Modeling and control of autonomous public transport vehicles

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1 Introduction

On busy urban arterials, especially during peak hours public transport delay is a prevailing problem. Fluctuation of passenger demand, intersection delays, changing traffic conditions and different driving styles of bus drivers bring several uncertainties into the system. Subsequently, due to the volatile nature of the system, any disparities in bus headways tend to increase over time, eventually resulting in bus bunching [NP64]. Due to bunching the periodicity of arrivals fail and homogeneous service cannot be maintained. In addition to bunching, punctuality and passenger delays are two important factors to be addressed by service providers. At frequent lines, if schedule cannot be held and a bus arrives at the stop late or with a large headway gap, the number of passengers is winding up. It leads to non-homogeneous utilization of buses and therefore degradation of level of service. Beside level of service, the need for energy efficiency via electrification is an emerging trend too. The advances of sensor technology (localization) and vehicle communication (V2X) along with larger computational capacity enable the emergence of advanced driver assistance systems such as eco-cruise control [SRDVdB13], [ANGH18].

Although there are numerous different goals a public transport service shall attain, these goals are intertwined. E.g. keeping a timetable also suggests periodic arrivals. Smooth accelerations are not only energy efficient but also mean ride comfort. Maintaining level of service and efficiency in the face of such a complex environment can only be achieved by careful, real-time trajectory planning.

Finding a compromise solution between energy consumption and level of service leads to a multi-objective optimization problem. The goal is merging four conflicting, public bus service related objectives:

- adherence to a predefined schedule,
- vehicles shall keep equidistant headways from each other,
- minimal waiting time of the passengers at stops, and
- energy efficiency.

The objectives are obtained solely based on velocity control. The main idea is to use short time horizon predictions and optimize the trajectory of every bus in real-time. The suggested rolling horizon policy is an adequate control solution to predict future obstacles along the route and incorporate reference trajectories from various sources. The control method focuses on network bunching, but in a distributed, overlapped way: every vehicle runs its own velocity controller and then they communicate their predicted trajectories among each other (Figure 1). The proposed control algorithms consider uncertainties such as varying dwell times and delays due to interaction with traffic and traffic lights too. The decentralized control can be recast into

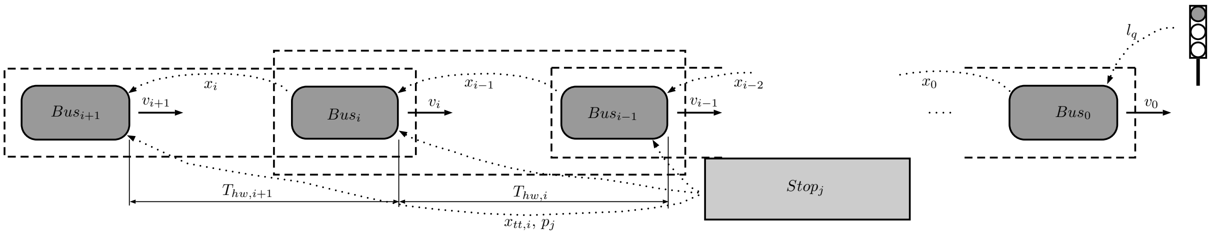

Figure 1: Overlapped, decentralized control strategy

a centralized network-level velocity control as well. In that case the speed advisory algorithm attempts to calculate public-transport network optimum for passenger wait times and headway adherence.

The proposed control algorithms can improve level of service in an energy efficient way. Depending on the prevailing traffic situation and public transport demand the system can be flexibly reconfigured by emphasizing certain objectives over others.

2 Contributions of the Thesis

Thesis 1

Discrete-time state-space based control oriented models were constructed for urban public transport trajectory planning. The models consider the vehicle dynamics of individual buses and passenger dynamics at bus stops. Using a first principle model, the energy consumption model of electrified buses was developed. The proposed models can serve as a basis of a decentralized, reference tracking velocity control. The modular nature of the models enables formulating different control strategies. The models can be extended to a whole public transport network and can be adapted to other modes of city public transport.
[KV16, VTK17, VTK18b, Var18, VTK18a, VTK19b, VPKT20]

