MARKOV MODELS FOR RADIO LINKS AND THEIR APPLICATION IN ATTENUATION TIME SERIES SYNTHESIS

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FADE AND INTERFADE DURATION MODELING WITH MARKOV CHAIN AND ITS APPLICATION IN TIME SERIES SYNTHESIS

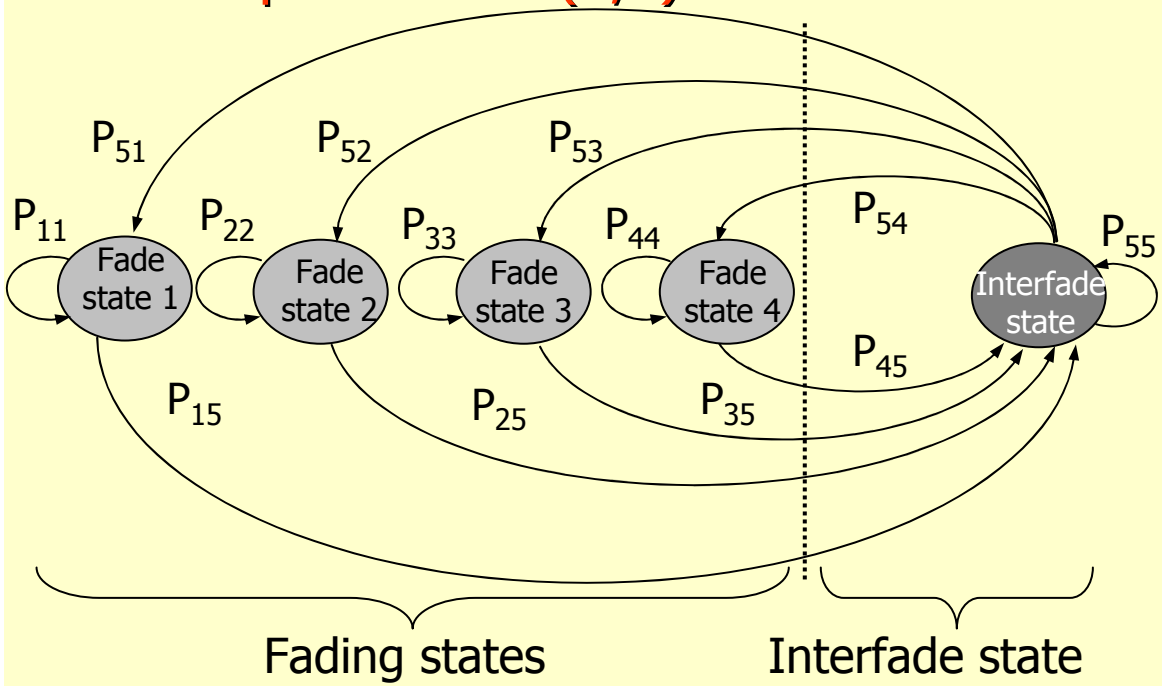
- The measurement data used in the modeling process:
  - Terrestrial link
  - Land mobile satellite link
- Stochastic modeling of fade and interfade duration with Markov-chains / the Fritchman model
- Model parameter estimation from measurement data
- Time series synthesis
  - Applying fade and interfade duration models
  - Synthesis with two-state and hidden Markov model
  - Synthesis with two-state and fade duration model

Measurement Conditions

Measurement data used in the modeling process:

Link	Terrestrial	Land Mobile Satellite
Path length	1.504 km	Geostationary (d=39150 km)
Frequency	38 GHz	1.54 GHz
Sampling rate	1 Hz	300.5 Hz
Duration	One year	81.2 min
Fading type	Mainly rain	Multipath propagation
Measured	2004, Hungary	1984-87, DLR

A partitioned (4,1) Fritchman's model for fade duration modeling



The complementary cumulative fade/interfade duration distribution (CCDF) can be calculated as:

$$CCDF_F(n) = \sum_{i=1}^{N-1} \frac{p_{Ni}}{p_{ii}} p_{ii}^n$$

$$CCDF_I(n) = \left(\sum_{j=1}^{k} \frac{Z_j p_{jN}}{Z_F p_{NN}}\right) p_{NN}^n$$

State 1 = longest fading interval
State N-1 = shortest fading interval

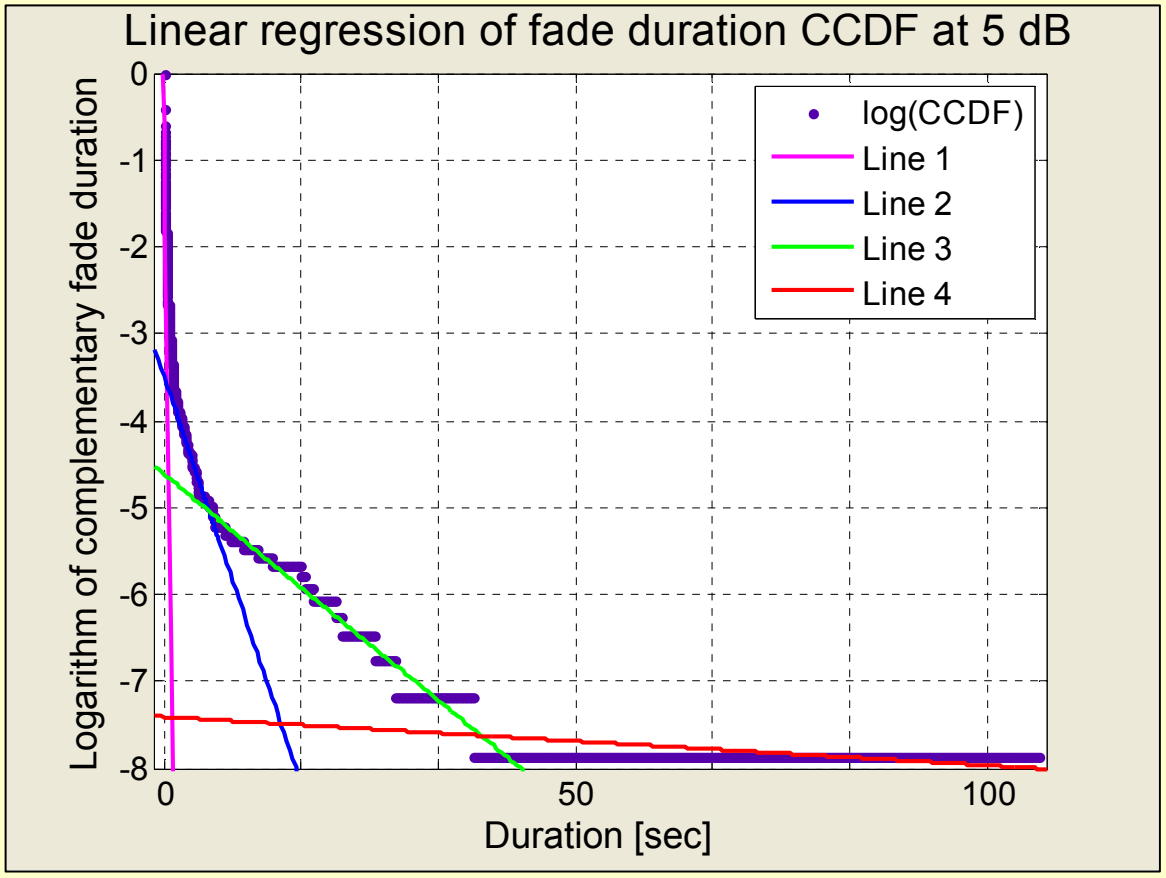
$$p_{11} > p_{22} > \ldots p_{N-1N-1}$$
$$p_{N1} < p_{N2} < \ldots p_{NN-1}$$

Steady state probabilities:

$$Z_N = \frac{1}{1 + \sum_{i=1}^{k} \frac{p_{Ni}}{p_{iN}}}$$

$$Z_i = \frac{p_{Ni}}{p_{iN}} Z_N$$

Fade duration model parameterization with gradient method (LMS link)



Linear regression of the log CCDF curve:

- Number of regression lines = 4
- Fading states in the model = 4
- Number of interfade state = 1

Equation of line: $f(x) = ax + b$

$a \rightarrow p_{ii}$

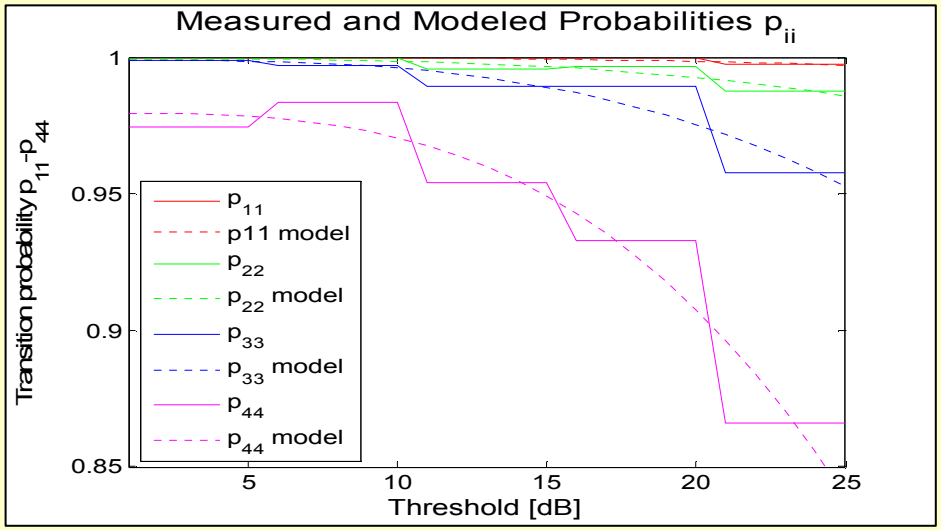
$b \rightarrow p_{Ni} / p_{ii}$

$$\begin{pmatrix} P_{11} & 0 & 0 & 0 & P_{15} \\ 0 & P_{22} & 0 & 0 & P_{25} \\ 0 & 0 & P_{33} & 0 & P_{35} \\ 0 & 0 & 0 & P_{44} & P_{45} \\ P_{51} & P_{52} & P_{53} & P_{54} & P_{55} \end{pmatrix}$$

