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Budapest University of Technology and Economics
Doctoral School of Informatics
Department of Telecommunications and Media Informatics

Modelling Various Extensions of the Time Dependent Traveling Salesman Problem Under Uncertain Conditions

PhD Thesis booklet

Ruba AlMahasneh
M.Sc. in Management Information Systems

Supervisor:
Prof. László T. Kóczy

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1. Introduction

The Traveling Salesman Problem (TSP) is a classical problem in combinatorial optimization and has been studied extensively in many forms due to its applicability in different domains. It is the problem of a traveling salesman who has n locations to visit, returning eventually to the starting point without visiting any location more than once [1]. The goal is to minimize the total cost for any distance traveled (in terms of length units, time, money or other). In the language of graph theory, it means finding the shortest Hamiltonian cycle. Finding the optimal solution of a given instance of the TSP is an NP-hard problem, as obtained by a simple reduction from the Hamilton-Cycle problem [2]. Even in the special case where distances between nodes are represented in the Euclidean plane, it is also proved to be an NP-hard problem. TSP has a polynomial time approximation scheme (PTAS) that means, for any $\varepsilon > 0$, there exists a polynomial time algorithm which guarantees an approximation of size at most $1 + \varepsilon$ times the optimal solution [3]. This explains the added complexity of the TSP when non-geometric (non-deterministic) variants taken in consideration (such as the rush hours periods and the jam regions effects on the tour's overall costs). In the literature, there have been many different extensions and variations of the original TSP [1]. Those approaches main goal was to present a more accurate and realistic costs estimate of any tour, however, those models assigned costs to the edges connecting the nodes (cities) based on Euclidean Distances as constant, which reveals the obvious limitations when simulating real-life cases. Indeed, any trip on the ground is subject not only to the physical distances between cities, but also to the non-deterministic natural and unexpected circumstances (such as the jam regions and the rush hours' periods during the day) which must be looked at as a modifying cost.

Moving towards a more realistic model, the TD TSP introduced a viable refinement for costs calculation, but still not efficient enough; since it is still a deterministic solution [4]. In the TD TSP, SA (simulated annealing) is used as part of an optimization problem, where the objective function is equated with a Hamiltonian distance between nodes (H). In the TD TSP, the SA usually starts with a random configuration, then series of moves ($\sigma_i \rightarrow \sigma_{i+1}$) are performed at each temperature (T) step. At each step the move is either accepted or rejected with the probability $p(\sigma_i \rightarrow \sigma_{i+1})= \min\{1, \exp(-\Delta H/T)\}$, T is the temperature, and $\Delta H = H(\sigma_{i+1})$- $H(\sigma_i)$ is the energy difference between the two participating moves (configuration steps). In case of rejection, σ_{i+1} is set to σ_i. By lowering the temperature (T) the probability for accepting a deterioration is reduced. On the other hand, to represent the jam factors effect in the TD TSP, there is a predefined area that represents the traffic jam region in the city center form (Augsburg was used) in a time dependent form, in which the jam function (f) is assumed to have different values according to its closeness from the city center, and thus, the distances vary in time due to the modified traffic jam cost. The time needed to drive from one end point to the other one is multiplied in this case by a traffic jam factor $f > 1$ (traffic jam factor = f). Fig. 1. shows optimum configurations for different strengths of the traffic jam f (L_0 equals the ground state energy). The node at which the traveling salesman starts his tour is marked by a filled circle. The traveling salesman is supposed to start his tour at one specific node inside this traffic jam area at 9 am. in the morning and return to it at 3 pm. in the afternoon. In case $f = 1$, then he moves along the optimum tour of the original Bier127 problem. Those crisp numbers are multiplied with the length of the corresponding edge between the nodes in the given rush hour times, to quantify the jam factor's effects on the respective costs [4].

Despite the TD TSP's ability to achieve good results in determining the overall modified cost of the jam factors, one main drawback is the use of concrete numbers (and fixed rigid area in the graph) for representing

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the proportional traffic jam region's effect on a tour (the city center). Indeed, those circumstances are rather uncertain, imprecise and can form at any place or time unexpectedly and gradually.

After the classical TD TSP, there have been many other attempts by researchers to quantify the rush hours and the jam regions' effects on the trip's overall cost. Some models used Ant Colony optimization to predict traffic for the TD TSP [5, 6]. Others used mixed approaches, they incorporated integer programming model to calculate the traveled time, then integrated both Ant Colony System (ACS) and Tabu Search (TS) algorithms [6] to calculate the overall cost. Other authors used linear constrains formulation techniques to present the jam factors' effect on any trip [7]. Also, dynamic programming algorithm was proposed to solve the TD TSP [8]. In fact, there is one common aspect in all the previously proposed approaches in the literature, that is, using crisp numbers to simulate those uncertain and imprecise road conditions, indeed, this is an insufficient formulation.

Although those models proposed good solutions, but without doubt they all have serious limitation of simulating real-life situations due to using crisp and deterministic approaches in modelling non-deterministic factors (intangible). In fact, it would be closer to reality to use fuzzy numbers to capture the uncertainties associated with those imprecise road conditions. In particular, using fuzzy sets [9] in representing the effects of those jam factors as cost coefficients. Thus, I have introduced, for the first time in the literature, five novel fuzzy-based approaches for the classic TD TSP; in pursuit of adequate modelling the jam regions and the rush hours periods in a more realistic environmental circumstances. Each one of those fuzzy-based models achieved one step closer to simulating more accurately the intangible costs associated with road uncertainties in a tour [10,11,12,13,14,15,16,17,18]. Those models ultimately offered practical and realistic formulations for the very first time by overcoming the rigidity presented in all the previously available approaches in the literature.

First, I introduced the novel 3FTD TSP (Triple Fuzzy Time Dependent Traveling Salesman Problem); a further extension towards including costs uncertainties using fuzzy notions, the first ever fully fuzzified model of the original TD TSP. The 3FTD TSP utilizes fuzzy values for determining the costs between any two nodes (cities, locations, shops. etc) within the traffic jam regions and/or during the rush hours periods more practically. It is called triple fuzzy as it has three aspects of (the time itself between nodes, the jam regions costs and the rush hours costs), all factors were represented using type-1 (classic) fuzzy sets. Since this model was characterized using two-dimensional membership functions, the rush hours times can be represented using piecewise linear normal fuzzy sets. On the other hand, the jam regions can be represented by various fuzzy membership functions depending on the area's topography (in the 3FTD TSP I used trapezoidal "oblong shaped" membership function for the city centres). This representation simulates the modified costs in case of crossing a specific jam region (jam regions here can be city centres, accidents, or road conjunction). This approach of modelling the jam regions and the rush hours periods effects on the overall trip's costs, led to fully fuzzified modified cost formulation for the classic TD TSP, and thus, expressed the uncertain conditions affecting the overall tour length (costs) more efficiently [11].

After achieving good results on small representative instances; I extended the novel 3FTD TSP (by developing a C++ program that handles different type of fuzzy numbers for the TD TSP) and applied it on the largest universal instances in the literature in pursuit of testing the generality and applicability of the model on real-life scenarios. The extended 3FTD TSP was optimized using the DBMEA (Discrete Bacterial Memetic Evolutionary Algorithm). The DBMEA has been successfully applied on various similar (although crisp) problems for finding the (quasi-) optimum solution. In order to support the claim about the efficiency of this selection, the classic GA (Genetic Algorithm) has been also applied on all universal instances, enabling the reader to compare the accuracy and the speed of the solutions [14].

However, realizing the possible enhancements on the 3FTD TSP representation, led me to introducing a more adequate and realistic formulation, the IFTD TSP (Intuitionistic Fuzzy Time Dependent Traveling Salesman Problem). In this model I employed type-2 Intuitionistic fuzzy sets which is characterized by three-dimensional membership functions, it involves the so-called hesitation part between the membership functions and the non-membership functions representing the same uncertain components in the previous model (costs, areas of traffic jams, rush hour times) allowing a higher level of vagueness representation [12]. In fact, adding the hesitation part to either the membership values or to the non-membership values (or to both) assigned to the local and temporal embedding of the costs (added from the jam factors) allowed a higher-level involvement of the complex nature of uncertainty, say, the secondary uncertainty originating from the random sampling of costs influencing factors.

This additional demonstration using the intuitionistic fuzzy elements to quantify the intangible jam factors and the rush hours resulted in creating an inference system that approximates any tour's costs in an even more adequate way. This model was tested on Bier127 instance and optimized using the DBMEA (Discrete Bacterial Memetic Evolutionary Algorithm) and then a complete comparison between the novel IFTD TSP and the classic TD TSP solutions on the same instance was performed [12].

Then, I moved forward by introducing another novel and even more accurate approach with the additional notion of interval-valued intuitionistic fuzzy sets; the IVIFTD TSP (Interval-valued Intuitionistic Fuzzy Time Dependent Traveling Salesman Problem). Type-2 fuzzy sets have the potential to provide better performance in modelling problems with higher uncertainties than the traditional type-1 (classic) fuzzy sets, thus, employing interval-valued intuitionistic fuzzy sets for quantifying the traffic jam regions, and the rush hours periods modified costs more precisely was achieved [13]. In addition, by employing the IIFWAA operator (in the IVIFTD TSP model) to aggregate the accumulative interval-valued fuzzy information from all the jam factors simultaneously decreased the information distortion (loss). Indeed, the IVIFTD TSP is the most complicated (yet practical) model that was capable of catering for all the external traffic factors affecting any edge collectively at any given time while least information loss was guaranteed. For applicability, the IVIFTD TSP model was optimized using the DBMEA on Bier127 instance (with the help of my colleague), and a complete comparison was conducted.

Afterwards, I introduced the IVFSTD TSP (Interval-Valued Fuzzy Soft Time Dependent Traveling Salesman Problem Model). In the IVFSTD TSP, I have applied the notion of interval-valued fuzzy soft sets to address the overlapping jam factors for an edge and thus, calculated the overall added costs efficiently. The case study has been taken to exhibit the simplicity of the technique and applicability. Then, I tested and validated this approach on small but representative example [16].

Lastly, in pursuit of proving the applicability and the generality of my models, and the fact that they can be applied on different real-world transportation and logistics problems [14]. I used the extended TOPSIS (The technique for ordered preference by similarity to the ideal solution) method using interval-valued intuitionistic fuzzy to solve the Dial-a-ride Problem with Time Window (a prime example of the TD TSP) as a multiple attribute group decision making (MAGDM) problem. This model can determine efficiently both the collective interval-valued intuitionistic positive-ideal solution (PIS) and the interval-valued intuitionistic negative-ideal solution (NIS) by constructing the weighted collective interval-valued intuitionistic fuzzy decision matrix. Based on different distance definitions, I calculated the relative closeness of each alternative to the PIS solution then ranked the alternatives accordingly (using scoring matrix), and ultimately selected the collective most desirable alternative(s).

Indeed, despite the different novel, fuzzy-based formulations I proposed, my work moved gradually in complexity, accuracy and applicability, starting from the simplest type-1 fuzzified version of the TD TSP with modified costs from the road conditions; the 3FTD TSP, to reach the most accurate yet general fuzzy type-2 based approach, the IVIFTD TSP.

2. Research objectives

Historically, effective representation for the TD TSP with the traffic regions and rush hours periods has been difficult to achieve. The reason for this insufficiency was the fact that all previous models used concrete numbers (except [19], where the first idea of applying fuzzy triangular numbers was raised) to model those uncertain vague factors. Overlooking the fact that they are non-deterministic, thus, they cannot be accurately represented using crisp numbers. In fact, the most promising and realistic approach is modelling the jam regions and rush hours' effects by using fuzzy sets. My goal was to introduce and compare several novel fuzzy-based models to accurately quantify the simultaneous presence of the rush hours and jam factors effects (unpredictable road conditions) in the original TD TSP with the least amount of information loss. My work has six main tightly connected goals:

First, incorporating classical type-1 fuzzy sets (characterized by a two-dimensional membership functions) in the first entirely fuzzified version for the TD TSP (the 3FTD TSP model) with the rush hours and the jam regions added costs [10].

Second, applying the intuitionistic fuzzy sets (IFTD TSP) to incorporate the hesitation part between the membership functions and the non-membership functions, which gave higher level of representation accuracy for the uncertainties associated with the roads conditions [11,12].

Third, formulating a model that calculates the effects of the jam factors on all edges simultaneously in the TD TSP [13] by using the interval-valued intuitionistic fuzzy sets (IVIFTD TSP). This model adequately considers multiple jam factors collectively by utilizing the IIFWAA (interval-valued intuitionistic fuzzy weighted arithmetic average aggregation) operator. Moreover, by facilitating Park's distance measure I was able to find the collective optimal solution and eventually recommend the most desirable routes formation (with the least possible collective costs) [13].

Fourth, incorporating different extensions of type-2 fuzzy sets (which is characterized by a three-dimensional membership functions) such as interval-valued fuzzy soft sets that allowed representing the uncertainties associated by the roads conditions in the TD TSP in an even more flexible, yet accurate way [16].

Fifth: to proving the applicability of my novel models, I applied them on a different transportation problem, namely, the Dial-a-ride problem with time window. This ultimately proves the applicability and scalability of the IVIFTD TSP (Interval-Valued Intuitionistic Fuzzy Time Dependent Traveling Salesman Problem) [14].

Sixth: developing and publishing a C++ tool box [15,17] to simulate the new models and integrate them with the DBMEA and the GA to calculate the (quasi-)optimum or semi optimum solution. Then, conducting a thorough comparison [15] between the novel models and the classical TD TSP on the largest instances accessible in the literature (Eil51, Bier127, Eil76, Eil101 s250_1 and s250_2) to prove the novel models' efficiency and generality compared with the state of the art solutions.

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3. Methodology

The essential methodology I followed in the research work is formulating and comparing different fuzzy-based novel models for the TD TSP (under non-deterministic road conditions influence) within two main categories; type-1 fuzzy sets and type-2 fuzzy sets.

Initially, I formulated a new concise mathematical model of the first fuzzy-based TD TSP extension using classic fuzzy type-1 sets. I tested the mathematical model on a small but representative instance. After achieving good results, I optimized the extended model with two different algorithms (the DBMEA and the GA) on the largest instances available in the literature (Eil51, Bier127, Eil76, Eil101 s250_1 and s250_2). Then, I compared the run results with the state of the art solutions.