The longitudinal motion of a bus along a corridor in discrete-time can be written as a double integrator:

$$\begin{bmatrix} v(k+1) \\ x(k+1) \end{bmatrix} = \begin{bmatrix} 1-\frac{\Delta t}{\tau} & 0 \\ \Delta t & 1 \end{bmatrix} \begin{bmatrix} v(k) \\ x(k) \end{bmatrix} + \begin{bmatrix} \frac{\Delta t}{\tau}(1-\beta) \\ 0 \end{bmatrix} v_{des}(k) + \begin{bmatrix} \beta \\ 0 \end{bmatrix} v_{mac}(k), \quad (1)$$

where position $x(k+1)$ and velocity $v(k+1)$ denote the states over the time period of $[k\Delta t, (k+1)\Delta t]$ with Δt being the discrete time-step length, and $k = 1,\ 2,\ 3, \ldots$ is the discrete time-step index. The acceleration is modeled with a linear relaxation term where $v_{des}(k)$ denotes the desired velocity (i.e. velocity setpoint) of the vehicle. τ is a model parameter capturing the sensitivity of drivers to the change of their desired velocity. Disturbance is introduced into the model via $v_{mac}(k)$. It is the macroscopic average velocity on the link the bus travels on. β describes relaxation of bus speed towards a traffic dependent equilibrium velocity.

Buses on a line shall achieve two conflicting objectives characterized via two error terms.

Timetable tracking: Buses shall follow a predefined, static timetable characterized by the reference trajectory $x_{tt}(k)$.

$$z_{tt}(k) = x(k) - x_{tt}(k). \quad (2)$$

Headway tracking: Buses shall keep equidistant headway from each other in order to avoid bus bunching. The headway reference trajectory is the past trajectory of the leading bus shifted by one ideal headway time T_{hw} ahead: $x_{hw}(k) = x_{i-1}(k - T_{hw}\Delta t)$. Subscript $i-1$ denotes the leading bus. The headway tracking error term is

$$z_{hw}(k) = x(k) - x_{hw}(k). \tag{3}$$

The two reference trajectories are depicted in Figure 2.

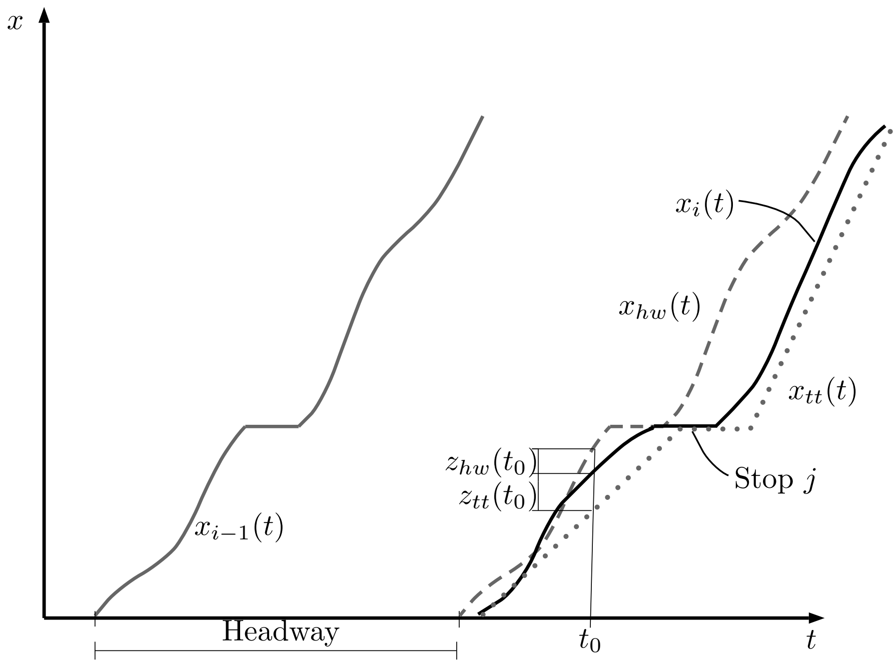

Figure 2: Reference tracking

The next model is describing the energy consumption $E_{cons}(k)$ of electrified public transport buses with regenerative braking. A physical-based model was constructed as

$$E_{cons}(k) = \left(\frac{P_{roll}(k) + P_g(k) + P_{drag}(k) + P_{acc}(k)}{\eta_{batt} \cdot \eta_{pe} \cdot \eta_{mot} \cdot \eta_{pt}} \right.$$
$$\left. + P_{regen}(k) \cdot \eta_{regen} \cdot \eta_{batt} \cdot \eta_{pe} \cdot \eta_{mot} \cdot \eta_{pt} \right) \Delta t, \tag{4}$$

where $P_{roll(k)}$, $P_g(k)$, $P_{drag}(k)$, $P_{acc}(k)$ and $P_{regen}(k)$ are powers to overcome rolling resistance, road inclination, air drag to accelerate or to regeneratively brake the vehicle, respectively. η_{regen}, η_{batt}, η_{pe}, η_{mot}, η_{pt} are efficiency parameters of the electric powertrain. Through the vehicle's acceleration (that also appears in Eq. 1) the model can be (piecewise-)linearized and can be incorporated into an energy minimizing optimization problem.

