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## Methods for Planning of Access Networks

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### Ph.D. Theses

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# 1 Introduction

The wider sense considered telecommunications networks were progressively established in the last decades. First they covered only smaller segments of the population, then the networks providing different services (telephone and cable TV network) gradually covered the majority of the population. Because of the gradual enlargement, more emphasis was on the planning of the backbone, the trunk or the core networks (in what follows, all are referred to as *core networks*), while the planning of the *access* and the last mile networks played a secondary role. Later the networks became larger and more complex, the core and the access network sharply separated from each other.

Furthermore, there are also significant changes in the purpose of the access networks. The role of the *traditional telephone networks* is changed because of the DSL technology settled over them. In addition to the cable TV networks, the telephone networks are the main medium of the Internet applications. The *cable TV networks* give opportunity to use virtual videotheques (e. g. video on demand) as well as other streaming and telemedical services (e-medicine). The evolution of *mobile networks* from GSM through GPRS and EDGE towards UMTS (3G) involves the enlargement of the data traffic in the networks.

The performance demands of the networks are also changed. In addition to the cost-optimal planning, the robustness and the fault tolerance of the networks are more and more important factors. These requirements may have a great influence on the structure of the networks.

The UMTS networks can be considered as an appropriate example for the planning of future mobile networks consisting of hundreds or thousands of *nodes*. The dimension of the networks indicates that the networks are built in a hierarchical way and the traffic in the network is aggregated in concentrator nodes placed in each level of the hierarchy. As the high quality access to the networks is very important (e. g. small delay), therefore the number of the hierarchy levels is limited.

In order to be cost-effective, the access networks usually have tree topology, where the connections between the nodes consist of wireline (or insensitive to interference in general) or microwave systems. The application of microwave systems is motivated by the fast and "cheap" settlement. In this case, however, there are stricter constraints on the network structure in order to avoid the disturbing interference between the microwave systems.

The trace of the network can be optimized according to how and which nodes are connected to each other in the tree. In this case, the length and the capacity of the connections can be optimized between three (weighted Fermat-problem) or more (Weber-problem) nodes. By the application of the so-called Fermat-points, the cost of the trace can be decreased (these points are the solution of the Fermat-Weber-problem).

Since the basic structure of the access networks is usually a tree, therefore the failure of an equipment may cause considerable outage in the network. The fault-tolerant topologies (e. g. ring or mesh) could be a solution to the above problem, but they may

greatly increase the cost of the network. A possible alternative solution is to keep the basic tree topology and expand it with some additional connections (in what follows referred to as *links*) at those parts of the network, where faults are critical. The main advantage of the network extension is that an acceptable equilibrium can be found between the increment of the cost and the increment of the fault tolerance capability of the network.

It is a general tendency that data traffic is becoming dominant and the *Internet Protocol* (IP) is more and more widely used. Where the operation of the network is IP-based, the most commonly used routing protocol recently is the *Open Shortest Path First* (OSPF) protocol. The OSPF is a link-state dependent routing protocol and uses so-called administrative link weights as metrics in the routing process. Finding the adequate setting of these weights is the most important part of the routing in order to obtain suitable network performance (e. g. maximal utilization, maximization of free capacities).

To sum up, the planning of access networks has many open and important questions in store including the topic of topology planning and routing as well.

## 2 Research Objectives

The objective of my dissertation is to analyze the routing, dimensioning and topology planning problems evolving in the field of access networks. My aim was to create effective network planning *methods* and *algorithms* that can be efficiently applied in the networks of our time and the immediate future.

I would like to note that I restrict the definition of access networks in such a way that the access network contains only fix-positioned network elements and the planning of the links between these elements is considered as the task of my work. Nevertheless this definition covers the most of the access networks of our time and the developed algorithms can be applied to them:

- traditional telephone networks, taking even each user/subscriber into consideration and for instance, independently from the DSL technology settled in the network
- cable TV networks including each user/subscriber
- in case of mobile networks such as GSM (GPRS/EDGE) and UMTS, the network between the base stations and the central controllers.

I have to note that the position of the base stations is a priori given in case of any cellular system. The evolving problems in this space are solved by a preliminary radio planning (e. g. a proper frequency allocation in order to avoid the interference between the stations).

The planning of the core network is out of the scope of the dissertation. Therefore the cost of the core network is taken into consideration as follows: the model of the access networks includes the central controllers, so the total cost contains their cost as well. Such a way the algorithms limit the number of the controllers. If their cost is virtually

increased (namely the cost of the core network is more emphatically represented), then it is indirectly made sure that the cost of the core network is kept "low".

My research work was focussed on the following topics:

- Cost-optimal planning of interference-insensitive hierarchical access networks: my goal was to propose a complex planning method that can solve this problem in reasonable running time and more efficiently than the existing methods.
- Cost-optimal planning of interference-sensitive hierarchical access networks: my goal was to propose heuristic algorithms, which take the special constraints of the interference-sensitive networks into consideration. Moreover, the algorithms take the advantage of the point-to-point and point-to-multipoint wireless systems.
- Solution of the weighted Fermat-problem and its applications to topology optimization: on the one hand, my goal was to analyze the solution of the weighted Fermat-problem as well as to give a compact and descriptive formula of this solution. On the other hand, my goal was to elaborate the way how the weighted Fermat-problem can be merged into the topic of traffic multiplexing as well as to analyze how much improvement can be reached with this technique in telecommunication networks.
- Optimization of OSPF administrative weights in access networks: with the knowledge of a priori given default and backup path system, an inverse shortest path problem can be formulated, where the OSPF-based routing finds the a priori given paths as the shortest ones based on the weights. My goal was to propose a weight setting algorithm, which is more efficient than the existing methods.

I set the following aims during the development of my algorithms and methods and I took the following conditions and requirements into consideration:

- I proposed algorithms, which are more *efficient* than the existing methods.
- In case of problems, where the running time was not critical and played a secondary role (e. g. planning of green field projects), my aim was to reach the best possible solution.
- In case of real-time problems, the fast operation was my basic aim besides that the quality of the solution should be satisfying.
- In those cases, when the size of the problem and its complexity allow it, I aimed at finding the exact solution.