Realizing the potential higher accuracy for type-2 fuzzy sets, I formulated, tested and validated my three novel type-2 fuzzy-based TD TSP extensions. Since each extension of type-2 fuzzy sets has various flexibility level in modelling and representing problems with high uncertainty, I was eager to apply three different extensions of type-2 fuzzy sets in sequence. First, I applied intuitionistic fuzzy sets and then, interval-valued intuitionistic fuzzy sets employing weighted average to cater for simultaneous jam factors affecting an edge, and finally, I applied interval-valued fuzzy soft sets for representing simultaneous road uncertainties in the TD TSP. Each model was tested and verified on a small sized problem containing 12 nodes (cities), then each model was extended, tested and compared on Bier127 instance. Moreover, to facilitate testing each model on larger scale, I developed a C++ program that can simulate any fuzzified model (type-1 and type-2 fuzzy-based). All these five extended fuzzified TD TSP models were integrated with two optimizations algorithms (the DBMEA and the GA), and tested on the largest instances in the literature for comparison purposes.

4. New Results

The new The original TSP was first formulated in 1930 [1], it involves a salesman who starts the journey from the company headquarters, visits all planned destination (cities or shops) exactly once, and then returns to the original starting point with the minimum overall travelled distance. TSP can be defined as a graph search problem with edge weights (costs) as in Eq. (1). To formulate the symmetric case with n nodes (cities) where $c_{ij} = c_{ji}$ represents the cost of going from city (vertex) i to city j. G_{TSP} in the graph on hand. Then, C is called cost (time) matrix, $C = \left[c_{ij}\right]$.

Let $G_{TSP} = \langle V_{cities}, E_{conn} \rangle$

where

$$V_{cities} = v_1 \cup \{v_2, \dots v_i \dots ., v_n\}, E_{conn} \subseteq \left\{\left(v_i, v_j\right) \middle| i \neq j\right\}, i = (2 \dots, n) \tag{1}$$

v_1 is the starting vertex. E_{conn} is the set of all edges. The cost matrix C is the mapping all subscripts of the vertices, thus:

$$C: V_{cities} \times V_{cities} \rightarrow R^+ \tag{2}$$

R^+ is the range of the costs from the set of edges to the positive real lines.

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The initial set of vertices in any tour is $\{1, k_2, \dots k_i \dots., k_n\}$ where $k_1 = 1$ the starting subscript, $i = (2 \dots, n)$. The goal is to find the path of edges that minimizes the sum of the costs (C_{sum}) by traveling through the vertices exactly once.
C_{sum} calculated in Eq. (3) and should be minimized as in Eq. (4). Here c_{1,k_2} is the cost of traveling the starting edge, and $c_{k_{n-1},1}$ is the cost of returning to the first vertex (traveling the last edge).

Thus, C_{sum} can be written as:

$$C_{sum} = c_{1,k_2} + \sum_{j=2}^{n} c_{k_j,k_{j+1}} \text{ , } (j = 2, \dots, n-1) \text{ , } c_{k_j,k_{j+1}} \epsilon \left[c_{ij}\right], k_2, k_n \epsilon \{2, \dots, n\} \quad (3)$$

Then to minimizes the sum of the costs:

$$\text{Minimize } C_{sum} \quad (4)$$

A valid permutation for vertices (the subscripts) is:

$$\left\{v_1, v_{k_2}, \dots v_{k_i} \dots., v_{k_n}\right\} \quad (5)$$

where $i = (2 \dots, n)$. v_1 is the starting vertex.

Let $\{k_1, \dots. k_n\}$ be a permutation of $\{1 \dots, n\}$, then a valid sequence of edges that forms a path that goes through all vertices (exactly once and goes back to the first one) can be expressed in the form:

$$e_{1,k_2}, e_{k_2,k_3}, \dots. e_{k_i,k_j} \dots \dots e_{k_{n-1},1} \text{ , } i, j \in \{2 \dots, n\} \quad (6)$$

where first edge is e_{1k_2} , $k_i \neq k_j$ so that $i \neq j$.
Considering an actual trip (for simplicity we will consider small sized tour Fig.1), clearly, it is subjected not only to the topography of the given route, but also to the unexpected circumstances and road conditions, which affect the overall timing (cost) of the tour. Naturally, in the city centers and during the rush hours periods, trips between nodes (vertices) take longer. Thus, those two factors must be looked at as vague or uncertain (non-deterministic but not in a random sense) values, and the relevant estimated costs for any tour traveled between two nodes are not constant themselves. Hence, it is more appropriate to represent those imprecise (uncertain) costs using fuzzy numbers. This was my main motivation for introducing the novel 3FTD TSP model, which is a more adequate and realistic version of the classic TD TSP (the crisp version fuzzy-based from multiple aspects).

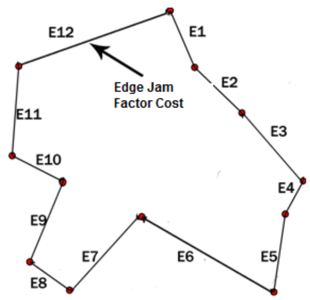

Fig. 1. Simple tour example

4.1. Thesis group 1: [C4, P32] The Triple Fuzzy Time Dependent Travelling Salesman Problem (3FTD TSP)

The 3FTD TSP is so far the only fully extended and published fuzzy version of the classic TD TSP. In the 3FTD TSP model, three “modifying parameters” of the edge costs that were represented using fuzzy sets. Each cost in the trip has its own membership function, namely, the fuzzy time (cost) needed for crossing edge (v_i, v_j) which is denoted by $\tilde{c}_{ij}$. The fuzzy jam region influenced cost with membership function $\mu_J(v_i, v_j)$ and the fuzzy rush hours cost with membership function $\mu_{RH}(h)$. Hence, according to the degree of belonging to the specified jam region or if the edge was being crossed during the fuzzy rush hours; the original fuzzified time of the related edge(s) will be influenced accordingly. It is worth mentioning that, in our fuzzy formulation, the cost of crossing the edges is defined by the time needed to cross them. The cost between edges is expressed by time, thus, from here on we will refer to time as cost. Moreover, cost here is a fuzzy set of the universe of the potential costs between two vertices (the universe of potential travelling time).

Thesis 1.1 [J5, C1, C7] *Using type-1 fuzzy sets, I introduced a novel mathematical formulation for the fuzzy time (costs) traveling between edges in a tour.*

Let c_{ij} be the cost of traveling between vertices v_i and v_j, $\tilde{C}$ is the fuzzy cost matrix, which can be defined as:

$$\tilde{C} = [\tilde{c}_{ij}] \tag{7}$$

where $\tilde{c}_{ij} : T \rightarrow [0, 1]$. $i \neq j$, and $i, j \in \{1, \dots . n\}$.
T is the universe of the costs $[0, \max\{T\}]$, such that $(\max\{T\})$ is the highest cost in the trip under study.

Thus,

$$\tilde{c}(v_i, v_j) = \tilde{c}_{ij} \tag{8}$$

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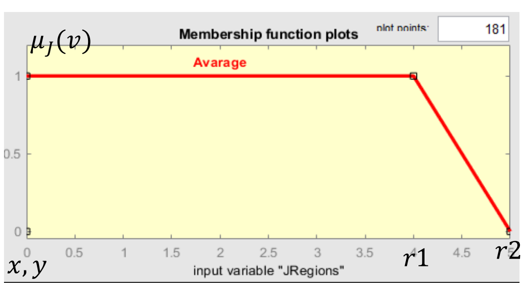
$$\tilde{c}_{ij} = \langle T, \mu_{ij}(t) \colon T \rightarrow [0,1] \rangle \text{ , such that } t \in T \tag{9}$$

In our illustrative example, $\mu_{ij}(t)$ defines $\tilde{c}_{ij}$ over T (the traveling time between vertices v_i and v_j). $\mu_{ij}(t)$ is a triangular membership function between vertices v_i and v_j. x, y, and z represent the break points of $\langle \mu_{ij}(x,y,z) \rangle$ in Eq. (11):

$$\tilde{c}(v_i, v_j) = \langle T, \mu_{ij}(x,y,z) \rangle \tag{10}$$

$$\mu_{ij}(x,y,z) = \begin{cases} 0 & (v_i, v_j) \leq x \\ \frac{(v_i,v_j)-x}{y-x} & x \leq (v_i,v_j) \leq y \\ \frac{z-(v_i,v_j)}{z-y} & y \leq (v_i,v_j) \leq z \\ 0 & z \leq (v_i,v_j) \end{cases} = \max\left\{\min\left\{\frac{(v_i,v_j)-x}{y-x}, \frac{z-(v_i,v_j)}{z-y}\right\}, 0\right\} \tag{11}$$

Thesis 1.2 [J5, C1, C7] *Using type-1 fuzzy sets, I introduced a novel mathematical formulation for the jam regions effects on a tour's costs simulating real-life circumstances.*



v

Fig. 2. Jam region membership function ($\boldsymbol{\mu_J}$)

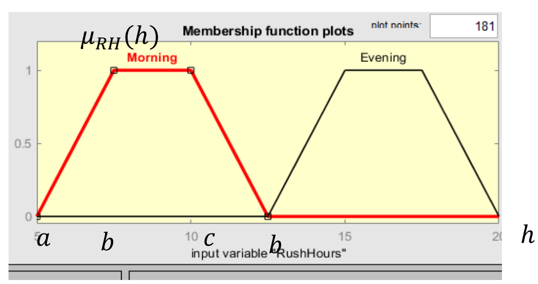


Fig. 3. Traffic rush hours' membership function ($\boldsymbol{\mu_{RH}}$)

To calculate the modified cost due to crossing the jam region, we assume (in the presented example) there is a center point at the city center, and the jam region has a truncated conic shaped membership function $\mu_J(v)$ (see Fig.2.). Where the membership degree depends on the distance measured from the center point. Since we assume a truncated conic type membership function for the jam region $\langle x, y; r1, r2 \rangle$, there are two radiuses ($r1$ and $r2$); one that of the conical kernel ($r1$) and the other is that of the support ($r2$).

To calculate the modified cost that affects edge (v_i, v_j) due to crossing the jam region, we must first find the membership degree μ_J of each vertex.

Let V be set of vertices that is $V = v_1 \cup \{v_2, \ldots v_i \ldots, v_n\}, v_1$ is the starting vertex. $i = (2, \ldots n)$. J denotes the fuzzy set of the jam region.

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$$J = \langle V, \mu_J(v): V \rightarrow [0,1] \rangle \qquad , v \in V \tag{12}$$

Then, to calculate the cost modified by being (partially) in the jam region for edge (v_i, v_j) we apply Eq.(13):

$$\mu_{CJ}(v_i, v_j) = \frac{\mu_J(v_i)+\mu_J(v_j)}{2} \tag{13}$$

Fig.2 represent the cut of the jam region membership function in two dimensions (for our illustrative example), that has trapezoidal shape with break point values [0,0, 4000, 5000].

We use Eq. (14) which defines a trapezoidal membership function$(v; \dot{a}, \dot{b}, \dot{c}, \dot{d})$, where $\dot{a}, \dot{b}, \dot{c}$ and $\dot{d}$ represent the break points.

$$\mu_J(v; \dot{a}, \dot{b}, \dot{c}) = \begin{cases} 1 & v \leq \dot{c} \\ \frac{\dot{d}-v}{\dot{d}-\dot{c}} & \dot{c} \leq v \leq \dot{d} \\ 0 & \dot{d} \leq v \end{cases} \tag{14}$$

where $\dot{a} = \dot{b} = \dot{c}$ (in our example).

This formulation eliminates (to some extent) one of Schneider's model inaccuracies, which ignores the intersections of the jam regions and the fact that such modified cost builds up gradually (which is better captured using fuzzy sets as we have shown).

Thesis 1.3 [J5, C1, C7] *Using type-1 fuzzy sets I introduced a novel mathematical formulation to quantify effectively the rush hours effects on a tour's costs simulating real-life circumstances.*

To show the effect of the modified fuzzy cost that might affect a trip during the rush hour periods. We propose a double trapezoidal membership function for the traffic rush hour periods during the day (in our simple illustrative example, see Fig. 4.4). It has the break point values {5 ,7.5, 10, 12.5, 12.5, 15, 17.5, 20}, and its membership values are: {0, 1, 1, 0, 0, 1, 1, 0, 0}.
$\mu_{RH}(h)$ was represented with two main rush hour periods during the day (with membership values equal to 1) the peaks are from 7.5 a.m. to 10 a.m. and from 3 p.m. to 5.5 p.m. Between the two periods the traffic intensity is lower but not zero.

Let H be the hours' time line (24 hours of the day), h the elements of the time line such as $h \in H$. The membership function for the rush hours is:

$$RH: \mu_{RH}(h): H \rightarrow [0,1], h \in H \tag{15}$$

$\mu_{RH}(h)$ is the degree of membership of belonging to the rush hour periods (whether the edge was crossed during rush hours periods).

Here a, b, c and d represent the break points, then the a trapezoidal $(h;\ a, b, c, d)$ membership function for the rush hours periods (in the morning or the evening in our example) can be written as in Eq. (16):

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$$\mu_{RH}(h) = \begin{cases} 0 & h \leq a \\ \frac{h-a}{b-a} & a \leq h \leq b \\ 1 & b \leq h \leq c \\ \frac{d-h}{d-c} & c \leq h \leq d \\ 0 & d \leq h \end{cases} \tag{16}$$

$\mu_{RH}(h) = \max\left\{\min\left\{\frac{h-a}{b-a}, 1, \frac{d-h}{d-c}\right\}, 0\right\}$.

Thesis 1.4 [J5, C1, C7] *I introduced for the first time, a fully fuzzified version of the classic TD TSP with jam regions, rush hours, and overall trip's costs; the 3FTD TSP model. I have shown that the costs between the nodes (cities) may depend on time (and location), due to the inherent uncertainty involved, they can be better translated as imprecise (fuzzy) values. Building on this concept, I mathematically validated the 3FTD TSP model on a representative size tour by modelling the uncertainties caused by road unpredicted conditions using fuzzy sets (convex and normal fuzzy sets of the respective universes). Using type-1 fuzzy sets I expressed the costs influenced by the rush hours times and the jam situations, and ultimately, calculated the overall tour length (costs) quantitatively.*

Finally, to calculate the resulting fuzzy cost that can possibility affect a trip (all edges in a trip), we must first sum the resulting fuzzy costs on each edge (v_i, v_j), then on the whole trip.

