FREQUENCY ANALYSIS OF STRUCTURES MODELLED BY THE CONTINUUM METHOD

Theses of the PhD Dissertation

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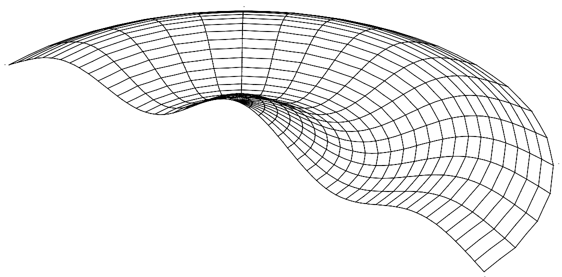


Budapest, June 2009.

INTRODUCTION

After finishing the post gradual course on structural dynamics at the ETH Zürich, I have done researches on the vibrations of different type of structures.

Using the continuum method I have analysed the vibrations of cracked reinforced [1], [12] and prestressed concrete beams [2], [16], of non-uniform cantilever beams [6], [11], of column subjected to non-uniform axial load [14], [18], of flat paraboloid shell [3], [4], and of thin-walled beam with an open cross-section [10], [15].

My dissertation contains the frequency analysis of two from the above structures which were theoretically the most interesting:

1. Paraboloid shell
2. Cracked reinforced and prestressed concrete beams

The dynamic properties of these structures were determined by the continuum method or by the difference method. Using the differential equations of the vibration, the natural frequencies and the mode shapes were calculated. For execution of these calculations MATLAB programmes were prepared.

1. VIBRATION ANALYZIS OF THE PARABOLOID SHELL

1.1. Background of the research

The theory of the shells developed to an independent science in the second part of the previous century. The authors of the most important monographs are *Girkmann (1954), Timoshenko (1966), Flügge (1973)*. Because of the complicated equations of equilibrium and compatibility the analytic treatment of the shell structures need excessive knowledge in mathematics. Previously the work of the designer was quite difficult. After a long period of development nowadays efficient programs are available in finite element analysis such as Ansys, Cosmos, Lusas which can be used also for complicated non-linear problems. Disadvantage is that the designer does not need detailed knowledge in mechanics and mathematics anymore.

The dynamics of the shell structures is a new field of the research. The static equations of the shell's theory, is complemented with the D'Alembert inertia forces and that makes the mathematical solution even more complicated especially in case of the non-linear behaviour. Several studies were published in this field recently. The authors *Godoy and De Sousa (1998)*, *Touze at al. (2008)* deal with vibration problems of special type of structures. The greatest part of the literature at the shells of revolution deals with the cylindrical shells due to the more intensive practical application. The interest on other types of shells of revolution like paraboloid or spherical shells is slightly smaller.

The literature of the flat shells and calotte deals mainly with static behaviour and stability. Much less of the publications discuss the dynamic behaviour of these shells.

1.2. Purpose of research

The first chapter of the dissertation deals with the vibration analysis of the flat paraboloid shell. The purpose of the research:

- Application of a new analytic method in the solution of the differential equation system of paraboloid shell's vibrations:
  - without shear deformations,
  - with shear deformations.
- Deriving the frequency equation of the shell and determination of the natural frequencies and the mode shapes.
- Calculation algorithm and program (MATLAB).
- Comparison of the analytic results with the results of alternative solution, obtained from the ANSYS finite element program.
- Calculation of the free vibration of a levitating antenna dish.
- Comparing the vibration properties of the paraboloid shell and of the circular plates.

1.3. Dynamic analysis without shear deformations

The thin, flat paraboloid shell of revolution is represented by the shape-function $z = \frac{r^2}{2R}$ in an r, ϑ, z cylindrical co-ordinate system. It has the boundary: $r = a$. This surface could be also assumed as a substituting paraboloid of a flat calotte that is cut out of a sphere at a circle of the radius a (Fig. 1.1).

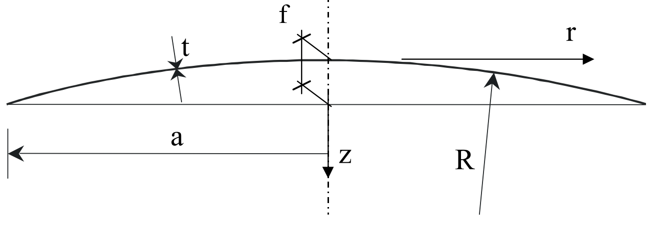


Fig 1.1. Geometric data of the shell.