In spite of the fact that each algorithm was developed to solve a particular planning task in a given network (see Section 5), my aim was to work out such algorithms, which can be applied without any major modification to similar planning tasks of other networks. The reached technology-neutrality lies in the constraints, which are handled as parameters inside broad bounds. Beyond that, the handling of the cost-functions as a "black-box" also serves the neutrality.

### 3 Methodology

According to the practice, I have modeled the telecommunication networks by graphs. In the solution of the above cited planning problems, I have applied two basic algorithms of the graph theory as building blocks: Prim's algorithm (for constructing minimum cost spanning trees) and Dijkstra's algorithm (for finding minimum cost shortest paths).

The arising network planning problems often can be formulated as *linear programming* tasks. If so, then the *optimal solution* can be found with an LP-solver program package (e. g. CPLEX or LP-solve). In the most of the cases, however, the size of the problems implies that the solution cannot be found in reasonable time.

Therefore, my aim was to develop *heuristic* methods that are efficiently applicable in case of practical planning problems. These methods are based on some well-known *general heuristic algorithms* as *K-means* (Thesis 1) and *Simulated Annealing* (Thesis 2), which were adapted to the particular planning problems and improved in order to obtain better solutions. In other cases, when the general heuristics cannot be applied efficiently, I developed *problem specific heuristics* (Thesis 4) that take the advantage of the peculiar properties of a problem in order to provide better solutions than general methods.

There are some geometrical subproblems of the network design tasks that can be solved in an analytical way. (In case of the Fermat-problem, the question is how to connect three points together through a fourth point (Fermat-point), where the "total cost" of the links is minimal.) The solution of the Fermat-problem can be found by a former heuristic solution [42]. Later on, some exact solutions were constructed [38, 39], however, exact solutions covering all possible cases arose only in the near past [41, 40]. Therefore, it was promising to analyze this research area and its applications (Thesis 3).

The most widely used technique to evaluate the performance of an algorithm is *simulation*. I used simulation based analysis in my theses, when the complexity of the design task and its state-space imply that there is no or just a very limited way to apply analytical evaluation. However, if it was possible to calculate lower bounds or even the exact optimum, then they were applied to measure the quality of the proposed algorithms. In spite of the fact that the development of the simulation environment and carrying out the performance analysis are considerable engineering tasks, I did not consider them as separate theses, since the performance analysis is an indispensable part of the introduction of a heuristic algorithm.

### 4 New Results

Different design tasks arising in the planning of access networks are analyzed and new solutions to them are proposed in my theses.

The planning of interference-insensitive and the interference-sensitive access networks were analyzed and presented separately. The former is discoursed in Thesis 1 and the results concerning the latter are presented in Thesis 2. Thesis 2 builds on the results

of Thesis 1 and applies them to plan the interference-insensitive parts of the network.

It could be important to know when and how it is possible to locally improve the topology of the network. This topic is analyzed in Thesis 3, whose results can be applied as a final part of a topology planning process or as an optional part of the planning algorithms presented in Thesis 1 and Thesis 2.

Access networks typically have tree structure, so they are quite sensitive to faults (e. g. cutting of links). The demand arose that some important nodes should have protection. By expanding the basic tree structure with some "protecting" backup links, higher availability can be obtained for these important nodes as well as for the whole network. This task was solved by my co-authors in [C2, J2]: they give the position of the backup links, while minimizing the cost of the network. The default paths, the backup paths, the capacity of the default and backup links are given in a dedicated way in their solution. After the topology is given, the question arise how the routing should be solved. Thesis 4 gives the answer to the question and proposes a solution building on the more and more widely used OSPF technique.

## Thesis 1 : Cost-optimal Planning of Interference-insensitive Hierarchical Access Networks [J4]

The planning of hierarchical access networks involves the task of finding the proper position for the central controllers and for the different concentrator nodes as well as the creation of the link-system (edge-system) connecting the nodes to each other. The resulting network topology should be a set of trees with minimal cost.

The difficulty of the task lies in the *cascading constraint* (limited depth of the trees) and in the *degree constraint*, which can be different even for each level of the hierarchy. The authors have proved in [8] that the construction of a tree like that is an  $\mathcal{NP}$ -hard problem in itself. Additional factors also increase the complexity of the planning problem. There are non-linear components of the *cost-structure* (e. g. the capacity dependent step-like cost functions of the links and the nodes); the *traffic is aggregated* level by level in the hierarchy (however, the solution is independent from the amount of the locally looped back traffic); and the costs of the network elements *have an effect on each other* (e. g. in case of more trees, more controllers have to be used, while the cost of the individual controllers is less).

Optionally, a level of the hierarchy can be allocated to each node, where the node has to be placed. Thus the "close" position to the controllers and the higher availability can be guaranteed for some very important nodes (e. g. ambulance and fire-stations).

I split the planning task into two phases and evaluated the proposed algorithms.

- Phase 1: giving a solution, which satisfies all the constraints and can be applied in itself. (Providing nearly the same solutions as the existing solutions in shorter time.) (Thesis 1.1)
- Phase 2: the systematic improvement of the solution given by Phase 1. (The algorithm can be applied to improve any valid solution.) (Thesis 1.2)
- Simulation-based Performance Analysis: developing the environment of the simulation and the analysis of the obtained solutions. Analysis of running times, comparison with existing algorithms, analysis of the results in case of special conditions (the effect of different topological constraints and cost-functions).

### Thesis 1.1 : An Algorithm for Finding a Valid Initial Solution

*I proposed a new heuristic algorithm based on clustering and graph algorithms. The algorithm determines the level of each node in the hierarchy and creates the connections between the nodes.*

The proposed planning algorithm builds on the *K-means* and the *Prim* algorithms, which are adaptively combined with each other according to the design task.

The main steps of the proposed algorithm (called *Top-Down*) are the following:

1. (*Creating Trees*) I divide the nodes into groups (clusters) by clustering, hereby partitioning the network into distinct areas. The centers of these clusters will be the central controllers placed into the roots of the evolving trees.
2. (*Second Level*) The clusters of the previous step are further divided into smaller clusters. The centers of these smaller clusters will be the secondary (second-level) concentrators. These concentrators are connected to the corresponding former centers (that is, to the corresponding central controllers), hereby creating the first two levels of the hierarchy.
3. (*Further Levels*) Like in the previous step, the remaining levels of the hierarchy are created.

