Budapest University of Technology and Economics
Department of Control Engineering and Information Technology
Budapest, Hungary

STRATIFIED MOTION AND MANIPULATION PLANNING ALGORITHMS IN ROBOTICS

(Rétegezett mozgás és manipuláció-tervezési algoritmusok a robotikában)

Summary of PhD thesis by

István Harmati

Supervisors:

Dr. Béla Lantos
Dept. of Control Eng. and Information Tech.
Budapest University of Technology and Economics
Budapest, Hungary

Dr. Shahram Payandeh
School of Engineering Science
Simon Fraser University
Burnaby, BC, Canada

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1 Introduction

Mankind is in strong relation with systems from his environment. In order to live a successful life, it is useful to get acquainted with them, especially with those that are created by our own work. The analysis leads through abstraction, modeling of these systems. These simplifications concentrate on some key characteristics making the model relevant. A better description of our world requires to set up more complex models. Different modelling levels are usually developed depending on which one can be handled within the frame of the actual synthesis methodology. For example, the kinematic and dynamic models are typical representations in mechanics. Another important property of a system that attracts the attention of system scientists nowadays appears when the equations of motion are nonlinear i.e. $\dot{x} = f(x) + g(x)u$ where x is the state vector, u is the input of the system. The model is kinematic (driftless), if there is no $f(x)$.

One creates systems, machines because they may make our life easier under suitable control. Control theory intends to find the proper input for a system that achieves a desired system output. The control design is subjected to specifications, restrictions, realization issues etc., hence, it is an important part of system synthesis. The technical development requires more and more sophisticated control systems in order to attain appropriate precision, reliability and robustness for the systems around us. One may distinguish controls on different levels with different complexities. The motion planning problem (MPP) for mechanical systems is a typical control issue. The question in this case is that how one can steer a (typically robotic) system in open-loop from an initial state x_I to a desired final state x_F. For example, the car parking in everyday life is a motion planning problem. MPP plays central role also in robotics, since an operation with robot arm needs to plan the robot motion from several reasons: it should avoid the collisions, comply with task specifications and so on. The problem is also important in the more sophisticated robotic manipulation where a robotic hand executes an object reposition and/or reorientation. This problem can be also transformed into MPP in the configuration space. Other, more advanced control techniques enable to feed back the actual output to the input making a closed-loop control architecture. Such an approach is able to consider the error between reference and output signals in the control strategy. In spite of process control where reference signals do not change often, the mo-

tion planning of complex physical system plays a fundamental role. Here, the designed motion generates the reference signal.

There exist several approaches for the control of nonlinear system. Some of them (e.g. numerous soft computing methods) do not exploits directly the system equations, but in order to reach the desired goal, they tune the parameters of a control structure in a manner specified by the actual method. Since such techniques do not rely on system equations, they provide mostly only an approximation. The linearization of nonlinear system by state feedback leads generally to hard symbolic computation which is often insolvable by the available computational tools hence it is useful to develop nonlinear control systems based on modern mathematics both for controller design and simulation. Several methods have been elaborated to solve this problem, most of them are divided into three classes. One may use geometric phase methods [16] which are especially useful to systems having cyclic motion in characteristic such as mobile robots. These methods exploit the periodicity of certain state variables (see a rotation of a wheel during motion). Another popular branch of motion planning (MP) is associated to optimal controls [17]. Optimality can be considered in different ways. A possible optimal control task is to find the shortest (straight-line) path between the initial and final state. The methods appeared in this work focus on differential geometry and differential algebraic techniques which make the geometric interpretation of control features possible.
Their main intention is to operate with Lie brackets in order to gain extra directions in the configuration space. Then MPP consists of finding inputs that systematically steers the system using possible directions in order to reach the desired final points. A more complicated problem expects the control strategy to follow a reference trajectory in the configuration space. However, this last problem is not always solvable. Motion planning algorithm (MPA) based on differential geometry were devised first for smooth systems [18], [19], [20], [21] since they are the easiest to be handled from mathematical point of view. Smoothness in this framework means that all the control vector fields of the system consist of smooth functions. A function is smooth (or of class C^∞) if its partial derivatives of any order with respect to the independent variables exist and are continuous. The most important smooth motion planning algorithm (MPA) in the framework of this dissertation follows the concept in [18] where the system reaches the final point x_F as a sequence of flows along the control

vector fields:

$$x(T) = \Phi(g_n, t_n) \circ \cdots \circ \Phi(g_2, t_2) \circ \Phi(g_1, t_1)x(0) \tag{1}$$