In the vibration problems, loads are inertia forces expressed using the second time derivative of w. When appropriate derivatives of z and time derivative of w are substituted into Marguerre's differential equation system, the following equations emerge:

$$K\Delta\Delta w - \frac{1}{R}\Delta F = -\rho t \frac{\partial^2 w}{\partial \tau^2}, \tag{1.1a}$$

$$\frac{1}{Et}\Delta\Delta F + \frac{1}{R}\Delta w = 0, \tag{1.1b}$$

in which w is the normal displacement of the middle surface, R is the radius of the replacing flat calotte cut out from a sphere, E is the Young’s modulus, ν is the Poisson’s ratio, F is the stress function of membrane forces, ρ is the density of the material of the shell, τ is the time variable.

For solving the partial differential equation system of the vibration problem, a new method, the generator function process was applied. This method is based on the generalization of determinants and co-factors of quadratic matrices. The main advantage of using the generator function is, that after solving the characteristic differential equation, all elements of the solution vector can be obtained by derivation, in this way the solution is free of the redundant constants of the integration *(Hegedűs (1986))*.

Separating the radial and the hoop variables an eighth-order common differential equation is obtained for the generator function. The solution consists of Bessel’s and power functions. Functions which are singular at point $r = 0$ have to be disregarded in case of the paraboloid or spherical cup.

The solution for the unknown w and F functions:

$$w = \frac{1}{Rl_k^2}\left[4C_2(k+1)\xi_k^k - C_5 J_k(\xi_k) + C_6 I_k(\xi_k)\right]\cos k\vartheta \tag{1.2}$$

$$F = -K\left[\frac{1}{l_\omega^4}\left(C_1\xi_k^k + C_2\xi_k^{k+2}\right) + \frac{1}{l_{stat}^4}\left(C_5 J_k(\xi_k) - C_6 I_k(\xi_k)\right)\right]\cos k\vartheta \tag{1.3}$$

where: $\xi_k = r/l_k$ is the dimensionless radial co-ordinate, l_k, l_ω and l_{stat} are characteristic length type quantities, C_i are integration constants to satisfy the boundary conditions.

The next task was the application of the boundary conditions to the w and F functions. The dissertation contains an analysis of a shell structure with free boundary, which can be considered as a model of a levitating antenna dish in the space. For the shell four independent boundary conditions could be stated for the $r = a$ boundary. The differential equation system and these boundary conditions define an eigenvalue problem. With the solution of this, eigenvalues and eigenmodes were obtained.

The $\omega_{k\ell}$ natural frequencies of the paraboloid shell could be calculated with the formula below:

$$\omega_{k\ell} = \sqrt{\frac{\alpha_{k\ell}^4 c^2 t^2}{a^4 12\left(1-\nu^2\right)} + \frac{c^2}{R^2}} \tag{1.4}$$

where: c is the velocity of the mechanic impulse in the shell, a, t, R are geometric properties according to Fig. 1.1. and the $\alpha_{k\ell}$ coefficients are the solutions of the eigenvalue problem. The parameter k means the number of waves in hoop direction and ℓ gives the number of antinodes of the surface lines in radial direction.

For modes without antinodes the $\alpha_{k\ell}$ coefficients and the l_k characteristic lengths belonging to lower k modes are complex. The $\alpha_{k\ell}$, l_k values change to real numbers, if the appropriate natural frequency exceeds the $\omega_0 = c/R$ frequency.