Hence, for calculating the resulting fuzzy cost $\tilde{C}_{sum}$ between vertex v_i and v_j we apply Eq.(17).

$$\tilde{C}_{sum}(v_i, v_j) = [\tilde{c}(v_i, v_j)] * [1 + \mu_{RH}(h)] * \left[1 + \frac{\mu_J(v_i) + \mu_J(v_j)}{2}\right] \tag{17}$$

Let $\{k_1, \ldots. k_n\}$ be a permutation of $\{1 \ldots, n\}$, then a valid sequence of edges that forms a path that goes through all vertices (exactly once and goes back to the first one) can be expressed in the form:

$$p = e_1, e_{k_{2,}k_3}, \ldots. e_{k_{i,}k_j} \ldots \ldots, e_1\ , i \neq j \text{ so that } k_i \neq k_j \text{ and } i, j \in \{1 \ldots, n\} \tag{18}$$

where the first edge is e_1.

To calculate the cumulative modified fuzzy cost for the first two edges e_1(with membership break points a, b, c) and $e_{k_{2,}k_3}$ (with membership break points p, q, r) whose triangular membership functions are shown in equation (11), we apply Eq.19) [87]:

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$$\bar{\bar{C}}_{sum}(e_1 + e_{k_{2},k_{3}}) = \mu_{e_1+e_{k_{2},k_{3}}(t)} = \begin{cases} \frac{t-(a+p)}{(b+q)-(a+p)}, & (a+p) \le t \le (b+q) \\ \frac{(c+r)-t}{(c+r)-(b+q)}, & (b+q) \le t \le (c+r) \end{cases} \quad (19)$$

We apply this addition to all edges of the path (p) to find $\bar{\bar{C}}_{sum}(p)$:

$$\bar{\bar{C}}_{sum}(p) = \begin{cases} \frac{t-(\sum_{ij\in n} a_{ij})}{(\sum_{ij\in n} b_{ij})-(\sum_{ij\in n} a_{ij})}, & (\sum_{ij\in n} a_{ij}) \le t \le (\sum_{ij\in n} b_{ij}) \\ \frac{(\sum_{ij\in n} c_{ij})-t}{(\sum_{ij\in n} c_{ij})-(\sum_{ij\in n} b_{ij})}, & (\sum_{ij\in n} b_{ij}) \le t \le (\sum_{ij\in n} c_{ij}) \end{cases} \quad (20)$$

In Fig.3 And Fig.4, the following three elements are described by fuzzy membership functions [zero values here mean the normal traffic, away from the city centre and not during rush hours]:

- Triangular fuzzy costs (time) between the edges; $\mu_{ij}(t)$.
- Membership function of the fuzzy jam region; $\mu_{CJ}(v_i, v_j)$.
- Membership function(s) of the traffic rush hour time period(s) $\mu_{RH}(h)$.

4.2 Thesis group 2: [C4, P46] Optimizing the Extended Triple Fuzzy Time Dependent Travelling Salesman Problem (3FTD TSP) on the Largest Universal Benchmarks with DBMEA (Discrete Bacterial Memetic Evolutionary Algorithm) and GA (Genetic Algorithm).

After achieving good results, I was eager to test it on the largest universal TD TSP benchmarks, to prove my model's efficiency and compare it with state-of-the-art solutions in the literature. Thus, my 3FTD TSP C++ software was integrated with various optimizations algorithms such as the DBMEA and the GA. The benchmarks we used are Eil51, Bier127, Eil76, Eil101 s250_1 and s250_2 (each instance's name reflects the number of cities (nodes) under study; Eil51has a total of 51 cities that a salesman must visit).

[J5, C7] *I built a dynamic C++ tool box that can simulate numerous variations of the jam regions and rush hour periods for the extended 3FTD TSP in a fuzzy environment. My C++ modules simulated my previously proven thesis group (section 4.1) on the largest instances (Eil51, bier127, eil76, eil101 and our own s250 cities instances) available in the literature, after it was integrated with two different optimization algorithms (DBMEA and GA). Leading to valuable comparative statistics to assess the novel 3FTD TSP's applicability, generality, and scalability.*

Tables 3 to 7 show the total time in hours required to visit each location for the extended 3FTD TSP with different jam factors for instances Eil51, Eil76, Eil101, s250, s250 respectively (our own instances s250_1 and s250_2 which consist of 250 cities each).

Testing our algorithm on the standard TD TSP benchmarks and achieving promising results (Tables 1 to 10, and comparisons shown in Figs. 4, 5 and 6) only proves that my new model is general enough to be applied for the extended problem. Indeed, the DBMEA was able to find high quality solutions for the 3FTD TSP problem, with different membership functions $\mu_J(v)$ and $\mu_{RH}(t)$, even with large rush hours and jam regions values. The obtained results are validated in the context of the known optimum solution for the original TSP, and Schneider's [2] solution for the TD TSP.

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Table I. Bier127 Extended tour length results for 3FTD TSP Vs TD TSP with DBMEA

jam factor	3FTD TSP best value	TD TSP best value
1.00	115929.0	118293.5
1.05	117498.6	119053.2
1.20	117997.2	119714.1
1.50	121076.4	120571.7
2.00	125083.8	121125.2
3.00	129369.0	121125.2
5.00	132220.2	121125.2
10.00	137813.4	121125.2
20.00	139286.4	121125.2
50.00	144363.1	121125.2
100.00	147520.2	121125.2

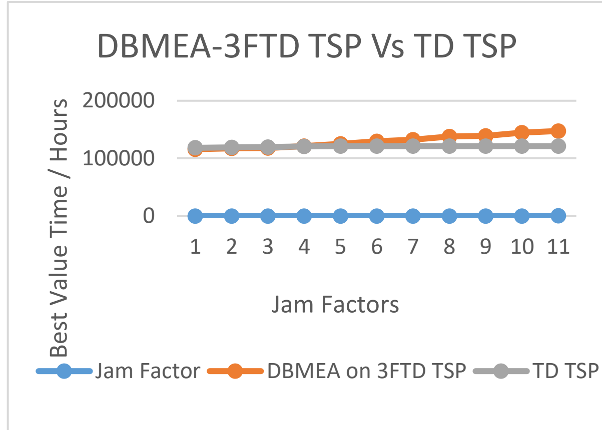

Fig. 4. Bier127 comparison run results for 3FTD TSP Vs TD TSP with DBMEA

Table 2 Shows different run results for the Bier127 benchmark problem applied on 3FTD TSP, with different jam factors, compared with our own instance of 250 cities and different jam factors (the detailed configuration is explained in the dissertation C4, P40).

Table II. Bier127 vs s250 computational results for 3FTD TSP with DBMEA

	multiple runs elapsed time			best elapsed time	average elapsed times
bier127	20.5317273	20.5807	20.5215	20.5215	20.5215
bier127	20.0984	20.3999		20.5215	20.2492
s250_1	21.08152503	20.71037	20.64352	20.64352184	20.81180495
s250_2	19.75256584	19.67563	19.64307	19.64307403	19.69042

Table III. Eil51 computational results for 3FTD TSP with DBMEA

elapsed time			best elapsed time	Average elapsed time
15.7147	15.7573	15.7147	15.7147	15.7289

Eil51-city problem (Christofides/Eilon) TYPE: TSP

Table V. Eil101 computational results for 3FTD TSP with DBMEA

elapsed time			best	average elapsed time
23.6234	23.5858	23.5537	23.5537	23.5876

Eil101
-city problem (Christofides/Eilon)

Table IV. Eil76 computational results for 3FTD TSP with DBMEA

elapsed time			best elapsed time	average elapsed times
21.4841	21.5433	21.7321	21.4841	21.5865

eil76.tsp - 76-city problem (Christofides/Eilon)

Table VI. s250_1 computational result for 3FTD TSP with DBMEA

elapsed time			best	average elapsed time
21.08152503	20.7104	20.6435	20.6435	20.8118

250-city

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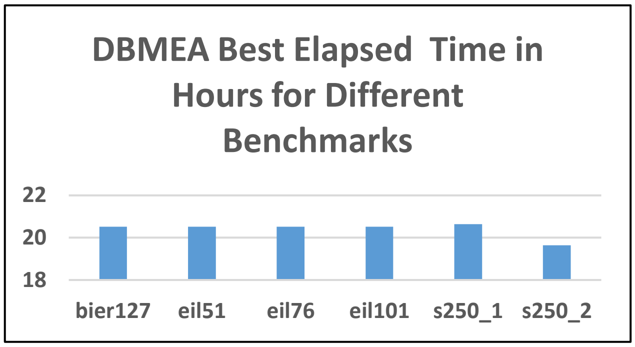

Fig. 5. Best elapsed time for different benchmarks with DBMEA

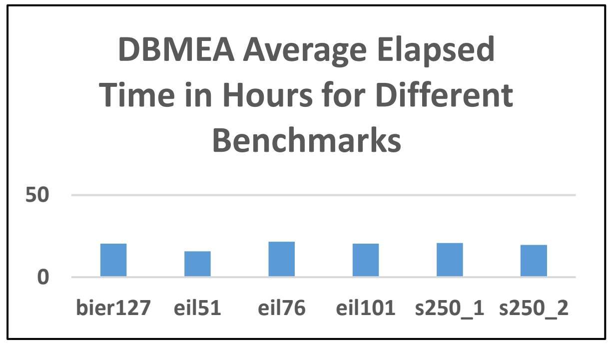

Fig. 6. Average elapsed time for different benchmarks with DBMEA

The comparison charts (Figs. 5. and 6.) for running the DBMEA on the 3FTD TSP model using universal instances, show that the DBMEA was able to get a robust result on different benchmarks continuously. In fact, this proves that our extension for the TD TSP with fuzzy sets (the 3FTD TSP) is giving a realistic run times and conversion speed. Concluding that the 3FTD TSP is closer and more general to simulating real-life scenarios with efficiency. Also, the conversion speed was consistent among different jam factors which supports the practicality of the extended model. Thus, the 3FTD TSP can be considered as a general robust solution for TDT SP with realistic representation of the rush hours and jam regions on the largest instances in the literature.

Table VII. s250_2 computational results for 3FTD TSP with DBMEA

s250_02 -hours	elapsed time			best	average elapsed time
	19.75256584	19.6756	19.6431	19.6431	19.6904
		250-city			

In Tables 9 and 10 (in bold), and Figs. 7 and 8, the results show that the DBMEA outperformed the GA although both approaches gave feasible solutions. We have run both algorithms on 300 generations. The Genetic Algorithm was faster than DBMEA after 100 generations. However, what makes the DBMEA generally outperforming the GA is the fact that – to some surprise - the GA gets stuck in a local optimum and seems to be unable to significantly improve the solution between 100 and 300 generations.

Table VIII. bier127 computational results for 3FTD TSP with DBMEA

	DBMEA 3FTD TSP	Classic TD TSP
jam factor	best value	best value
1.00	115929.0	118293.5
1.05	117498.6	119053.2
1.20	117997.2	119714.1
1.50	121076.4	120571.7
2.00	125083.8	121125.2
3.00	129369.0	121125.2
5.00	132220.2	121125.2
10.00	137813.4	121125.2
20.00	139286.4	121125.2
50.00	144363.1	121125.2
100.00	147520.2	121125.2

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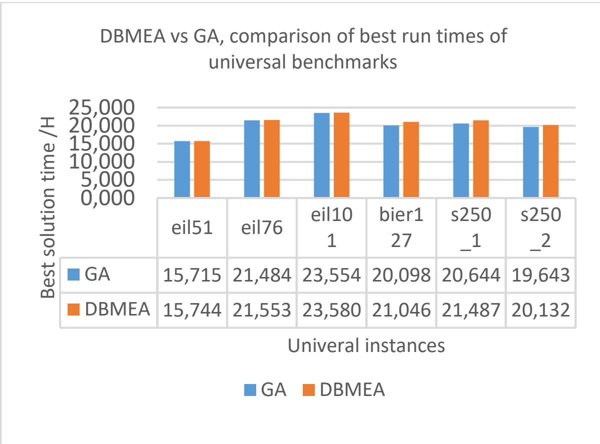

Fig.8. DBMEA vs GA, comparison of best elapsed times of universal benchmarks

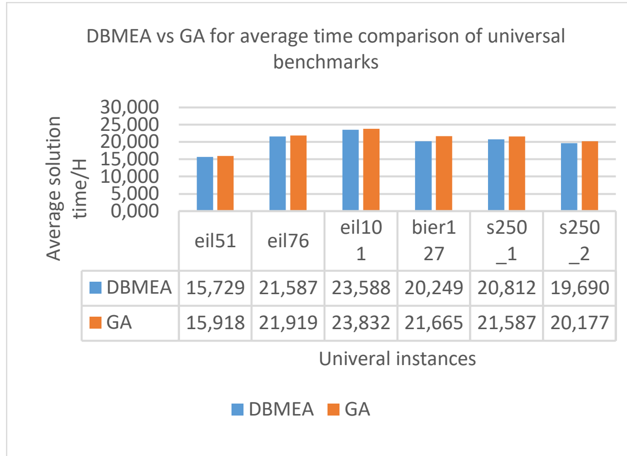

Fig. 7. DBMEA vs GA for average elapsed time comparison of universal benchmarks

Table IX. 100 Generation comparison of results for the 3 FTD TSP and the TD TSP

DBMEA		
Benchmark	best value	average value
eil51	**15.715**	**15.729**
eil76	**21.484**	**21.587**
eil101	**23.554**	**23.588**
bier127	**20.098**	**20.249**
s250_1	**20.644**	**20.812**
s250_2	**19.643**	**19.690**

Table X. 100 Generation comparison of results for the 3 FTD TSP and the TD TSP

Genetic algorithm		
Benchmark	best value	average value
eil51	15.744	15.918
eil76	21.553	21.919
eil101	23.580	23.832
bier127	21.046	21.665
s250_1	21.487	21.587
s250_2	20.132	20.177

4.3 Thesis group 3: [C5, P53] The Intuitionistic Fuzzy Based Model of the Jam Regions and the Rush Hours for the Time Dependent Traveling Salesman Problem.