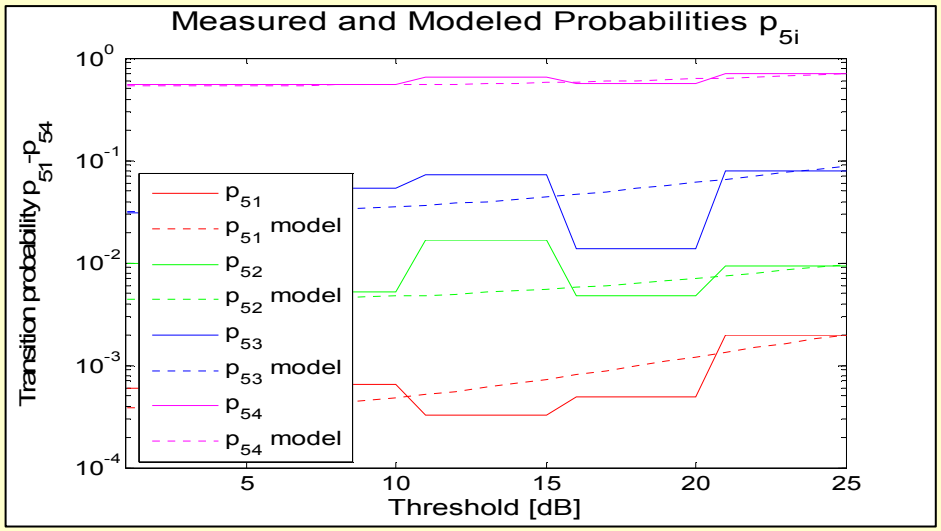
State transition matrix

The CCDF of fade duration can be calculated

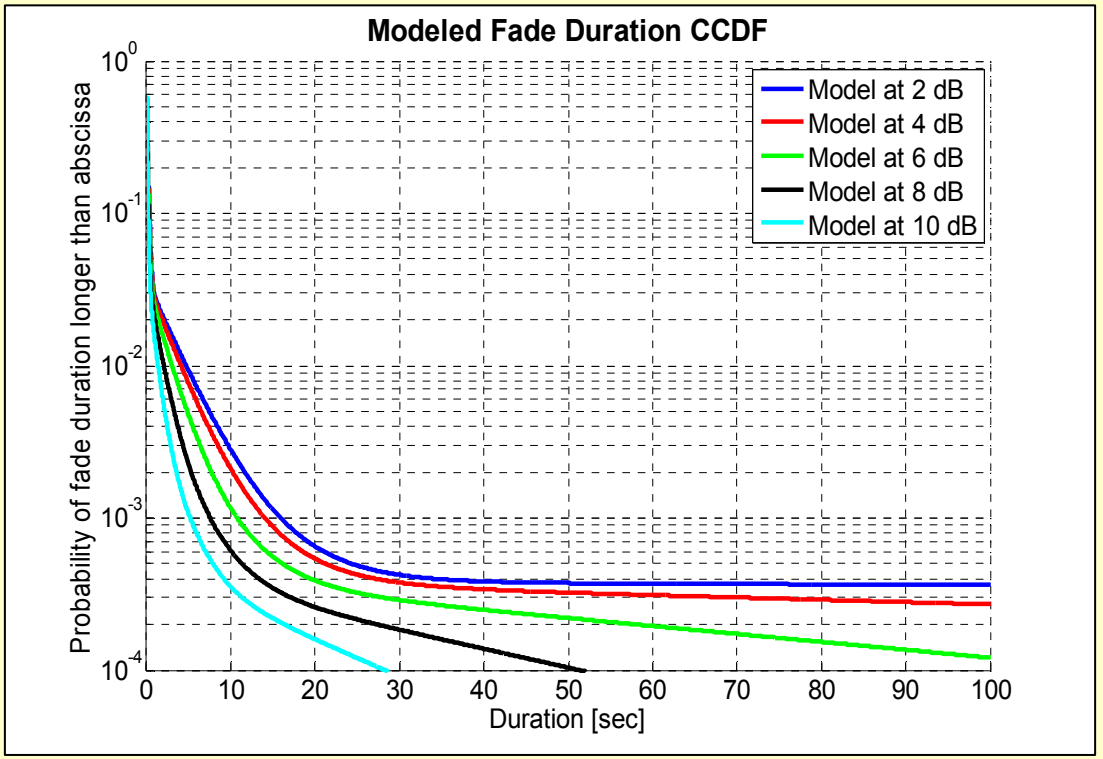
Threshold Dependency Of Model Parameters (LMS)



$$p_{ii}(A) = a_{p_{ii}} A^3 + b_{p_{ii}}$$



$$p_{5i}(A) = a_{p_{5i}} A^3 + b_{p_{5i}}$$



Modeled CCDF diagrams

Investigated methods for attenuation time series synthesis:

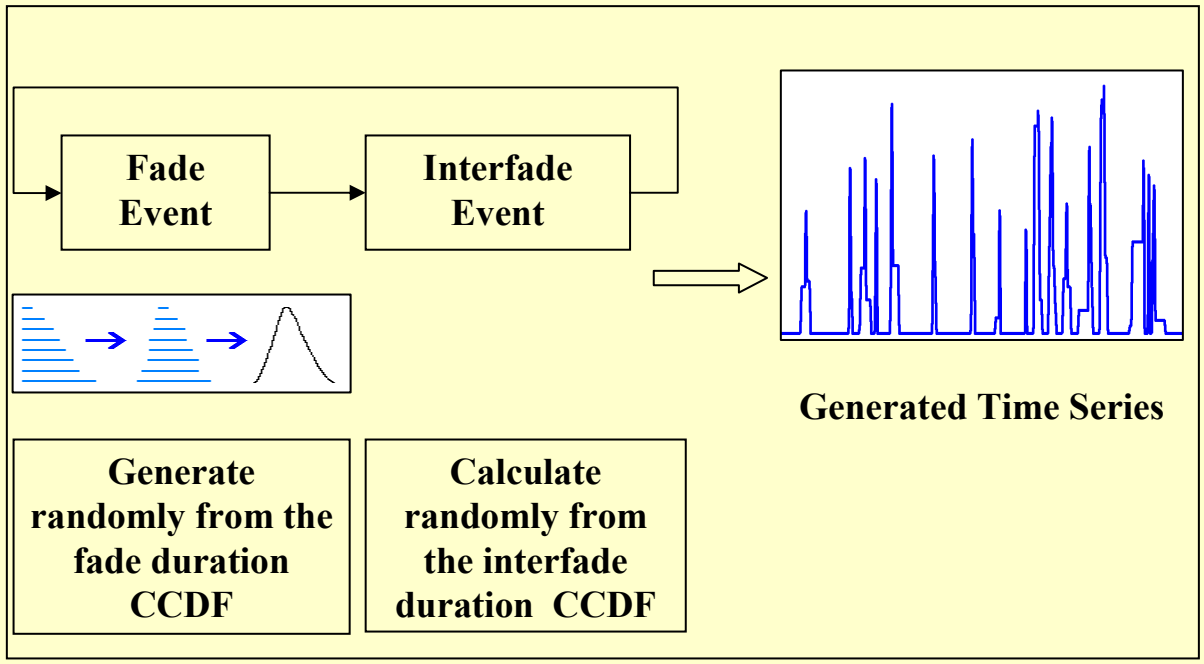
1) Applying fade and interfade duration models

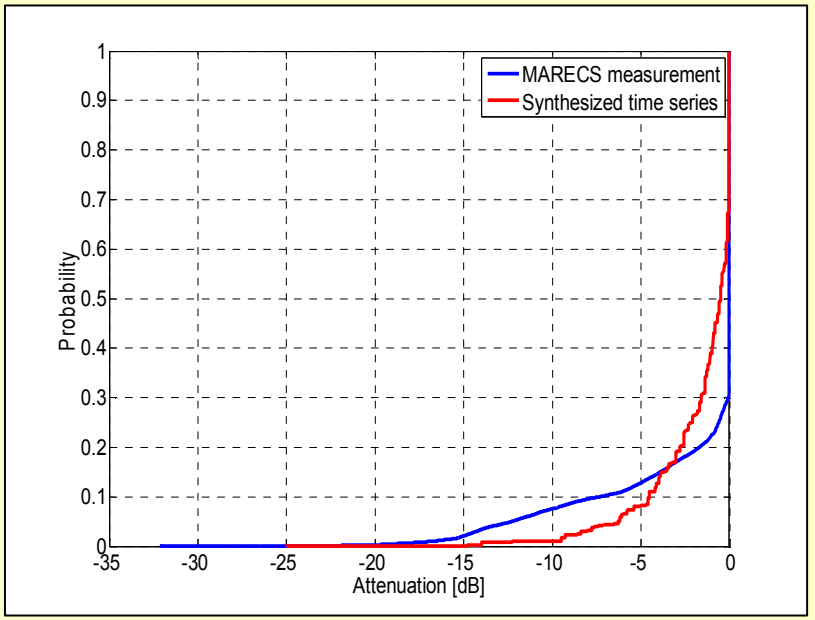
2) Two-state fade/non-fade model combined with hidden Markov model synthesized fading events

3) Two-state fade/non-fade model combined with fade duration model synthesized fading events

Time series synthesis with fade and interfade duration model

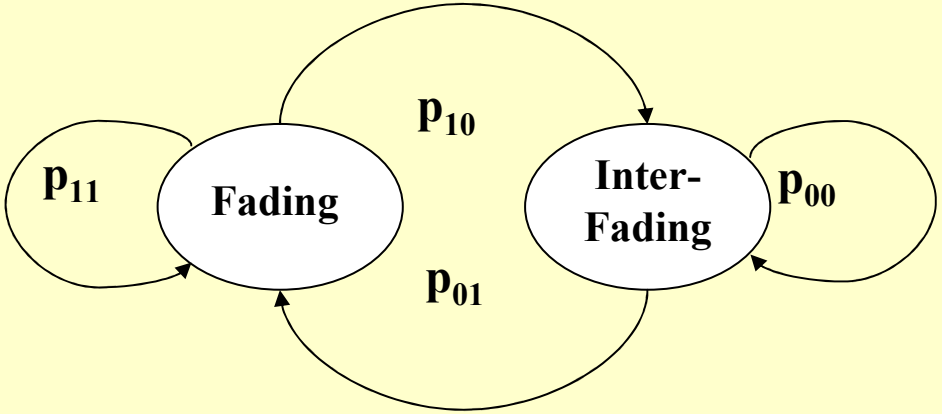
- Create the Markov model of the fading and inter-fading process of the radio channel
- Calculate the model parameter dependence on the attenuation threshold
- Model fade and interfade periods for the desired fade depth with the fade duration CCDF
- Synthesize single fading events