During the algorithm, the cost of the network is calculated from two factors. The first factor is the cost of the already planned parts (levels), where the cost calculation can be based on the exact capacity values. The second factor is the cost of the not yet fixed levels, which practically equals to the cost of the clusters created in the actual step. In order to calculate the cost of these clusters in a realistic way, all clusters are transformed into trees. That is, the network is temporarily extended with these trees and their cost is considered as the second factor.

### **Thesis 1.2 : An Enhanced Algorithm for Iterative Improvement of a Solution**

*I proposed an enhanced variant of the algorithm presented in [31]. The algorithm is extended to efficiently handle any number of hierarchy levels and to systematically improve any valid solution by advancing in the hierarchy level by level. The enhancement includes new and improved operations (which are the building-blocks of the algorithm) and a new conception to control the complexity and effectiveness of the operations.*

The improvement starts from a valid solution and systematically revises the connections between the nodes and the level of the nodes in the hierarchy (by applying clustering and local improvement techniques).

The proposed iterative algorithm for improvement (*Full-Iteration*) examines all levels of the hierarchy in each round of the iteration. The rounds are repeated until the solution can be improved. One round of the algorithm is the following:

1. (*New round*) Improve the actual solution by going through the hierarchy level by level. To achieve this a loop is applied with  $l = 0, \dots, L - 1$  (where  $L$  is the cascading constraint and  $l = 0$  denotes the highest level of the hierarchy) and:
  - (a) (*Clustering*) Apply clustering to re-plan level  $l$  of the hierarchy by using nodes of level  $l$  and  $l + 1$ .
  - (b) (*Inter-level Optimization*) Apply local optimization to optimize the connections between the nodes of level  $l$  and  $l + 1$ .

2. (*Thorough Optimization*) Revise all connections between the nodes, that is, each node can be connected to any other node in order to further improve the solution.
3. (*Evaluation*) Calculate the network cost and evaluate the actual solution:
  - (a) (*Advance Complexity*) If the improvement of the cost between the actual and the previous round is not satisfactory, then advance the complexity of the applied operations (for details see below) if it is possible and go to Step 1.
  - (b) (*Termination*) If the solution improved, then go to Step 1 else stop.

It is important to see that a valid solution is available at all steps of the algorithm, so the cost of the whole network can be exactly calculated even in Step 1a.

The *advance of the complexity* of the applied operations allows us to find an equilibrium between the running time of the algorithm and the quality of the solution. For that purpose, I propose the following strategy. First local optimization is applied tree by tree at a time and I try out only those change possibilities, which can be applied on nodes close to each other. Then local optimization is also applied within the clustering operation. Afterwards, the nodes can be moved to other trees. Finally, the movements of the nodes is allowed in the entire network also to far positions. Instead of the number of rounds, the advance of the complexity is based on the efficiency of the rounds measured by the decrement of the cost of the network.

In order to speed up the algorithm, the *cost calculation* in each operator is restricted to those parts of the network, which are modified.

The application of *local optimization operators* is extended to revise the connections between any hierarchy levels and distinct trees. Therefore, a *constraint-checking* step is applied by the operations in order to satisfy the various node position restrictions (a node can be fixed or forbidden to be in a given level), cascading- and degree constraints.

In the *clustering step*, the nodes of level  $l$  and  $l + 1$  ( $\# \text{ nodes} = n$ ) are divided into two groups: upper level nodes in level  $l$  and nodes in level  $l + 1$ . Instead of the  $1 \dots n$  interval, only the theoretically possible interval (based on the constraints and restrictions) is scanned to find the best number of upper level nodes. The allocation of the nodes to an upper level node is cost-based. Two strategies are proposed: (i) connect the nodes one by one to that upper level node, at which the cost of the connection would be the least; (ii) go connection by connection: choose the least-cost connection from the possible ones. Version (ii) is proposed to be applied in the second phase of the algorithm, when local optimization is applied within the clustering.

### Simulation-based Performance Analysis of the Proposed Algorithms

The algorithms presented in Thesis 1.1 (*Top-Down*) and Thesis 1.2 (*Full-Iterate*) were implemented as a part of a network planning tool (see Section 5).

The proposed algorithms were compared with the methods of [C1, 26] as well as with the results of a system (*TreePlan*) combining these methods. Furthermore, I developed a randomized greedy method (*Random*) in order to analyze the effectiveness of the algorithm presented in Thesis 1.2. (For further details see [J4].)

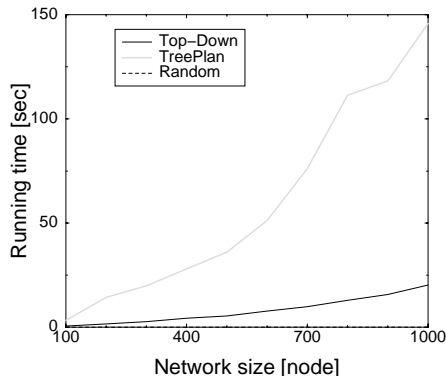


Figure 1.1: The running times of the algorithms.

The algorithm of Thesis 1.1 provides solutions of the same quality in much less ( $< 20\%$ ) time than other algorithms. The running times are presented in Figure 1.1 and it can be seen that the running time of *Top-Down* is just a fraction of the running of *TreePlan*.

The aim of the algorithm presented in Thesis 1.2 is to improve a valid solution and to obtain solutions of good quality starting from any valid initial solution. Table 1.1 shows the results obtained by different algorithms. The sign  $\checkmark$  denotes the best solution found to the given network and the difference from the bests are shown in percentage. The results shows that *Full-Iteration* can considerably improve other solutions ( $> 10\%$ ) and can provide good solutions starting from any initial solution (e. g. after *Random*, the solution lags behind the bests only with 1%).

