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Displacement response of bridge decks to flutter in turbulent wind

Summary of the PhD thesis

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1 Introduction

1.1 Wind engineering definitions

The thesis is mainly based on the civil engineering knowledge, thus the declaration of the aims requires the review of some basic wind engineering definitions.

Wind engineering is defined as the science field treating the interactions between the atmospheric boundary layer wind and man and his works on the Earth's surface. The domain requires knowledge in meteorology, fluid dynamics and structural dynamics. Its application domain covers the analysis of a wide variety of buildings and structures (high rise buildings, bridges, towers), and is tightly connected to the energy industry (wind turbines).

The movement of the atmospheric air is call wind, the aerodynamic effects are the actions of the wind on structures, the forces are called *aerodynamic forces* or *wind forces.* Davenport (1961, 1967) split the aerodynamic forces of the turbulent wind by statistical tools into mean value *(static wind force)* and to the fluctuation around it, the *buffeting forces.* The structural responses be divided similarly.

The *aeroelasticity* refers to the interaction of the aerodynamic forces and the structural flexibility. Aeroelasticity can lead to important structural displacements and even to the failure of the structure. The *aerodynamic instability* with periodic displacement of the bridge decks is called the *flutter* to which the *self-feeding forces* contribute. The history of the flutter analysis of bridges goes back to the failure of the Tacoma Narrows Bridge in 1940. Selberg (1961) defined the flutter with the admissible rotation of the deck, he and – independently – Rocard (1963) gave empirical formulae to the critical *flutter speed* of the wind to the mainly torsional branch flutter phenomenon. Klöppel and Thiele (1967) treated the aerodynamic instability as a complex eigenvalue problem. In aeronautics Theodorsen (1935) gave an analytical, frequency dependent and linear model of the self-feeding forces of a two degree-of-freedom (2 DOF) thin airfoil cross section. His results were reformulated by Scanlan and Tomko (1971) to bridge decks based on flutter *derivatives* of the cross section to be measured in wind tunnel. Their formulae are as follows (fig. 1)

$$L_{se}(t) = \frac{1}{2}\rho U^2 B \left(K H_1^* \frac{\dot{h}}{U} + K H_2^* \frac{\dot{\alpha} B}{U} + K^2 H_3^* \alpha + K^2 H_4^* \frac{h}{B} \right) \tag{1a}$$

$$M_{se}(t) = \frac{1}{2}\rho U^2 B^2 \left(K A_1^* \frac{\dot{h}}{U} + K A_2^* \frac{\dot{\alpha} B}{U} + K^2 A_3^* \alpha + K^2 A_4^* \frac{h}{B} \right) \tag{1b}$$

where L and M are respectively the self-feeding lift force and torsional moment, h and α are the two displacements, the heave and the rotation, resp., of the cross section of width B, U if the laminar wind speed, $K = \frac{2\pi}{U_{\mathrm{red}}} = \frac{\omega B}{U}$ is the reduced circular frequency, U_{red} is the reduced wind speed, ω is the circular frequency of the movement, ρ is the air density, t is the time, H_i^* and A_i^* $(i = 1, \ldots, 4)$ are the derivatives and $(\dot{})$ represents the derivation respect to time. The flutter instability occurs at the flutter speed designed by U_F. Chen X., Matsumoto, et al. (2000) gave a rational function approximation of the derivatives which resulted a frequency independent description of the self-feeding forces and thus made them able to be taken into account with the buffeting forces in time domain applications.

The stability analysis of streamlined sections accentuates the importance of the interaction of the self-feeding and the buffeting forces.

1.2 The aims of the study

A wind tunnel investigation and the accompanying aerodynamic instability analysis made at the department call attention to the need of investigate the effects of the measurement errors. This represented the first aim of the present study in the course of which I have developed a model to investigate the errors in the derivatives and in the flutter speed attributable to the geometric imperfections of the set-up.

In parallel with this manifested the practical need to estimate the effects of the self-feeding forces on the structural displacements due to turbulent wind. The analyses revealed the tight relation of these effects to the derivative of the total damping respect to wind speed, where the total damping is the sum of the structural and the aerodynamic damping. The analysis of this relation connected the two aims.

The analysis on a 2 DOF, symmetric cross section model was enough to achieve the goals.

