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Booklet of thesis statements

Stabilizing and destabilizing effects of time delays in nonlinear dynamical systems

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Introduction

Motivation

Time delay is present in a wide variety of dynamical systems originating in traveling wave phenomenons, in regenerative effects, in feedback control loops or in other phenomenons yielding that the current behavior of the system is influenced by its history. It is a rule of thumb that time delay has a destabilizing effect, however, it may also provide certain damping. This yields that the presence of time delay often leads to intricate dynamical behavior in oscillatory systems, especially, when relevant nonlinearities are also present.

Nowadays, as industries develop increasingly complex and sensitive products, understanding a system's nonlinear behavior is becoming more and more important. For example, to reduce the weight of the products and to save energy costs, today's structures are often slender, which amplifies nonlinear effects. Nonlinear dynamical systems, unlike linear ones, may have several equilibria and/or limit cycles, even chaotic or transient chaotic behaviors, which are crucial to be identified. Limit cycles emerge at the dynamic stability boundary through the so-called Hopf bifurcations. We speak about supercritical Hopf bifurcation if a stable limit cycle coexists with the unstable equilibrium, while in case of subcritical Hopf bifurcation, an unstable limit cycle coexists with the stable equilibrium.

From an engineering point of view, the subcritical case is more relevant since large enough perturbations can cause the system to transition from the desired locally stable equilibrium to an undesired one, leading to failures and accidents. The presence of time delay often leads to subcritical Hopf bifurcations, however, by the appropriate tuning of the delay and/or the nonlinearity parameters, the criticality of the arising limit cycles can be changed.

Finally, the sampling effect in feedback control implies that the system is subjected to time periodic time delay. This sawtooth-like time delay variation together with the rounding effect or other type of nonsmooth dynamics (such as fuzzy control) also leads to chaotic, or transient chaotic motions. Sometimes, transient chaos cannot be avoided; then it is beneficial to determine the expected lifetime of the transient chaotic oscillations and tune the parameters to an optimal value that provides the shortest settling time to the desired state.

Overview of the dissertation

In the thesis, the dynamics of five similar oscillatory mechanical systems are investigated, all of which are subjected to some kind of time delays.

In engineering practice, the control of certain mechanical systems is often designed with a feedback that neglects time delay, and the high-frequency dynamics also remains unmodeled. Both issues may cause difficulties and require a lengthy experimental gain tuning procedure due to the occurrence of unexpected and undesired vibrations. The thesis discusses two representative examples: the position control of an elastic robotic arm and the position control of a trolley that carries a pendulum. The dimensionless linearized dynamics of the two systems are identical; the corresponding analysis showed alternately disappearing and reappearing stable parameter domains. Considering the presence of Coulomb friction in the drive chain of the actuator of the robotic arm, an optimal control parameter setup is proposed, which minimizes the positioning error (*Thesis statement 1*).

The nonlinear dynamics differ essentially in the two models. For both systems, an infinite dimensional center manifold reduction is carried out, which is followed by a subsequent Hopf bifurcation calculation. The calculated vibration amplitudes and frequencies of the self-excited oscillations support the fast and precise tuning of the control gains (*Thesis statement 2*).

The delay effects are even more significant in human-controlled processes, for example, in case of human-controlled vehicle towing or in case of the automated guidance of a human driver. In these examples, the systems are subjected to two separate control laws simultaneously, which lead to intricate dynamics. A simplified mechanical model was constructed for the investigation of the human-controlled vehicle towing in which the relevant reaction times of the drivers were also taken into account. The system consist of two blocks connected with a spring that refers to the elastic drawbar. The driver of the front car controls the velocity of the towing procedure, while the driver of the towed vehicle aims to brake in a way that constant tension is generated in the drawbar. The linear stability analysis led to the conclusion, that there is a preferable region of drawbar stiffness, which yields stable motion (*Thesis statement 3*).