CDF of the measured and generated time series (LMS)

Two-state model for fading/non-fading event generation



The transition matrix:

$$\overline{\overline{P}} = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix} = \begin{pmatrix} 1-p_{01} & p_{01} \\ p_{10} & 1-p_{10} \end{pmatrix}$$

State probability vector at time n :

$$Z(n) = [p_0(n) \quad p_1(n)] = [1-p_1(n) \quad p_1(n)]$$

Discrete time/state first order Markov chain:

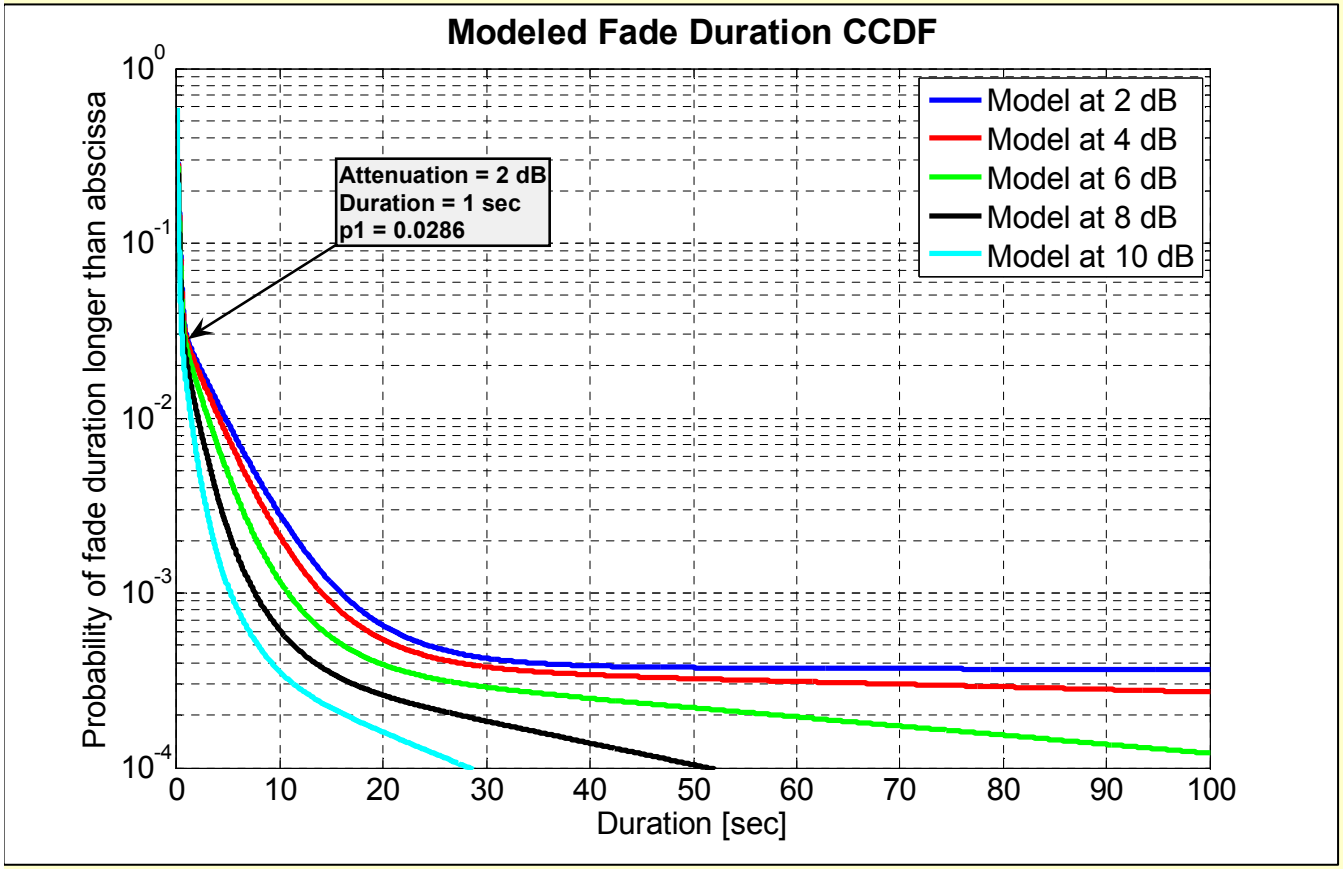
$$Z(n+1) = \overline{\overline{P}} \cdot Z(n)$$

$$(1-p_1, p_1) = \begin{pmatrix} 1-p_{01} & p_{01} \\ p_{10} & 1-p_{10} \end{pmatrix} \cdot (1-p_1, p_1)$$

The Markov chain determined by P_1 and P_{10}:

$$p_{01} = \frac{p_1}{1-p_1} \cdot p_{10}$$

Determine the two-state model parameters from the fade duration CCDF (LMS)



CCDF's calculated from the Fritchman model

At threshold 2 dB and duration 1 sec the fade duration probability:

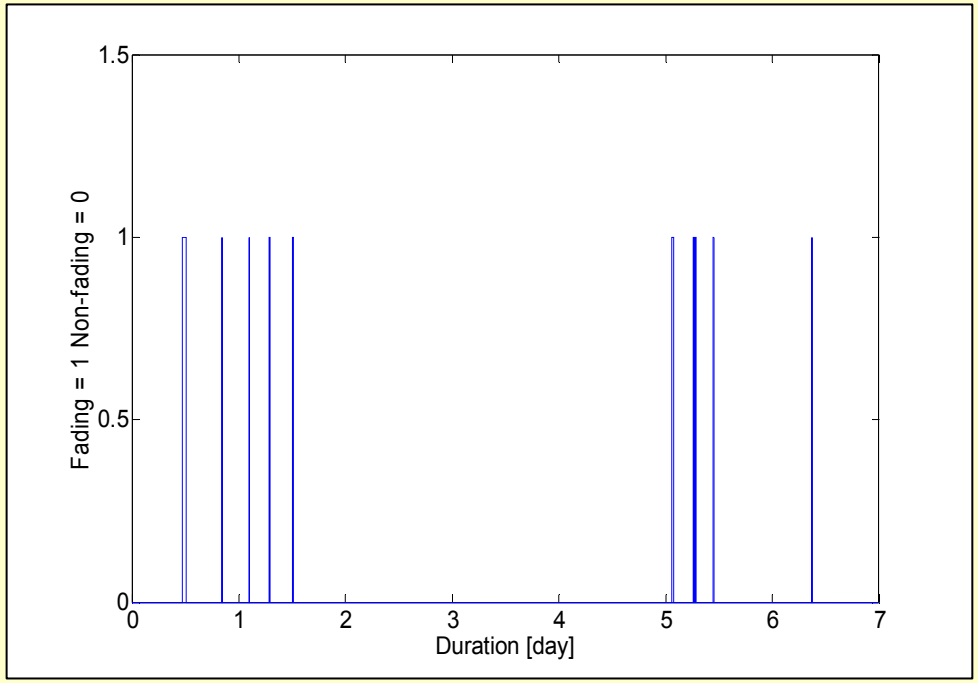
p_1 =0.0286

Calculate p_{10} probability from the Fritchman model:

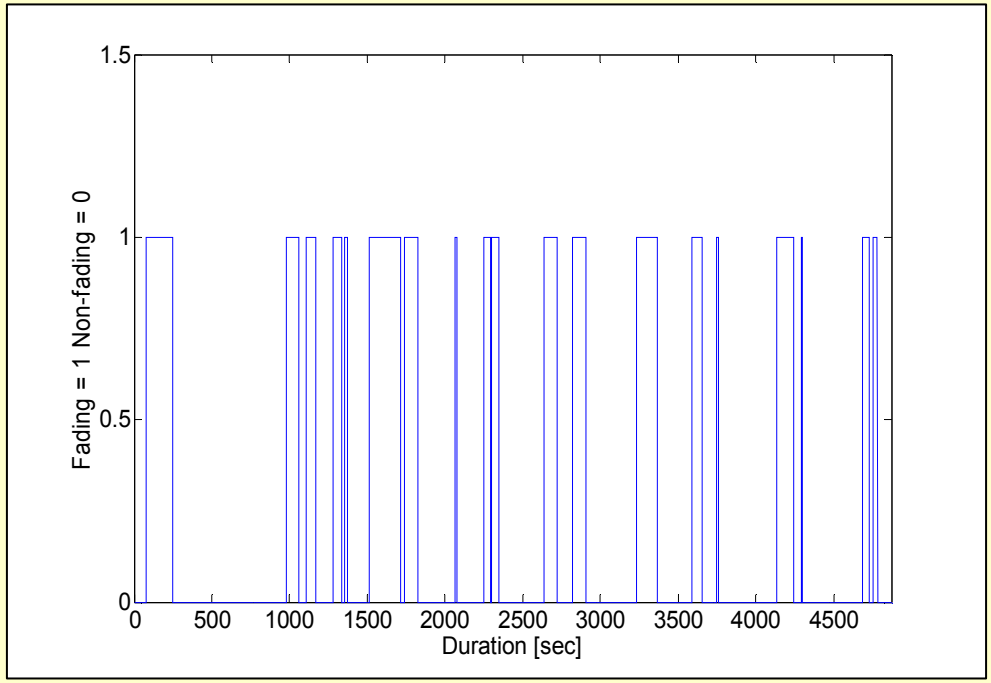
$$p_{10} = \sum_{i=1}^{N-1} p_{iN} \cdot \frac{Z_i}{Z_F}$$