1.3 The structural parameters

The analysis of the fundamental equation of motion of the structure to the wind forces is accomplished through the following dimensionless structural parameters, as introduced by Klöppel and Thiele (1967). The B wide cross section has a relative radius $r_\alpha = \sqrt{I/m}\,2/B$ of the torsional inertia where m is the mass and I is the torsional inertia respect to the centre of gravity. $\epsilon = \omega_{0h}/\omega_{0\alpha}$ is the ratio of the ω_{0h} bending and $\omega_{0\alpha}$ torsional eigenfrequencies, and $\epsilon_F = \omega_{0\alpha}/\omega_F$ is the flutter frequency ratio. The relative mass is designated by $\mu = 4m/\left(\pi\rho B^2\right)$.

2 The pitch–pitch section model

The 2 DOF cross-section model with h heave and α rotational displacements is referred to as the heave–pitch model (fig. 1). Its coupled flutter manifests as a periodic rotation around a quasi rotational axis in the horizon.

The flutter derivatives of the bridge of the motorway M43 over Tisza were measured by a force driven wind tunnel test and some measurements errors were present. I investigated the errors of the derivatives cause by supposed geometric imperfection of a small scale section model. I defined the errors as the difference of the erroneous values and the correct values. I supposed small movements in the degree-of-freedom supposed to be fix during the two stages of the wind tunnel test. I analysed the errors thus involved in the derivatives and in the flutter speed.

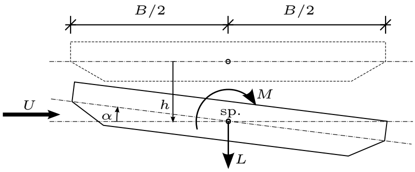


Figure 1: Heave–pitch section-model

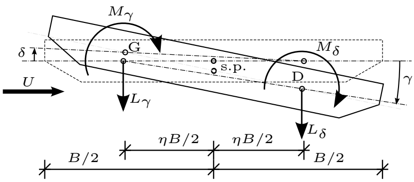


Figure 2: Pitch–pitch section-model, where L and M are the self-feeding lift force and torsional moment

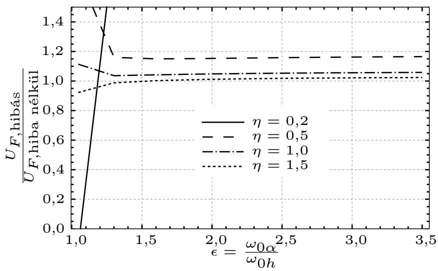


Figure 3: Ratio of the erroneous and error-less flutter speed on the pitch–pitch model. Thin airfoil derivatives, $r_\alpha = 0{,}5$, $\mu = 30$, 10% torsional imperfection. The error in the flutter speed can be decreased by incrementing the η eccentricity

I elaborated a 2 DOF section-model of two rotational displacements around two eccentric axes (fig. 2). Based on the equivalence of the displacements and the self-feeding forces on both models I developed the relation between the derivatives of the heave–pitch and the newly defined pitch–pitch model. The determination step of the derivatives are analogous to the "traditional" way.

I defined the geometric imperfections and the above mentioned errors originating from them on the pitch–pitch model. Using the thin airfoil derivatives I showed the dependency of the errors in the derivatives on the applied eccentricity and the reduced wind speed. I highlighted the derivatives H_2^* and A_2^* where the error is inversely proportional to the eccentricity.

At first sight it seems obvious that the optimal eccentricity respect to minimizing the errors coincides with or is at around the quasi rotational axis of the coupled flutter. The investigation of the errors in the flutter speed contradicted this (fig. 3).

2.1 The unimodal model

The literature revealed several approximative relations between the derivatives, which emphasizes the redundancy of them. The coupled characteristic of the flutter and this redundancy motivated the investigation of a 1 DOF, eccentric rotational unimodal section-model. The ϕ ratio of the amplitudes of the two displacement degrees and their φ phase shift can be written as

$$\phi = -\frac{2A_2^*}{\cos(\varphi)A_1^* + \sin(\varphi)A_4^*} \qquad \text{and} \qquad \mathrm{tg}(\varphi) = \frac{\phi \sec(\varphi)H_1^* + 2H_2^*}{2H_3^*}. \tag{2}$$

These two relations show no direct dependency on the structural parameters, but only on the reduced wind speed through the derivatives.