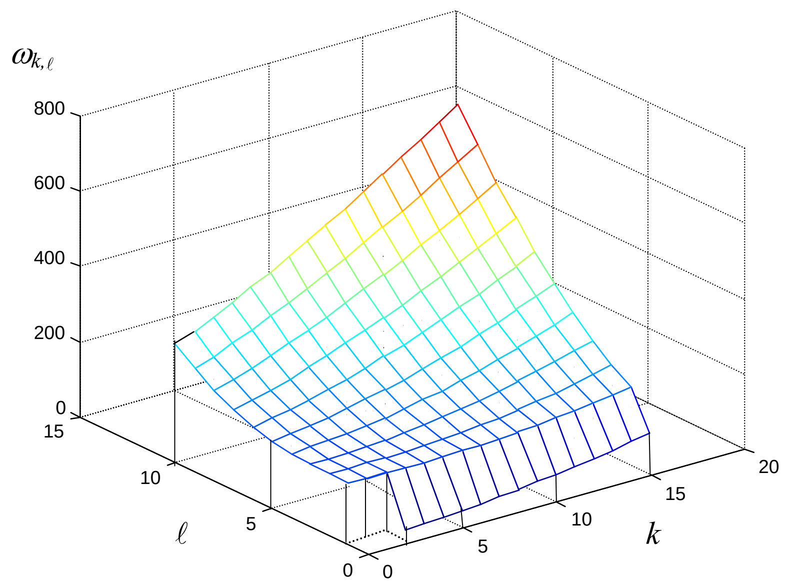
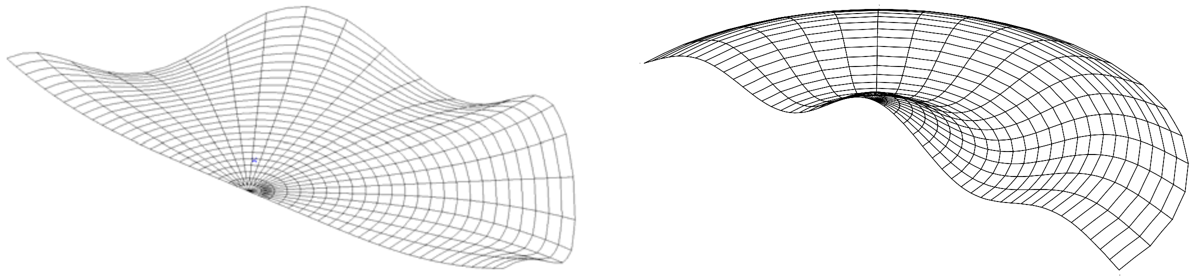


Fig. 1.2. Natural frequencies of the analysed paraboloid shell.

Typical mode shapes of the analysed antenna dish can be seen in Fig. 1.3.



a) Relief of w for $\ell = 0$ and $k = 6$ b) Relief of w for $\ell = 3$ and $k = 0$

Fig. 1.3. Relieves of the paraboloid shell.

Practical use of the presented analytical method needs an efficient code for finding solution of the frequency equation. For that purpose MATLAB has been used. The analytic results were verified with ANSYS finite element program.

First, the natural frequencies and mode shapes of a levitating antenna dish were calculated.

For the sake of a deeper insight, comparisons have been made between:

- levitating paraboloid shell,
- an unsupported circular plate having the same boundary radius, thickness and material properties as the paraboloid,
- and the same circular plate resting on a fictitious elastic foundation with the Winkler coefficient $C = \frac{Et}{R^2}$.

The natural frequencies of the paraboloid shell belonging to modes with and without antinodes in radial direction differ by magnitudes and can be estimated using basically different models. Modes without antinodes ($\ell = 0$) are similar to inextensional deformations and the corresponding natural frequencies are close to those of a replacing unsupported circular plate. On the other hand modes with antinodes in radial direction can be estimated as those of a circular plate resting on a fictitious elastic foundation. It is in accordance with the theory of *Rayleigh (1945)* about the two groups of the vibration modes of the spherical caps.

The effect of the geometric properties on the natural frequencies of the shell was also investigated.

The presented method can be extended to an open paraboloid shell fixed at the internal circular boundary.

1.4. Dynamic analysis with shear deformations

The total deflection w was decomposed into two parts similarly to the case of the discussion of Thimoshenko-rods:

$$w = w_B + w_S \tag{1.5}$$

where: w_B is the deflection due to bending, w_S is the deflection due to shear. The equation system of the free vibration of the shell (1.1) will be complemented by a third equation as follows:

$$\begin{bmatrix} K\Delta\Delta - \rho t\omega^2 & -\rho t\omega^2 & -\frac{1}{R}\Delta \\ \frac{1}{R}\Delta & \frac{1}{R}\Delta & \frac{1}{Et}\Delta\Delta \\ K\Delta\Delta & S\Delta & 0 \end{bmatrix} \cdot \begin{bmatrix} w_B \\ w_S \\ F \end{bmatrix} = \underline{0}\,, \tag{1.6}$$

where: S is the shear stiffness of the structure.