Bus stops are modeled as piecewise affine systems:

$$\begin{bmatrix} t_w(k+1) \\ p(k+1) \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} t_w(k) \\ p(k) \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta t \end{bmatrix} \lambda(k) + \begin{bmatrix} 0 \\ -\frac{\Delta t}{t_b} \end{bmatrix} \xi(k), \tag{5}$$

where the system states are the number of passengers waiting at a stop $p(k)$ and their cumulated waiting time $t_w(k)$. Passengers arrive at a stop based on an arrival rate function $\lambda(k)$ [JH75] and get on the bus with a fixed boarding rate t_b. Boarding is enabled by the integer variable $\xi(k)$. A bus is performing passenger exchange if it is at the stop, its velocity is zero and there are passengers at the stop. In mathematical form:

$$\begin{aligned} \xi(k) = \sum_{i=1}^{M_B} \exists i : (|x_i(k) - x_{stop}|) < \epsilon \\ \& \ v_i(k) \leq \epsilon. \ \& \ p(k) \geq \epsilon, \end{aligned} \tag{6}$$

where M_B is the number of buses in the network and ϵ are numerical tolerances.

Finally, the bus stop and bus dynamics models can be combined into a single public transport network model to serve the basis of a centralized control algorithm. The headway reference trajectory indirectly couples the system as $x_{hw,i}(k) = x_{i-1}(k - T_{hw,i}(x_i)\Delta t)$, where i denotes the i^{th} bus. Similarly, the boarding state $\xi_j(k)$ (for stop j) couples buses and bus stops. In addition, an extra logical constraint shall be prescribed, similarly to $\xi_j(k)$ that forces the buses to stop at a bus stop:

$$\eta_i(k) : \exists j : (|x_i(k) - x_{stop,j}|) < \epsilon \ \& \ p_j(k) \geq \epsilon. \tag{7}$$

Thesis 2

The classic shockwave theory was extended with stochastic description of traffic flow. Vehicle arrivals to a link is modeled with its distribution function instead of its average value (hourly flow). Analytical description was given for the distribution functions of shockwave profiles and the queue length evolution. The proposed stochastic model was validated in microscopic traffic simulation. The stochastic model is suitable for predicting traffic flow states and queue length in signalized intersections. The model was extended to multiple intersections, thus it is capable of modeling urban networks with signalized intersections in a stochastic way.
[VTK18c, VTK19a, VT19, VTKQ20]

The shockwave profile model (SPM) [WL11] is efficient in modeling networks with signalized intersections, where signal cycles and shockwaves shape traffic flow. The model distinguishes three different traffic states.

i) Traffic flows freely towards the queue at the intersection, vehicles travel at their desired speed $v_A(t)$ and slow down when they approach the queue.

ii) Traffic is jammed inside, assuming jam density ρ_J and zero velocity $v_J = 0$. This traffic state is denoted by $R_J(t, \omega)$.

iii) When the queue starts dissipating, the traffic flow state goes from jammed to its critical capacity. The model calls this state queue discharge region $R_C(t, \omega)$. Here, the traffic flow is assumed to be saturated, the average velocity of vehicles is the critical velocity v_C.

Stochasticity (denoted with the symbol ω) is injected through the following assumptions: Traffic flow $\mathcal{Q}_A(t)$ is usually given in vehicles per hour, i.e. traffic signal program is planned based on hourly average traffic demand. However, a traffic light cycle time is only a fraction of an hour. Vehicle arrivals within that hour are not uniformly distributed, therefore scaling down traffic flow from *veh/h* to *veh/cycle* introduces uncertainty to the model. This uncertainty is grasped with the help of the probability distribution function of $Q_A(t, \omega)$: $F_{Q_A}(t, \varphi)$. The link fundamental diagram is assumed to be known too. Thus, traffic flow states and the shockwave profiles can be explicitly determined using the classic shockwave theory (Figure 3). The location of the traffic light l_l $[m]$ and the signal program shall be known as well. It is represented by three variables: t_{cyc} $[s]$ is the cycle time, $t_{1,c}$ $[s]$ is the start of red phase and $t_{2,c}$ $[s]$ is the end of red phase in the cycle. Subscript c denotes the c^{th} traffic light cycle. The model is capable of handling traffic-responsive signal control as well.