In case of the automated guidance of a human driver, the drawbar is replaced by virtual connection. In the simplified car following model, the automated lead vehicle is not only responding to a reference velocity but it also takes into account the speed of the subsequent human-driven vehicle, while the human driving behavior is described with the generalized Bando model, which takes into account the headway and the velocity difference. The dynamics of the system was studied first analytically, which was followed by human-in-the-loop experiments: a graphical interface illustrates the automated vehicle ahead, while the human operator controls the velocity of the following vehicle via gas and brake pedals. The theoretical results are in good agreement with the measurements; based on them, advisable control gain combinations were proposed for the automated lead vehicle (*Thesis statement 4*).

The digital control of unstable dynamical systems often leads to chaotic or transient chaotic motions. The fifth chapter of the thesis work investigates the dynamics of a fuzzy-controlled polishing machine; both chaotic and transient chaotic behaviors are identified for certain control parameter combinations. The transient chaotic domain is studied in detail, and it is obtained that the mean kickout number has a minimum at the boundary of transient chaos and the even more intricate, so-called embedded transient chaos. Closed form expressions are given for the corresponding control gains, which support the process of control parameter tuning (*Thesis statement 5*).

Finally, the concept of spectral submanifolds are extended to nonlinear time delay systems, and closed-form expressions are derived for the invariant submanifolds and for the corresponding reduced dynamics. These reduced models enable us to investigate the essential nonlinear dynamics, and thus, it allows the direct and accurate approximation of limit cycles exactly at the parameter point of interest (*Thesis statement 6*). This makes the algorithm a powerful tool for the estimation of the robustness of locally stable engineering systems with respect to external perturbations.

Main results

Position control of oscillatory mechanical systems

I have analyzed the simplified model of the delayed position control of an elastic robotic arm. The mechanical model consists of two blocks connected with a spring, which refers to the first bending vibration mode of the arm. A saturating collocated proportional-derivative control force is considered, while an end-effector at the other end of the arm introduces high frequency dynamics to the system. The linear stability analysis showed alternately disappearing and reappearing stable domains in the plane of the proportional and derivative control gains both for increasing time delay and for increasing spring stiffness. Modeling the friction that always occurs in the drive chain of the actuator, I determined an optimal control gain combination for fixed time delay, which minimizes the positioning error. After the linear analysis, I investigated the effect of the saturation in the control force, and I obtained that the Hopf bifurcations are always supercritical.

Thesis statement 1

Consider the delayed collocated proportional-derivative position control of a robotic arm when the actuator force is orthogonal to the elastic arm. If the modal stiffness k of the first bending vibration mode is fixed, then the proportional gain K_p can be increased by decreasing the time delay. With a fixed time delay τ, the following selection of the spring stiffness and the proportional gain minimizes the positioning error δ_min in the presence of the Coulomb friction force C:

$$k = 12.4\frac{m_2}{\tau^2}\,, \quad K_\mathrm{p} = 1.16\frac{m_1}{\tau^2} \quad \Rightarrow \quad \delta_\mathrm{min} = 0.86\frac{C}{m_1}\tau^2\,,$$

where m_1 and m_2 denote the modal masses of the elastic robotic arm.

Related publication: [1]

From the point of view of linearized dynamics, I obtained an analogous system, when I investigated the collocated position control of a trolley carrying a pendulum. The position and velocity of the trolley were fed back with a proportional-derivative controller and the delayed force acted on the same body. The corresponding dimensionless characteristic equation is identical to the one obtain for the robotic arm; therefore, the stability charts show alternately disappearing and reappearing stable domains in the plane of the control gains. I carried out an infinite dimensional center-manifold reduction, and a subsequent Hopf bifurcation calculation, which lead to a closed form algebraic expression for the amplitude of the self-excited oscillations. The results can be summarized as follows.