After achieving remarkable results in the 3FTD TSP, I was more motivated to enhance the model, by introducing, validating, and comparing other newly extended fuzzy-based models [11,12,15]. Since type-2 fuzzy sets have three-dimensional membership functions, while type-1 sets have only two; then, the intuitionistic fuzzy sets are more flexible to fuzzify the TD TSP under the influence of the jam regions and the rush hours periods.

Thesis 3.1: [C3, C2, J1] *I formulated a novel mathematical model for the jam regions effects on any tour using the intuitionistic fuzzy sets.*

Thesis 3.2: [C3, C2, J1] *I formulated a novel mathematical model for the rush hours effects on any tour using the intuitionistic fuzzy sets.*

Let, there be n number of edges E that form a whole tour for a salesman (basic equations and operations are fully explained in chapter 3 in the dissertation). $e_i \in E$. Let R be an IFR $(J \times C)$ and construct an IFR

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Q of $(E \times J)$. Clearly, the composition T of IFRs R and Q $(T = R \circ Q)$ give the costs of any edge e_i in terms of E to C given by the membership functions in Eqs. (21 and 22).

$$\boldsymbol{\mu_T(e_i, c) = \vee_{j \epsilon J}\left[\mu_Q(e_i j) \wedge \mu_R(j, c)\right]} \tag{21}$$

and the non-membership function given by

$$\boldsymbol{v_T(e_i, c) = \wedge_{j \epsilon J}\left[v_Q(e_i j) \vee v_R(j, c)\right]} \tag{22}$$
$$\forall e_i \in E \text{ and } c \in C$$

For given R and Q, the relation $(T = R \circ Q)$ can be computed [25]. From the knowledge of Q and T, an improved version of the IFR R can be computed, for which the following holds valid:

(i) $J_R = \mu_R - v_R \cdot \pi_R$ is greatest. Thus, calculating J_R will lead to find the additional cost that must be added from the jam regions effect on any edge, and ultimately, quantify the intangible cost of the such uncertain factor with higher degree of accuracy. Lastly, it will be added to the overall cost for the edge that crosses that specific jam region.

(ii) The equality $T = R \circ Q$ is retained.

As seen in Fig.9, it is clear that this proposed concept improved version of R will be a more significant IFR translating the higher degrees of association and lower degrees of non-association of J as well as lower degrees of hesitation to the cost evaluation C. If almost equal values for different C in T are obtained, then we consider the case for which hesitation is least. From a refined version of R one may infer the costs from the jam factors in the sense of a paired value, one being the degree of association and other the degree of non-association.

Thesis 3.3: [C3, C2, J1, J2] *I introduced a complete novel intuitionistic fuzzy sets based model; The Intuitionistic Fuzzy Time Dependent Traveling Salesman Problem model (IFTD TSP), to translate the higher degrees of association and the lower degrees of non-association for trip's costs as well as the lower degrees of hesitation to edges costs and ultimately, I introduced a more realistic calculation for the travelled routes under vague and unpredicted road conditions.*

I have applied the IFTD TSP on simple tour with 12 nodes (Table 10 to Table 13), to test the methodology first [11]. Then we used the DBMEA on both the classic TD TSP, and the extended IFTD TSP on Bier127 instance for comparison [12].

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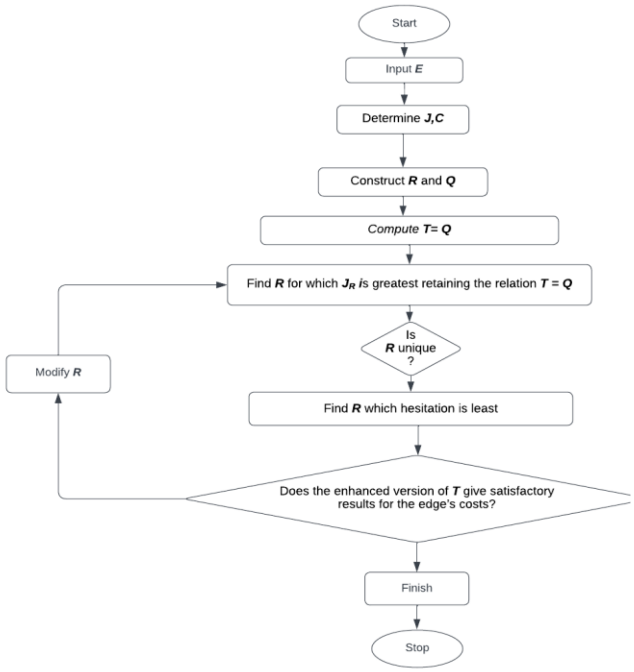

Fig.9. IFTD TSP flow chart

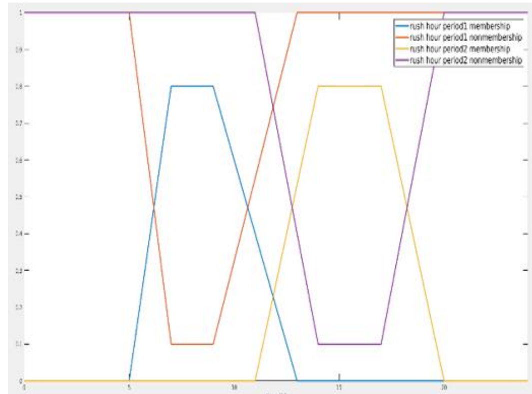

Fig.10. IFTD TSP rush hours and jam regions costs membership and non-membership functions

4.4 Thesis group 4: [C5, P56] Quasi-optimization of the Time Dependent Travelling Salesman Problem by Intuitionistic Fuzzy Model and Memetic Algorithm.

Building on the previous thesis group, let IFR set of jam factors costs $R(J \to C)$ be given in Table 11. Let IFR $Q(E \to J)$ be given as shown in Table 12 [24,25]. Therefore, the composition $(T = R \circ Q)$ is given in Table 13. We calculated the jam region costs factors on an edge e (c_{jam_e} is the total jam cost affecting an edge e) as given in Table 14, where the four cost factors are ($c_1 = 1.2, c_2 = 1.5, c_3 = 2, c_4 = 5$) with weighted average calculations:

$$\boldsymbol{c_{jam_e}} = \frac{\sum_{i=1}^{n}[c_{ij} * j_{ij}]}{\sum_{i=1}^{n} j_{ij}} \tag{23}$$

The rush hour factors costs of each tour edge (c_{rush}) are determined in similar intuitionistic model. The relations between the time and the rush hour periods ($\grave{Q}$) are described with intuitionistic fuzzy functions Fig.10. An intuitionistic fuzzy relation ($\grave{R}$) is given between the rush hour periods and the jam factors costs was computed similarly ($\grave{T} = \grave{Q} \circ \grave{R}$) .

Thesis 4.1: [J2] *My novel extended IFTD TSP model was optimized using the DBMEA on Bier127 instance. High quality solutions were found for different jam factor values affecting any edge in a tour using the weighted average operator*.

Clearly, this enhanced model was able to translate the higher degrees of association and lower degrees of non-association of the jam factor and rush hours as well as lower degrees of hesitation to any edge cost and,

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ideally, a more realistic calculation for the travelled routes on universal instances such as the Beier127 (Fig.11. and Table 16).

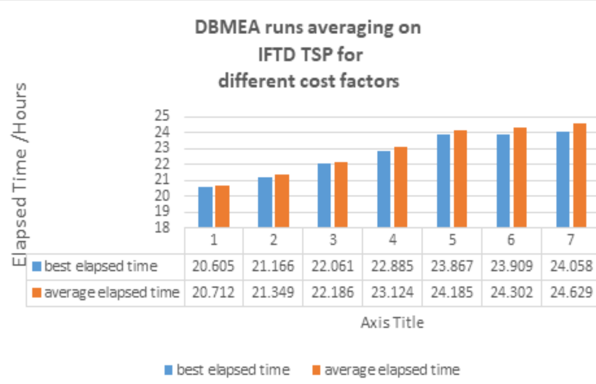

Fig. 11. DBMEA bier127 runs for IFTD TSP with different rush hours and jam regions' cost factors

Table XI. Jam regions costs (R)

Jam area (*R*)	Cost factor 1 (c1)	Cost factor 2 (c2)	Cost factor 3 (c3)	Cost factor4 (c4)
Jam Region1	(0.4, 0)	(0.7, 0)	(0.3, 0.3)	(0.1, 0.7)
Jam Region2	(0.3, 0.5)	(0.2, 0.6)	(0.6, 0.1)	(0.2, 0.4)
Jam Region3	(0.1, 0.7)	(0, 0.9)	(0.2, 0.7)	(0.8, 0)
Jam Region4	(0.4, 0.3)	(0.4, 0.3)	(0.2, 0.6)	(0.2, 0.7)

Table XII. Edges belonging and non-belonging to different jam regions costs (Q)

(*Q*)	Jam Region1	Jam Region2	Jam Region3	Jam Region4
E1	(0.8, 0.1)	(0.6, 0.1)	(0,1)	(0,1)
E2	(0,1)	(0,1)	(0.2, 0.8)	(0.6, 0.1)
E3	(0.8, 0.1)	(0.8, 0.1)	(0,1)	(0,1)
E4	(0,1)	(0,1)	(0, 0.6)	(0.2, 0.7)
E5	(0.8, 0.1)	(0.8, 0.1)	(0, 0.6)	(0.2, 0.7)
E 6	(0, 0.8)	(0.4, 0.4)	(0,1)	(0,1)
E 7	(0,1)	(0,1)	(0.6, 0.1)	(0.1, 0.7)
E 8	(0, 0.8)	(0.4, 0.4)	(0.6, 0.1)	(0.1, 0.7)
E9	(0.6, 0.1)	(0.5, 0.4)	(0,1)	(0,1)
E10	(0,1)	(0,1)	(0.3, 0.4)	(0.7, 0.2)
E11	(0, 0.8)	(0.4, 0.4)	(0.6, 0.1)	(0.1, 0.7)
E12	(0.4, 0.4)	(0.6, 0.1)	(0,1)	(0.2, 0.8)

Table XIII. Edges added costs from jam factors (T)

Jam Cost (*T*)	Cost factor1	Cost factor2	Cost factor3	Cost factor4
E1	(0.4, 0.1)	(0.7, 0.1)	(0.6,0.1)	(0.2, 0.4)
E2	(0.4,0.3)	(0.4,0.3)	(0.2, 0.6)	(0.2, 0.2)
E3	(0.4, 0.1)	(0.7, 0.1)	(0.6,0.1)	(0.2, 0.4)
E4	(0.2,0.7)	(0.2,0.7)	(0.2, 0.7)	(0.2, 0.6)
E5	(0.3, 0.1)	(0.7, 0.1)	(0.6, 0.1)	(0.2, 0)
E6	(0.3, 0.5)	(0.2, 0.6)	(0.4, 0.4)	(0.2,0.4)
E7	(0.1,0.7)	(0.1,0.7)	(0.2, 0.7)	(0.6, 0.1)
E8	(0.3, 0.5)	(0.2, 0.6)	(0.4, 0.4)	(0.6, 0.1)
E9	(0.4, 0.1)	(0.6, 0.1)	(0.5, 0.3)	(0.2, 0.4)
E10	(0.4,0.3)	(0.2,0.6)	(0.2, 0.6)	(0.3, 0.4)
E11	(0.3, 0.5)	(0.2, 0.6)	(0.4, 0.4)	(0.6, 0.1)
E12	(0.4,0.4)	(0.4,0.4)	(0.6, 0.1)	(0.2, 0.4)

Table XIV. Edges added costs from jam factors (J_R)

J_R	(J_R1)	(C1)	(J_R2)	(C2)	(J_R3)	(C3)	(J_R4)	(C4)	*Total Jam region cost factors*
E1	0.35	1.2	0.68	1.5	0.57	2	0.04	5	1.695
E2	0.31	1.2	0.31	1.5	0.08	2	0.08	5	1.791
E3	0.35	1.2	0.68	1.5	0.57	2	0.04	5	1.695
E4	0.13	1.2	0.13	1.5	0.13	2	0.08	5	2.151
E5	0.24	1.2	0.68	1.5	0.57	2	0.2	5	2.04
E6	0.2	1.2	0.08	1.5	0.32	2	0.36	5	2.917
E7	0	1.2	0	1.5	0.13	2	0	5	2
E8	0.2	1.2	0.08	1.5	0.32	2	0.57	5	3.291
E9	0.35	1.2	0.57	1.5	0.44	2	0.04	5	1.682
E10	0.31	1.2	0.08	1.5	0.08	2	0.18	5	2.388
E11	0.2	1.2	0.08	1.5	0.32	2	0.57	5	3.291
E12	0.32	1.2	0.32	1.5	0.57	2	0.04	5	1.763

Table XV. DBMEA average run on bier127 with different costs values

DBMEA average run for cost factors (c_1, c_2, c_3, c_4)		
Best elapsed time	**Average elapsed time**	**average runtime [s]**
20.605	20.712	170.535
21.166	21.349	176.691
22.061	22.186	192.313
22.885	23.124	207.919
23.867	24.185	195.417
23.909	24.302	207.109
24.058	24.629	124.910
$c_1 = 1.2, c_2 = 1.5, c_3 = 2, c_4 = 5$		

Table XVI. Computational results for IFTD TSP

		DBMEA		
		best elapsed time	**average elapsed time**	**average runtime [s]**
		20.605	20.712	170.535
cost factors	$c_1 = 1.01,$	$c_2 = 1.05,$	$c_3 = 1.1,$	$c_4 = 1.2,$
		21.166	21.349	176.691
cost factors	$c_1 = 1.05$	$c_2 = 1.1$	$c_3 = 1.2$	$c_4 = 1.5$
		22.061	22.186	192.313
cost factors	$c_1 = 1.1$	$c_2 = 1.3$	$c_3 = 1.5$	$c_4 = 2$
		22.885	23.124	207.919
cost factors	$c_1 = 1.2$	$c_2 = 1.5$	$c_3 = 2$	$c_4 = 5$
		23.867	24.185	195.417
cost factors	$c_1 = 1.5$	$c_2 = 2$	$c_3 = 5$	$c_4 = 10$
		23.909	24.302	207.109
cost factors	$c_1 = 2$	$c_2 = 5$	$c_3 = 10$	$c_4 = 15$
		24.058	24.629	124.910
cost factors	$c_1 = 5$	$c_2 = 10$	$c_3 = 20$	$c_4 = 50$

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4.5 Thesis group 5: [C6, P63] Optimizing The Extended Interval-Valued Intuitionistic Fuzzy Time Dependent Travelling Salesman Problem (IVIFTD TSP) Using the DBMEA.