Algorithm \ number of nodes	100	200	400	500	600	800	1000	Average
<i>Top-Down + Full-Iteration</i>	$\checkmark$	0.02	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	0.003
<i>TreePlan + Full-Iteration</i>	0.34	$\checkmark$	0.54	0.47	1.06	0.22	0.6	0.46
<i>Random + Full-Iteration</i>	0.61	0.6	1.26	1.06	1.23	0.89	0.64	0.9
<i>Top-Down</i>	3.98	17.51	12.68	15.92	15.91	18.54	23.77	15.47
<i>TreePlan</i>	4.14	8.51	10.36	17.93	20.56	17.89	20.33	14.25
<i>Random</i>	54.49	73.39	135.64	168.38	187.72	218.14	247.7	155.07

Table 1.1: The quality of the solutions for different approaches and instance sizes.

## Thesis 2 : Cost-optimal Planning of Interference-sensitive Hierarchical Access Networks [C3, C4]

The general purposes in case of planning of a hierarchical access network also holds in the interference-sensitive case and the cost-structure remains also the same. That is, the center controllers and the concentrator nodes have to be positioned and the link-system between the nodes have to be created. In other words, the aim is to find a topology of a set of trees, which is cost-optimal and it also satisfies all additional constraints derived from the interference-sensitivity (see below).

The links connecting the nodes together are (can be) sensitive to interference. Besides the general cascading constraint (limited depth of the trees) and degree constraint

(which can be different even for all levels of the hierarchy), further constraints have to be taken into consideration. In case of point-to-point connections, a minimal pointing angle separation have to be satisfied in order to avoid the damaging interference [C3]. Moreover, the length of these kind of links is limited. In case of additional point-to-multipoint connections (e. g. antennas with sector coverage), the coverage range and the sector width of the antennas is also limited (these two factors determine the size of the area, which can be covered by a point-to-multipoint connection) [C4]. Furthermore, the so-called Line of Sight (LOS) constraint on the visibility is another important limiting factor. This factor restricts the circle of nodes, which can be connected to each other.

I have to note that the cascading and the degree constraints can be different in case of interference-sensitive links from the values valid in the interference-insensitive areas.

I split the planning problem into two parts and evaluated the proposed algorithms.

- Task 1: solution to the case where only point-to-point connections are allowed in the network, however, the network may contain both interference-sensitive and interference-insensitive elements. (Thesis 2.1)
- Task 2: planning of a given hierarchy level by applying both point-to-point and point-to-multipoint connections. Completing the solution with the planning of the whole network and the optimization of the point-to-point connections (by applying the solutions of Task 1). (Thesis 2.2)
- Simulation-based Performance Analysis: developing the environment of the simulation and the analysis of the obtained solutions. Analysis of running times, comparing different approaches with each other and with a lower bound of a special case.

### **Thesis 2.1 : Settlement of Point-to-point Connections**

*I proposed a new heuristic algorithm, which satisfies the constraints on the point-to-point connections in order to avoid the damaging interference. The algorithm minimizes the cost of the network by applying the methods of Thesis 1.*

The kernel of the proposed algorithm is a loop. The planning task is divided into two phases. In the first phase, the interference-sensitive parts are planned. Then in the second phase, these interference-sensitive parts of the network are joined together with interference-insensitive links. The solution will be a valid solution satisfying all the constraints.

*The interference-sensitive parts:* planning of smaller trees with interference-sensitive links, where the roots of these trees constitute the basis of the planning. First a root is chosen from the nodes, to which the most nodes can be connected. Then a new root or

an already connected node is picked, to which the most of the yet unconnected nodes can be connected. Finally the previous step is repeated until there is a node, which can be connected to the network with an interference-sensitive link. (Let us suppose, that the cost of the interference-sensitive links is less than the cost of the interference-insensitive links. Otherwise the interference-sensitive links are needless, since the constraints on the interference-insensitive links are always weaker in practice.)

*The interference-insensitive parts:* new trees are created from the unconnected nodes and from the roots of the previous phase. This task is solved by the algorithm proposed in Thesis 1.

The framework of the proposed algorithm is as follows.

1. (*Initialization*) Pick as many nodes to be a root in the interference-sensitive parts as many needed at least to cover the whole network.
2. (*Iteration*):
  - (a) (*Sensitive parts*) Plan the interference-sensitive parts according to the above described method.
  - (b) (*Insensitive parts*) First compute the "new" constraints, which will be valid for input nodes of the method. Then plan the interference-insensitive parts of the network in the above described way.
  - (c) (*Evaluation*) Calculate the cost of the network and update the best configuration found so far if necessary. Modify the number and the position of the roots of the interference-sensitive parts. Start a new iteration if the predefined terminal condition is not met (for details see below).
3. (*Post Processing*) Improve the best solution found in the *Iteration* (Step 2.) by applying local changes.

In the *Evaluation*, the simulated annealing technique was applied to decide whether a new configuration is acceptable or not. To modify the roots of the interference-sensitive parts, three possibility was considered: a) adding a new, b) deleting or c) moving an existing root to another position (that is the application of a) and b) together). The types of the modification are selected adaptively based on their past performance. The targets of the modification are selected with respect to the position of unutilized roots, existence of concentration areas and the position of nodes, which are not covered by interference-sensitive links. According to the practice, the terminal condition of the *Evaluation* (Step 2c) can be a limit on the number of iterations or a limit on the maximal number of unsuccessful iterations, in which the best-so-far solution failed to improve.

## Simulation-based Performance Analysis of the Proposed Algorithm

I developed a planning tool based on the proposed algorithm and investigated the efficiency of the algorithm with this tool.

Since the exact solution of the problem is not known, thus a simplified case is considered in the analysis. In this case, the goal is to maximize the number of the interference-sensitive links (e. g. the cost of a microwave link hardly depends on its length in practice) and a lower bound can be given to the cost of the network. (Further details of the analysis are presented in [C3].)

number of nodes	1 length		2 length		3 length		any length	
	a)	b)	a)	b)	a)	b)	a)	b)
49	4.17%	14.58%	optimal	4.17%	optimal	2.08%	optimal	2.08%
100	6.06%	21.21%	2.02%	7.07%	2.02%	6.06%	2.02%	6.06%
225	6.64%	26.55%	2.65%	7.96%	2.65%	6.64%	2.65%	6.64%
400	7.88%	25.62%	2.71%	7.88%	1.72%	4.93%	1.72%	4.68%

Table 2.1: The quality of the solutions for different networks.