p10 = 0.0018

Simulated fade and non-fade event series



Simulated fading event series for terrestrial link, one week duration

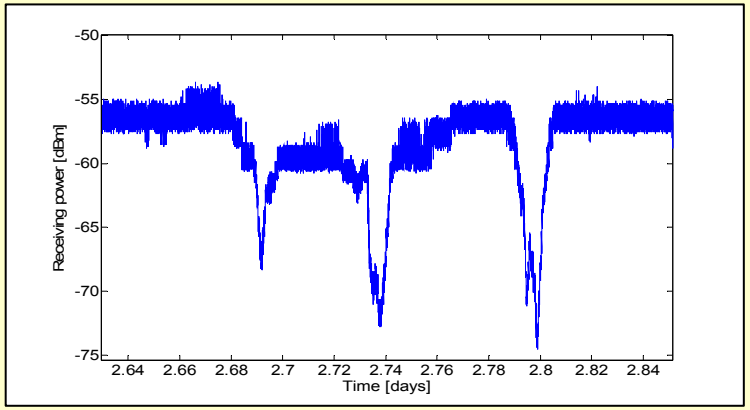


Simulated fading event series for LMS link, 80 min

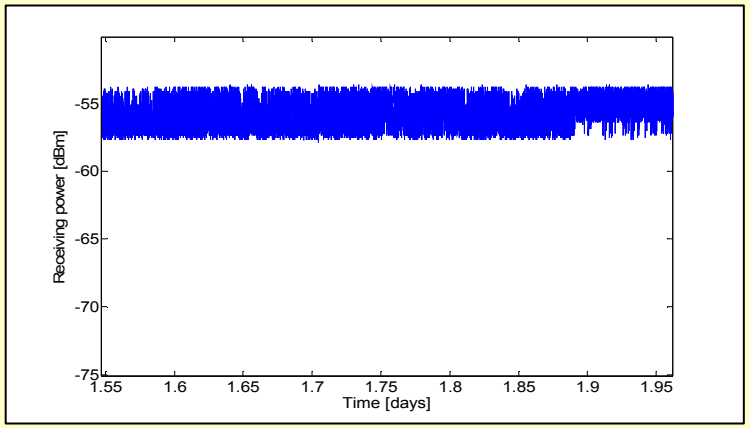
Fade event synthesis with hidden Markov model

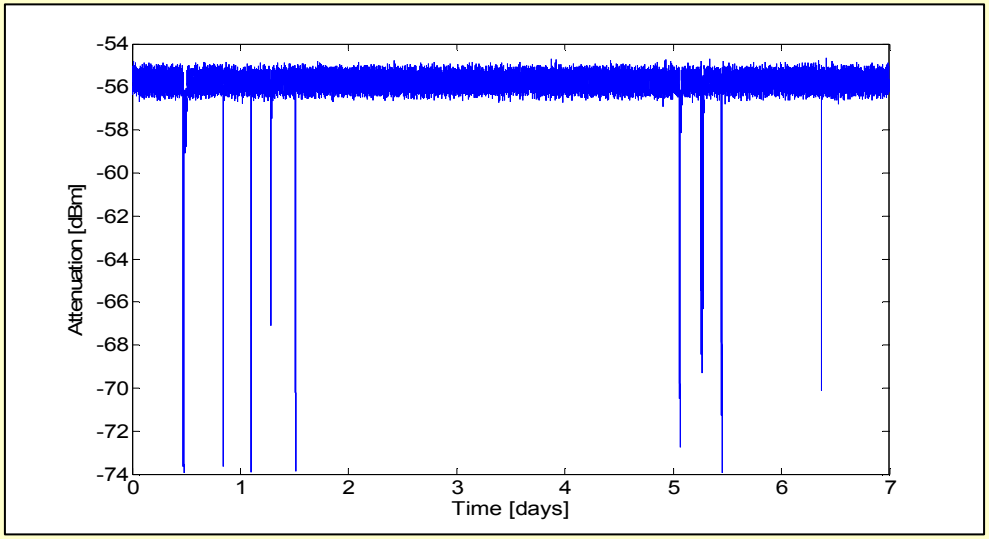
- Parameterization with Baum-Welch algorithm
- Training data from the measurement

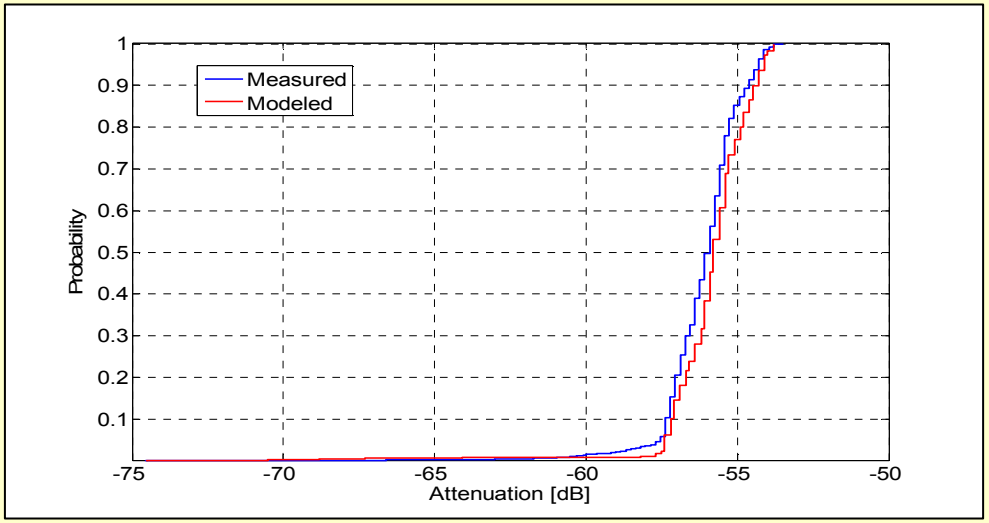
Training period for fading event



Training series for non-fading periods







One week synthesized attenuation time series and CDF for terrestrial link

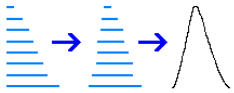
Fade event synthesis with fade duration CCDF

Set the required duration D_t and maximal fade depth A_{max}

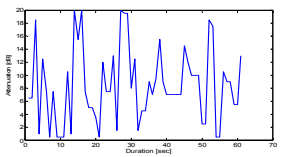
↓

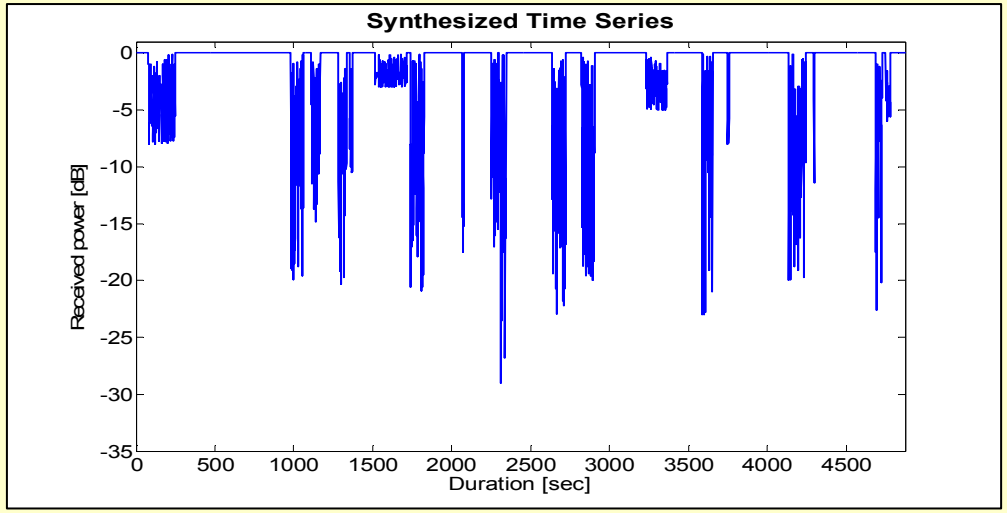
Calculate the modeled fade duration CCDF's from 0 to A_{max} with the required A_{step}

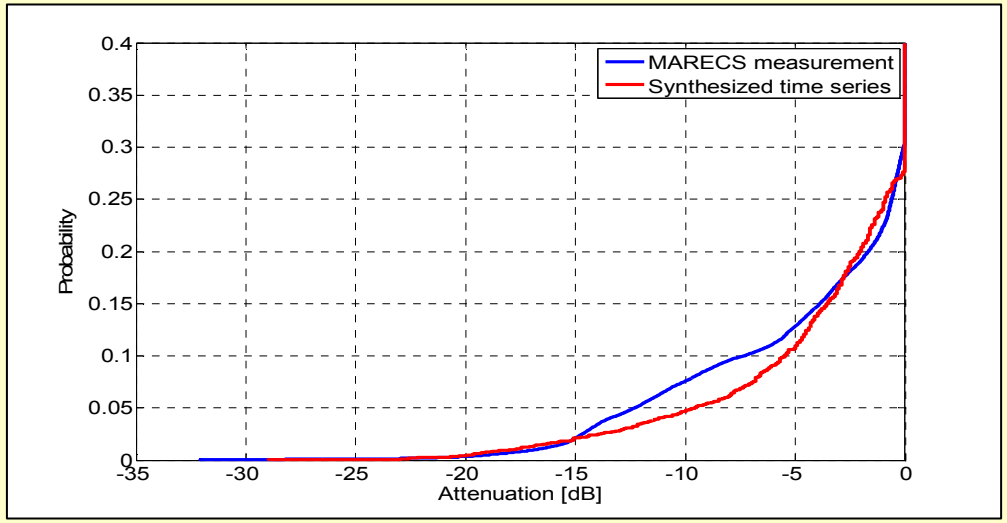
↓

Synthesize one single event 

↓

Concatenate the single events while D_t achieved 





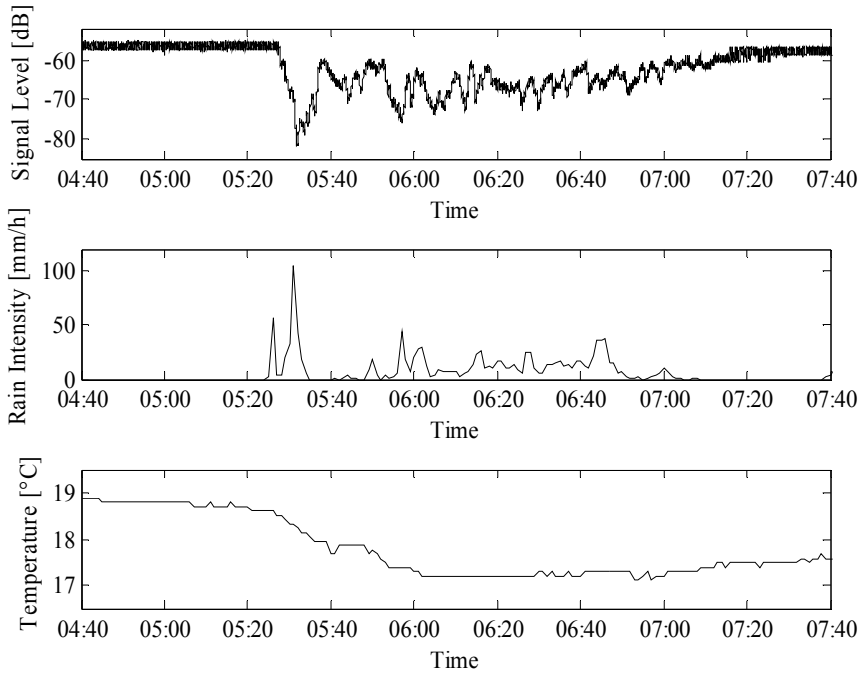
80 min. synthesized attenuation time series and CDF