The unimodal section-model could result a simpler flutter analysis if the ratio of the amplitudes and the phase shift could be estimated in advance (based on experience). The model was derived from the pitch–pitch model with retaining only the windward DOF. The newly defined derivatives are subscripted with α. The real part of the complex eigenvalue problem of the undamped unimodal model resulted the ϵ_F flutter frequency ratio in the form

$$\epsilon_F = \sqrt{\frac{r_\alpha^2 + \phi^2 + \frac{8}{\pi\mu}A_{3\alpha}^*}{r_\alpha^2 + \frac{\phi^2}{\epsilon^2}}}. \tag{3}$$

The flutter criteria is given by the root of the derivative $A_{2\alpha}^*$, which is formally equivalent to the formula used for the uncoupled torsional flutter.

The simplification of the formulae by the approximative interrelations of the derivatives resulted in equations already present in the literature, but has determined in an independent way. However, the examination revealed that neither the ϕ ratio of the amplitudes, nor the φ phase shift show no extremal value at flutter, their preliminary estimations requires more investigation. Even though the unimodal model did not attain its final goal, the revealed relations and results seemed to be valuable and founded the need of further research. In spite of this fact the utility of the model is not questionable, as shown by the example of Chen A. and Rujin (2011) where the first steps of the development of an unimodal model to study the nonlinear behaviour of the flutter is presented.

3 The softness of the flutter

One of the characteristic of the flutter speed can be measured by the derivative of the logarithmic decrement of the total damping (δ) respect to the wind speed. Chen X. and Kareem (2006) defined the *softness* of the flutter based on it. They define the flutter to be soft-type at small value of the derivative of the decrement, and hard-type at its high value. (The "small" and "high" adjectives are subjective, and are used only to compare two cases.) The alteration effect of the r_α relative radius of the torsional inertia on the flutter speed and the softness is shown on fig. 4. My examinations showed the presence of a relation between the phase shift of the two displacement components and the damping. Using a one-term polynomial approximation of the derivatives I worked out a method to investigate the softness and the effects of the structural parameters on it.

Several algebraic simplifications expressed the derivative of the damping in a dimensionless form as $\delta' B f_{0\alpha}$, where $f_{0\alpha} = \omega_{0\alpha}/(2\pi)$ is the torsional eigenfrequency and δ' is the softness.

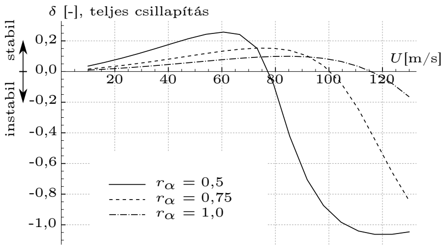


Figure 4: Total damping vs. wind speed at different torsional inertia. Thin airfoil derivatives, $\mu = 30$, $\epsilon = 3$, $B = 20\,\mathrm{m}$, $\omega_{0h} = 1\,\mathrm{rad/s}$

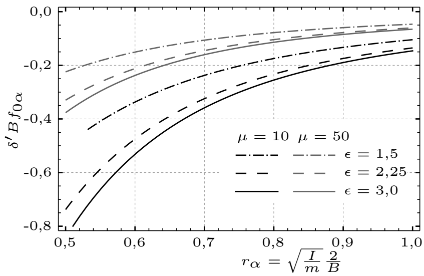


Figure 5: Flutter softness of a thin airfoil

Fig. 5 shows the softness in function of r_α, where the softness was generated by a purely second order approximation of the derivatives of the thin airfoil. The further investigations of the softness based on the derivatives of the Hardanger Bridge found in the literature showed that the structural parameters can be sorted in order in respect of the importance of their impact on the softness. The most important impact is caused primarily – naturally – by the flutter derivatives, then by the relative radius r_α of the torsional inertia, followed by the ratio ϵ of the eigenfrequencies and finally by the relative mass μ. The examinations revealed that even the derivatives can be arranged in order: H_1^*, H_3^*, A_2^* and A_3^* influenced the most the softness, while H_4^* was practically negligible.

Nevertheless the procedure resulted only a qualitative review (according to the precision of the used approximation of the derivatives), it can be utilized in practice as a help in the decision for an eventual bridge design: what kind of structural changes are needed to increase the flutter speed. E.g. in the case of a soft type flutter (case of footbridges) the slight changes in the structural damping influence highly the flutter speed, while for hard type flutter other structural changes are also need to obtain the same results.