I investigated four cases, which differ from each other in the limit of the length of an interference-sensitive link ("length" = the distance between the farthest two adjacent points of the quadratic lattice). I differentiated two types of networks: a) the nodes were deployed according to a regular quadratic lattice, b) the nodes were uniformly distributed in a square area. The results were compared to the lower bound and difference between them is shown in Table 2.1. The comparison shows that the algorithm is very efficient (in average, the results are 6% above the bound), especially in the regular case (when the results lag behind the bound only with 3%).

## Thesis 2.2 : Combined Planning of Point-to-point and Point-to-multipoint Connections

*I proposed new algorithms based on clustering, which combine the point-to-point and the point-to-multipoint connections with each other in a cost-effective way. In order to construct the point-to-point connections and to create constraint-trees, the algorithms of Thesis 2.1 and Thesis 1 were applied, respectively.*

I split the planning task into two phases. In the first phase, the lower levels of the hierarchy are created. I propose a clustering based algorithm to find the optimal position of those nodes (hubs), which are able to handle the point-to-multipoint (PMP) connections (e. g. the planning of sectors in case of a radio network). At the same time, the rest of the nodes are assigned to the hubs (*allocation* task) and point-to-point (PTP) connections are applied to create the interference-sensitive parts of the network (that is the creation of the hub areas). In the second phase, the former planned parts of the network are connected to each other with interference-insensitive links. Nevertheless, the resulting network topology corresponds to all requirements.

First the frame of the algorithm is presented, then three *allocation* strategies are described, finally the creation of the sectors (hub areas) is outlined.

The framework of the proposed algorithm is as follows.

1. (*Initialization*) Pick as many nodes to be a hub as many minimally required in order to cover the other nodes. Let the number of hubs denoted by  $k$ .
2. (*Iteration of sensitive parts*) For the case of the actual value of  $k$ , determine the optimal positions for the hubs and allocate the other nodes to the hubs.
  - (a) (*Clustering loop*) Iterate the following steps to create the best possible hub areas.
    - (*Allocation of nodes*) According to the actual positions of the hubs, allocate the nodes to the hubs with PTP or PMP connections.
    - (*Relocation of hubs*) Inside the hub areas, find better position for each hub. At the same time, set new sectors in order to optimize the PMP connections.
  - (b) (*Evaluation*) If the result at the actual value of  $k$  is the best found so far, then store it. Then increase the value of  $k$  (if it is possible or it is worth it, for details see [C4]) and start a new *Iteration*.
3. (*Local improvement*) In order to obtain lower network cost, modify the connections. That is the nodes can be connected to another hubs and the PMP connections can be replanned.
4. (*Point-to-point links*) Reconstruct the star-like topology given by the above steps. The algorithm of Thesis 2.1 is applied to replan the PTP links (of the interference-sensitive part) in order to avoid the damaging interference between the links.
5. (*Insensitive part*) Create tree(s) from the hubs of the sensitive parts with the algorithms of Thesis 1.

The main difference of the allocation strategies lies in the evaluation of the solution. The evaluation can be distance-based (*DIST*) or cost-based (*COST*). The strategies can be further divided based on the way of allocation. An obvious possibility is to connect the nodes to the "cheapest" positions (*CLU*). Another possibility is to create a minimal spanning tree from the nodes and split the tree at the "most expensive" links to get the initial hub areas (*TREE*). I analyzed three combinations of the above strategies: *DIST-TREE*, *DIST-CLU* and *COST-CLU*. The results showed that strategy *COST-CLU* is the most effective in case of practical problem instances.

The planning of the sectors is based on an increasing-decreasing strategy in order to cover the most of the nodes with the least number of sectors. Thus the more expensive PTP links can be replaced by PMP connections. The most important property of

the method is that the sectors can overlap each other in an arbitrary fixed degree.

### Simulation-based Performance Analysis of the Proposed Algorithms

The core of the three algorithm is the *Iteration of sensitive parts*. I compared the effectiveness of these iterations with each other. I analyzed the algorithm in case of several instance networks of two types (described in [C4]). In the first case, the nodes were uniformly distributed and strategy *COST-CLU* gave the best results: *DIST-TREE* lagged behind with 10 to 15 % and *DIST-CLU* lagged behind with 2 to 5 %. In the second case, there were some concentration areas with higher density of nodes. In that case, *COST-CLU* was the best as well, however, the two other strategies lagged behind only with 4 to 5 %.

### Thesis 3 : Solution of the Weighted Fermat-problem and its Applications to Topology Optimization [J3]

In the field of network planning, local optimization techniques are frequently applied. The local optimization can be the building-block of each step of a planning algorithm or an independent final phase of a given method. In case of topology planning, local optimization means the improvement of the quality of the link structure between the nodes and many times it is restricted to small parts of the entire network. In these circumstances, not only is it possible to determine between which nodes should be a connection, but also links can be merged at extra nodes (at the so-called Fermat-points) in order to save cost.

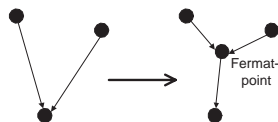


Figure 3.1: Application of the Fermat-problem

Exact and heuristic algorithms also exist to find these Fermat-points, however, the general formulation of the solutions (the optimal position of the Fermat-points) are rather complex. (The planning task belongs to the *weighted Fermat-problem* in case of merging two links /see Figure 3.1/ and to the *weighted Weber-problem* in case of merging several links.) Moreover, the problem of deciding in advance whether or not the application of such Fermat-points results in cost saving is an open question. Existing solutions first determine the optimal position of the Fermat-point and then calculate the improvement.