I applied The interval-valued intuitionistic fuzzy sets to construct a more accurate extension of The TD TSP. This model (IVIFTD TSP) was able to calculate multiple uncertain additional costs coming from the jam regions and the rush hours periods. To efficiently combine the collective factors using the interval-valued intuitionistic fuzzy values representation, I utilized the interval-valued intuitionistic fuzzy weighted arithmetic average operator for aggregation [26,31]. I also proposed a measure based on Park's distance [28,29,30] between IVIFS for the extended TD TSP costs under unpredicted road circumstances. On the other hand, optimizing and comparing the IVIFTD TSP run results on Bier127 with the classic TD TSP can be seen in Table 24. and Fig 12., they indicate that the new model is effective while it simulates real-life conditions and successfully quantifies the traffic effects without significant information loss on large instances. This model [13,15] gives a more tangible treatment of such uncertain factors, and thus, it offers a better model in terms of accuracy and flexibility than all my previous models [12].

Thesis 5.1: [J3, C4, BC1] *Using the Interval-Valued Intuitionistic Fuzzy sets, I have mathematically fuzzified the jam regions costs in the classic TD TSP. I employed the weighted average operation to take in consideration all the rush hours costs that are affecting any edge simultaneously and thus, I decreased the accumulative information loss.*

Thesis 5.2: [J3, C4, BC1] *I formulated a mathematical fuzzified model for the rush hour periods effects in the classic TD TSP using the Interval-Valued Intuitionistic Fuzzy sets. To cater for all simultaneous rush hours costs affecting any edge at any given time; I employed the weighted average operation which decreased the information loss and offered an accurate tangible model.*

Let X be a set (space), with a generic element of X denoted by x, that is $X = \{x\}$. For a set X, an IFS A is given by (all definitions and operations are fully explained in chapter 3 in the dissertation):

$$\tilde{A} = \{\langle x; \tilde{\mu}_A(x), \tilde{v}_A(x)\rangle | x \epsilon X\} \tag{24}$$

where $\tilde{\mu}_A(x) \subset [0,1]$ and $\tilde{v}_A(x) \subset [0,1]$ are connected intervals such that for all $x\epsilon X$ with the condition :

$$0 \leq \sup(\tilde{\mu}_A(x)) + \sup(\tilde{v}_A(x)) \leq 1, \forall x \in X$$

The intervals $\tilde{\mu}_A(x)$ and $\tilde{v}_A(x)$ denote the degrees of membership and non-membership of x to $\tilde{A}$, respectively. Clearly, if $\inf(\tilde{\mu}_A(x)) = \sup(\tilde{\mu}_A(x))$ and $\inf(\tilde{v}_A(x)) = \sup(\tilde{v}_A(x))$, then the IVIFS $\tilde{A}$ is reduced to traditional IFS.

Let $J = \{J_1, J_2, J_3, \ldots\ldots J_m\}$, $C = \{C_1, C_2, C_3, \ldots\ldots C_n\}$ and $E = \{E_1, E_2, E_3, \ldots\ldots E_q\}$ denote the sets of jam factors, costs and edges, respectively, where $n, m, q \epsilon \{1,2 \ldots, n\}$. We will introduce two fuzzy relations: Q that is defined over set $E \times J$ Eq. (25), and R that is defined over set $J \times C$ as in Eq. (26).

C and P stand for the chapter and page number in the dissertation respectively on **thesis group** level.
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$$\boldsymbol{Q} = \{\langle(\boldsymbol{e},\boldsymbol{j}), \boldsymbol{\mu_Q}(\boldsymbol{e},\boldsymbol{j}), \boldsymbol{v_Q}(\boldsymbol{e},\boldsymbol{j}) \rangle \mid (\boldsymbol{e},\boldsymbol{j}) \in \boldsymbol{E} \times \boldsymbol{J}\} \quad (25)$$

$$\boldsymbol{R} = \{\langle(\boldsymbol{j},\boldsymbol{c}), \boldsymbol{\mu_R}(\boldsymbol{j},\boldsymbol{c}), \boldsymbol{v_R}(\boldsymbol{j},\boldsymbol{c}) \rangle \mid (\boldsymbol{j},\boldsymbol{c}) \in \boldsymbol{J} \times \boldsymbol{C}\} \quad (26)$$

Where $\mu_Q(e,j)$ indicates the degree to which jam factor j affects edge e, while $v_Q(e,j)$ indicates the degree to which jam factor j does *not* affect edge e. Similarly, $\mu_R(j,c)$ and $v_R(j,c)$ are the relationships between jam factors and their respective costs. (This is what we refer to as *confirmability degree* in the coming sections). Thus, $\mu_R(j,c)$ and $v_R(j,c)$ represent the degrees to which jam factor j confirms and does not confirm the presence of cost c, respectively. Since Q is defined over set $E \times J$ and R over set $J \times C$, the composition T of R and Q $(T = R \circ Q)$ for predicting the costs for a specific edge can be represented by FR from E to C given the membership function in Eq. (27) and non-membership function in Eq. (28)

$$\boldsymbol{\mu_T}(\boldsymbol{e},\boldsymbol{c}) = \mathbf{max}_{\boldsymbol{j}}\{\mathbf{min}[\boldsymbol{\mu_Q}(\boldsymbol{e},\boldsymbol{j}), \boldsymbol{\mu_R}(\boldsymbol{j},\boldsymbol{c})]\} \quad (27)$$

$$\boldsymbol{v_T}(\boldsymbol{e},\boldsymbol{c}) = \mathbf{min}_{j}\{\mathbf{max}[\boldsymbol{v_Q}(\boldsymbol{e},\boldsymbol{j}), \boldsymbol{v_R}(\boldsymbol{j},\boldsymbol{c})]\} \quad (28)$$

for all $e \in E$ and $c \in C$

Let us consider that E_1 is the first edge of a 12 nodes simple tour in Fig.1. [11]. In the case study [11,13], there are four different traffic factors assumed (which may be jam regions and/or rush hour periods) affecting E_1 simultaneously. In the illustration we use $\langle \mu_Q(e,j), v_Q(e,j)\rangle$ estimated to define the degree on how a jam factor j affects edge e as in Eqs.(27 and 28). The jam factors of E_1 affecting that edge in the example are (1.1, 1.2, 1.3, 1.4, 2.1 and 2.2), which are all related to a specific cost (delay) as shown in Table 17.

Table XVII Knowledge base for rush hour traffic factor costs

Traffic Factors	**Cost1**		**Cost 2**		**Cost 3**	
[jam regions or rush hours]	μ_R	v_R	μ_R	v_R	μ_R	v_R
Factor 1.1	[0.6,0.7]	[0.1,0.2]	[0.2,0.3]	[0.5,0.6]	[0.1,0.3]	[0.4,0.6]
Factor 1.2	0.6,0.7]	[0.2,0.3]	[0.2, 0.4]	[0.4,0.6]	[0.4,0.6]	[0.1,0.2]
Factor 1.3	[0.5,0.6]	[0.1,0.2]	[0.1, 0.2]	[0.6,0.7]	[0.3,0.4]	[0.3,0.5]
Factor 1.4	[0.7,0.8]	[0.1,0.2]	[0.1, 0.2]	[0.6,0.8]	[0.1,0.2]	[0.7,0.8]
….	….	…	….	….	….	…..
Factor 2.1	[0.5,0.6]	[0.2,0.3]	[0.2,0.3]	[0.4,0.6]	[0.2,0.3]	[0.5,0.6]
Factor 2.2	[0.7,0.8]	[0.1,0.2]	[0.1,0.2]	[0.6,0.7]	[0.1,0.2]	[0.6,0.7]
Factor 2.3	[0.5,0.6]	[0.2,0.3]	[0.3,0.4]	[0.4,0.6]	[0.1,0.3]	[0.3,0.5]
….	….	…	….	….	….	…..
Factor $n.n$	[0.6,0.8]	[0.1,0.2]	[0.1,0.2]	[0.6,0.7]	[0.2,0.3]	[0.6,0.7]

Table 18 shows the jam factors assigned to E_1. Table 19 gives the confirmability and non-confirmability degrees with jam factors (1.1, 1.2, 1.3, 1.4, 2.1 and 2.2), according to their degrees of belonging to the rush hour periods and jam regions (referred to from here as "traffic factors", as a generalized term). Table 20 gives confirmability degrees for E_1, which is extracted from the general knowledge base represented in Table 17.

C and P stand for the chapter and page number in the dissertation respectively on **thesis group** level.
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Table XVIII E_1 degrees of jam factors $\mu_Q(E_1,j)$, $v_Q(E_1,j)$

E_1 **Traffic Factors**	**1.1**	**1.2**	**1.3**	**1.4**	**2.1**	**2.2**
$\mu_Q(E_1)$	[0.5, 0.6]	[0.5, 0.6]	[0.4, 0.6]	[0.7, 0.8]	[0.5,0.6]	[0.5,0.7]
$v_Q(E_1)$	[0.2, 0.3]	[0.1, 0.3]	[0.1, 0.2]	[0.1, 0.2]	[0.1,0.2]	[0.2,0.3]

Table XIX Confirmability degrees $\mu_R(j,c)$, $v_R(j,c)$

Traffic Factors	**IF degree**					
	Cost1		**Cost2**		**Cost 3**	
Factor 1.1	**[0.6,0.7]**	***[0.1,0.2]***	[0.2,0.3]	[0.5,0.6]	[0.1,0.3]	[0.4,0.6]
Factor 1.2	**[0.6,0.7]**	***[0.2,0.3]***	[0.2, 0.4]	[0.4,0.6]	[0.4,0.6]	[0.1,0.2]
Factor 1.3	**[0.5,0.6]**	***[0.1,0.2]***	[0.1, 0.2]	[0.6,0.7]	[0.3,0.4]	[0.3,0.5]
Factor 1.4	**[0.7,0.8]**	***[0.1,0.2]***	[0.1, 0.2]	[0.6,0.8]	[0.1,0.2]	[0.7,0.8]
Factor 2.1	[0.3,0.4]	*[0.4,0.5]*	[0.6,0.7]	[0.1,0.2]	[0.2,0.3]	[0.5,0.6]
Factor 2.2	[0.2,0.4]	*[0.4,0.6]*	[06,0.7]	[0.1,0.3]	[0.1,0.3]	[0.5,0.6]

To apply the IVIFTD TSP the below sequential steps must be followed:

Step 1 Based on Table 18 and 19 we calculate the results in Table 20 and 21 by applying the IIFWAA [30,31,32] operator in Eq. (29). on μ_R, v_R. For example, [0.61, 0.71], an IIFWAA μ_R of Table 20, is calculated as follows (***bolded*** in the first two columns of Table 19) :

$$\text{IIFWAA}(A) = \left(\left[1-\prod_{i=1}^{n}(1-\mu_{AL}(x_i))^{\omega_i}, 1-\prod_{i=1}^{n}(1-\mu_{AU}(x_i))^{\omega_i}\right], \left[\prod_{i=1}^{n}(v_{AL}(x_i))^{\omega_i}, \prod_{i=1}^{n}(v_{AU}(x_i))^{\omega_i}\right]\right) \quad (29)$$

The confirmability membership degrees of the edge's jam factors (1.1,1.2,1.3 and 1.4) are ([0.6, 0.7], [0.6,0.7], [0.5, 0.6], and [0.7, 0.8]), and here the first edge is affected by four jam factors, $w_i = 1/4$, and so, the distributed weight is calculated as $w = [\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}]$ as follows (in Table 21):

$$0.61 = 1 - \left\{\left(1-0.6\right)^{\frac{1}{4}}\right\} * \left\{\left(1-0.6\right)^{\frac{1}{4}}\right\} * \left\{\left(1-0.5\right)^{\frac{1}{4}}\right\} * \left\{\left(1-0.7\right)^{\frac{1}{4}}\right\}$$
$$0.71 = 1 - \left\{(1-0.7)^{\frac{1}{4}}\right\} * \left\{(1-0.7)^{\frac{1}{4}}\right\} * \left\{(1-0.6)^{\frac{1}{4}}\right\} \left\{(1-0.8)^{\frac{1}{4}}\right\}$$

Then IIFWAA v_R in Table 21. values for the non-membership degrees of the jam factors [0.1, 0.2] [0.2, 0.3][0.1, 0.2][0.1, 0.2] and apply IIFWAA, as below :

$$0.12 = \left\{\left(0.1^{1/4}\right) * \left(0.2^{1/4}\right) * \left(0.1^{1/4}\right) * \left(0.1^{1/4}\right)\right\}$$
$$0.22 = \left\{\left(0.2^{1/4}\right) * \left(0.3^{1/4}\right) * \left(0.2^{1/4}\right) * \left(0.2^{1/4}\right)\right\}$$

Table XX *E1* IVIF degrees (IIFWAA $\boldsymbol{\mu_Q}$,**IIFWAA**$\boldsymbol{v_Q}$)

Q	**Factor1**	**Factor2**
Edge 1	([0.54,0.66], [0.12,0.24])	[0.5,0.65],[0.14,0.24]

Table XXI *E1* confirmability degrees (IIFWAA $\boldsymbol{\mu_R}$, **IIFWAA** $\boldsymbol{v_R}$)

R	Cost1	Cost 2	Cost3
IIFWAA Factor1	*([0.61,0.71],[0.12,0.22])*	([0.15,0.28],[0.52,0.67])	[0.24,40],[0.30,0.47]
IIFWAA Factor2	[0.37,0.51],[0.24,0.39]	[0.21,0.31],[0.35,0.46]	[0.55,70],[0.10,0.24]

Step 2 Calculate the distance by applying Eq. (30), taking values from Tables 20 and 21, gives.