The equation system (1.6) was solved again by the method of the generator function. The deflection and stress functions of the vibration problem consist of Bessel's and power functions too.

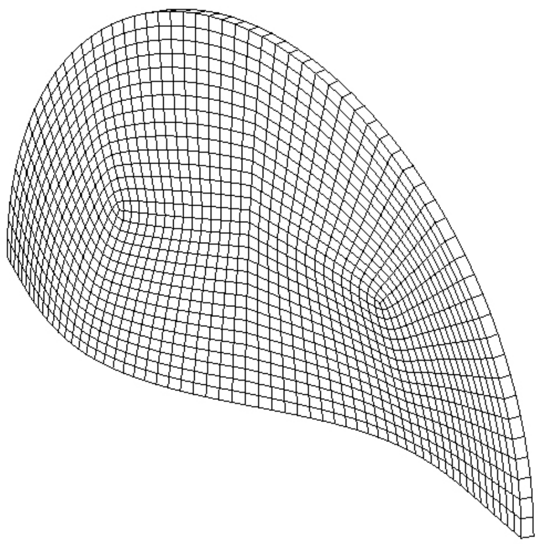


Fig. 1.4. Relief of ANSYS model built up of volume elements for $\ell = 1$ and $k = 1$.

For the verification of the analytic solution the ANSYS finite element program was used again. Both thin shell and volume elements were applied.

The effect of the shell thickness on the natural frequencies was analysed as well. According to expectations shear deformations has effect only in case of high modes.

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2. VIBRATION ANALYSIS OF CRACKED REINFORCED AND PRESTRESSED CONCRETE BEAMS

2.1. Background of the research

The anticipated age limit of the important engineering structures covers approximately 50-75 years. During this lifetime the constructions are sometimes exposed to extraordinary effects that could cause certain damages.

Damage detection methods of the structures, is a subject that has received considerable attention not only in civil engineering but also in mechanical engineering, especially in the aeronautical industry. One of the most important non-destructive damage detection techniques is based on the change of the dynamic properties of the structures.

The main methods for damage (crack) detection of reinforced concrete structures based on dynamic behaviour are the followings:

- methods based on frequency change, *(Salawu and Wiliams (1993))*
- methods based on damping change,

– methods based on mode shape change, *(Rizos (1990))*
– methods based on change of the modal curvature*, (Pandey (1991))*
– methods based on change in flexibility matrix.

Several publications appeared in the last two decades that intend to determine the grade of the defects and sometimes the position of the cracks of the concrete structures on bases of their dynamic behaviour. Greatest part of this works is based on considering linear behaviour, the smaller part is based on assuming non-linear vibration.

2.2. Purpose of research

The second chapter of the dissertation deals with the non-linearity following from the cracking of the reinforced and prestressed concrete beams. The details of this are:

- Calculation model for linear and non-linear analysis of cracked reinforced concrete beams.
- Preparation of computer algorithm and program,
- Linear and non-linear analysis of a cracked beam model,
- The spectrum of the investigated cracked beams contained double peak at the place of the first eigenfrequency. Finding explanation for that phenomenon.
- Analysis of varying virtual natural frequency during the vibration process.
- Comparison of the experimental and analytic results,
- Simplified new models to determine the virtual natural frequencies.
- Linear and non-linear approximate model for the cracked prestressed beams.
- Analysis of the connection between the natural frequencies and the prestressing force.

2.3. Vibration analysis of cracked reinforced concrete beams

The dynamic behaviour of the bent reinforced concrete beams in elastic range is significantly influenced by cracks caused by former loads. Experiments have shown that the features of vibration differ from the results obtained by the well-known linear model, if cracked zones exist. The cause of this phenomenon is that the bending rigidity of the cross-sections in the cracked range depends on the sign of the actual bending moment. The flexural stiffness of the cracked reinforced concrete beam, in bending vibration, changes in time. Therefore the vibration shows non-linear characteristics in the elastic range as well.

In my dissertation the quasi periodic opening and closing of cracks were analysed for vibrating reinforced concrete beams by laboratory experiments and by numeric simulation.