My thesis focuses on the weighted Fermat-problem, because only its solution can be given by an exact and closed formula. I analyze the geometrical properties of the above described extra nodes and propose a simple formula to describe the position of these nodes. I worked out a mapping system, which helps to reduce the cost of the links in the network to the weighted Fermat-problem, when the cost of the links is not only distance-based, but capacity-dependent as well. Thus the multiplexing gain can be

taken into account in case of merging two links. Finally I give some formulae to directly decide, whether or not the topology optimization is worth to be applied. These formulae give the attainable amount of the cost saving (gain) without determining the optimal position of the Fermat-point and calculating the new cost of the network.

I split the results into two groups.

- Group 1: the analysis of the solution of the weighted Fermat-problem and a new description of its properties. (Thesis 3.1)
- Group 2: the analysis of the applications of the solution and the approximation of the attainable gain without solving the problem. (Thesis 3.2)

### Thesis 3.1 : A General Solution to the weighted Fermat-problem

*I analyzed the geometrical properties of the point, which is the solution of the weighted Fermat-problem: a) I gave new conditions to decide when the weights of the nodes determines the position of the solution in advance, b) I gave a simple mathematical description of how to find the position of the solution in general and special cases, c) I showed how the solution point divides the area of the "original triangle" in special symmetrical cases, d) I showed the connection between the value of the objective function and the sides of the triangle.*

**Definition 3.1.1** The weighted Fermat-problem can be formulated as follows. Let  $\triangle ABC$  be a given triangle with positive weights  $w_A$ ,  $w_B$  and  $w_C$  associated with the three vertices. For any point  $X$  in the plane, let  $|AX|$ ,  $|BX|$  and  $|CX|$  be the Euclidean distances between  $X$  and  $A, B, C$ . Then the weighted Fermat-problem is to find a point  $P$  such that  $F(P) = \min(F(X) \in \mathcal{R}^2)$ , where  $F(X) = w_A|AX| + w_B|BX| + w_C|CX|$ .

If any of the weights is less than the sum of the other two weights, then  $P$  can be constructed [38, 39, 41]. In other cases, the weights determine the position of  $P$  in advance [J3]. The subject of my analysis is the former case, of course.

It is easy to see, that point  $P$  cannot be outside the triangle. Thus viewing angles can be given, under which the sides of the triangle are seen from  $P$ . In the following, these angles are referred to as *Fermat-angles*. Let them be denoted by  $\hat{\alpha} = \angle BPC$ ,  $\hat{\beta} = \angle APC$  és  $\hat{\gamma} = \angle APB$ . Of course, point  $P$  can be constructed as the intersection of the three viewing circles according to Fermat-angles. This technique is referred to as *Angle-technique*.

**Theorem 3.1.1** If the angles of  $\triangle ABC$  are less or equal to the corresponding Fermat-angles (i.e.  $\alpha \leq \hat{\alpha}$  and  $\beta \leq \hat{\beta}$  and  $\gamma \leq \hat{\gamma}$ ), then  $P$  will either be an interior point in  $\triangle ABC$  or a vertex of it. Furthermore, only one of the angles can be equal to its corresponding Fermat-angle, otherwise the weights determine  $P$  in advance.

**Theorem 3.1.2** If one angle of  $\triangle ABC$  is greater than its corresponding Fermat-angle, then  $P$  will be the corresponding vertex of  $\triangle ABC$  (e.g. if  $\alpha > \hat{\alpha}$ , then  $P = A$ ). If there is more such angle, then the weights determine  $P$  in advance.

**Theorem 3.1.3** Independently from  $\triangle ABC$ , point  $P$  is uniquely determined by the Fermat-angles, so  $P$  can be given in a *Fermat-angle coordinate system* as  $P = P(\hat{\alpha}, \hat{\beta}, \hat{\gamma})$  and these angles can be expressed by the weights as follows.

$$\hat{\alpha} = 2\operatorname{arccot}\sqrt{\frac{w_A^2 - (w_B - w_C)^2}{(w_B + w_C)^2 - w_A^2}} \quad (3.1a)$$

$$\hat{\beta} = 2\operatorname{arccot}\sqrt{\frac{w_B^2 - (w_A - w_C)^2}{(w_A + w_C)^2 - w_B^2}} \quad (3.1b)$$

$$\hat{\gamma} = 2\operatorname{arccot}\sqrt{\frac{w_C^2 - (w_A - w_B)^2}{(w_A + w_B)^2 - w_C^2}} \quad (3.1c)$$

**Theorem 3.1.4** If  $a = b$  and  $w_A = w_B = 1$  in the triangle, then the optimal  $P$  point divides the area of the  $\triangle ABC$  into the following areas.

$$T_{ABP} = \frac{c^2 \cdot w_C}{4\sqrt{4 - w_C^2}} \quad (3.2a)$$

$$T_{CAP} = T_{BCP} = \frac{c}{4} \left[ \sqrt{a^2 - \left(\frac{c}{2}\right)^2} - \frac{c \cdot w_C}{2\sqrt{4 - w_C^2}} \right] \quad (3.2b)$$

Note that the areas of the small triangles equal to the barycentric coordinates of the  $P$  point, so  $P$  can be exactly given in this coordinate system as well!

**Claim 3.1.1** If all the weights are equal in a general triangle, then the sum of the distances ( $S$ ) from the optimal  $P$  to the vertices of the triangle ( $A, B, C$ ) can be expressed as

$$S = \frac{a^2 - b^2}{|BP| - |AP|} = \frac{a^2 - c^2}{|CP| - |AP|} = \frac{b^2 - c^2}{|CP| - |BP|} \quad (3.3)$$

(In case of isosceles triangles, the equation with the equal sides cannot be used. In case of equilateral triangles, the equations can be reduced as:  $S = a\sqrt{3} = b\sqrt{3} = c\sqrt{3}$ .)

### Thesis 3.2 : Application of the Weighted Fermat-point to Topology Optimization

*I analyzed the possible application areas of the weighted Fermat-problem in telecommunications environment: a) connection between the weights and the capacity dependent cost of the traffic demands, b) connection between the weights and the multiplexing gain*

applied at merging of links. On the other hand, I analyzed the attainable gain in case of topology optimization: c) analysis of the efficient applicability of the solution against some parameters of the network, d) the analysis of the attainable gain provided by the solution (maximum, upper bound).