Thesis statement 2

Consider the time-delayed collocated position control of a trolley carrying a pendulum. The system can be stabilized with the appropriate selection of the control gains if the dimensionless time delay $\bar{\tau}$ satisfies

$$\bar{\tau} \in \left[2k\pi - \frac{\pi}{2}\,, 2k\pi + \frac{\pi}{2}\right)\,, \quad k \in \mathbb{N}^+\,, \quad \textbf{or} \quad \frac{\tan\bar{\tau}}{\bar{\tau}} > 1\,.$$

The corresponding critical values of the dimensionless proportional gain k_p and the dimensionless differential gain k_d assume the form:

$$k_{\mathrm{p,cr}}(\omega) = \left(1 - \frac{m\bar{\tau}^2}{(1+m)\omega^2 - \bar{\tau}^2}\right)\omega^2\cos\omega\,,$$
$$k_{\mathrm{d,cr}}(\omega) = \left(1 - \frac{m\bar{\tau}^2}{(1+m)\omega^2 - \bar{\tau}^2}\right)\omega\sin\omega\,,$$

where ω denotes the dimensionless angular frequency of the self-excited oscillations and m refers to the mass ratio of the payload and the trolley. In case of a pendulum with fixed length l, the approximate amplitude of the stable or unstable self-excited oscillations are determined by:

$$A = \sqrt{-\frac{\mathrm{Re}\lambda'}{\Delta}(k_\mathrm{p} - k_{\mathrm{p,cr}})}\,,$$

where

$$\mathrm{Re}\lambda' = \frac{b(\omega)\sin\omega - a(\omega)\cos\omega}{2(a^2(\omega)+b^2(\omega))}\,,$$

$$\Delta = \frac{m\omega^9\left(\bar{\tau}^2-\omega^2\right)\left(\omega-\frac{\sin(2\omega)}{2}\right)\left(3\frac{\bar{\tau}^2}{1+m}-4\omega^2\right)}{32l^2\left(\frac{\bar{\tau}^2}{1+m}-\omega^2\right)^5\left(a^2(\omega)+b^2(\omega)\right)}\,,$$

and

$$a(\omega) = \frac{1}{2}\omega\left(1-\frac{m\bar{\tau}^2}{(1+m)\omega^2-\bar{\tau}^2}\right)\left(\frac{1}{2}\sin(2\omega)-\omega\right)\,,$$

$$b(\omega) = \omega + \frac{m\omega\bar{\tau}^4}{\left((1+m)\omega^2-\bar{\tau}^2\right)^2} - \frac{1}{2}\left(1-\frac{m\bar{\tau}^2}{(1+m)\omega^2-\bar{\tau}^2}\right)\omega\sin^2\omega\,.$$

Related publications: [1], [2], [3]

Human-controlled towing of vehicles

I have analyzed the simplified mechanical model of human-controlled towing of a vehicle when an elastic drawbar is considered between the vehicles. The driver of the front car controls the velocity of towing, while the driver of the rear vehicle aims to keep constant tension in the drawbar by braking. Both control forces are subjected to constant time delays representing the reaction times of the drivers. Routh reduction is applied to obtain the most compact structure of the governing equations that consist of a partially coupled system of second and first order DDEs. The linear analysis showed alternately disappearing and reappearing stable domains both when the reaction times of the drivers were increased or when the natural frequency of the system was increased. The results are summarized as follows.

Thesis statement 3

In the simplified (undamped) mechanical model of human-controlled towing of a vehicle, there exists no practically relevant stable domain for the critical parameter combinations when the reaction time of the drivers is equal to one half of the time period of oscillation of the uncontrolled mechanical system, or it is an odd multiple of that half period. In case a soft drawbar is not advisable, its stiffness k should be tuned to

$$\frac{\pi^2}{\tau^2}\frac{m_1 m_2}{m_1+m_2} \ll k \lessapprox k_{\mathrm{opt}} = \frac{4\pi^2}{\tau^2}\frac{m_1 m_2}{m_1+m_2}\,,$$

where τ is the reaction time of the drivers, while m_1 and m_2 refer to the masses of the towed vehicle and of the towing tractor, respectively.

Related publications: [4], [5], [6]

Automated guiding of a human driver

I have analyzed the guided control of a human-driven vehicle (HV) via an automated lead vehicle (AV), when the AV is not only responding to a reference velocity but it also takes into account the speed of the subsequent HV. Considering the relevant time delays, the plant and string stability properties of the underlying dynamical system were determined and stability charts were presented. To verify the theoretical results and to estimate the parameters of the human driver model, I developed a human-in-the-loop simulation environment, with which 9 human subjects carried out the same driving task, all of them, for 79 control gain combinations of the automated vehicle. Utilizing the measurement data, the human driver parameters were estimated with the help of the sweeping least squares method, after which, the experiment showed good agreement with the theoretical plant stability boundaries.