4 The interaction of the self-feeding forces and the turbulence

I analysed the effects of the self-feeding forces in turbulent wind by time domain analysis. The time history of the turbulent wind was modelled by the proper orthogonal decomposition (POD) (Carassale and Solari, 2002) of the power spectral density function. The self-feeding forces were modelled with rational function approximation from Chen X., Matsumoto, et al. (2000), which resulted the forces as the frequency independent sum of steady and unsteady terms. The equation of motion of the system resulted in an extended equation in the form

$$\mathbf{M}_1\ddot{\mathbf{z}} + \mathbf{C}_1\dot{\mathbf{z}} + \mathbf{K}_1\mathbf{z} = \mathbf{q} \tag{4}$$

where $\mathbf{M}_1$, $\mathbf{C}_1$ and $\mathbf{K}_1$ are the mass, damping and stiffness matrix, respectively, of the system extended with the unsteady terms of the approximation. $\mathbf{z}$ is the extended state vector (displacements + unsteady variables), $\mathbf{q}$ represents the buffeting forces. In the equation the effects of the static wind forces are omitted.

To obtain statistical representations, the displacements variances, I used 20 runs of 655 s time length each with time steps $\Delta t = 0{,}04\,\mathrm{s}$ at every parameter settings. The turbulent flow required a new definition of the flutter which I identified with the statistically constant value of the variance in function of the time interval length considered. At wind speeds smaller than the flutter speed the variances of the rotation get higher values than those of the heaving. The latter increased suddenly in the small environment of the flutter speed. Torsional branch flutter occurred at every configuration considered, similarly to the typical case of bridges.

The additional effects of the self-feeding forces on the displacements due to the buffeting forces was expressed as a ratio of the variance as

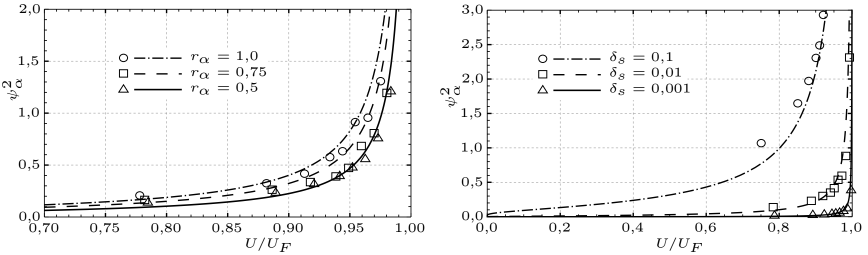
$$\psi_x^2 = \frac{\sigma_{x,b,se}^2}{\sigma_{x,b}^2} \tag{5}$$

where σ^2 is the variance, $x = h, \alpha$ is the displacement considered, the subscript $(\,)_b$ is for the displacements due to the buffeting forces only, $(\,)_{b,se}$ stands for the interaction of the self-feeding and the buffeting forces. I only examined this ratio for torsional branch flutter, which is the case for most bridges. Based on theoretical assumptions, the parametric examination resulted the equation

$$\psi_\alpha^2(U) = \frac{A_3^*}{\mu\pi r_\alpha}\,\frac{1}{U_F}\,\frac{1}{\left(\frac{U}{U_F}\right)^{0{,}02\,\delta'/\delta_s} - 1} \tag{6}$$

as a good fit on the data simulated with the flutter derivatives of a thin airfoil (fig. 6). In the equation U and U_F are the wind speed and flutter speed in [m/s], resp., δ_s stands for the logarithmic decrement of the structural damping, δ' is the softness of the flutter in [s/m], A_3^* is the value of the derivative at the flutter reduced wind speed.

The fig. 6a shows the effect of the relative radius r_α of the torsional inertia, which in accordance with the fig. 5 is the effect of the flutter softness. The fig. 6b



(a) The effect of the relative radius of the torsional inertia, $\epsilon = 2{,}5$, $\delta_s = 0{,}01$

(b) The effect of the structural damping, $\epsilon = 2{,}0$, $r_\alpha = 0{,}5$

Figure 6: Simulated data and the fitted curve for the ratio ψ_α^2 of the variance of the rotation. Common values: $\mu = 10$, $B = 5\,\mathrm{m}$, $\omega_{0h} = 1\,\mathrm{rad/s}$

displays the effect of the structural damping, where it is remarked that the gap between the data and the curve increases with decreasing wind speeds. This results the lower limit of the applicability of the proposed formula.

The additional effects of the self-feeding forces represent themselves with different intensity in the two displacement components. The proposed, fitted formula (6) accentuate that this difference is mainly determined by the flutter softness and the structural damping. The calculations verified the observation that through the aerodynamic damping caused by the flow the self-feeding forces decrease importantly the variance of the displacement at wind speeds lower than the flutter speed.