General N-State Markov model for Rain Attenuation Time Series Generation

- Rain Attenuation Measurement
- N-State Markov Chain Model for attenuation time series generation
- Model parameterization from fade slope
- First and second order statistics of generated time series
- Comparison of measured and generated time series with
  - First order statistics (attenuation CCDF)
  - Second order statistics (Fade duration)
- Conclusions, Future Work

Rain attenuation measurement

Measured signal level rain intensity and temperature



Measurement parameters

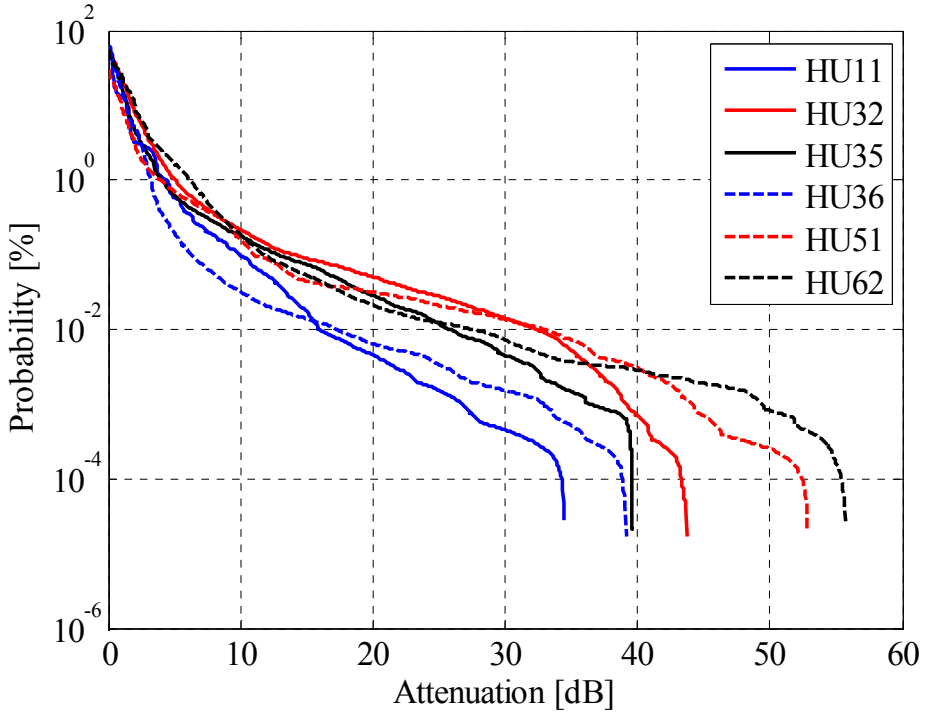
Site Name	Location	Freq. [GHz]	Pol.	Length [km]	Applied data interval	
					From	To
HU11	Budapest	38	H	1.50	1st January 2005	31th December 2005
HU32	Pécs	23	V	10.03		
HU35	Pécs	15	V	23.36		
HU36	Pécs	38	V	1.69		
HU51	Győr	15	V	12.30		
HU62	Szeged	23	V	7.79		
Hyp.	Miskolc	23	V	1.00	-	-

Transformation to hypothetical link

- First order statistics of attenuation could be very diverse because of the different link parameters.
- Transformation based on ITU-R P. 530 recommendation:

$$A_h(t) = \frac{k_h \cdot L_h}{1 + L_h / d_0} \cdot \left[\frac{A_m(t) \cdot \left(1 + \frac{L_m}{d_0}\right)}{k_m \cdot L_m} \right]^{\frac{\alpha_h}{\alpha_m}}$$

$$d_0 = 35 \cdot e^{(-0.015 \cdot R_{0.01})}$$



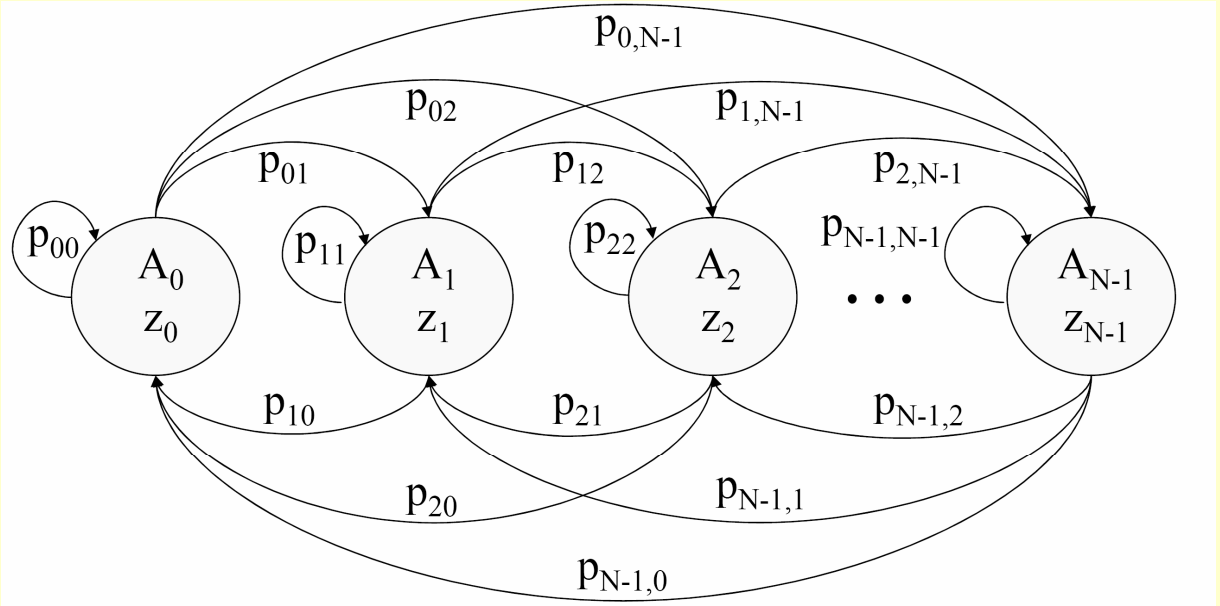
Attenuation CCDFs of measured data

$A_h(t)$ and $A_m(t)$ are time dependent att. on the hypothetical and measurement links.
k_m, α_m, k_h, α_h are polarization and freq. dependent variables for the hypothetical and measurement links
d_0 is the path reduction factor
The geographical location dependent rain intensity R is higher than or equal to $R_{0.01}$ in 0.01 percent of the year.

The N-State Markov Chain Model

- Each state represents an attenuation level with 0.05 dB resolution

- N: number of states



$$\overline{\overline{P}} = \begin{bmatrix} p_{00} & p_{01} & \cdots & p_{0,N-1} \\ p_{10} & p_{11} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ p_{N-1,0} & \cdots & p_{N-1,N-2} & p_{N-1,N-1} \end{bmatrix}$$

$$\bar{z} = [z_0, z_1, z_2 \ldots, z_{N-1}]$$

$$\bar{z}(t) = \overline{\overline{P}}^{T} \cdot \bar{z}(t-1)$$

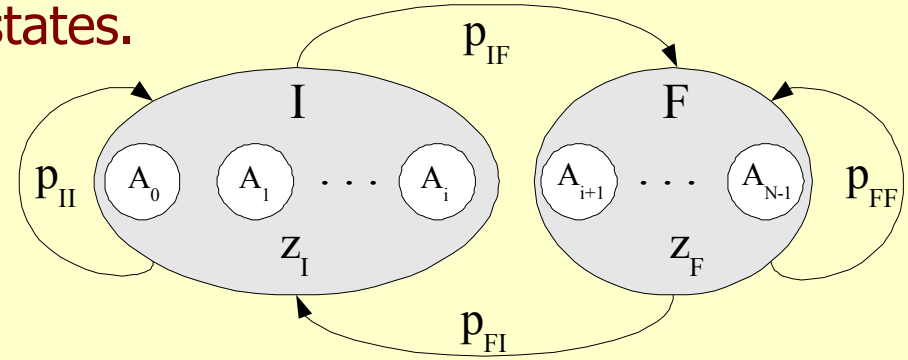
$$\bar{z} = \overline{\overline{P}}^{T} \cdot \bar{z}$$

First and second order statistics of generated time series

- Attenuation CCDF of the time series generated with the Markov chain model:

$$P(A \geq A_i) = \sum_{j=i}^{N-1} z_j$$

- Fade duration: the states of the N-state Markov model must be sorted into two classes, into the fading (F) and interfading (I) states.



z_F : fading state probability

z_I : interfading state probability

$p_{II}, p_{FF}, p_{IF}, p_{FI}$:transition probabilities

The two-state Markov chain model if fade duration is considered at attenuation level A_i.

