those operational conditions can be identified (primer and seconder side mass flows, evaporation temperature, condensation temperature, etc.), by which the system's operational and/or investment costs are minimal. These costs are possible based on relations, which I introduced in the 4th chapter of my dissertation, whereby I described the transformation equations, both for basic and for inverse tasks. For the applicability of the inverse task I provide an example in Appendix 6 of my dissertation.

By the creation of system theory models it is important to define the heat capacity extractable from the ground and the temperature of the upcoming fluid. I expose in tables and graphs, the amount of extractable heat capacity by different conditions (in the function of temperature of the descending fluid) in the function of operation time, change of months, distance of U-tube legs and the primer mass flow of the circulated fluid. The results obtained can be utilised by design of heat pump heating systems with U-tube installations and also for finding the optimum in system theory models.

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