Table XXII Distance for $\boldsymbol{E_1}$ with traffic factors $\boldsymbol{l_h}$

T	Cost1	Cost 2	Cost3
Edge 1	0.16	0.26	0.24

For any two IVIFSs A and B, where $A = \{\langle x_i, \mu_A(x_i), v_A(x_i)\rangle \,|x\epsilon\ X\}, (i = 1,2 \ldots \ldots n)$ and $B = \{\langle x_i, \mu_B(x_i), v_B(x_i)\rangle \,|x\epsilon\ X\}, (i = 1,2 \ldots \ldots n)$, the normalized Hamming distance considering the hesitation part is defined as in [34,35]:

$$l_h(A,B) = (1/4n)\sum_{i=1}^{n}[|\mu_{AL}(x_i) - \mu_{BL}(x_i)| + |\mu_{AU}(x_i) - \mu_{BU}(x_i)| + |v_{AL}(x_i) - v_{BL}(x_i)| + |v_{AU}(x_i) - v_{BU}(x_i)| + |H_{AL}(x_i) - H_{BL}(x_i)| + |H_{AU}(x_i) - H_{BU}(x_i)|] \quad (30)$$

Step 3 Find the lowest distance, because it points out the jam factor that is causing the highest delay and thus the one worth considering (in our case this is 0.16, as shown in Table 22). If we carry on with the same calculations for all the edges, we reach the data presented in Table 16; that shows all the jam factor costs for the edges, depending on their respective confirmabilities.

Thesis 5.3*:* [J3, C4, BC1] *I have introduced a complete novel fuzzified costs formulation for the classic TD TSP using the Interval-Valued Intuitionistic Fuzzy sets. I utilized the IIFWAA (interval-valued intuitionistic fuzzy weighted arithmetic average) operator to aggregate accumulative information from all the jam factors (depending on their weight/importance). Also, I calculated the hesitation part by incorporating the modified Park's distances between IVIFSs. Finally, ranking the factors according to their closeness lead to less information loss and more accurate tangible calculation for all traffic non-deterministic vague factors.*
This model was able to consider not only all the jam factors affecting an edge at any time, but also, to cater for the severity/conformability of the edge's belonging and not-belonging for a specific jam region or rush hour period.

Thesis 5.4: [J3, C4, BC1] *The IVIFTD TSP was optimized on universal instances Bier127 using the DBMEA and compared with the classical TD TSP's average and best elapsed times.*

The DBMEA algorithm was modified (by my colleague) and tested on Bier127 for the new IVIFTD TSP problem. The results were calculated by averaging five runs for different jam costs values. Tables 23 and 24 and Fig. 12. clearly indicate that the DBMEA was able to find high quality solutions for the proposed problem even with big jam factors values [13]

Table XXIII Distances for $E_1, E_{2,\ldots,}E_{12}$ with traffic factors l_h

	Cost1	Cost 2	Cost 3
Edge 1	0.16	0.26	0.24
Edge 2	0.13	0.20	-
Edge 3	0.15	-	0.24
Edge 4	0.16	0.14	0.44
Edge 5	0.3	0.6	0.2
Edge 6	0.16	-	0.24
Edge 7	0.16	0.30	0.20
Edge 8	0.16	0.06	0.44
Edge 9	0.04	0.36	-
Edge 10	0.1	0.23	0.34
Edge 11	0.6	-	0.3
Edge 12	-	0.27	0.23

Table XXIV DBMEA vs Simulated Annealing results on the Bier127 IVIFTD TSP extension

	DBMEA			Simulated Annealing		
Jam costs	**Best elapsed time**	**Average elapsed time**	**average runtime [s]**	**Best elapsed time**	**Average elapsed time**	**Average runtime [s]**
c1=1, c2=1.20, c3=1.50, c4=2	19.8315	19.8404	2358.75	20.8090	21.5924	23.89
c1=1, c2=1.5, c3=2, c4=5	19.8352	19.8609	2115.33	21.6720	21.9057	23.13
c1=1, c2=2, c3=5, c4=10	19.8537	19.8741	2249.31	22.0331	22.7257	26.25
c1=1, c2=5, c3=10, c4=25	19.8951	19.9395	2384.19	22.1808	22.8266	27.24

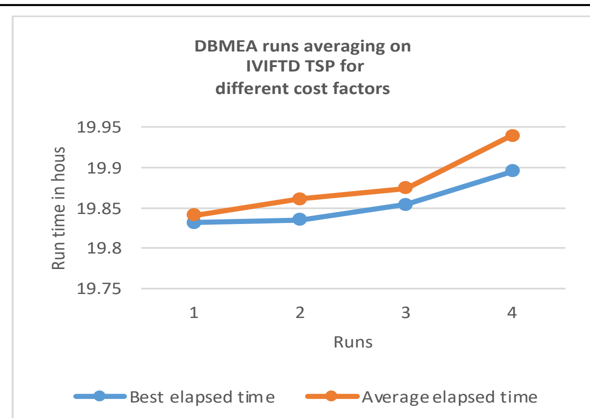

Fig. 12. DBMEA bier127 runs for IVIFTD TSP with different rush hours and jam regions' cost factors

4.6 Thesis group 6: [C7, P78] Extension of The Time Dependent Travelling Salesman Problem Model with Interval-Valued Fuzzy Soft Sets (IVFSTD TSP)

Since we established the fact that the jam regions and the rush hours are insufficiently representation in the TD TSP, in this model, I was able to employ interval-valued soft fuzzy sets to simulate the effects of the jam regions and the rush hours in any tour in a simple formulation.

C and P stand for the chapter and page number in the dissertation respectively on **thesis group** level.
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Thesis 6.1: [C6] *I introduced a complete novel Interval-Valued Fuzzy Soft Time Dependent Traveling Salesman Problem (IVFSTD TSP), by applying interval-valued fuzzy soft sets on the costs between nodes to realistically quantify the traffic jam regions, and the rush hours periods effects on any tour, which offered a closer-to-reality calculation for the travelled routes under vague and unpredictable road conditions.*

Suppose J is the set of jam factors (universe). C is the set of the modified costs (parameters), and E for set of the edges in a tour. Construct an interval-valued fuzzy soft set (F, C) over J. F representing a mapping given by:

$$F : C \rightarrow \tilde{\mathrm{F}}(J) \tag{31}$$

A relation matrix R_1 is constructed from the interval-valued fuzzy soft set (F, C) and called jamfactors-costs matrix.
Similarly, its complement $(F, C)^c$ gives another relation matrix, R_2, called non-jamfactors-costs matrix.

Again, we construct another interval-valued fuzzy soft set (F_1, J) over E, where F_1 is a mapping given by:

$$F_1 : J \rightarrow \tilde{\mathrm{F}}(E) \tag{32}$$

This interval-valued fuzzy soft set gives another relation matrix Q called edges-jamfactors matrix.

Then we obtain two new relation matrices: $T_1 = Q \circ R_1$, called jamfactors-edges matrix and $T_2 = Q \circ R_2$, non-jamfactors–edges matrix. we calculate the edges score $\boldsymbol{S_{T_1}}$and $\boldsymbol{S_{T_2}}$for and against the costs respectively:

$$S_{T_1} = \left[max_{i,j}\left\{ST1^{\mathrm{U}}\left(e_i, j_j\right), ST1^{\mathrm{L}}\, R_1\left(j_j, c_j\right)\right\}, \forall(i = 1,2, \ldots . n), (j = 1,2, \ldots \ldots , n)\ \right] \tag{33}$$

$$S_{T_2} = \left[max_{i,j}\left\{ST2^{\mathrm{L}}\left(e_i, j_j\right), ST2^{\mathrm{U}}\, R_1\left(j_j, c_j\right)\right\}, \forall(i = 1,2, \ldots . n), (j = 1,2, \ldots \ldots , n)\ \right] \tag{34}$$

Then we find $maxCost_{ei}$ for each edge:

$$maxCost_{ei} = \max_j [ST1\left(e_i, j_j\right) - ST2\left(e_i, j_j\right)]$$

In this model, I have applied the notion of interval-valued fuzzy soft sets on the TD TSP to be able to calculate the jam factors and rush hours' additional costs that affect each edge in a tour and how the additional delays are being interpreted in terms of trip costs respectively (full example and walkthrough is applied in chapter 7 in the dissertation).

4.7 Thesis group 7: [C8, P78] Extended TOPSIS Method for the Dial-a-ride Problem with Time Window Using Interval-Valued Intuitionistic Fuzzy Sets.

C and P stand for the chapter and page number in the dissertation respectively on **thesis group** level.
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The Dial-a-ride problem with time window (DARPTW) [35,36,37] is a widely applied logistics problem in real life and it is a prime example of the TD TSP. It specifies transportation requests between pairs of origins and destinations within a specific time window. The transportation is provided by a fleet of vehicles based on a common depot. The aim is to design a set of least cost vehicle routes capable of accommodating all requests. There are some constraints such as the vehicle capacity, route duration and the maximum tolerated (accepted) delay time by passengers [38].

***Thesis 7.1**:* [J4] *I introduced a novel mathematical formulation for the extended Dial-a-ride problem as multiple attributes decision making under the assumption of an interval-valued intuitionistic fuzzy environment. I incorporated the extended Technique for Ordered Preference by Similarity to the Ideal Solution (TOPSIS) method to determine the most beneficial alternative for all group members (the "best" ride). To avoid information distortion; I used the collective interval-valued intuitionistic fuzzy hybrid geometric mean (IIFHG) operator to build the collective decision matrix and to calculate each alternative's closeness from the interval-valued intuitionistic positive-ideal solution (PIS) using Park's extension for Hamming distance. Afterwards, I ranked the alternatives according to their closeness coefficients using scoring matrix to ultimately select the unanimous most desirable alternative(s) efficiently.*

The novelty of this model the fact that it handles passengers' demands with uncertainties while fulfilling the tour's requirements, which is a vital but completely overlooked aspect in the literature. It facilitates all individuals' interval-valued intuitionistic fuzzy decision matrices into the collective interval-valued intuitionistic fuzzy decision matrix. Employing the interval-valued intuitionistic fuzzy hybrid geometric mean (IIFHG) operator [31], leading to less information loss by aggregating the information pairs of the degrees of belonging and of not belonging of the attributes obtained from the users which are represented by intuitionistic fuzzy sets by. Calculating the collective interval-valued intuitionistic fuzzy decision matrix to find the interval-valued intuitionistic positive-ideal solution and negative-ideal solution. Lastly, calculating the relative closeness of each alternative using different extended distances measures from the interval-valued intuitionistic positive-ideal solution (PIS) [37,38,39,40], and thus, rank the alternatives according to their closeness using scoring matrix to choose the most desirable alternative(s) for all passengers.

Let $J = \{u_{1,} u_2, u_3, u_4, u_5\}$ be the set of five attributes, and assume that $u_{1,} u_2, u_3$ and u_5 are benefit attributes and u_4 is the cost attribute. That is, $J_1 = \{u_{1,} u_2, u_3, u_5\}$ and $J_2 = \{u_4\}$. There are four passengers who are involved in evaluating the situation: d_1, d_2, d_3 and d_4 (four was chosen, since it is the average number of passengers in a medium capacity car). The passengers' weight vector is $\lambda = (0.3, 0.2, 0.3, 0.2)^T$. The passengers d_k $(k = 1,2,3,4)$ represent, respectively, the characteristics of the alternatives O_j $(j = 1,2,3,4)$ by the IVIFNs $r_{ij}^{(k)}$, $(i = 1,2,3,4,5$ and $j = 1,2,3,4)$ with respect to the attributes u_i $(i = 1,2,3,4,5)$, listed in Tables 15 to 18 (i.e., interval-valued intuitionistic fuzzy decision matrices $R^{(k)} = \left(r_{ij}^{(k)}\right)_{5\times 4}$, $(k = 1,2,3,4)$).
Let us assume that the information on the attribute weights given by the passengers is shown as follows:

- d_1: $w_1 \leq 0.3$, $0.2 \leq w_3 \leq 0.5$;
- d_2: $0.1 \leq w_2 \leq 0.2$, $w_5 \leq 0.4$;
- d_3: $w_3 - w_2 \geq w_5 - w_4$, $w_4 \geq w_1$;
- d_4: $w_3 - w_1 \leq 0.1$, $0.1 \leq w_4 \leq 0.3$;

Then the set H of the known information on the attribute weights provided by the passengers is:

$H = \{w_1 \leq\ 0.3, 0.2 \leq\ w_3 \leq\ 0.5, 0.1\ \leq w_2 \leq\ 0.2\ , w_5 \leq\ 0.4, w_3 - w_2\ \geq\ w_5 - w_4,$
$w_4 \geq w_1, w_3 - w_1\ \leq 0.1\ , 0.1 \leq w_4\ \leq 0.3\}$.

Based on the analysis and definitions in the above sections, in the following section I will apply the presented model on multiple attribute interval-valued intuitionistic fuzzy group decision making in the pursuit of choosing the accumulative aggregated PIS alternative for all the passengers:

- **Step 1**. Utilize the IIFHG operator (let α =(0.155,0.345,0.345,0.155)^T be its weight vector derived by the normal distribution [41] based method Eq. (37)) which aggregates the individual interval-valued intuitionistic fuzzy decision matrices $R^{(k)} = \left(r_{ij}^{(k)}\right)_{5\times 4}$ $(k\ =\ 1,2,3,4)$ into the collective interval-valued intuitionistic fuzzy decision matrix $R = \left(r_{ij}\right)_{5\times 4}$ (see Table 7.7).

Let $R^{(k)} = \left(\tilde{r}_{ij}{}^{(k)}\right)_{m\times n}$ $(k = 1,2 \ldots . l\,)$ be the collective interval-valued intuitionistic fuzzy decision matrix $R = \left(\tilde{r}_{ij}\right)_{m\times n}$.