It is a composition of symbolic mappings. Solving a differential equation in this form is not elementary. Observe, in this form only one active input is present at a time, namely the input which activates the vector field of actual flow.
Control engineering applies efficient software packages where the machinery of control theory with its mathematical tools may be implemented. Some highly-developed softwares such as MATLAB, Mathematica or MAPLE are able to realize complex control algorithms. However, it is worth remarking that these tools have also boundaries that influences the efficiency or/and the possibility of the realization. It is especially true for the techniques where symbolic computations are extensively used such as for determination of (1). Symbolic computations are very common in differential geometry, hence the question of efficiency and realization may arise in some complicated applications.

The dissertation addresses motion and manipulation problems of stratified kinematic systems. Stratified systems compose a more complex subclass of nonlinear systems than smooth systems. To see the differences, let be given a nonlinear system which may be subjected to several kinematic constraints. Every combination of constraints points out a submanifold in the configuration space which is called stratum. The equations of motions are different but smooth in each stratum. Denoting the strata by S_1, S_2, S_3 etc., one may imagine a stratified system as a combination of smooth systems with the equations of motion

$$\begin{aligned}
S_1 \ : \ \dot{x} &= g_{1,1}u^{1,1} + \cdots + g_{1,n_1}u^{1,n_1} \\
S_2 \ : \ \dot{x} &= g_{2,1}u^{2,1} + \cdots + g_{2,n_1}u^{2,n_2} \\
&\vdots \\
S_I \ : \ \dot{x} &= g_{I,1}u^{I,1} + \cdots + g_{I,n_I}u^{I,n_I}
\end{aligned}$$

where x is the state vector, $g_{i,j}$ is the jth vector field in the ith stratum and $u^{i,j}$ is its corresponding input. The equations of motion may change discontinuously at strata boundaries making the control and the motion planning much more intricate.
A stratified system typically appears where intermittent constraints affect the system. Legged robots and manipulation with robotic hand using finger relocations represent the most common examples for such a system. A biped

robot, for instance, moves in a manner if the first leg is on the terrain while the another one is in the air and it moves another manner if both the legs are on the ground. Every combination of the contacting legs results different motions even if the inputs remain the same. Obviously, the motion planning of a stratified system needs extra considerations. The basic stratified control was derived originally in [22]. Its main idea consists of introducing a fictitious, so called bottom stratified system (BSS). The BSS includes all the vector fields which are decoupled from the vector fields that switch between strata. The first group of vector fields contains the moving on vector fields while the switching vector fields define the moving off vector fields. All the moving on vector fields are considered in bottom stratum (S_1 in our notation now) when one forms the BSS:

$$\begin{aligned} S_1 &: \dot{x} = g_{1,1}u^{1,1} + \cdots + g_{1,n_1}u^{1,n_1} \\ S_2 &: \dot{x} = g_{2,1}\mid_{S_1} u^{2,1} + \cdots + g_{2,n_1}\mid_{S_1} u^{2,n_2} \\ &\vdots \\ S_I &: \dot{x} = g_{I,1}\mid_{S_1} u^{I,1} + \cdots + g_{I,n_I}\mid_{S_1} u^{I,n_I} \end{aligned}$$

The sequence of flows in smooth MP applied to BSS also solves the MPP of stratified systems if suitable flows of moving off vector fields are additionally inserted between any two flows defined in different strata.
Another important application of stratified control can be found in manipulation problems. Here, the robotic hand is equipped with fingers and the goal is to move an object by means of fingers from the initial position and orientation into a desired position and orientation, possibly following a trajectory in configuration space. Developing manipulation strategies for the problem is an active research area. The problem is quite complicated and different approaches exist using different assumptions [23], [24], [25], [26]. Most of them rely on finger rolling, sliding, relocation or their combinations depending on the contact model. The stratified manipulation aims to solve the problem by finger relocations (or finger gaiting) resembling the manner when a spider rolls a pellet. The foundation of this approach can be found in [27]. The main difficulty in the case of stratified manipulation comes from the complexity of the system. In many cases, the decoupling assumption is not satisfied therefore stratified MP cannot be applied directly but requires new strategies.