Thesis statement 4

Consider the guidance of a human driver via an automated lead vehicle, when the automated vehicle is not only responding to a reference velocity but it also takes into account the speed of the subsequent human-driven vehicle. The cruise control gain $\hat{\beta}$ should be tuned to a slightly larger value than the positive backward looking gain β_{-1}, while a necessary condition for stability is that

$$\hat{\beta} + \beta_{-1} < \frac{\pi}{2\sigma},$$

where σ is the time delay of the automated vehicle. According to human-in-the-loop measurements, large gain ratios $\hat{\beta}/\beta_{-1}$ imply that the automated vehicle accurately follows the prescribed reference velocity, while the human driver can not follow the automated vehicle smoothly; on contrary, small gain ratios yield smooth guidance for the human-driven vehicle but significant deviations from the reference velocity.

Related publications: [7], [8]

Fuzzy controlled polishing machine

I have analyzed the possible microchaotic behavior of a fuzzy controlled polishing machine when the effect of sampling was also taken into account. Both chaotic and transient chaotic behaviors were identified for certain control parameter combinations. For the membership function parameters of the fuzzy control, critical values were derived at which the dynamics of the system changes qualitatively.

In case of the transient chaotic behavior, the zero equilibrium remains globally stable but the system goes through a chaotic-like transient motion before it settles at the equilibrium. I gave closed-form expressions for the expected value and for the standard deviation of the kickout number, and I proved that the mean kickout number is independent of the dimensionless Coulomb friction. The numerical simulations presented a good agreement with the analytical predictions of the mean kickout number and its standard deviation for the transient chaotic parameter domain, while it also showed that the mean kickout number, and consequently, the mean escape time has a minimum at the boundary of the parameter domains of transient chaos and embedded transient chaos.

Thesis statement 5

Consider a trapezoidal membership function based fuzzy control of a polishing machine with temporal sampling. The trivial equilibrium point is globally stable if

$$a - S - 1 < p \leq a - S \,, \quad q < \frac{S}{a-1} \,,$$

where a **is the dimensionless system instability parameter originated in the Stribeck-type damping characteristics,** S **refers to the dimensionless Coulomb friction in the drive chain of the actuator,** p **denotes the dimensionless proportional gain in the feedback loop, and** q **is the membership function parameter.**

When the membership function parameter is set for the large values $q > S/(a-1)$ **while the dimensionless proportional gain is in the interval**

$$\frac{a}{a-1}(a - S - 1) < p \leq a - S \,,$$

chaotic, transient chaotic, or even more intricate, so-called embedded transient chaotic motions coexist with the zero equilibrium. The mean escape time of transient chaos is the shortest if the membership function parameter is tuned to

$$q = \frac{S}{a-1} + \frac{p}{a^2}\,. \tag{1}$$

Related publication: [9]

Spectral submanifolds in time delay systems

I have extended the concept of spectral submanifolds (SSMs) to carry out model-order reduction of nonlinear time delay systems, that is, to project the dynamics from the infinite dimensional phase space to a low dimensional invariant manifold that holds the essential part of the dynamics. The SSMs and the relevant reduced dynamics are obtained both for a real dominant eigenvalue and for a pair of complex conjugate dominant eigenvalues of the linearized system. In the latter case, the method allows the direct and accurate approximation of self-excited oscillations exactly at the parameter point of interest, which may be further away from possible Hopf bifurcation boundaries.