Where

$$\tilde{r}_{ij} = \text{IIFHG}_{\alpha,\lambda}\left(\tilde{r}_{ij}^{(1)}, \tilde{r}_{ij}^{(2)}, \ldots . \tilde{r}_{ij}^{(l)}\right) =$$
$$\left(\left(\dot{\tilde{r}}_{ij}^{(\sigma(1))}\right)^{\alpha_1} \otimes \left(\dot{\tilde{r}}_{ij}^{(\sigma(2))}\right)^{\alpha_2} \otimes \ldots . \left(\dot{\tilde{r}}_{ij}^{(\sigma(l))}\right)^{\alpha_l}\right) =$$
$$\left\langle\left[\prod_{k=1}^{n}\left(\dot{a}_{ij}^{(\sigma(k))}\right)^{\alpha_k}, \prod_{k=1}^{n}\left(\dot{b}_{ij}^{(\sigma(k))}\right)^{\alpha_k}\right], \left[1-\prod_{k=1}^{n}\left(1-\dot{c}_{ij}^{(\sigma(k))}\right)^{\alpha_k}, 1-\prod_{k=1}^{n}\left(1-\dot{d}_{ij}^{(\sigma(k))}\right)^{\alpha_k}\right]\right\rangle \tag{35}$$

where $\alpha = (\alpha_1, \alpha_2, \ldots \alpha_l)^T$ is the weight vector of IIFHG operator and $\alpha_k > 0\ (k = 1,2 \ldots . l)$ and $\sum_{k=1}^{l} \alpha_k = 1$,

and $\dot{\tilde{r}}_{ij}^{(\sigma(k))} = \left\langle\left[\dot{a}_{ij}^{(\sigma(k))}, \dot{b}_{ij}^{(\sigma(k))}\right], \left[\dot{c}_{ij}^{(\sigma(k))}, \dot{d}_{ij}^{(\sigma(k))}\right]\right\rangle$ is the k^{th} largest of the weighted IVIFNs $\dot{\tilde{r}}_{ij}^{(k)}, j = (1,2, \ldots . n)$; $i = (1,2, \ldots . m)$. In the situations where the information about attribute weights is completely known, that is, the weight vector $w = (w_1, w_2, w_3 \ldots w_m)^T$ of the attribute $u_i, (i = (1,2, \ldots . m)$ can be completely determined in advance, then we can construct the weighted collective interval-valued intuitionistic fuzzy decision matrix $R^* = \left(\tilde{r}_{ij}^*\right)_{m\times n}$.

- **Step 2.** Calculate the score matrix $S = \left(s_{ij}\right)_{5\times 4}$ of the collective interval-valued intuitionistic fuzzy decision matrix *R* (Table 30):

Let $R = \left(\tilde{r}_{ij}\right)_{m\times n}$ be the collective interval-valued intuitionistic fuzzy decision matrix. Then, we call

$S = \left(s_{ij}\right)_{m\times n}$ the score matrix of $R = \left(\tilde{r}_{ij}\right)_{m\times n}$

where

$$\boldsymbol{s_{ij} = s(\tilde{r}_{ij}) = 1/2\left(a_{ij} - c_{ij} + b_{ij} - d_{ij}\right)}, (\boldsymbol{i = 1, 2, 3, .. m}), (\boldsymbol{j = 1, 2, 3, .. n}) \tag{36}$$

and s($\tilde{r}_{ij}$) is called the score of $\tilde{r}_{ij}$. Based on the score matrix, we present the overall score values of each alternative O_j $(j = 1,2, \dots . n)$ as:

$$\boldsymbol{s_j(w) = \sum_{i=1}^{m} w_i s_{ij}} \ , \boldsymbol{(j = 1, 2, \dots n)} \tag{37}$$

- **Step 3**. Use the model **(F)** in Eq. (48) to obtain the optimal weight vectors $w^{(j)} = \left(w_1^{(j)}, w_2^{(j)}, w_3^{(j)}, w_4^{(j)}, w_5^{(j)}\right)^T, (j = 1,2,3,4)$ corresponding to the alternatives $O_j = (1,2,3,4)$

$$\textbf{(F)} \ \rightarrow \textbf{Maximize } \boldsymbol{s_j(w)} = \boldsymbol{\sum_{i=1}^{m} w_i s_{ij}} \tag{48}$$

subject to : $w = (w_1, w_2, w_3 \dots w_m)^T \in H$, $w_i \geq 0$, $i = 1 \dots m$, $\sum_{i=1}^{m} w_i = 1$.

$w^{(1)}$=$(0.2667, 0.1, 0.3667, 0.2667, 0)^T$
$w^{(2)} = \ (0.16, 0.1, 0.26, 0.16, 0.32)^T$
$w^{(3)}$ = $(0.1, 0.2, 0.2, 0.25, 0.25)^T$
$w^{(4)}$ = $(0.3, 0.1, 0.2, 0.3, 0.1)^T$

Table XXV Interval-valued intuitionistic fuzzy decision matrix $\boldsymbol{R}^{(1)}$

	$\boldsymbol{O_1}$	$\boldsymbol{O_2}$	$\boldsymbol{O_3}$	$\boldsymbol{O_4}$
u_1	[[0.5, 0.6], [0.2, 0.3]	[0.3, 0.4], [0.4, 0.6]]	[0.4, 0.5], [0.3, 0.5]	[0.3, 0.5], [0.4, 0.5]
u_2	[0.3, 0.5], [0.4, 0.5]	[0.1, 0.3], [0.2, 0.4]	[0.7, 0.8] , [0.1, 0.2][	[0.1, 0.2], [0.7, 0.8]
u_3	[0.6, 0.7], [0.2, 0.3]	[0.3, 0.4], [0.4, 0.5]	[0.5, 0.8], [0.1, 0.2]	[0.1, 0.2], [0.5, 0.8]
u_4	[0.5, 0.7], [0.1, 0.2]	[0.2, 0.4], [0.5, 0.6]	[0.4, 0.6]] [0.2, 0.3]	[0.1, 0.2], [0.5, 0.8]
u_5	[0.1, 0.4], [0.3, 0.5]	[0.7, 0.8], [0.1, 0.2]	[0.5, 0.6], [0.2, 0.3]	[0.2, 0.3], [0.5, 0.6]

Table XXVI Interval-valued intuitionistic fuzzy decision matrix $\boldsymbol{R}^{(2)}$

	$\boldsymbol{O_1}$	$\boldsymbol{O_2}$	$\boldsymbol{O_3}$	$\boldsymbol{O_4}$
u_1	[0.4, 0.5], [0.2, 0.4]	[0.3, 0.5], [0.4, 0.5]	[0.4, 0.6], [0.3, 0.4]	[0.3, 0.4], [0.4, 0.6]
u_2	[0.3, 0.4], [0.4, 0.6]	[0.1, 0.3], [0.3, 0.7]	[0.6, 0.8], [0.1, 0.2]	[0.1, 0.2], [0.6, 0.8]
u_3	[0.6, 0.7], [0.1, 0.2]	[0.3, 0.4], [0.4, 0.5]	[0.7, 0.8], [0.1, 0.2]	[0.1, 0.2], [0.7, 0.8]
u_4	[0.5, 0.6], [0.1, 0.3]	[0.2, 0.3], [0.6, 0.7]	[0.4, 0.6], [0.3, 0.4]	[0.3, 0.4], [0.4, 0.6]
u_5	[0.1, 0.3], [0.3, 0.5]	[0.6, 0.8], [0.1, 0.2]	[0.5, 0.6], [0.2, 0.4]	[0.2, 0.4], [0.5, 0.6]

Table XXVII Interval-valued intuitionistic fuzzy decision matrix $\boldsymbol{R}^{(3)}$

	$\boldsymbol{O_1}$	$\boldsymbol{O_2}$	$\boldsymbol{O_3}$	$\boldsymbol{O_4}$
u_1	[0.4, 0.7], [0.1, 0.2]	[0.4, 0.5], [0.2, 0.4]	[0.2, 0.4], [0.3, 0.4]	[0.3, 0.4], [0.2, 0.4]

u_2	[0.3, 0.5], [0.3, 0.4]	[0.2, 0.4], [0.4, 0.5]	[0.6, 0.8], [0.1, 0.2]	[0.1, 0.2], [0.6, 0.8]
u_3	[0.6, 0.7], [0.1, 0.2]	[0.4, 0.5], [0.3, 0.4]	[0.5, 0.7], [0.1, 0.3]	[0.1, 0.3], [0.5, 0.7]
u_4	[0.5, 0.6], [0.1, 0.3]	[0.1, 0.2], [0.7, 0.8]	[0.5, 0.7], [0.2, 0.3]	[0.2, 0.3], [0.5, 0.7]
u_5	[0.3, 0.5], [0.4, 0.5]	[0.6, 0.7], [0.2, 0.3]	[0.6, 0.8], [0.1, 0.2]	[0.1, 0.2], [0.6, 0.8]

Table XXVIII Interval-valued intuitionistic fuzzy decision matrix $\boldsymbol{R}^{(4)}$

	$\boldsymbol{O_1}$	$\boldsymbol{O_2}$	$\boldsymbol{O_3}$	$\boldsymbol{O_4}$
u_1	[0.6, 0.7], [0.2, 0.3]	[0.4, 0.5], [0.4, 0.5]	[0.4, 0.5], [0.3, 0.4]	[0.3, 0.4], [0.4, 0.5]
u_2	[0.3, 0.4], [0.3, 0.4]	[0.1, 0.2], [0.2, 0.3]	[0.6, 0.7], [0.1, 0.3]	[0.1, 0.3], [0.6, 0.7]],
u_3	[0.7, 0.8], [0.1, 0.2]	[0.3, 0.4], [0.5, 0.6]	[0.5, 0.8], [0.1, 0.2]	[0.1, 0.2], [0.5, 0.8]
u_4	[0.5, 0.6], [0.1, 0.3]	[0.2, 0.3], [0.4, 0.6]	[0.4, 0.5], [0.2, 0.3]	[0.2, 0.3], [0.4, 0.5]
u_5	[0.1, 0.2], [0.5, 0.7]	[0.6, 0.7], [0.1, 0.2]	[0.5, 0.6], [0.3, 0.4]	[0.3, 0.4], [0.5, 0.6]

Table XXIX Collective interval-valued intuitionistic fuzzy decision matrix $\boldsymbol{R}$

	$\boldsymbol{O_1}$	$\boldsymbol{O_2}$	$\boldsymbol{O_3}$	$\boldsymbol{O_4}$
u_1	[0.4385, 0.6199], [0.1549, 0.2848]	[0.3502, 0.4797], [0.3114, 0.4681]	[0.3516, 0.4906], [0.2940, 0.4214]	[0.3000, 0.4170], [0.3114, 0.4887]
u_2	[0.3000, 0.4573], [0.3404, 0.4710]	[0.1138, 0.3010], [0.2511, 0.4773]	[0.6395, 0.7711], [0.0980, 0.2263]	[0.1000, 0.2103], [0.6012, 0.7678]
u_3	[0.6116, 0.7117], [0.1089, 0.2083]	[0.3379, 0.4387], [0.3872, 0.4887]	[0.5213, 0.7804], [0.0980, 0.2083]	[0.1000, 0.2366], [0.5577, 0.7569]
u_4	[0.5000, 0.6395], [0.0980, 0.2567]	[0.1758, 0.3134], [0.5305, 0.6496]	[0.4387, 0.6252], [0.2263, 0.3262]	[0.2103, 0.3109], [0.4050, 0.5613]
u_5	[0.1323, 0.3623], [0.3747, 0.5482]	[0.6395, 0.7521], [0.1089, 0.2083]	[0.5452, 0.6502], [0.1770, 0.3005]	[0.1849, 0.3121], [0.5031, 0.6118]

Table XXX Collective score matrix *S*.

	$\boldsymbol{O_1}$	$\boldsymbol{O_2}$	$\boldsymbol{O_3}$	$\boldsymbol{O_4}$
u_1	0.3093	0.0252	0.0634	-0.0415
u_2	- 0.0270	-0.1568	0.5431	- 0.5294
u_3	0.5030	-0.0496	0.4977	- 0.4890
u_4	0.3924	-0.3454	0.2557	- 0.2225
u_5	- 0.2141	0.5372	0.3589	- 0.3089

Table XXXI Weighted collective interval-valued intuitionistic fuzzy decision matrix $\boldsymbol{R}^{(*)}$

	$\boldsymbol{O_1}$	$\boldsymbol{O_2}$	$\boldsymbol{O_3}$	$\boldsymbol{O_4}$
u_1	[0.1125, 0.1814], [0.6798, 0.7711]	[0.0853, 0.1264], [0.7855, 0.8547]	[0.0857, 0.1303], [0.7762, 0.8363]	[0.0711, 0.1056], [0.7855, 0.8623]
u_2	[0.0437, 0.0737], [0.8738, 0.9100]	[0.0150, 0.0438], [0.8411, 0.9116]	[0.1199, 0.1685], [0.7476, 0.8303]	[0.0131, 0.0291], [0.9383, 0.9675]
u_3	[0.2183, 0.2766], [0.5614, 0.6646]	[0.1018, 0.1396], [0.7811, 0.8299]	[0.1746, 0.3261], [0.5461, 0.6646]	[0.0271, 0.0679], [0.8589, 0.9300]
u_4	[0.1560, 0.2210], [0.5664, 0.7169]	[0.0462, 0.0879], [0.8563, 0.8998]	[0.1318, 0.2135], [0.6952, 0.7602]	[0.0561, 0.0871], [0.8016, 0.8682]
u_5	[0.0228, 0.0706], [0.8524, 0.9068]	[0.1530, 0.2030], [0.6972, 0.7747]	[0.1203, 0.1571], [0.7545, 0.8223]	[0.0327, 0.0591], [0.8942, 0.9232]

Here we construct the weight matrix

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$$W = \begin{pmatrix} 0.2667 & 0.16 & 0.1 & 0.3 \\ 0.1 & 0.1 & 0.2 & 0.1 \\ 0.3667 & 0.26 & 0.2 & 0.2 \\ 0.2667 & 0.16 & 0.25 & 0.3 \\ 0 & 0.32 & 0.25 & 0.1 \end{pmatrix}$$

Then

$$(S^T W)^T S^T W = \begin{pmatrix} 0.3455 & 0.2621 & 0.2835 & 0.2848 \\ 0.2621 & 0.2634 & 0.2683 & 0.2266 \\ 0.2835 & 0.2683 & 0.2808 & 0.2423 \\ 0.2848 & 0.2266 & 0.2423 & 0.2365 \end{pmatrix}$$