The four shockwave velocities characterizing the shockwave profile model are the following:

i) **Queuing shockwave velocity** is formed by vehicles accumulating at the red light. Vehicles stopping at the tail of the queue from their actual velocity to zero form a shockwave. This process is stochastic due to the randomness of vehicle arrivals affecting the queue length. $W_1(t,\omega) = -\frac{Q_A(t,\omega)}{\rho_J - \rho_A(t,\omega)}$.

ii) **Discharge shockwave velocity** is assumed to be deterministic, as it is not affected by vehicle arrivals. In the fundamental diagram the discharge shockwave velocity is the slope of the line connecting the jam density and the critical density: $W_2 = -\frac{Q_C}{\rho_J - \rho_C}$.

iii) **Departure wave velocity** is generated as the queue dissipates as vehicles leave the intersection at green. It starts at the intersection of the queuing and discharge shockwaves. In addition, newly arrived vehicles feed the queue, hence $W_3(t,\omega) = \frac{Q_C - Q_A(t,\omega)}{\rho_C - \rho_A(t,\omega)}$.

iv) **Pressure wave velocity** separates a critical density and a jam density region, and it has the same speed as the discharge wave W_2. The pressure wave is only present if there is a residual queue, i.e. the queue cannot fully discharge during a green interval: $W_4 = -\frac{Q_C}{\rho_J - \rho_C}$.

Shockwave velocities can be represented in time-space diagram, see Figure 4. Shockwave profiles represent the temporal building and dissipating of queues in a signalized intersection, thus it can be used as a model for predicting traffic states. The stochastic process for the queue length is:

$$l_q(t,\omega) = \begin{cases} W_1(t,\omega)(t - t_{r,c-1}(t,\omega)) + l_{r,c-1}(t,\omega), \\ if\ t_{r,c-1}(t,\omega) \leq t < t_{s,c}(t,\omega)\ (queuing), \\ \\ max(l_l, W_3(t,\omega)(t - t_{s,c}(t,\omega)) + l_{s,c}(t,\omega)), \\ if\ t_{s,c}(t,\omega) \leq t < t_{r,c}(t,\omega)\ (departure). \end{cases} \tag{8}$$

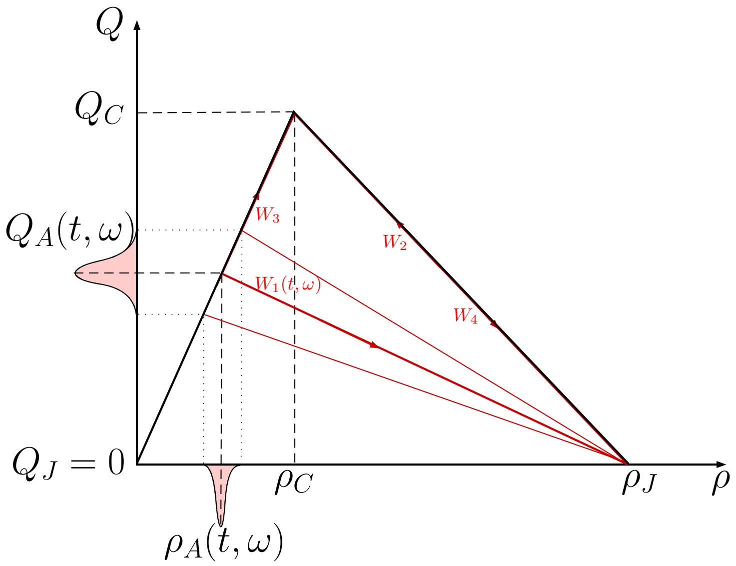

Figure 3: Triangular link fundamental diagram of traffic flow with shockwave speeds ($W_1 ... W_4$). $Q_A(t, \omega)$ is represented with a probability density function, showing how it affects the slope of the queuing shockwave $W_1(t, \omega)$.