2 Research objects

The aim of dissertation is to develop new stratified motion and manipulation planning algorithms that extend the earlier methods, improve the efficiency of basic approaches and make the realization easier. The methods we devised in the dissertation can be divided into two main groups.

One of the groups deals with stratified motion planning algorithms as a basic frame which can be also used for manipulation planning. Here, some techniques are used to consider the convergence and time scaling issues of a moving robotic system. Another goal of the research is to extend the point to point stratified motion planning to a restricted trajectory tracking problem. Two methods are devised for holonomic and nonholonomic stratified systems.

The second group involves stratified manipulation problem. The main motivation here is to reduce the symbolic computational cost, develop a software which is able to solve the manipulation problem and implements the algorithm. Another goal is to make the final trajectory interpretable which helps us to recognize easier the object and fingertips motion from the solution of the motion planning algorithm. An additional goal is to guarantee force closure stability if it is possible.

Another aim of the research may be to extend the stratified motion planning concept to stair-like problems. Such a problem occurs for example when a legged robot should climb a stair instead of moving in a plane terrain as before.

3 Research methodology

The theoretical background of the new methods is based on the differential geometry which is an active research area of nonlinear control systems. Since some part of the developed algorithms relies on new theoretical results, we proved the correctness of these results by invoking the tool of differential geometry. In these parts of the dissertation, the new algorithms apply theorems which are proved mostly in constructive way.

Efficiency and characteristics of the algorithms are investigated by simulation. On the one hand, it is an exigency, since a six-legged robotic locomotion and a four-fingered robotic hand were not available for us. On the other hand,

simulation is an efficient tool to study the algorithms. Additionally, it prepares the practical applications equipped already with proper sensors, actuators.

When the trajectories of legged robot or robotic hand were computed, we used the MATLAB, its Control System and Symbolic toolboxes. We also tested similar intelligent mathematical tools such as Mathematica and MAPLE. We decided for MATLAB and Control System Toolbox because they are well-tried in control engineering and support several high-level functions on this area. Since stratified motion planning is based on differential geometry and differential geometry requires mostly symbolic computations, we invoked the Symbolic toolbox, as well. This toolbox is adopted from MAPLE and shows similar properties as Mathematica. From the point of view of the investigated problems, the efficiency of these programs is similar.

Own algorithms are also implemented in MATLAB using especially the service of Control System and Symbolic toolboxes. For this purpose, we developed own algorithms and a software package. One of the most important part of this package is an own toolbox of differential geometry on the foundation of the symbolic toolbox. This new toolbox can be useful for general nonlinear control purpose if the control philosophy is based on differential geometry.
The highly oscillatory sequences in certain methods are based on abstract theoretical results. When we implemented the algorithm with an engineering software (MATLAB), it was needed to approximate the solution. Fortunately, this approximation did not cause considerable issues in realization. Another (symbolic) approximation is needed when operations (e.g. inversion) with symbolic exponential map are used because the symbolic toolboxes are not very robust at this stage of development, yet.

The simulations were executed on models. We used the kinematic model of hexapod robot and the egg-shaped object manipulation with four-fingered robotic hands. Some investigations were performed on a modified model of hexapod robot where the modification makes the model holonomic. As a consequence, holonomic MP methods became testable.

4 Summary of new scientific results

1st thesis group

We have developed a new stratified motion planning algorithm which extends and improves the standard approach. The new method is able to reach the final point at a desired time and the convergence property was also improved by adaptive insertion of reference points (Chapter 3).

The publications related to the thesis group are: [1], [2], [3], [4].