$$z_I = \sum_{k=0}^{i} z_k \qquad z_F = \sum_{k=i+1}^{N-1} z_k$$

$$p_{FI} = \sum_{k=i+1}^{N-1} \sum_{m=0}^{i} \frac{z_k}{z_F} \cdot p_{km} \qquad p_{IF} = \sum_{k=0}^{i} \sum_{m=1+i}^{N-1} \frac{z_k}{z_I} \cdot p_{km}$$

$$p_{II} = 1 - p_{IF}$$

$$p_{FF} = 1 - p_{FI}$$

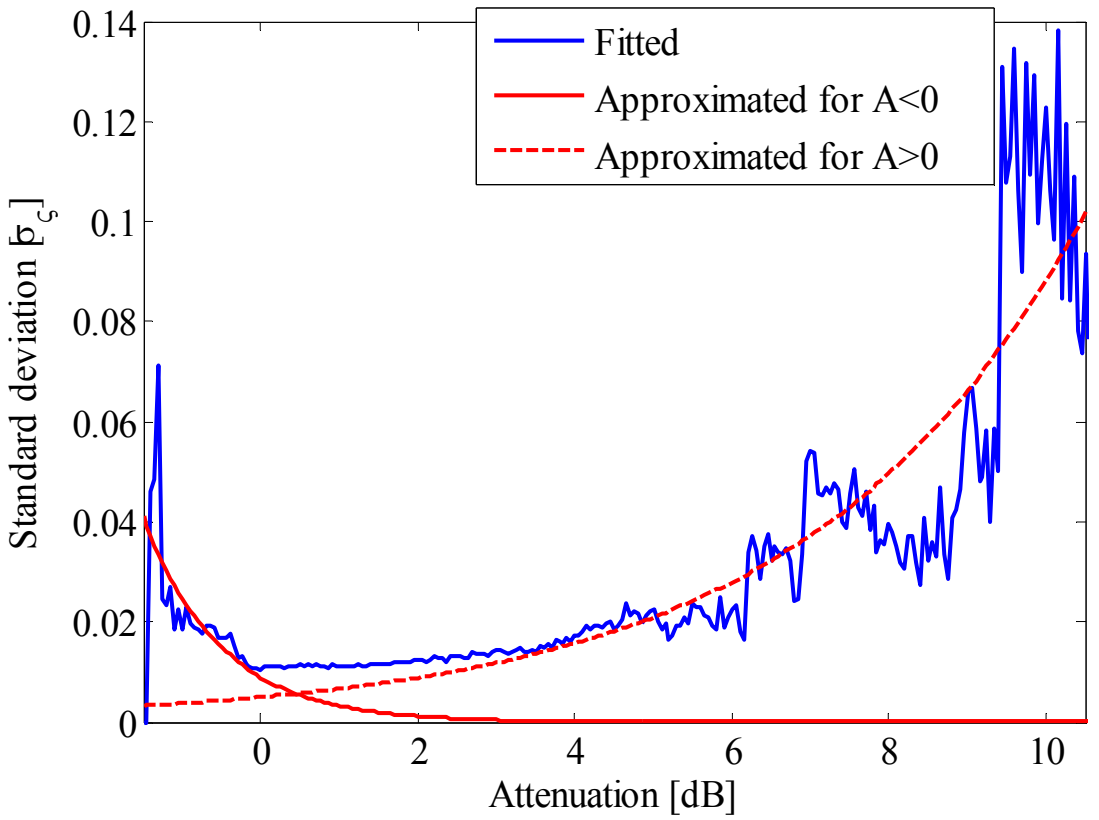
$p_d(A_i, t) = p_{FF}^{t} \cdot p_{FI}$:the probability of a fading at A_i attenuation level which duration is exactly *t* seconds

Parameterization of the model

Fade slope (ς) gives the gradient (in dB/s) of the fading at a given A_i attenuation level:

$$\varsigma^{[dB/s]}\Big|_{A_i} = \frac{A(t_{n+1}) - A(t_{n-1})}{2}, \; where \; A(t_n) = A_i$$

CPDF of fade slope can be estimated by a 0 mean Gaussian distribution function:



$$P(\varsigma|A) = \frac{1}{\sqrt{2\pi}\,\sigma_\varsigma(A)} \cdot e^{-\frac{1}{2}\cdot\left(\frac{\varsigma}{\sigma_\varsigma(A)}\right)^2}$$

A :att. level in dB

$\sigma_\varsigma(A)$:standard deviation

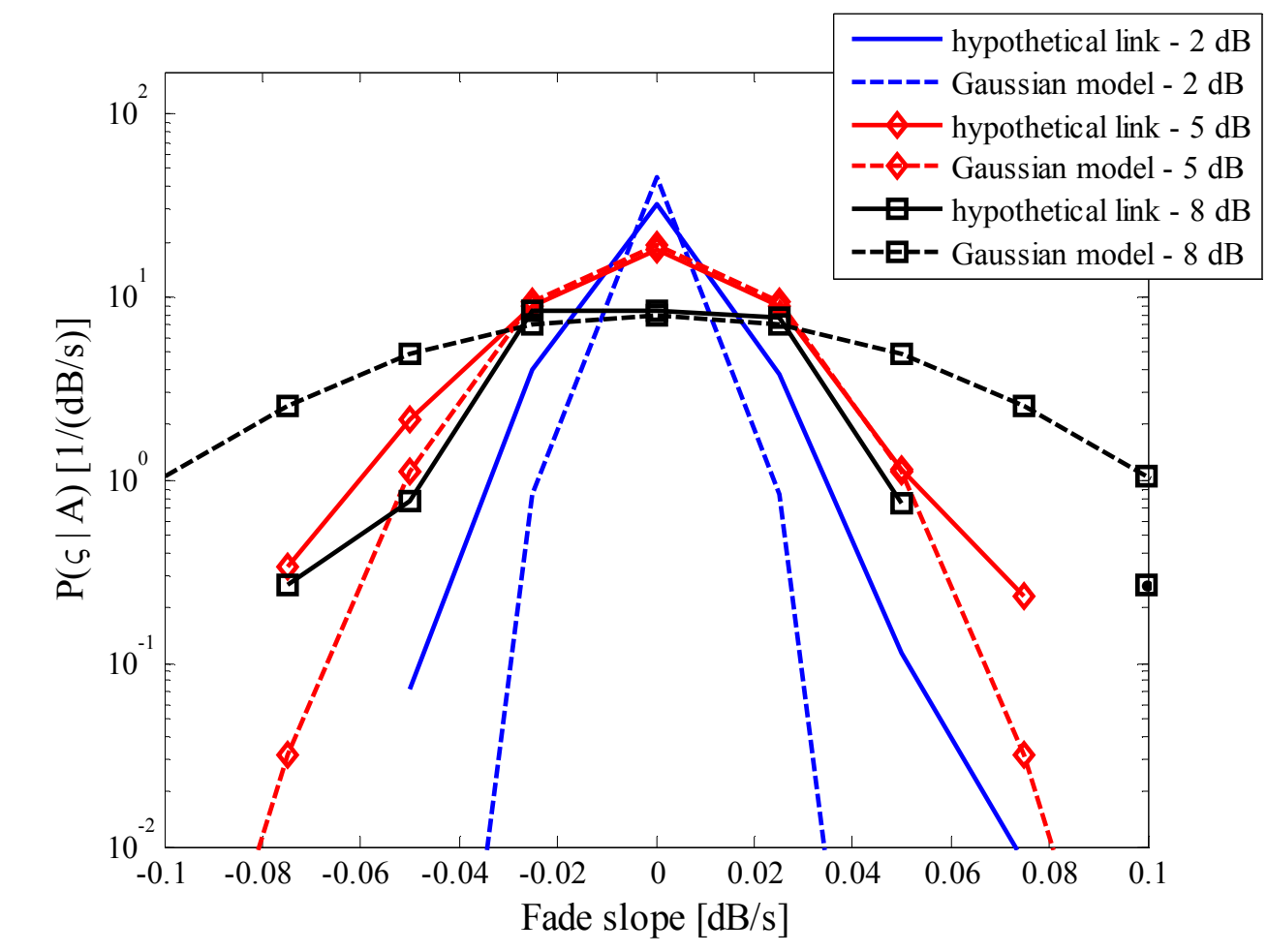
ς :fade slope [dB/s]

$$\sigma_\varsigma(A) = \begin{cases} a \cdot e^{b \cdot A} & , A < 0 \text{ dB} \\ c \cdot e^{d \cdot A} & , A \geq 0 \text{ dB} \end{cases}$$