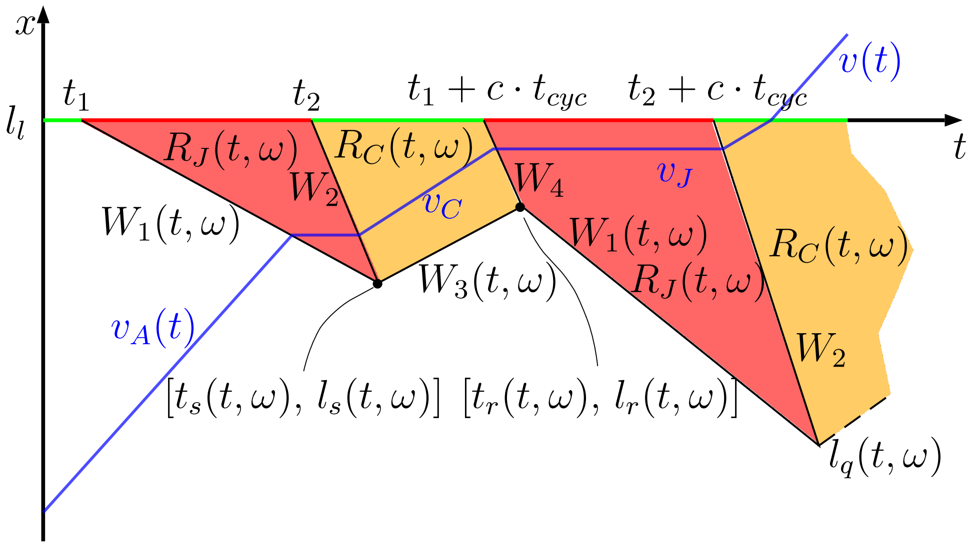

Figure 4: Shockwave profile of a single intersection

Thesis 3

Decentralized, multi-objective model predictive control strategies were formulated for public transport velocity control. The four (often) conflicting objectives considered in the optimization are timetable adherence, equidistant headways, energy efficiency, and minimal passenger waiting time. The controller operates in a shrinking horizon way meaning that trajectories are predicted until the vehicle reaches the next stop. The four objectives can be cast into control strategies via different weighting of the cost function.
[VTK18c, VTK18b, Var18, VTK19b]

The control oriented models in *Thesis 1* can be used for model predictive control design. The goal of the controller is calculating an optimal velocity profile between the actual position of the vehicle and the next stop, while taking into account several uncertainties. The proposed MPC classifies as shrinking horizon, meaning a fixed state shall reached (the next stop) by the end of the horizon. The time interval between the current time t_0 and the desired arrival time t_{arr} is discretized with Δt splitting it into N equidistant steps, see Figure 5. In every time step the prediction horizon decreases by one.

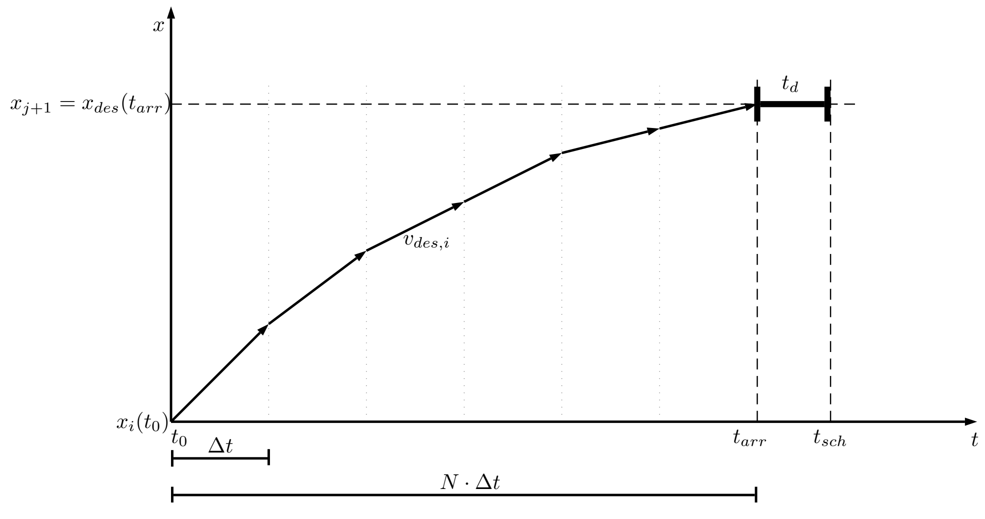

Figure 5: Shrinking horizon control strategy

Each model in *Thesis 1* serves as a component of the final multi-objective cost function and is extended for N horizon via stacking the system equations. The timetable and headway tracking errors $z_{tt}(k)$ and $z_{hw}(k)$, are used for penalizing deviation from the timetable and unequidistant headways through cost terms $J_{tt}(k)$ and $J_{hw}(k)$, respectively. Energy consumption of the vehicles are penalized through the piecewise-linear cost J_e. Finally, passenger waiting times are embedded into J_p. Finally, the multi-objective cost function becomes:

$$\min_{v_{des}(k)...v_{des}(k+N)} J(k) = J_{tt}(k) + J_{hw}(k) + J_e(k) + J_p(k),$$

subject to:

$$\underline{\hat{\chi}} = \underline{\underline{\Lambda}}\,\underline{\chi} + \underline{\underline{\mathcal{E}}}\,\underline{\lambda} + \underline{\underline{\Upsilon}}\,\underline{\xi}, \tag{9}$$

$$|z_{tt}(k+N|k)| < \epsilon, \tag{10}$$

$$|v(k+N|k)| < \epsilon, \tag{11}$$

$$v_{min} \leq v_{des}(k+\kappa|k) \leq v_{max},\ \forall\kappa = 1...N, \tag{12}$$

$$a_{min} \leq \frac{1}{\tau}(v_{des}(k+\kappa|k) - v(k+\kappa|k)) \leq a_{max},\ \forall\kappa = 1...N, \tag{13}$$

$$p(k+\kappa|k) \geq 0,\ \forall\kappa = 1...N. \tag{14}$$

The constraints suggest that the vehicle shall stop at a bus stop and collect passengers. The velocity and acceleration of the vehicle are bounded and passengers at a stop cannot be negative.

Among the four objectives several weighting strategies can be formulated, considering each with different importance. The proposed weighting strategies are summarized in Figure 6.

a) Timetable tracking with J_{tt} being the only considered cost. Only the reference trajectory x_{tt} is tracked by the bus, obeying the prescribed timetable and disregarding every other objective.

b) Headway tracking, where only J_{hw} is taken into account. The goal is to mimic the trajectory of the leading bus via reference trajectory x_{hw}.

c) Balanced, where headway and timetable tracking are equally important, i.e. $J = 0.5J_{tt} + 0.5J_{hw}$.

d) Passenger demand driven: on frequent lanes passengers usually do not consult the timetable [DHZS03]. In order to avoid bunching (causing increased waiting times, [FSL15]) and minimize passenger waiting time. $J = J_{hw} + J_p$.

e) Cheap service driven. From the service providers' perspective minimizing energy consumption of their fleet is crucial as it has direct impact on their expenses. In addition, running buses based on a periodic timetable is the simplest in terms of planning. $J = J_{tt} + J_e$.

f) The balanced strategy (*c*)) is augmented with the two nonlinear objectives, taking into account all four: $J = J_{tt} + J_{hw} + 0.5J_e + 0.5J_p$.

g) Adaptive control, incorporating varying control weights, depending on the magnitude of timetable and headway errors: $J = \phi(k)J_{tt} + J_{hw}$. Where $\phi(k)$ is a state dependent coefficient.