1.1. We have developed a new algorithm that improves the convergence property of the point-to-point stratified motion. The convergency problem may come if the system is not nilpotent. In this case, stratified MP provides only an approximation and not exact reaching of the final point. This may lead divergence in some cases. Earlier works [18] have proved the existence of a critical distance between the initial and the final states under which convergence is guaranteed, however there is no theoretical result how to determine it generally. The proposed algorithm tunes the refinement of reference points if divergence is identified. This algorithm starts with an initial distance D_i between x_i and y_i where x_i is the desired final point, y_i is the reached final point in the ith subsegment. If the $\parallel x_i - y_i \parallel > FD_i$ where $F = 1/3$ is applied, then $k = 3$ extra reference points are inserted between y_i and x_{i+1} along a straight-line which lead back the system to the next desired final points with a better refinement than $i$the subsegment provided. If reference trajectory contains a few reference points, such a philosophy resulted convergent motion planning. On the other hand, if motion planning achieves small error, i.e. $\parallel x_i - y_i \parallel < SFD_i$ then one may delete a reference point in order to reduce computational cost. In simulation, $S = 1/2$ was applied which keeps the error in the near of critical distance. The algorithm offers an experimental compromise to solve the trade-off between the computational cost and the refinement of the trajectory.

1.2. We have developed a new algorithm that performs a time-scaling along the trajectory. This algorithm modifies the planned inputs in such a way that not only the location of final point but also the time when the system reaches that point can be defined.

Stratified MP obtains the piecewisely constant input sequence which makes possible to connect x_I and x_F. If the Lie algebra associated to bottom stratified system fails to be nilpotent, the solution is an approximation as was

discussed above. Consider a motion planning problem on smooth nonlinear system with initial state x_I and desired final state x_d . Let the solution to motion planning problem be given in the form

$$x_d \approx x_F = \Phi(B_z, t_z, u^z) \circ \Phi(B_{z-1}, t_{z-1}, u^{z-1}) \circ \cdots \circ \Phi(B_1, t_1, u^1)(x_I) \qquad (2)$$

where B_i are the Philip Hall basis, t_i are the (forward) Philip Hall coordinates. The active input in the ith time interval is $\left|u^i\right| = |u_j| = 1$, $j \in \{1, \ldots, m\}$ for every $i = 1, \ldots, z$. If $x(0) = x_I$ is the initial condition and $x(T) = x_F$ is the desired end condition to be achieved with respect to time, then

$$\begin{aligned} x_F = x(T) \;\; = \;\; & \Phi(B_z, \frac{T}{T_s} t_z, u_T^z) \circ \Phi(B_{z-1}, \frac{T}{T_s} t_{z-1}, u_T^{z-1}) \circ \cdots \circ \\ & \Phi(B_1, \frac{T}{T_s} t_1, u_T^1)(x(0)), \end{aligned} \qquad (3)$$

where

$$u_T^i = \frac{T_s}{T} \text{sign}(u^i), \qquad T_s = \sum_{i=1}^{z} t_i \qquad (4)$$

for every $i = 1, \ldots, z$. It is seen that reaching final state needs the desired time T instead of time T_s provided by the basic stratified motion planning. Since only one input is active at a time, the input rescaling algorithm is able to perform prescribe the time for other points from trajectory, as well. However, the active input fails to be constant in this case.

We have demonstrated the efficiency of the new algorithms on the motion planning problem of hexapod robot.

2nd thesis group

We have developed new fitted stratified and semi-stratified manipulation planning methods which reduce the sometimes very complicated symbolic computations to almost pure numerical ones. They provide easily interpretable trajectory. The first one can be applied for dextrous manipulation in a restricted workspace while the second one performs an object manipulation and assures force closure stability and collision avoidance (Chapter 4).

The publications related to the thesis group are: [5], [7], [6], [8], [9].

2.1. We have developed a new fitted stratified manipulation planning algorithm that describes the stratified system in a new aspect using a special