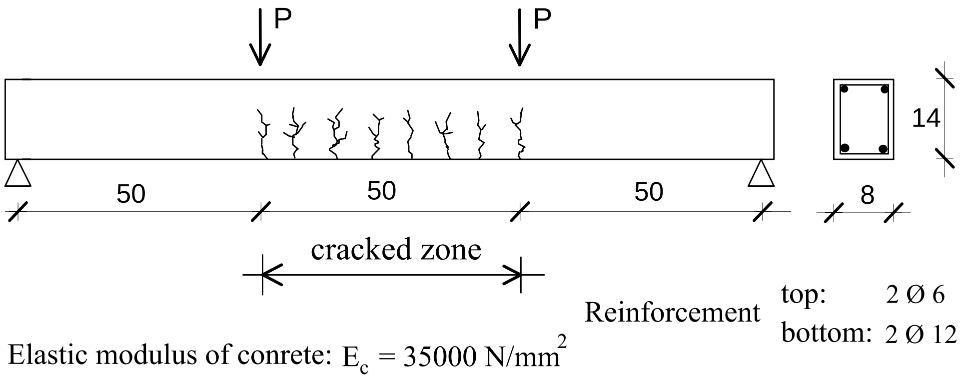


Fig. 2.1. Model used at the experiments

In the laboratory the cracked beam was brought into vibration with an exterior impact load, by a rubber hammer blow. According to the spectral decomposition of the time-acceleration series, a double peak appeared in the spectrum in the place of the first eigenfrequency.

With the MATLAB program based on the differential equation of the vibrating beam *(Clough and Penzien (1975))* there were numerical simulations carried out on the linear and non-linear computing model of the beam in Fig. 2.1.

The linear analysis supplied lower and upper bounds for the natural frequencies. The solution of the non-linear vibration problem needs discretising in space and in time. The discretising in the axial direction was made with the difference method. Considering the non-linear property, makes it necessary discretising in time, by applying a time-step algorithm. For that purpose the Wilson method was used *(Bathe and Wilson (1976))*.

For the sake of a detailed analysis of the dynamic behaviour of the beam the non-linear analysis was performed both without and with considering the opening of cracks due to self-weight. If the crack opening due to self-weight is not considered the flexural stiffness of the beam in the cracked zones can be described with the formula below:

$$EI_i = \begin{cases} E_b I_{i,I} & \text{if} \quad M_{din,i} < 0 \\ E_b I_{i,II} & \text{if} \quad M_{din,i} \geq 0 \end{cases} \tag{2.1}$$

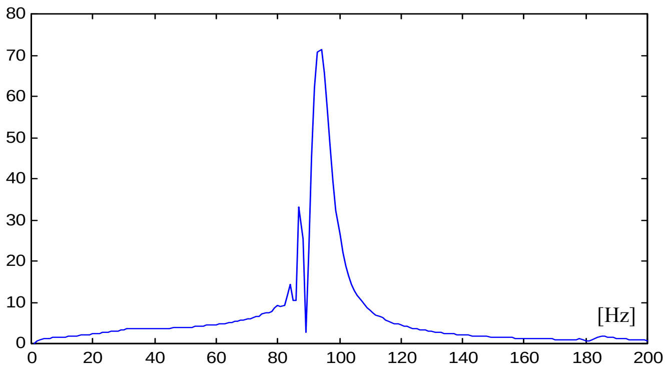
The computation based on (2.1), without considering the crack-opening due to self-weight, does not show the spontaneous separation of the first natural frequency, namely the existence of the secondary peak. To the first eigenmode belongs only one virtual natural frequency.

However, when taking into consideration the crack-opening due to self-weight, the situation changes. Due to the self-weight, cracks open in the middle region already in the static condition. Thus the stepwise change of the flexural stiffness is the following:

$$EI_i = \begin{cases} E_b I_{i,I} & \text{if} \quad M_{din,i} + M_{stat,i} < 0 \\ E_b I_{i,II} & \text{if} \quad M_{din,i} + M_{stat,i} \geq 0 \end{cases} \tag{2.2}$$

This means a shifting of the base line compared with relation (2.1). The double peak can appear in the spectrum if there is an appropriate relation among the starting impulse, the self-weight and the damping.