- **Step 4**. Calculate the normalized eigenvectors ω of the matrix $(S^T W)^T S^T W$

$$\omega = (0.2764, 0.2390, 0.2519, 0.2326)^T$$

- **Step 5.** Use Eq. (39) to derive the weight vector *w***:**

$$w = W\omega = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & .. & w_1^{(n)} \\ w_2^{(1)} & w_2^{(2)} & .. & w_2^{(n)} \\ . & .. & .. & .. \\ w_m^{(1)} & w_m^{(2)} & .. & w_m^{(n)} \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ .. \\ \omega_n \end{pmatrix} = \omega_1 w^{(1)} + \omega_2 w^{(2)} + \dots + \omega_n w^{(\mathrm{n})} \tag{39}$$

$$w = W\omega = \begin{pmatrix} 0.2667 & 0.16 & 0.1 & 0.3 \\ 0.1 & 0.1 & 0.2 & 0.1 \\ 0.3667 & 0.26 & 0.2 & 0.2 \\ 0.2667 & 0.16 & 0.25 & 0.3 \\ 0 & 0.32 & 0.25 & 0.1 \end{pmatrix} \begin{pmatrix} 0.2764 \\ 0.2390 \\ 0.2519 \\ 0.2326 \end{pmatrix} =$$

$$(0.2069, 0.1252, 0.2604, 0.2447, 0.1627)^{\mathrm{T}}$$

Table XXXII Separation measures for the example. Here, the separation measures are based on the Hamming distance

	$S_{j^*}^{d_1}$	$S_{j^-}^{d_1}$	$S_{j^*}^{d_2}$	$S_{j^-}^{d_2}$	$S_{j^*}^{d_H}$	$S_{j^-}^{d_H}$
O_1	0.4385	0.3877	0.5201	0.4300	0.4982	0.4251
O_2	0.3339	0.4655	0.4002	0.5395	0.4002	0.5395
O_3	0.2337	0.5657	0.2687	0.6713	0.2687	0.6713
O_4	0.6447	0.1551	0.7445	0.1846	0.7441	0.1846

Table XXXIII The relative closeness of each alternative in the example

	Value	Rank	Value	Rank	Value	Rank
	$C_j^{d_1}$		$C_j^{d_2}$		$C_j^{d_H}$	
O_1	0.4692	3	0.4526	3	0.4604	3
O_2	0.5823	2	0.5741	2	0.5741	2
O_3	0.7076	1	0.7141	1	0.7141	1
O_4	0.1939	4	0.1987	4	0.7141	4

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- **Step 6.** Calculate the collective weighted interval-valued intuitionistic fuzzy decision matrix $R^* = \left(\tilde{r}_{ij}^*\right)_{m\times n}$ (see Table 31).
- **Step 7**. Utilize Eqs. (40 and 41) to determine the interval-valued intuitionistic PIS O^* and NIS O^-:

$$\boldsymbol{O}^{*} = \left\{\langle \boldsymbol{u_i}, \left(\max_j \tilde{\boldsymbol{r}}_{\boldsymbol{ij}}{}^* \mid \boldsymbol{i} \in \boldsymbol{J_1}\right), \left(\min_j \tilde{\boldsymbol{r}}_{\boldsymbol{ij}}{}^* \mid \boldsymbol{i} \in \boldsymbol{J_2}\right)\rangle \mid \boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}, \dots \boldsymbol{m}\right\}^{\boldsymbol{T}} = \left\{\tilde{\boldsymbol{r}}_{\boldsymbol{1}}{}^+, \tilde{\boldsymbol{r}}_{\boldsymbol{2}}{}^+ \dots, \tilde{\boldsymbol{r}}_{\boldsymbol{m}}{}^+\right\}^{\boldsymbol{T}} \quad (40)$$

$$O^{-} = \left\{\langle u_i, \left(\min_j \tilde{r}_{ij}{}^* \mid i \in J_1\right), \left(\max_j \tilde{r}_{ij}{}^* \mid i \in J_2\right)\rangle \mid i = 1,2, \dots m\right\}^{T} = \{\tilde{r}_1{}^-, \tilde{r}_2{}^- \dots, \tilde{r}_m{}^-\}^T \quad (41)$$

where

$\tilde{r}_i{}^+ = \langle [a_i^+, b_i^+], [c_i^+, d_i^+]\rangle$ and $\tilde{r}_i{}^- = \langle [a_i^-, b_i^-], [c_i^-, d_i^-]\rangle$, $i = 1,2, \dots m$.

$$O^* = \begin{Bmatrix} \langle[0.1125, 0.1814], [0.6798, 0.7711]\rangle, \\ \langle[0.1199, 0.1685], [0.7476, 0.8303]\rangle, \\ \langle[0.1746, 0.3261], [0.5461, 0.6646]\rangle, \\ \langle[0.0462, 0.0879], [0.8563, 0.8998]\rangle, \\ \langle[0.1530, 0.2030], [0.6972, 0.7747]\rangle \end{Bmatrix}^T$$

and

$$O^- = \begin{Bmatrix} \langle[0{:}\,0711;\ 0{:}\,1056], [0{:}\,7855;\ 0{:}\,8623]\rangle, \\ \langle[0{:}\,0131;\ 0{:}\,0291], [0{:}\,9383;\ 0{:}\,9675]\rangle, \\ \langle[0{:}\,0271;\ 0{:}\,0679], [0{:}\,8589;\ 0{:}\,9300]\rangle, \\ \langle[0{:}\,1560;\ 0{:}\,2210], [0{:}\,5664;\ 0{:}\,7169]\rangle, \\ \langle[0{:}\,0327;\ 0{:}\,0591], [0{:}\,8942;\ 0{:}\,9232]\rangle \end{Bmatrix}^T$$

- **Step 8. C**alculate the separation measures S_{J^*} and S_{J^-} (see Eqs. from 8.17 to 8.22 in the dissertation) for each alternative O_j $(j = 1,2,3,4)$ from the interval-valued intuitionistic PIS O^* and the interval-valued intuitionistic NIS O^-, respectively, based on the Hamming distance [42,43,44] as in Table 32.
- **Step 9**. Utilize Eq. (42) to calculate the relative closeness C_{j^*} of each alternative O_j $(j = 1,2,3,4)$ to the interval-valued intuitionistic PIS O^* with the chosen separation measures, thus, $C_j^{d_1}, C_j^{d_2}$ *and* $C_j^{d_H}$ in Table 33.

The relative closeness of an alternative O_j with respect to interval-valued intuitionistic PIS O^* is defined as follows:

$$\boldsymbol{C}_{\boldsymbol{j}^*} = \frac{\boldsymbol{S}_{\boldsymbol{j}^-}}{\boldsymbol{S}_{\boldsymbol{j}^*} + \boldsymbol{S}_{\boldsymbol{j}^-}}, (\boldsymbol{j} = \boldsymbol{1}, \boldsymbol{2}, \dots \boldsymbol{n}) \quad (42)$$

Finally, rank the preference order of alternatives O_j $(j = 1,2,3,4)$ in Table 33, according to the relative closeness to the interval-valued intuitionistic PIS O^*. Obviously, the most desirable alternative is O_3.

Indeed, this model is leading to an effective and practical way to deal with dial-a-ride problem, it made it possible to solve instances with several hundreds of users within reasonable times. Furthermore, it is

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(somewhat) computationally not as expensive as the meta-heuristics and heuristics approaches presented in the literature.

5. Applicability

The presented results are applicable in both academic and industrial fields. The results were tested and validated on the largest universal benchmarks and compared to the state-of-the-art solutions such as Schneider's approach. The results have shown applicability and universality where the models were able to simulate real life cases with different rush hours and jam factors affecting the tours and offered more accurate solutions that converged in a reasonable time even with large values (rush hours and jam regions) and thus, offering the (quasi) optimal path for the TD TSP (Time Dependent Traveling Salesman Problem) under uncertainty. Also, the models were optimized using DBMEA and compared to GA which only proves its applicability and generality.

In addition, my main purpose of applying the interval-valued intuitionistic fuzzy sets with the extended TOPSIS method on the Dial-a-ride Problem with Time Window (which is a prime example of the TD TSP) was to simulate the most complicated model I have proposed (the IVIFTD TSP) on another variation of a common transportation problem. The results only proved the applicability and practicality of my proposed model on general cases within different application areas. Hence, my proposed extended models proven to be beneficial, applicable, and efficient on numerous real-life problems.

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1. Publications

7.1 International Journals

J5. **Almahasneh, Ruba** ; Tüű-Szabó, Boldizsár; Kóczy, László T., *Optimization of the Extended Triple Fuzzy Time Dependent Travelling Salesman Problem Model with Discrete Bacterial Memetic Algorithm*, Fuzzy Sets and Systems journal, under review

J4. **Almahasneh, Ruba** ; Kóczy, László T., *Extended TOPSIS Method for the Dial-a-ride Problem with Time Window Using Interval-Valued Intuitionistic Fuzzy Sets*, Fuzzy Sets and Systems journal, under review

J3. **Almahasneh, Ruba** ; Tüű-Szabó, Boldizsár ; Kóczy, László T. ; Földesi, Péter, *Optimization of the Time-Dependent Traveling Salesman Problem Using Interval-Valued Intuitionistic Fuzzy Sets*, AXIOMS 9 : 2 p. 53 (2020), DOI DOI (WoS Scopus)

J2. **Almahasneh, Ruba** ; Tuu-Szabo, Boldizsar ; Foldesi, Peter ; Koczy, Laszlo T., *Quasi-Optimization of the Time Dependent Traveling Salesman Problem by Intuitionistic Fuzzy Model and Memetic Algorithm*, STUDIES IN COMPUTATIONAL INTELLIGENCE 872 pp. 239-253. Paper: Chapter 14 , 15 p. (2020), DOI (WoS Scopus)

J1. **Ruba, S.Almahasneh** ; László, T. Kóczy, *Intuitionistic Fuzzy Rule-base Model for the Time Dependent Traveling Salesman Problem*, INTERDISCIPLINARY DESCRIPTION OF COMPLEX SYSTEMS 18 : 3 pp. 352-359. , 8 p. (2020), (DOI WoS)

7.2 Book Chapters

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BC2. Zargar, Hisham ; **Almahasneh, Ruba** ; Kóczy, László T., *Automatic Recognition of Handwritten Urdu Characters*, In: István, Á. Harmati; László, T. Kóczy; Computational Intelligence and Mathematics for Tackling Complex, Problems 3 ,Cham, Svájc : Springer, Cham (2022) 220 p. pp. 165-175. Paper: Chapter 19, 11p, DOI

BC1. **Almahasneh, Ruba** ; Tuu-Szabo, Boldizsár ; Foldesi, Peter ; Koczy, Laszlo T., *Fuzzy Set Based Models Comparative Study for the TD TSP with Rush Hours and Traffic Regions*, In: Information Processing and Management of Uncertainty in Knowledge-Based Systems , Springer International Publishing (2020) pp. 699-714. Paper: Chapter 55 , 16 p., DOI Scopus

7.3 International Conference Papers

C7. Tüű-Szabó, B., **Ruba Almahasneh**,Földesi, P.,Kóczy, L.T.: A software tool for solving the Traveling Salesman Problem and related non-fuzzy and fuzzy optimization problems, Proceedings of the 14th European Symposium on Computational Intelligence and Mathematics (ESCIM 2022), October 2nd-5th, 2022, in publication

C6. **Almahasneh, R**. ; Koczy, L.T., *Extension of The Time Dependent Travelling Salesman Problem Model with Interval-Valued Fuzzy Soft Sets*, 14th European Symposium on Computational Intelligence and Mathematics (ESCIM 2022), October 2nd-5th, 2022, in publication

C5. **Almahasneh, R**. ; Foldesi, Peter ; Koczy, L.T., *Mixed Arabic and English Reviews Sentiment Analysis with LSTM Recurrent Neural Network*, 12th IEEE International Conference on Cognitive Infocommunications (CogInfoCom 2021),pp. 179-185 (2021)

C4. **Almahasneh, R**. ; Tuu-Szabo, B. ; Foldesi, P. ; Koczy, L.T., *Extension of the Time Dependent Travelling Salesman Problem with Interval Valued Intuitionistic Fuzzy Model Applying Memetic Optimization Algorithm*, In: ACM, 2020 4th International Conference on Intelligent Systems, Metaheuristics and Swarm Intelligence (ISMSI 2020) , New York (NY), Amerikai Egyesült Államok : Association for Computing Machinery (ACM) (2020) pp. 111-118. Paper: 3396490 , 8 p., DOI (WoS Scopus)

C3. **Almahasneh, Ruba**; Tuu Szabo, Boldizsar ; Foldesi, Peter ; Koczy, T. Laszlo, *Quasi-optimization of the Time Dependent Travelling Salesman Problem by Intuitionistic Fuzzy Model and Memetic Algorithm1*, In: Computational, Intelligence in Emerging Technologies for Engineering Applications , Springer International Publishing (2020) pp. 239-253., 15 p.

C2. **Almahasneh, Ruba** ; Tuu-Szabo, Boldizsar ; Foldesi, Peter ; Koczy, Laszlo T., *Intuitionistic Fuzzy Model of Traffic Jam Regions and Rush Hours for the Time Dependent Traveling Salesman Problem*, ADVANCES IN INTELLIGENT SYSTEMS AND COMPUTING 1000 pp. 123-134. Paper: Chapter 12 , 12 p. (2019), DOI (WoS Scopus)

C1. Kóczy, T László ; Foldesi, Peter ; Tuu, Szabo Boldizsar ; **Almahasneh, Ruba,** *Modeling of Fuzzy Rule-base Algorithm for the Time Dependent Traveling Salesman Problem*, In: IEEE (szerk.) 2019 IEEE International Conference on Fuzzy Systems (FUZZ -IEEE) , New York (NY), USA, IEEE (2019) , 6 p., DOI Scopus

7.4 Awards and Grants

1. *Fuzzy youth Award 2021* by Hungarian Fuzzy Association Youth Award
2. International Fuzzy Systems Association World Congress (IFSA),2019, *Outstanding paper award*, *"*Intuitionistic Fuzzy Model of Jam Regions and Rush Hours for the Time Dependent Traveling Salesman Problem, International Fuzzy Systems Association World Congress (IFSA), pp 123-134.*"*.

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3. *EUSFLAT* Grants for IPMU 2020.
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7.5 Collaboration Research Groups

1. SAP Hungary Ltd., BlockChain for ERP
2. AIMS5.0

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