In the field of network planning, the weighted Fermat-problem can be applied to merge two links of a network at an extra point if it results in cost saving. Namely, the cost of the original links ( $AC$  and  $BC$ ) is greater than the cost of the links to the extra point ( $AP$  and  $BP$ ) and the cost of the merged link ( $PC$ ). In practice, the cost of the links can be calculated according to the following formula:  $C_{\text{link}} = l \cdot f(t)$ , where the first component  $l$  denotes the length of the link and the second component  $f(t)$  denotes the capacity related cost of the link. The so-called multiplexing gain gives how much capacity can be spared by merging links and the multiplexing gain indirectly determines the required capacity on link  $PC$  and the cost of this link.

Let us consider the weight of the links to be equivalent to their traffic related cost:  $w_A \equiv f(t_A)$  and  $w_B \equiv f(t_B)$  (where  $t_A$  and  $t_B$  are the traffic of node  $A$  and  $B$ , respectively). Then a multiplexing gain  $M$  can be defined ( $0 \leq M \leq 1$ ), which guarantees that the cost of the multiplexed traffic ( $w_C$ ) satisfies the following assumptions:  $w_C \leq f(t_A) + f(t_B)$  and  $w_C \geq \max(f(t_A), f(t_B))$ .

**Definition 3.2.1** According to the above demands, the formula for calculating  $w_C$  by the multiplexing gain  $M$  is

$$w_C = (1 - M) \min(w_A, w_B) + \max(w_A, w_B), \quad (3.4)$$

Note that *Angle-technique* presented in Thesis 3.1 is just a way to find the solution of the weighted Fermat-problem. So if *Angle-technique* cannot be used in the topology optimization, then the weights trivially determine  $P$  and there is no need for any other methods to "calculate"  $P$ .

**Theorem 3.2.1** The *Angle-technique* is applicable only if the multiplexing gain satisfies the following inequality

$$M \geq \frac{w_A + w_B - \frac{\sqrt{(w_B - w_A)^2 + (w_B + w_A)^2 \cot^2(\frac{\gamma}{2})}}{\sqrt{1 + \cot^2(\frac{\gamma}{2})}}}{\min(w_A, w_B)} \quad (3.5)$$

where  $\gamma$  is the angle between  $AC$  and  $BC$ .

Another approach is to consider  $M$  and the weights to be known. Then we get the following condition.

**Theorem 3.2.2** The *Angle-technique* is applicable if the angle  $\gamma$  between link  $AC$  and  $BC$  is at most

$$\gamma_{max} = 2 \operatorname{arccot} \sqrt{\frac{2 - M}{M} \cdot \frac{2 \max(w_A, w_B) - M \min(w_A, w_B)}{2 \max(w_A, w_B) + (2 - M) \min(w_A, w_B)}} \quad (3.6)$$

**Theorem 3.2.3** If  $\gamma > \frac{2}{3}\pi$ , then there is no legal multiplexing gain  $0 \leq M \leq 1$ , for which *Angle-technique* is applicable.

**Theorem 3.2.4** If  $\gamma < \frac{1}{2}\pi$ , then there always exists a legal multiplexing gain  $0 \leq M \leq 1$  for which *Angle-technique* is applicable.

**Theorem 3.2.5** The maximal gain ( $G = 1$  denotes 100 %) that can be achieved by *Angle-technique* is

$$G_{max} = M \frac{\min(w_A, w_B)}{w_A + w_B} \quad (3.7)$$

**Theorem 3.2.6** An upper bound for the gain can be given as a linear function of  $\gamma$  in the following form

$$G(\gamma) \leq G_{max} \left( 1 - \frac{\gamma}{\gamma_{max}} \right) \quad (3.8)$$

If  $\max(w_A, w_B)$  is much greater than  $\min(w_A, w_B)$ , then the above upper bound gives a practically satisfying estimation.

**Claim 3.2.1** If  $\frac{\max(w_A, w_B)}{\min(w_A, w_B)} < 10$ , then the gain can be estimated by the following formula:

$$\max_{\alpha, \beta} G(\gamma) \approx G_{max} \left[ \vartheta_1 \left( \frac{\gamma}{\gamma_{max}} \right)^2 - \vartheta_2 \frac{\gamma}{\gamma_{max}} + 1 \right] \quad (3.9)$$

where

$$\begin{aligned} \vartheta_1 &= 1 - M \left( 0.0846 + 0.0679 \frac{\max(w_A, w_B)}{\min(w_A, w_B)} \right) \\ \vartheta_2 &= 2 - M \left( 0.0503 + 0.0691 \frac{\max(w_A, w_B)}{\min(w_A, w_B)} \right) \end{aligned}$$

and the cost of the network is given with 99.2 % accuracy at the attainable maximal gain (at any given  $\gamma$ ).

## Thesis 4 : Optimization of OSPF Administrative Weights in Access Networks [J5]

The demand arose that some important areas of the access networks should remain in working order in case of failures. For that purpose, the basic tree topology has to be extended with some extra links.

The dedicated assignment of the default paths and the backup paths is of help to the planning and to the operation as well. In our days, however, more and more networks are based on the IP technology, where the most commonly used routing protocol is the OSPF. This protocol routes the data packets in the shortest path according to the

administrative weights of the links. If there are multiple shortest paths, then the Equal-Cost MultiPath (ECMP) principle is applied, which divides the traffic between these paths.

The above requirements yield an inverse shortest path problem, where the OSPF-based routing finds the a priori given default paths and backup paths as the shortest ones based on the weights. I have to note, that a uniform weight system not always can be given. For all the cases (default and failure cases), however, an individual weight system always can be given.

I proposed a new heuristic algorithm to solve the problem and the efficiency of the proposed algorithm was backed up with detailed tests.

- Algorithm: a heuristic solution to the planning task. The algorithm takes the topology information into consideration and decreases the deviation from the predefined path system step by step. (Thesis 4)
- Simulation-based Performance Analysis: solve the task with linear programming and with known heuristics in case of many different sample networks. Then compare their results with the results of the proposed algorithm.