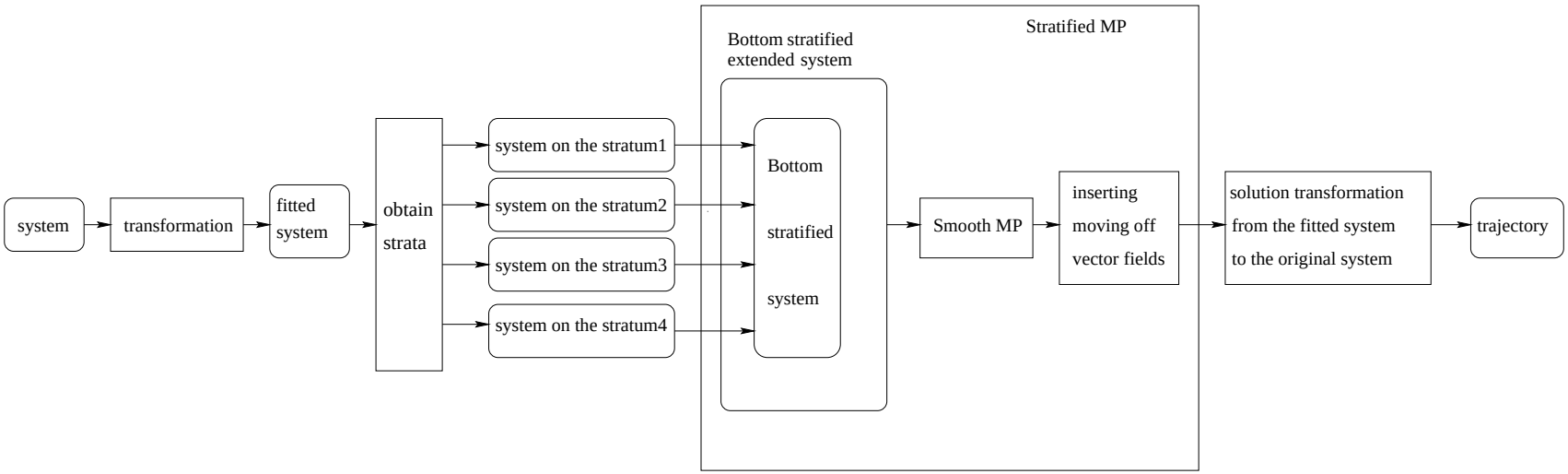

Fig. 1: The steps of fitted stratified manipulation planning.

parametrization by means of a transformation. In the new description, the fingertip position is described by a vector $p^o_{pf_i} = (u_i \ v_i \ z_i)^T$ where u_i and v_i describes the projection of the fingertip position onto the object along the normal vector of the surface. The u_i and v_i are given as the explicite parameters of the object surface. The z_i denotes the distance between the ith fingertip and the object. This transformation divides vector fields into moving on and moving off vector fields. We have proved that it allows to define a new, fictitious model called fitted system which satisfies the decoupling condition of stratified MP. For example, in the case of 3D smooth object, 4 fingered robotic hand, each finger with 3 degrees of freedom, the fitted BSS has a form

$$\begin{pmatrix} v_o \\ \omega_o \\ \dot{u}_1 \\ \dot{v}_1 \\ \vdots \\ \dot{u}_4 \\ \dot{v}_4 \end{pmatrix} = \begin{bmatrix} I_6 & 0 & 0 & 0 & 0 \\ 0 & I_2 & 0 & 0 & 0 \\ 0 & 0 & I_2 & 0 & 0 \\ 0 & 0 & 0 & I_2 & 0 \\ 0 & 0 & 0 & 0 & I_2 \end{bmatrix} \begin{pmatrix} v^d_o \\ \omega^d_o \\ \dot{u}^d_1 \\ \dot{v}^d_1 \\ \vdots \\ \dot{u}^d_4 \\ \dot{v}^d_4 \end{pmatrix}, \tag{5}$$

where v_o and ω_o are the linear and angular velocities of the object, $\dot{u}^d_i$, $\dot{v}^d_i$ define the desired fingertip velocities in the new frame. By structure, it is computationally easy to perform a stratified MPA on the fictitious system. Since MP is completed on fictitious (fitted) system, its inputs are fictitious, as well. As a result, the proper inputs (the joint variables) can be obtained via new transformation using the inverse kinematics.

$$q_i = \left(J^v_{pf,i}\right)^{-1} J^v_{po,i}(\text{object2palm}(\mathit{objectshape}(u,v), p^o_{pf_i})) \begin{pmatrix} v^d_o \\ \omega^d_o \end{pmatrix}$$

where $J^v_{pf,i}$ and $J_{po,i}$ define the ith finger kinematics and its role in the object kinematics. The function object2palm transforms the object coordinates into palm coordinates. The method produces a dextrous manipulation planning in a restricted workspace. The steps of the method can be seen in Fig. 1.