Taking the crack opening due to self-weight into assumption, the vibration spectrum was produced by the Wilson type time-step integral, which is shown in Fig. 2.2.



Dampings: $\zeta_1 = 1.5$ %, $\zeta_3 = 8$ %

$[C] = 8.5[K] + 2.55 \cdot 10^{-5}[M]$

Impulse: $\Im = 7.0$ kgm/s

$w_{din}/w_{stat} = 9.1$

$f_1 = 93.5$ Hz $\quad f_1^* = 87.1$ Hz

Fig. 2.2. Double peak in the spectrum of the beam model

The virtual natural frequencies of the cracked beams are influenced by several effects. It was showed on diagrams obtained from parametric studies, how the spectrum of the beam depends on the starting impulse and the damping.

The changing of the first virtual natural frequency in time was analysed by the method of the moving window combined with the discrete Fourier transformation.

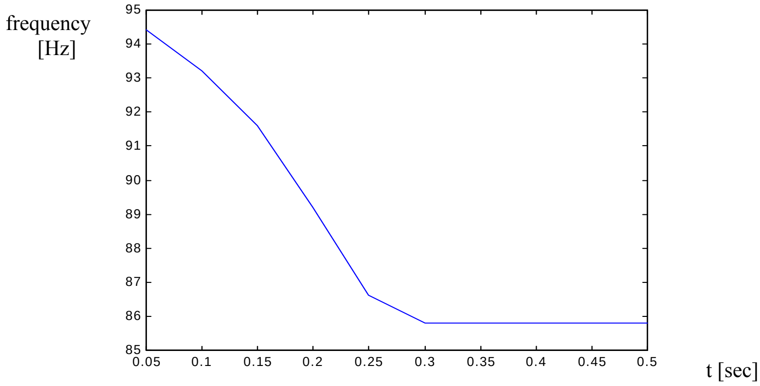


Fig. 2.3. Decreasing of the virtual natural frequency of the beam in time.

Linear models were worked out for an approximate determination of the first two virtual natural frequencies of the cracked beams.

Table 2.1. Measured and calculated natural frequencies.

Experiment and computations	f_1 [Hz]	f_2 [Hz]
Experiment, cracked beam	89, 98	
Linear computation, uncracked beam	109	436
Linear computation, weakened beam	86	397
Non-linear computation, without the crack-opening due to self weight	96	414
Non-linear computation, with the crack-opening due to self weight	87, 93	
New approximate linear models	96.1	417.5

2.4. Vibration analysis of cracked prestressed concrete beams

The bending vibration of the cracked prestressed concrete beam is influenced not only by the crack pattern but by the prestressing force as well. The effective curvature in the cracked zone depends besides on the static and dynamic moment and on the normal force due to prestressing as well (Fig. 2.4).

In this chapter the virtual natural frequencies of the prestressed beam are determined on the basis of bending stiffness with linear and nonlinear analysis. Besides, the connection is investigated between the effective prestressing force and the virtual natural frequency.

The bending stiffness of the cracked prestressed reinforced concrete beam is changing in time and similarly to the case of the non-prestressed beam this results in a nonlinear vibration. In case of the non-prestressed beam the moment–bending stiffness function changes stepwise (2.2), while for the prestressed beams it is continuous. The bending stiffness of the section under eccentric compression (2.3-2.4) can be defined with the aid of the curvature's concept *(Dulácska, E. (1978))*.

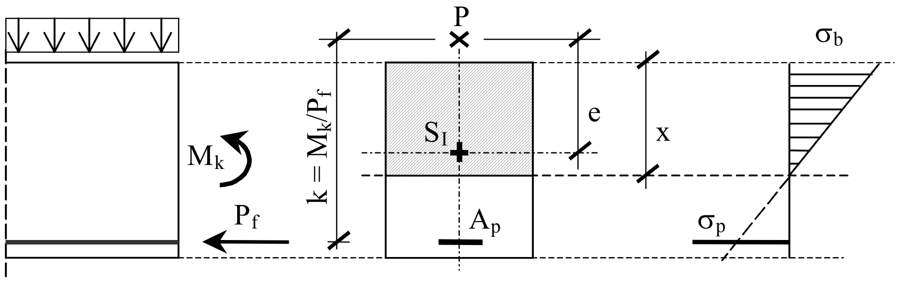


Fig. 2.4. Prestressed, cracked section subjected to bending

The curvature of the prestressed beam's section loaded by bending moment *M*:

$$M = M_k - M_f = M_{k,din} + M_{k,stat} - M_f = P_f k - P_f (k - e) = P_f e \tag{2.3a}$$

$$g = \frac{\varepsilon_b}{x} = \frac{\sigma_b}{E_b x} = \frac{M}{E_b I} \quad , \tag{2.3b}$$

where: σ_b is the concrete stress, ε_b is the concrete strain in the extreme fibre; $E_b I$ is the bending stiffness, in which I is the wanted moment of inertia:

$$I = I_g = S_d e \quad , \tag{2.4}$$

where: S_d is the static moment of the section's active part to the neutral axis, e is the eccentricity of the resultant force P to the neutral axis of the uncracked section $(P = P_f)$.

Considering an undamped free vibration of a prestressed "I" beam of 10 m span, three calculations were performed:

a) The first linear calculation was done with neglecting the cracks, using a constant $E_b I_{iI}$ bending stiffness along the beam.
b) The second linear calculation was performed, on the cracked beam without considering the dynamic moment. The bending stiffness was constant in time,
c) The third solution was obtained from a non-linear calculation. The non-linearity follows from taking the dynamic moment into consideration. The bending stiffness of the beam is changing in time as well.

In case "a" simple elementary calculation was performed with the well known formula of the uniform vibrating beam.

In the calculation "b" first, the moment of inertia of curvature was developed as function of the eccentricity of the P resultant force. The Fig. 2.5 shows that the bending stiffness of the prestressed beam is changing continuously between the extreme values $E_b I_{iI}$ and $E_b I_{iII}$.

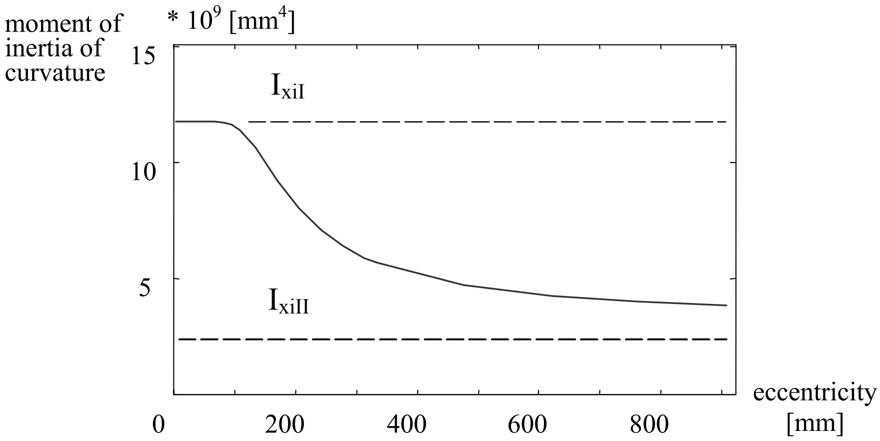


Fig. 2.5. Moment of inertia of the curvature
in function of the eccentricity of the P resultant force

After this, using the effective prestressing force and the static moment due to the dead load, the moment of inertia of curvature was calculated in function of the eccentricity in each point of the discretised beam model. Using the function in Fig. 2.5, the distribution of the I_g bending stiffness was obtained along the beam (Fig. 2.6).

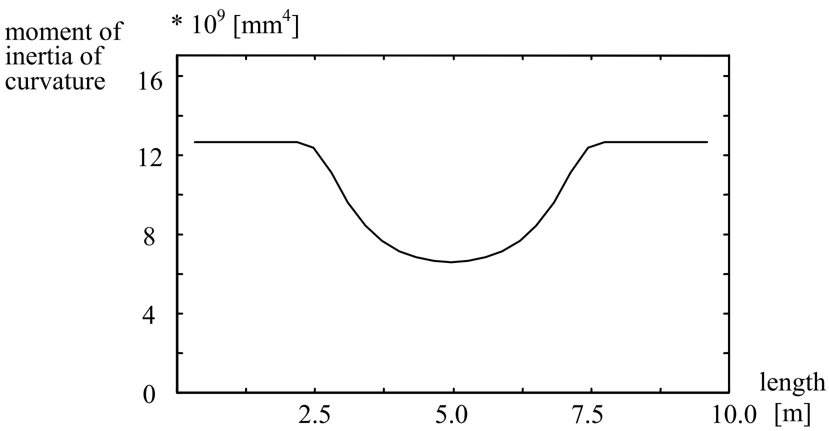


Fig. 2.6. Variation of the moment of inertia of curvature along the beam
(for calculation "b")