$a = 8.914 \cdot 10^{-3}$ $\quad c = 4.983 \cdot 10^{-3}$

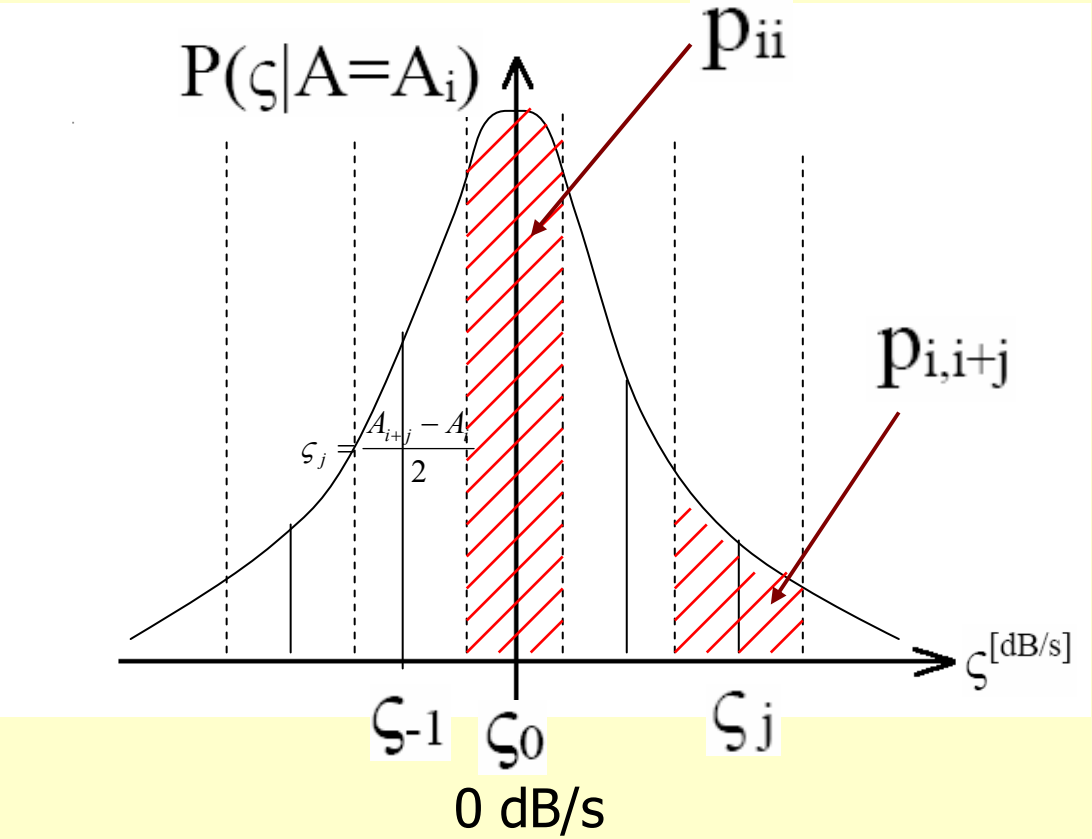
$b = -1.018$ $\quad d = 0.2874$

Estimating the fade slope density functions with Gaussian distribution function

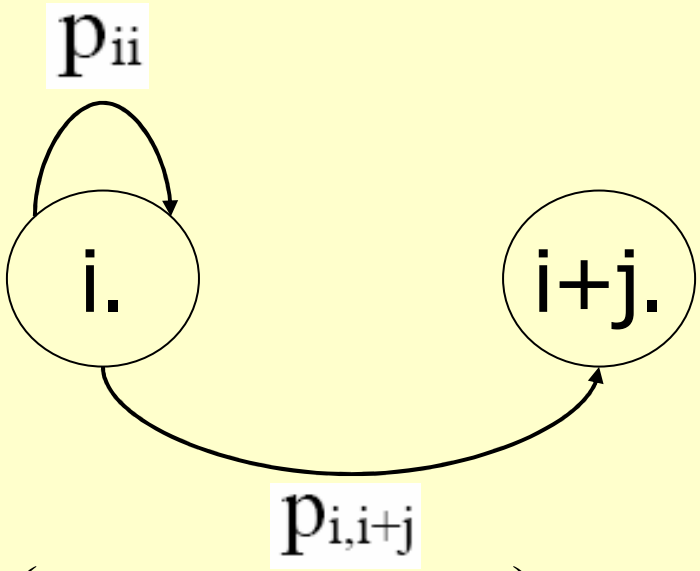


CPDF of fade slope for different attenuation levels calculated on the hypothetical link and from the Gaussian fade slope model

From fade slope to transition probability



$$\varsigma^{[dB/STU]} = \frac{A(t_{n+1}) - A(t_{n-1})}{2}, if\ A(t_n) = A_i$$



Conditional Probability Density of Fade Slope at i. att. level (i. state)

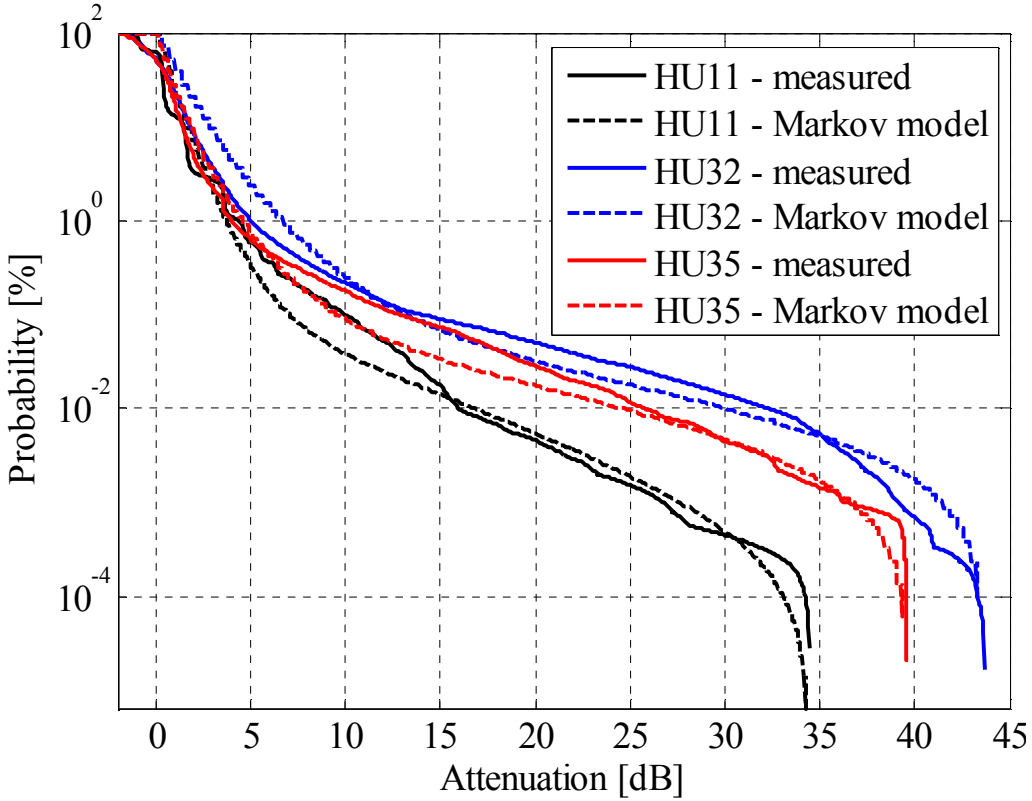
$$p_{ij} \Leftrightarrow P\left(\varsigma = \frac{A_{i+j} - A_i}{2} \middle| A = A_i\right) = P\left(\varsigma_j \middle| A = A_i\right)$$

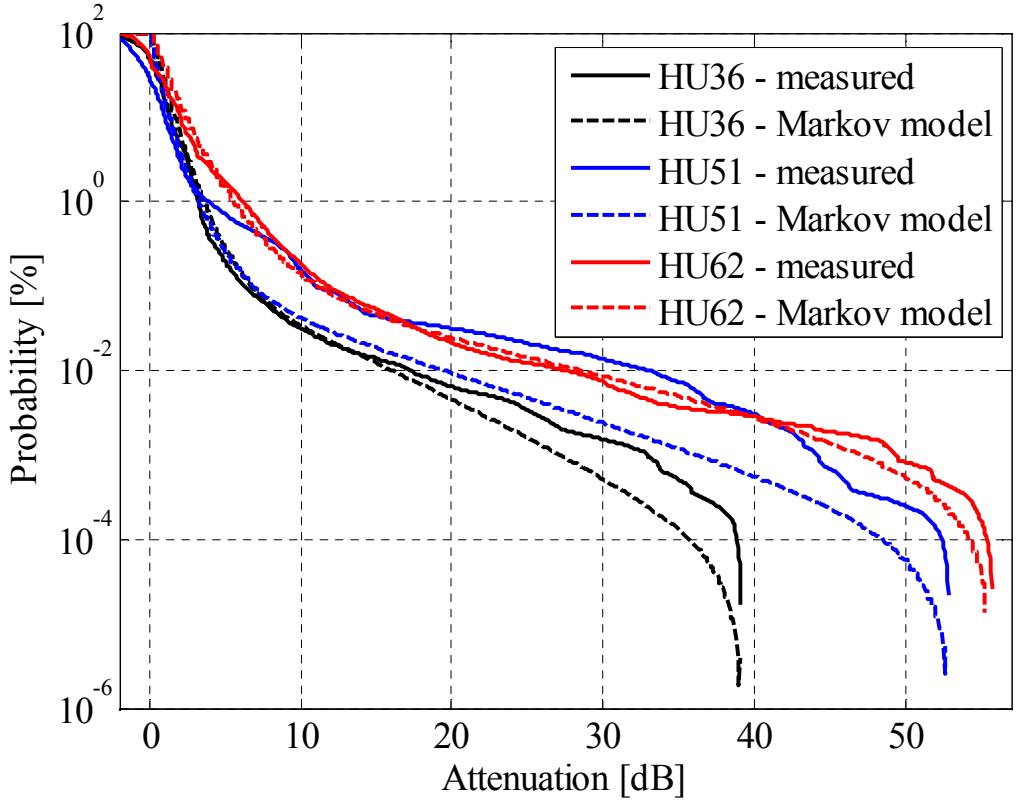
First order statistics of the measured links

$$\overline{\overline{P'}} = \{P_{ij}; m < i, j < M\}$$

$$m = \frac{A_{m,\min} - A_{h,\min}}{0.05} \qquad M = \frac{A_{m,\max} - A_{h,\min}}{0.05}$$