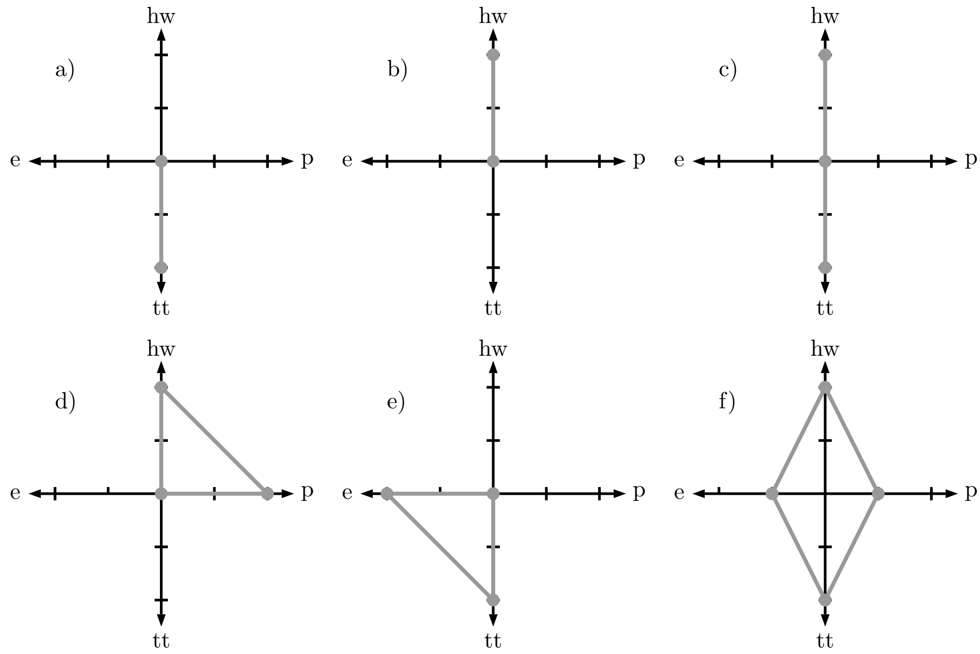

Figure 6: Weighting strategies (*hw* - headway objective, *tt* - timetable objective, *p* - passenger wait objective, *e* - energy consumption objective)

Thesis 4

To improve the performance of green-wave cruise control systems, a trajectory planning algorithm was augmented with the stochastic shockwave profile model. The controller considers traffic signal states and queue lengths at signalized intersections in a stochastic way. In the optimization, stochasticity arises in the form of chance-constraints which are alleviated with the sampling and discarding method.
[VTK18b, VTK18c, VTK19a, VT19, VTKQ20]

The proposed model predictive trajectory control, presented in *Thesis 3* can be augmented with the adverse effect of vehicle queues at signalized intersections. It is done by implementing the model in *Thesis 2* into the optimization via chance-constraints. According to *Thesis 2*, the vehicle being in the stationary queue or in the queue discharge can be written as follows:

$$x(k) \in R_J(t,\ \omega)\ if\ \begin{cases} x(k) < l_l, \\ x(k) > l_l + W_2(t - (t_2 + ct_{cyc})), \\ x(k) < l_{s,c}(t_{s,c}, \omega) + W_3(t, \omega)(t - t_{s,c}(t, \omega)), \\ x(k) > l_l + W_4(t - (t_1 + (c+1)t_{cyc})). \end{cases} \tag{15}$$

Similarly,

$$x(k) \in R_C(t,\ \omega)\ if\ \begin{cases} x(k) < l_l, \\ x(k) > l_l + W_4(t - (t_1 + ct_{cyc})), \\ x(k) > l_{r,c-1}(t_{r,c}, \omega) + W_1(t, \omega)(t - t_{r,c-1}(t, \omega)), \\ x(k) < l_l + W_2(t - (t_2 + ct_{cyc})). \end{cases} \tag{16}$$

If the vehicle is inside the stationary queue $x(k) \in R_J(t,\ \omega)$, its velocity is zero $v_J = 0$. If it is in the queue discharge $x(k) \in R_C(t,\ \omega)$ it is determined by the average velocity of the surrounding traffic v_C. These two assumptions can be incorporated into the optimization as chance-constraints. For simplicity, in the chance-constrained optimization, only the timetable tracking objective is considered.

$$\min_{v_{des}(k)\ldots v_{des}(k+N)} J_{tt}(k), \tag{17}$$

subject to:

$$\underline{\hat{\chi}} = \underline{\underline{\Lambda}}\,\underline{\chi} + \underline{\underline{\mathcal{E}}}\,\underline{\lambda} + \underline{\underline{\Upsilon}}\,\underline{\xi}, \tag{18}$$

$$|z_{tt}(k+N|k)| < \epsilon, \tag{19}$$

$$|v(k+N|k)| < \epsilon, \tag{20}$$

$$v_{min} \leq v_{des}(k+\kappa|k) \leq v_{max},\ \forall\kappa = 1...N, \tag{21}$$

$$a_{min} \leq \frac{1}{\tau}(v_{des}(k+\kappa|k) - v(k+\kappa|k)) \leq a_{max},\ \forall\kappa = 1...N, \tag{22}$$

$$v(k+\kappa|k) = v_J = 0,\ if\ x(k+\kappa|k) \in \hat{R}_J(t,\ \omega),\ \forall\kappa = 1...N, \tag{23}$$

$$v(k+\kappa|k) = v_C,\ if\ x(k+\kappa|k) \in \hat{R}_C(t,\ \omega),\ \forall\kappa = 1...N. \tag{24}$$

Next, the probability sampling and discarding approach in [CG11] is translated into the problem of stochastic queue lengths in the trajectory optimization. Queue lengths are discretized by the vehicle arrival rate $Q_A(t,\omega)$ according to $F_{Q_A}(t,\varphi)$ (i.e. discrete number of vehicles). Then, the continuous $R_J(t,\omega)$, $R_C(t,\omega)$ regions turn into discrete ones denoted by $\hat{R}_J(t,\omega)$, $\hat{R}_C(t,\omega)$ respectively. The spatially discretized traffic flow state regions $\hat{R}_J(t,\ \omega)$ and $\hat{R}_C(t,\ \omega)$ impose finite number of nonlinear constraints on the optimization through fixed probability levels for every prediction step. By selecting a probability level for the optimization, feasibility of the predicted trajectory can be guaranteed in a probabilistic way.