With the aid of the longitudinal bending stiffness diagram (Fig. 2.6.) solving the eigenvalue problem, the approximate natural frequencies was calculated in function of the prestressing force. The approximation is the consequence of neglecting the oscillation of the stiffness in time.

In case "c" the time history of the beam was created with the Wilson method and the oscillation of the bending stiffness in time was taken into account. The natural frequencies were obtained by Fourier transformation.

The results of the linear calculation "b" differ only slightly from that of the non-linear method "c". Results showed that the virtual natural frequencies can be calculated with an adequate accuracy, using a linear algorithm in which the bending stiffness is constant in time. This approximation can be applied for larger prestressed beams or bridges, where the dead loads are considerably larger than the dynamic loads.

With the aid of the above simplified linear method the curve of the natural frequency of the cracked, prestressed reinforced concrete beam can be plotted against the prestressing force. The diagram could also be used for temporary inspection of prestressed bridges.

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3. THE THESES OF THE PHD DISSERTATION

I. Vibration analysis of paraboloid shell

The presented research and its scientific results can be summarized as follows:

I.1. The differential equation system of the vibration of the thin, flat paraboloid shell or of the flat calotte with free edge was derived according to the flat shell theory without shear deformations using a new method, the generator function process. It is based on the generalization of determinants and co-factors of quadratic matrices. With this method the partial differential equation system was reduced to an eighth-order common differential equation [4].

I.2. The frequency equation obtained from the analytic solution – based on the generator function process – is capable to determine the natural frequencies and the mode shapes at all the modes of vibration of thin flat paraboloid shell or of the flat calotte with free edge [4].

I.3. On the basis of the analytic solution and numeric results it was shown that the $\alpha_{k,0}$ eigenvalues and the l_k characteristic vibration lengths are complex in the modes of the paraboloid shell without antinodes ($\ell = 0$) in case of small k parameters. The $\alpha_{k,0}$ and l_k values change to real number if the appropriate natural frequency exceeds the $\omega_o = c/R$ frequency [3], where c is the velocity of the mechanic impulse in the shell, R is the radius of the replacing flat calotte cut out from a sphere.

I.4. Taking the shear deformation into consideration the frequency equation of the flat paraboloid shell or calotte with free edge, was also derived with the aid of the generator function method.

II. Vibration analysis of cracked reinforced and prestressed concrete beams

The presented research and its scientific results can be summarized as follows:

II.1. It was shown by numerical simulation of the vibration of symmetrically cracked beams (time history analysis, Fourier transformation) that a double peak could appear in the spectrum in place of the first natural frequency if there was an appropriate relation among the starting impulse, the self-weight and the damping. That follows from the periodic opening and closing of the cracks [1].

II.2. Applying the moving window in the Fourier analysis of the cracked beam it was shown that the virtual natural frequency of the elastic vibration decreases in time if the spectrum contained double peak.

II.3. It was shown by numerical simulation that in the case of periodical closing of cracks of the beam the virtual natural frequency depends on the starting impulse as well.

II.4. In case of symmetrically cracked beams introducing a new linear model a simple formula was derived for the upper bound of the first virtual eigenfrequency of the cracked beam. Also a model was worked out for finding the upper and lower bounds of the second virtual natural frequency of the cracked beam [1].

II.5. Using the expression of the moment of inertia of curvature at the cracked prestressed beams it was shown that the bending stiffness due to the periodic opening and closing of the cracks has a smaller oscillation, than that of the non prestressed beams. The non-linear vibration of prestressed beams this way could be well approximated with a linear model. In this model the bending stiffness of the cracked beam is taken into assumption in static condition, if the dynamic amplitudes were small compared with the static deflection. It was also determined the connection between the prestressing force and the virtual natural frequencies [2].

THE CANDITATE'S MAIN PUBLICATIONS ON THE SUBJECT OF THE THESES

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