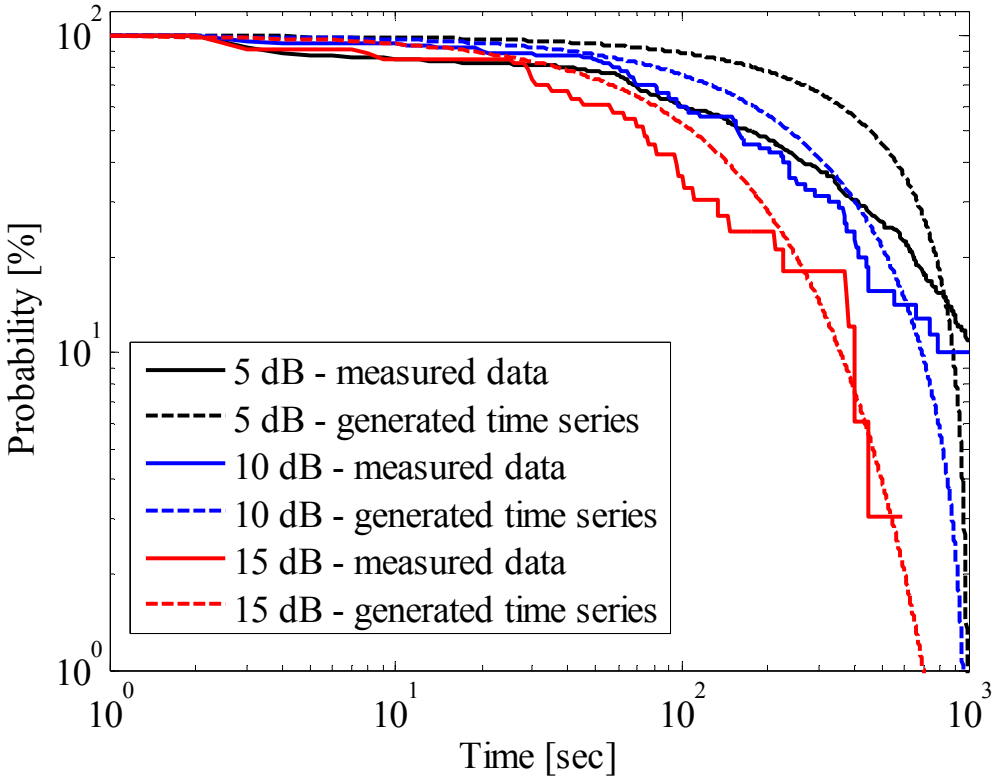
m, *M* :minimum and maximum state number
$A_{m,min}$, $A_{m,max}$ min. and max. transformed attenuation level in the measurement
$A_{h,min}$, $A_{h,max}$ min. and max. attenuation level in the hypothetical link data

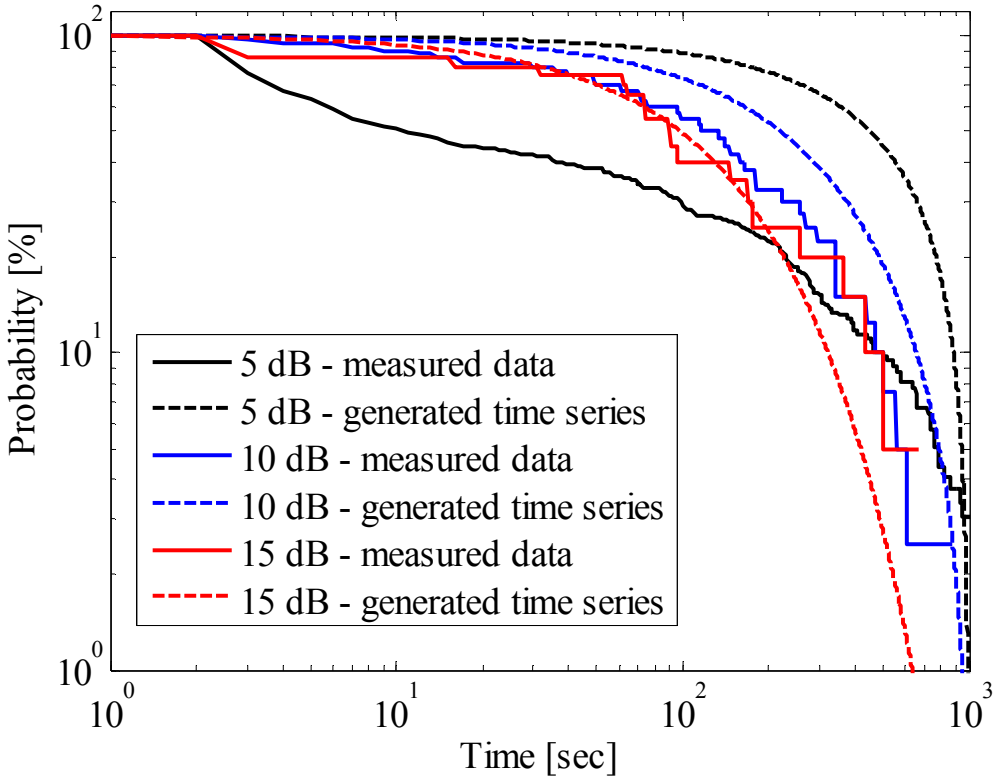




The yearly attenuation CCDFs of the measured data and of the generated time series.

Second order Statistics of the Measured Links



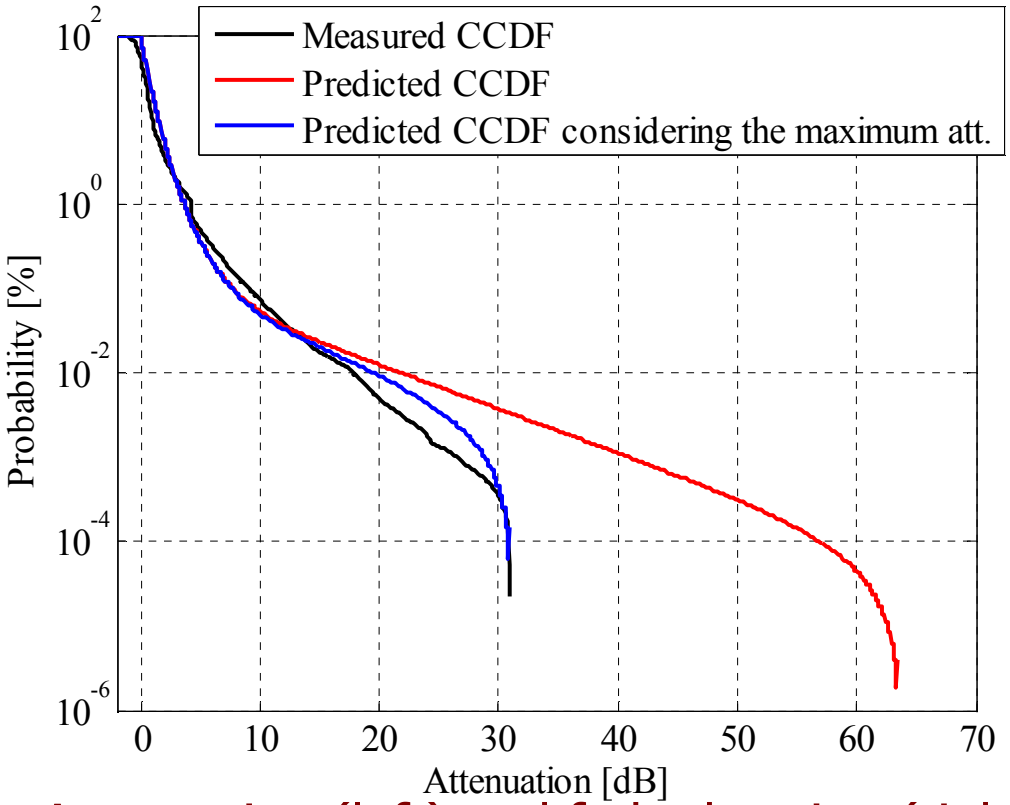


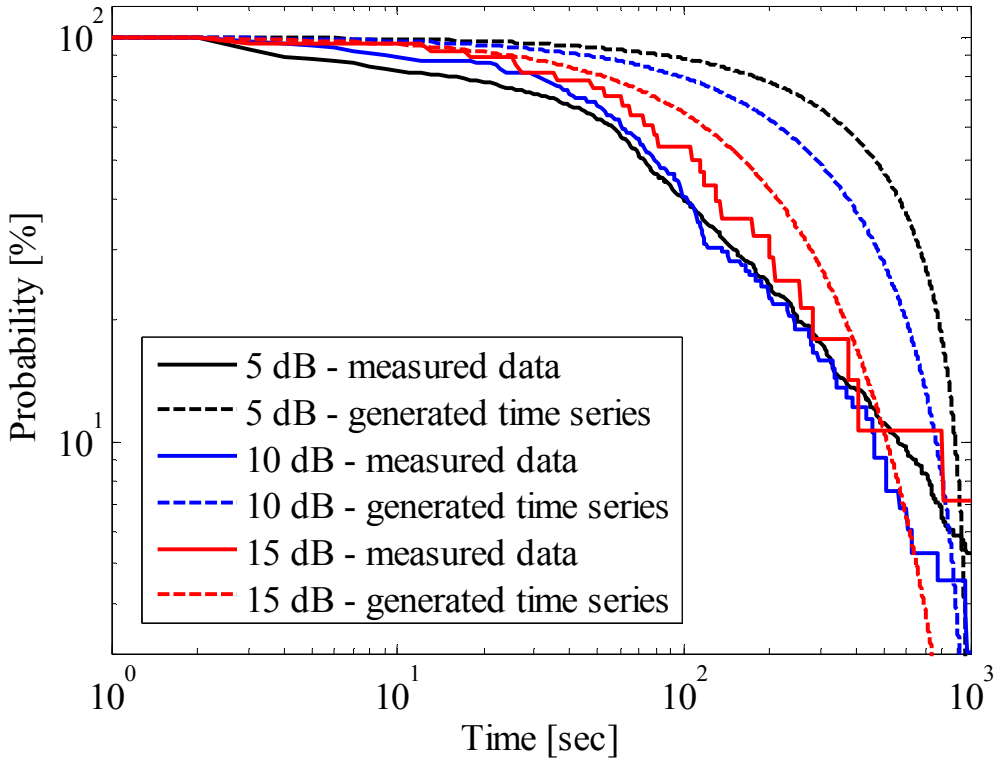
Fade duration statistics of the measured data and of the generated time series belong to HU11 (in the left) and HU36 (in the right)

Estimating first and second order statistics of arbitrary links

Location: Szeged Frequency = 15 GHz Length = 15.17 km

$R_{0.01}$ = 35.97 mm/h Vert. polarization





Attenuation (left) and fade duration (right) CCDF prediction Fade duration statistics prediction

Summary, Conclusions

- Fade and interfade duration process can be modeled with attenuation threshold dependent Fritchman's Markov chain
- The CCDF of fade duration is applicable to parameterize a two-state fade/non-fade model and simulate the fine structure of a fade event
- General N-State Markov model parameterized from fade slope statistics is applicable for attenuation time series generation
- The attenuation CCDF and the fade duration statistics can be directly determined from the model parameters
- Future works:
  - Introduce the long-term variance in the model (weather impact in case of rain attenuation, change of the environment geometry for multipath propagation)
  - Apply the model for valid satellite links, or consider satellite links for model parameterization
  - Complete the model with correlation angle dependency

Thank you for your attention!