In case of dominant complex conjugate roots $\lambda_{1,2}$, the time delayed SSM algorithm allows to calculate the spectral submanifold in closed form, while the corresponding reduced dynamics is governed by the normal form

$$\begin{bmatrix} \dot{z} \\ \dot{\overline{z}} \end{bmatrix} = \begin{bmatrix} \lambda_1 z + \beta_{21} z^2 \overline{z} \\ \overline{\lambda}_1 \overline{z} + \overline{\beta}_{21} z \overline{z}^2 \end{bmatrix} .$$

A limit cycle exists if

$$\frac{\mathrm{Re}\lambda_1}{\mathrm{Re}\beta_{21}} < 0\,;$$

the corresponding self-excited oscillation is unstable for $\mathrm{Re}\lambda_1 < 0$ and stable for $\mathrm{Re}\lambda_1 > 0$ while $\mathrm{Re}\lambda_{3,4,\ldots} < 0$. Furthermore, the amplitude of the limit cycle $z(t) = \hat{\rho}\mathrm{e}^{\mathrm{i}\omega t}$ can be approximated by

$$\hat{\rho} = \sqrt{-\frac{\mathrm{Re}\lambda_1}{\mathrm{Re}\beta_{21}}}\,,$$

while the corresponding approximate angular frequency assumes the form

$$\omega = \mathrm{Im}\lambda_1 - \frac{\mathrm{Im}\beta_{21}}{\mathrm{Re}\beta_{21}}\mathrm{Re}\lambda_1\,.$$

Thesis statement 6

Consider the n-dimensional smooth nonlinear dynamical system given by

$$\dot{\mathbf{x}}(t) = \mathbf{L}\mathbf{x}(t) + \mathbf{R}\mathbf{x}(t-\tau) + \mathbf{N}(\mathbf{x}(t), \mathbf{x}(t-\tau))$$

with constant time delay τ when the dominant eigenvalues of the linearized system form the complex conjugate pair of λ_1 and $\lambda_2 = \overline{\lambda}_1$.

The developed delayed SSM calculation is a closed form algorithm that carries out the projection of the infinite dimensional dynamics to the spectral submanifold that holds the essential finite dimensional dynamics corresponding to the relevant eigenvalues $\lambda_{1,2}$.

In case of a one degree of freedom dynamical system

$$\ddot{x}(t) = f_{1000}x(t) + f_{0100}\dot{x}(t) + f_{0010}x(t-\tau) + f_{0001}\dot{x}(t-\tau) + \sum_{2 \le j+k \le 3} f_{kl} x^j(t-\tau)\dot{x}^k(t-\tau)$$

with delayed nonlinearities, the normal form coefficient β_{21} can be expressed as

$$\beta_{21} = \frac{1}{2\lambda_1 - f_{0100} + (\tau f_{0010} + f_{0001}(\lambda_1\tau - 1))\mathrm{e}^{-\lambda_1\tau}} \times \left(\frac{\mathrm{e}^{-(4\lambda_1+\overline{\lambda}_1)\tau}}{\det(\boldsymbol{\Delta}(2\lambda_1))}(2f_{20} + f_{11}(2\lambda_1 + \overline{\lambda}_1) + 4f_{02}\lambda_1\overline{\lambda}_1)(f_{20} + f_{11}\lambda_1 + f_{02}\lambda_1^2) + \frac{\mathrm{e}^{-(3\lambda_1+2\overline{\lambda}_1)\tau}}{\det\big(\boldsymbol{\Delta}(\lambda_1 + \overline{\lambda}_1)\big)}(2f_{20} + f_{11}(2\lambda_1 + \overline{\lambda}_1) + 2f_{02}(\lambda_1^2 + \lambda_1\overline{\lambda}_1)) \times (2f_{20} + f_{11}(\lambda_1 + \overline{\lambda}_1) + 2f_{02}\lambda_1\overline{\lambda}_1) + \mathrm{e}^{-(2\lambda_1+\overline{\lambda}_1)\tau}(3f_{30} + f_{21}(2\lambda_1 + \overline{\lambda}_1) + f_{12}(2\lambda_1\overline{\lambda}_1 + \lambda_1^2) + 3f_{03}\lambda_1^2\overline{\lambda}_1) \right),$$

where $\boldsymbol{\Delta}(\lambda) = \lambda\mathbf{I} - \mathbf{L} - \mathbf{R}\mathrm{e}^{-\lambda\tau}$ is the characteristic matrix of the linearized system.

The developed algorithm is also applicable for higher degrees of freedom delayed oscillators and provides information about the existence and stability of possible self-excited oscillations together with their amplitude and frequency.

Related publications: [10], [11]

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