5 Scientific results

5.1 New scientific results

I summarize hereafter the new scientific results detailed in the thesis.

5.1.1 1st new result

I worked out a section-model with two rotational displacements around axes eccentric to the centre of gravity (the pitch–pitch model) for the investigation of the self-feeding forces acting on bridge decks.

- Using this model I analysed the errors of the flutter derivatives due the geometric imperfections of the section-model.
- Similarly, I determined the errors of the flutter derivatives due to the geometric imperfections of the section-model with vertical displacement and rotational degrees of freedom (heave–pitch model).
- Based on the flutter derivatives of the thin airfoil I demonstrated that the implied errors in the derivatives and in the flutter speed are less on the pitch–pitch model than on the heave–pitch model with the same degree of imperfection.

– I concluded that the optimal eccentricity applied in the pitch–pitch model in respect to minimise these errors does not coincide with the quasi rotational axis of the coupled flutter.

Related publication: Hunyadi and Hegedűs (2012b).

5.1.2 2nd new result

By a one-term polynomial approximation of the flutter derivatives I worked out such an investigation method of the softness of the flutter defined by Chen X. and Kareem (2006) that the softness can be expressed in function of the derivatives and the structural dimensionless dynamic properties.

– The method is appropriate for the qualitative analysis of the flutter softness.
– Using the available derivatives (thin airfoil and Hardanger Bridge) I sorted the structural dimensionless dynamic properties in order in respect of the importance of their impact on the flutter softness. In descending order
  i. the strongest impact is caused by the flutter derivatives,
  ii. the relative radius of the torsional inertia in the case of torsional branch flutter, which is the typical case for bridges,
  iii. the ratio of the eigenfrequencies,
  iv. the relative mass has the weakest effect.
– I sorted the flutter derivatives in order in accordance of their impact on the softness: the derivatives H_1^*, H_3^*, A_2^* and A_3^* have the strongest impact, while the softness seemed to be practically insensitive to the derivative H_4^*.
– The results of the analysis made the softness usable in practice as a help in the decision for an eventual bridge design: what kind of structural changes are needed to increase the flutter speed.

Related publication: Hunyadi and Hegedűs (2012a).

5.1.3 3rd new result

I worked out a procedure to determine the additional effect of the self-feeding forces on the structural displacements due to the buffeting forces. I analysed this effect as a scaling factor.

– I rearranged the fundamental equation of motion including the buffeting and the self-feeding effects in such a way that only the damping matrix extended with the unsteady terms is responsible for the coupled characteristic of the extended system.
– Using a parametric analysis I demonstrated that the magnitudes of the two displacements of a two degree-of-freedom section model, in accordance with the flutter branch, differ in function of the wind speed.
– Based on data generated for the thin airfoil I formulated an empirical formula for the estimation of the above mentioned additional effects of the self-feeding forces.

– I demonstrated the significant impact of the ratio of the flutter softness and the structural damping on these addition effects.
– I concluded that the relation between the buffeting forces and the self-feeding forces is not analogous to the multiplication factor of Southwell used in the structural stability.

Related publication: Hunyadi and Hegedűs (2012a).

5.2 Main conclusions of the researches currently in progress

Due to formal reasons I do not present the results of my researches currently in progress and summarised hereafter as new results, but I presume them to be important.

– I worked out an unimodal, one degree-of-freedom model which I used to give an approximation to the phase shift and the ratio of the amplitudes of the displacements of a 2 degree-of-freedom model. By an independent way I developed the formula of the flutter frequency known in the literature.
– I demonstrated that coupled flutter can occur both when the heaving motion leads and lags the torsional rotation, depending the flutter derivatives.
– I expanded the definition of the flutter to the statistics of the structural displacements in turbulent flow.
– Analysing the extended state of motion I demonstrated that the flutter phenomenon is dependent of the unsteady terms of the self-feeding forces.
– The relation between the phase shift of the two displacement components and the total damping merits further investigations.

5.3 Outlook of the research

The definition of the flutter speed should be modified to include a specific time interval related to the dynamic properties of the structure. The so defined flutter speed could demand the redetermination of the safety factors given in standards relative to the maximum admissible wind speed. Beside its theoretical feasibility, practical and economical reasons claim this.

The structural displacements due to turbulent wind near the flutter speed are higher than those assumed by the linear model of the self-feeding forces. The analysis of these nonlinear effects gains importance. The importance of their study and the creation of adequate models is emphasized by the application of streamlined cross sections. These new model could found the elaboration of wind energy harvesting systems.

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