2.2. We have developed a new semi-stratified manipulation planning algorithm on the foundation of fitted stratified approach still using the new parametrization. This method includes an extra systematic finger relocation procedure using appropriate reference points. The method combines a fitted stratified manipulation planning with free finger relocation tasks. In the case of fitted

stratified manipulation of smooth, 3D object (with four, 3DOF fingers), final configuration can be obtained as a sequence of 14 connected flows

$$\begin{aligned} x_F &= \Phi(v^{o,d}_{pf_4}(2), t_{14}) \circ \Phi(v^{o,d}_{pf_4}(1), t_{13}) \circ \cdots \circ \Phi(v^{o,d}_{pf_1}(2), t_8) \circ \Phi(v^{o,d}_{pf_1}(1), t_7) \circ \\ &\circ \Phi(\omega^{o,d}_{o}(3), t_6) \circ \cdots \circ \Phi(\omega^{o,d}_{o}(1), t_4) \circ \\ &\circ \Phi(v^{o,d}_{o}(3), t_3) \circ \cdots \circ \Phi(v^{o,d}_{o}(1), t_1)(x_I) \end{aligned} \tag{6}$$

The first 6 flows (i.e. the last 6 in writing order) manipulate the object and every 2 flows following it belong to fingers. It may occur already in the object manipulation part realized by flows $\Phi(\omega^{o,d}_{o}(3), t_1), \ldots, \Phi(v^{o,d}_{o}(1), t_1)$ that the fingertip positions exceed their allocated workspace. To avoid this phenomena, the semi-stratified manipulation inserts unconstrained finger relocations (where actual finger is not in contact with the object) which relocates back the fingers one after other to new reference points ($x = \pm y$ line in the plane of $z = 0$, see Fig. 2). The unconstrained finger relocations keep fingers in separated workspaces (avoiding finger collisions) and assists to achieve force closure stability. However, it restricts the dexterity to object manipulation. (One cannot realize arbitrary contact points on the surface.)

2.3. We have developed an extension of stratified manipulation planning where one may prescribe not only the final point for the manipulation but one may choose a special class of reference trajectories that can be followed or approximated. It exploits the benefits of the new parameter description mentioned above. We have proved that considering the bottom stratified fitted manipulation system like (5), one can exploit its simple structure and in spite of basic stratified manipulation, it is possible to activate more than one inputs at the same time. We have also shown that the inputs cannot be related to different fingers because it makes impossible the switching between two strata. The new method is able to rotate (and reposition) the object around arbitrary axis which was impossible in the case of basic stratified approach.

We have demonstrated the efficiency of the new algorithms on the manipulation planning problem of an egg-shaped object.

3rd thesis group

We have developed a new holonomic stratified motion planning method which provides a greater freedom in designing motion planning for stratified system.

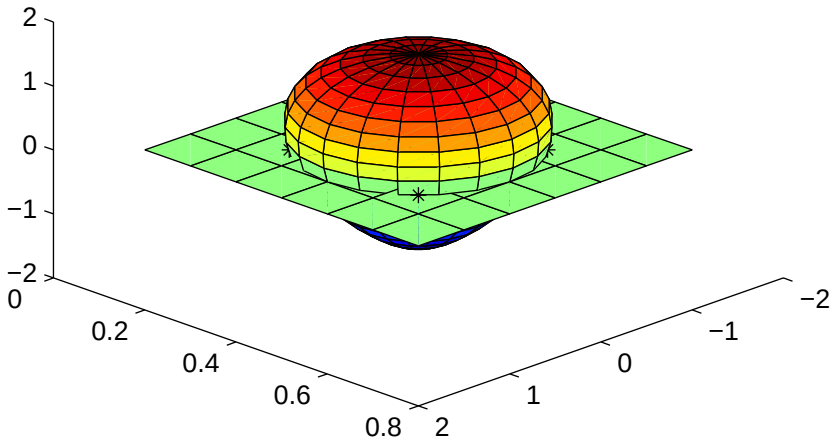

Fig. 2: The fingertips are relocated back to $x = \pm y$ line in the plane of $z = 0$, e.g. if they are about to leave their workspace during the object motion.