### **Approximate heuristic solution**

*After an initial weight setting, the proposed algorithm analyzes the network and finds those demands in case of each link failure, which demands are not routed in the predefined paths. Then it corrects these errors demand by demand, which means the modification of the weights in order to obtain the predefined path system.*

An important limitation of the task is that in order to satisfy the maximal delay constraint in the network, the backup paths can be at most 1 hop longer than the default paths [C2, J2]. Consequently, the backup paths contain exactly 1 backup link, which connects nodes together in the same hierarchy level or steps forward 1 level.

As a solution, I propose to combine the strategy of path "restoration" with the strategy of overload decrement. Thus if the predefined path system cannot be reproduced for some reason, then the proposed solution can still minimize the overload of the links. The framework of the proposed *OSPF Weight Setting Algorithm (OWSA)* is the following:

1. (*Initialization*) Initial settings and analysis of the network.
  - (a) (*Initial Weights*) Set the weight of each default link to 1 and set the weight of the backup links to 3. This provides that all backup paths contain exactly 1 backup link. (This procedure is referred to as *IWS*).
  - (b) (*Decomposition*) Search for infeasible situations caused by circles of backup links. If it is possible to decompose the circles, then decompose them by deleting redundant backup links, else STOP.

2. (*Weight Adjustment*) Iterative adjustment of the link weights.
  - (a) (*Basic Adjustment*) Eliminate the link overload in the network by modifying the weight of the links. (Increase the weight of the overloaded links in order to divert the traffic to the predefined paths.) Repeat this step until the overload decreases.
  - (b) (*Advanced Adjustment*) Eliminate the difference between the predefined and the actual path system by modifying the weight of the links. (Node by node examine whether the routing uses the same paths as the predefined ones. If not, then modify the weight of the links in the forbidden paths to get the predefined paths for that given node.) Repeat this step until the difference decreases.
  - (c) (*Special Adjustment*) There can be paths correlating to each other, so they cannot be corrected one after the other in Step 2b. Correct these paths (by discovering them and setting the weights of their links correctly) until the difference between the predefined and the actual path system decreases.
  - (d) (*Termination*) If all repairable errors are corrected, then STOP. Else go to Step 2a.

If it is required to decompose the circles, then the same backup path system cannot be reproduced as in the predefined case. In most of the cases, however, the overall performance of the network still remains the same after the decomposition.

### Simulation-based Performance Analysis of the Proposed Algorithm

The problem of weight setting can be often formulated as an *integer linear programming* (ILP) task. The solution of this task can be exact, of course. In case of a larger network, however, this solution cannot be applied, since extreme long time is needed to find this solution. By comparing ILP [60] and OWSA, one can find that the running time of ILP is hundredfold in case of networks with 50 nodes and many thousandfold in case of networks with 100 nodes compared to the running time of OWSA. Table 4.1 illustrates the difference of the running times. (The phrase "*uncertain*" refers to that the ILP cannot reach solution even for 100 nodes in three days.)

algorithm	number of nodes			
	50	100	150	200
OWSA	4.16 sec	8.03 sec	18.6 sec	35 sec
ILP	≈ 1 hour	uncertain	uncertain	uncertain

Table 4.1: The average running time of the algorithms for different networks.

Table 4.1 backs up that heuristic solution have to applied in case of practical network sizes. In the rest of the thesis, OWSA is compared with other heuristics.

The weight setting algorithms [56, 45] usually aim at abolishing the overload of the links and at maximizing the free capacities. As reference, two simulated annealing based heuristics are applied.

The first heuristic (RefSol-1) picks an overloaded link and increases its weight by 1. After evaluating the new solution, it steps forward in the optimization, which is the minimization of the overload in that case. This indirectly guarantees that the predefined path system is reproduced if there is no overload in the network, since the capacities of the links are set according to the predefined path system.

The second heuristic (RefSol-2) not only increases the weights, but also decreases them in case of the links, which are not fully utilized. In that case, the weights are modified according to a probability in proportion to the underload or overload of the link.

The algorithms were compared to each other in case of several sample networks. Different network sizes were evaluated (50 . . . 200 nodes) and the number of the backup links was also varied (10 . . . 40 % compared to the number of default links).

Figure 4.1 shows the results of the comparison. It can be seen that the Initial Weight Setting (IWS) (see Step 1a of OWSA) cannot solve the problem in itself. RefSol-1 can solve the problem in case of 10 % backup links, but in case of higher protection, it fails. RefSol-2 cannot solve the problem even in case of 10 % backup links if the network is larger. Nevertheless, both reference methods can reduce the overload of the network under 0.6 %. The proposed OWSA always solves the problem perfectly and its running time was less with an order than the reference solutions’.

The tests showed that OWSA solves the planning problem more efficiently than the existing algorithms and approaches.

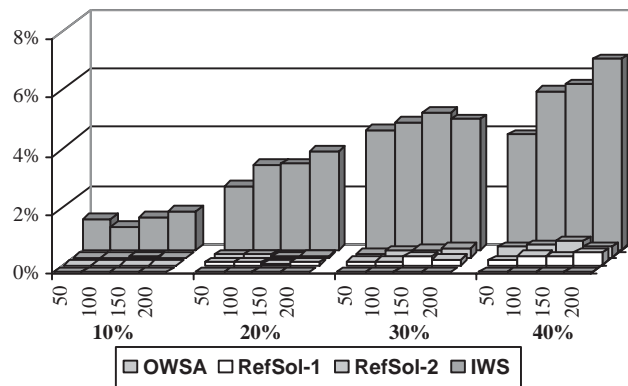


Figure 4.1: Average overload in the network.

## 5 Application of the Results

The algorithms presented in *Thesis 1* (for the planning of interference-insensitive access networks) are used in the UMTS Network Planning and Analysis Tool of Ericsson Telecommunication Ltd.

The research work related to the interference-sensitive access networks (*Thesis 2*)

and to the optimization of OSPF administrative weights (*Thesis 4*) was carried out in the projects of Ericsson Research Hungary in cooperation with the Product Units of Ericsson.

The theoretical and applied research results presented in *Thesis 3* can be widely used in the field of network planning and geometry-based optimization.

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