The explicit connection of stochastic properties between the arrival rate probability distribution function $F_{Q_A}(t,x)$ and the predicted trajectories fades away and can only be recovered with numerical simulations. According to Monte-Carlo simulations [VTKQ20], there is negative correlation between the queue length and the predicted trajectory samples. Resultantly, longer queues mean the bus is farther from the desired stop, as it tries to avoid the queue and approach it slower.

According to simulations in a microscopic traffic simulator, the proposed controller tries to avoid coming to full stops in front of signalized intersections, vehicles try to avoid the queue. This suggests improvement in energy efficiency. Chance-constraints are therefore adding efficient probabilistic guarantees to velocity control algorithms.

Thesis 5

Methods for feasibility analysis of large-scale bus networks have been investigated. A centralized rolling horizon algorithm was formulated and analyzed for controlling the vehicles in a public transport network. The subsystems describing individual buses and stops can be rewritten into a single piecewise-affine system. The performance and feasibility of the bus network was analyzed with Monte Carlo simulation and set theory.
[VTK18a, VPKT20]

The public transport network model from *Thesis 1* is used for a centralized MPC. The centralized controller calculates the velocity profile for every bus in the network simultaneously. The control differs from the decentralized control introduced earlier. Handling the system in a centralized way significantly increases the dimension of the state space model. Furthermore, the controller design is based on an NP-hard mixed integer optimization. The decentralized control in the previous sections pinpointed a fixed point in the state-space, serving as a terminal set. This shrinking horizon approach, i.e. the bus shall be at the next stop by the end of the prediction horizon does not work. To this end, the horizon length N is chosen as large as computational capacity permits. This means, the proposed MPC does not have a terminal set, therefore, closed-loop behavior shall be checked separately.

The centralized controller was compared to the decentralized approach too. The main conclusions are that due to the NP-hard nature of the problem, prediction can only made for a few steps ahead, curbing the advantages of the look-ahead control. In terms of performance, it is on par with the balanced control. Buses with the centralized control arrive at stops earlier due to the cost on the passenger waiting time objective. In addition, when the flow of traffic is disrupted, public transport service can be recovered 17% faster with the centralized controller.

Due to the numerical challenges of the centralized control, the feasibility of the results was analyzed with random simulations [CGP09]. A probabilistic measure is given on the upper bound of the total passengers waiting at stops at a network. In addition, with the help of set theory [Bor03], infeasible regions of the controlled system were sought in an indirect way. The algorithm involves designing an explicit MPC (EMPC) for the system [AB09]. If the EMPC cannot cover a partition of the state space, that means the solution is infeasible. To this end, a hypercube $\mathcal{X}$ is defined bounding the valid system states. Then, the intersection of the hypercube and the EMPC controller is calculated. If the intersection is equal to the initial polytope, the original MPC controller can cover the whole relevant state space. The steps of this analysis is summarized below.

Algorithm 1: Feasibility analysis with EMPC

Define: $\mathcal{X}$ as a polytope of relevant states.
Compute an EMPC for the network model.
Define: $\mathcal{E}$ as the feasibility domain of the EMPC .
$\mathcal{I} = \mathcal{X} \cap \mathcal{E}$.
if $\mathcal{E} \setminus \mathcal{I} = \varnothing$ **then**
  there are no infeasible states.
else
  There are infeasible states.

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3 Future work

There are two research directions that seem interesting for future research. The proposed stochastic shockwave profile can be extended to a whole network with signalized intersections. Considering joint probabilities emerging from the interconnection of traffic links can lead to more accurate queue length descriptions. Some limitations, such as queue spillover or gridlock shall also be further studied. In addition, benchmarking the SSPM network model against other traffic models (e.g. store-and-forward) is a logical next step.

The analysis of the centralized public transport network control is difficult to analyze due to its high dimension and non-convex nature. Analysis of such a high dimensional piecewise affine system is not well established. Reformulating the model in such a way it is better suited for analysis could lead to new conclusions on the stability of public transport networks.

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