With this method, one may follow a special class of trajectories called decomposed trajectories instead of reaching only a final state in the configuration space (Chapter 5).

The publications related to the thesis group are: [10], [11], [15].

3.1. We have developed the theoretical foundation of the holonomic stratified motion planning using a new representation for the case if moving on vector fields span the whole configuration space of BSS. The new representation is called completely involutive representation (CIR). Roughly speaking, CIR is a sequence of involutive subdistributions (involutive interiors) $\underline{\Delta}_{I(l)}^{J(k_1,\ldots,k_l)} = \{\underline{\Delta}_{i_1}(J_{k_1}), \ldots, \underline{\Delta}_{i_l}(J_{k_l})\}$ where the control vector fields are bundled in a way such that $\underline{\Delta}_{i_j}(J_{k_j})$ represents moving on vector fields that are selected by the J_{k_j} multi-index from the i_jth stratum. We have proved that this representation generalizes the basic stratified MP based on sequence of single flows and the order of vector fields defined by CIR allows to steer the stratified system along a decomposed trajectory i.e. along a trajectory that

evolves on connecting integral submanifolds (slices) of $\underline{\Delta}_{i_j}(J_{k_j})$, $j = 1, \ldots, l$. We have given a method how to construct CIR from vector fields.

3.2. We have provided a method how to find optimal representations considering two points of view. The first method achieves maximal independency among distribution of CIR. We have shown that such a CIR

$$\underline{\Delta}_{I_{obr}(L)}\{J_{obr}(k_1, \ldots, k_L)\}$$

(called optimal basis representation or OBR) minimizes the number of fixed, unchangeable points (called basis points) in the decomposed trajectory which also minimizes restrictions in trajectory tracking from this point of view.
The second method shows that putting as many vector fields in CIR as many possible can reach maximal redundancy which maximizes the aggregated dimensions of $\underline{\Delta}_{i_j}(J_{k_j})$, $j = 1, \ldots, l$. It minimizes the restrictions due to the subtrajectories evolving in integral submanifolds, hence this CIR is called optimal dimension representation and denoted by

$$\underline{\Delta}_{I_{odr}(l)}\{J_{odr}(k_1, \ldots, k_l)\}.$$

We have shown that the two criteria above leads to a trade off, hence we have provided a procedure to find a heuristically optimal CIR from the set

$$R_o : S_{obr} \cup S_{odr} \tag{7}$$

where S_{obr} is the set of optimal basis representations and S_{odr} is the set of optimal dimensional representations for every $l \in l_{odr}$, $l_{odr} = \{l : l \Rightarrow \exists \underline{\Delta}_{I_{odr}(l)}\{J_{odr}(k_1, \ldots, k_l)\}\}$. The procedure establishes a compromise between the two criteria.

3.3. We have developed an algorithm which provides an optimal set of basis points for completely involutive representations. We have proved that it is possible to find a $\pi(o)(I(l))$ permutation of $\underline{\Delta}_{i_j}(J_{k_j})$, $j = 1, \ldots, l$ for any (not only decomposed) reference trajectory that minimizes the deformation between the desired and the computed trajectories. This procedure is beneficial especially for the optimal completely involutive representation based on basis points.

3.4. We have developed a new algorithm for reference trajectory generation which uses the symbolic approximation of exponential map. The approximation of exponential map can be applied for flow evaluation:

$$\Phi(g, t)x = e^{g(x)t} \approx \sum_{k=1}^{p} \frac{L_g^k(x)t^k}{k!} \tag{8}$$

where p is a final integer in the case of approximation. This evaluation allows to determine a mapping Ψ (used in Frobenius theorem) that consists of symbolical decompositions of exponential maps. The mapping Ψ is used for solving the partial differential equation related to CIR which also solves the trajectory problem. This algorithm shows advantages especially for those stratified systems whose vector fields are given in complicated symbolic expression.

We have demonstrated the efficiency of the new algorithms on the motion planning problem of hexapod robot, if the model is stratified holonomic.

4th thesis group

We have developed a new nonholonomic stratified motion planning method which works on a special class of stratified controllable systems where the moving on vector fields do not span necessarily the configuration space of BSS. Using this method, the nonholonomic stratified system is able to follow an approximation of reference decomposed trajectories. Similarly to holonomic stratified motion planning, the system may follow a trajectory instead of reaching only a final state in the configuration space (Chapter 6).

The publications related to the thesis group are: [12], [13], [14], [15].

4.1. We have developed the theoretical foundation of nonholonomic stratified motion planning using a new representation. The new representation is called decoupled nonholonomic representation (DNR). Using the notations above, the sequence of involutive closures $\bar{\Delta}_{I(l)} = \{\bar{\Delta}_{i_1}, \ldots, \bar{\Delta}_{i_l}\}$ is a Decoupled Nonholonomic Representation (DNR) of the BSS where s denotes the number of strata. We have proved that if the stratified system is decomposed controllable i.e. there exists a decomposed trajectory between the initial and final states then the system satisfies the Decoupled Lie Algebra Condition DLARC i.e.

$$\sum_{j=1}^{l} \bar{\Delta}_{i_j} = T_x\mathcal{C}_0$$

where $T_x\mathcal{C}_0$ is the tangent space of BSS at any $x \in \mathcal{C}_0$. We have also proved a weakener version of the reverse statement. It asserts that if the DNR satisfies DLARC then it is weakly decomposed controllable. In this case, for any x_I (at $t = 0$) and x_F (at $t = T$) satisfying $\| x_F - x_I \| < d_{cr}$, there exists a sequence of inputs u^j,$j \to \infty$ involving a sequence of trajectories $x^j(t)$ that

converges to a decomposed trajectory $x_{dec}(t) := x^{\infty}(t)$ along $\bar{\Delta}_{I(l)}$ such that $x_{dec}(0) = x_I$ and

$$\| x_F - x^{\infty}(T) \| < \frac{1}{2} \| x_F - x_I \| . \tag{9}$$

The positive number d_{cr} is the critical distance to DNR. Trajectory $x_{dec}(t)$ is said to be an *approximated decomposed trajectory* between x_I and x_F.

4.2. We have provided a new algorithm that uses the new results for nonholonomic stratified systems and steers this system along an approximated decomposed trajectory using highly oscillatory sequence (HOS) of inputs. The algorithm carries out first a basic stratified MP, however in the order of vector fields which is determined by DNR. Then HOS inputs are used on every integral submanifold of $\bar{\Delta}_{i_j}$ which generate a sequence of subtrajectories. We have proved that the sequence of trajectories converges to the reference decomposed trajectory. Since the method can be used for a restricted trajectory tracking, it is possible to handle the object avoidance problem. This feature may become especially beneficial if a stair-like environment is given (e.g. a robot climbs a stair).

We have demonstrated the efficiency of the new algorithms on the motion planning problem of hexapod robot.

5 Applications

The methods devised above can be used for an important class of nonlinear systems, especially, for those systems where intermittent constraints (e.g. contacts) determine the exact form of the equations of motion. Walking robots, certain kind of manipulation arrangements and other multi-agent systems belong typically to this system class. The algorithms work on kinematic models. Such approaches for motion planning are also popular on other areas of complex mechatronics systems (e.g. aircraft, helicopter, rockets). The methods presented in the thesis provide a good foundation for the extension of stratified motion and manipulation planning to stair-like environment. For the better and more robustious control properties, the extension of methods to more complex dynamic models and implementation of reference motion in closed-loop will designate the main track of research in the future. The simulation results makes worthwhile the algorithms be implemented on physical systems.

The limit of this is only a financial question: based on the study of Fraunhofer Institute Magdeburg, a six-legged robot where each leg is equipped by three segments and sensors costs around 40000 EUR. The research results have become known in scientific society as conference presentations and journal articles. The results have been built into the research projects of the Control Systems Research Group of the Hungarian Academy of Sciences, the Hungarian National Research Program (OTKA) T 02972 and T 042634, the Hungarian Research and Development Program for Higher Education (FKFP) 0417/1997, the Natural Sciences and Engineering Research Council (NSERC) of Canada under Research Grant 611205 and proceed in the project EU GIRT-CT-2002 (CLAWAR2 - CLimbing And WAlking Robots) in which our department also takes a part.

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