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Mixing power requirement determination of granular material in horizontal paddle mixer

Thesis Booklet of PhD Dissertation

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INTRODUCTION

The operation of mixing plays an important role in many industries. It is a fundamental process in the production of agricultural, pharmaceutical, food, mining, oil, and chemical products. In addition, the primary or secondary function of many household appliances is mixing. Mixing can be categorised according to the state of matter of the materials being mixed. Furthermore, the mixing of liquids with liquids, gases with gases, solids with solids, or different phases can be discussed. The theory of mixing, the material movement during mixing, the tendency to mix and the energy required for mixing differ for each phase.

In contrast to liquids, there is still no general relationship for mixing solid particulate materials that can be used to determine the power requirement for mixing. Consequently, no such correlation exists for the horizontal paddle mixer investigated in this thesis. The description of this phenomenon is complicated by the existence of different layouts and mixing element designs, depending on the characteristics of the material to be mixed and the purpose of the mixing. Furthermore, in order to optimise mixing operations in terms of energy efficiency, it is necessary to know the mixing power requirement. This is because the mixing power requirement and the operating time are used to determine the energy requirement of the mixer.

The drying process in a batch-type horizontal paddle mixer has already been developed at the Department of Building Services and Process Engineering. A computational algorithm for the description of the drying operation has been created, but the description of the mixing power requirement of the mixing motor has not been addressed before.

In my doctoral thesis, I proposed a series of contact models for the simulation of a horizontal paddle mixer based on the discrete element method (DEM) with the aim of accurately simulating the mixing power requirement. In addition, I established a dimensionless equation for the calculation of the mixing power requirement through the use of dimensional analysis. I have introduced a new dimensionless number, the cohesive power number, which replaces the classical power number also used in liquid mixing. Laboratory measurements were conducted to validate the simulation results and to determine the unknown parameters of the dimensionless equation. The measurements were carried out using a state-of-the-art, mechanically and instrumentation-wise equipped, batch-type horizontal paddle mixer, located at the Stokes Laboratory and illustrated in *Figure 1*.

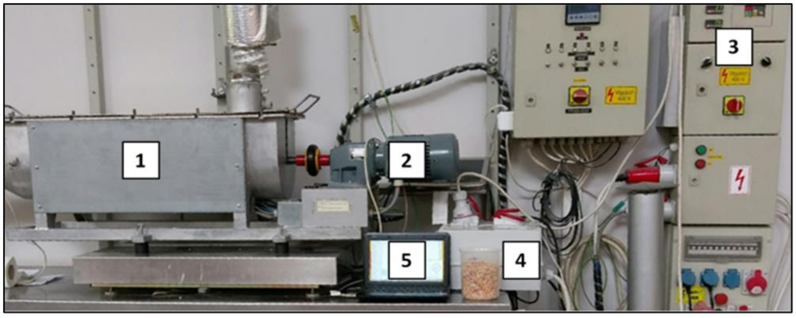


Figure 1. Measurement apparatus for the investigation of mixing power requirement (1. drum; 2. geared motor; 3. frequency inverter; 4. power transmitter, 5. notebook)

LITERATURE REVIEW

The energy requirement of the mixing operation can be traced back to the mixing power requirement under the given operating parameters. The main operating parameters are the rotational speed for the mixing of granular materials, the drum loading factor for batch-type mixers and the hold-up for continuous mixers. These operating parameters have a significant influence on the mixing power requirement. Both the rotational speed ([1–12], [S1, S2]) and the drum loading factor ([2,3,7,10,12–14], [S1, S2]) have a significant influence on the mixing power requirement. Additionally, the mass and/or volume flow rate [4,11] contribute to an increase in the mixing power requirement. Moreover, it is important to consider the idling power requirement of the mixer [S1]. This is created due to the geometry of the mixing elements, their mass, the friction of the components and the air resistance during the mixing [1,2,6,14]. The mixing power requirement is also dependent on the physical and material properties of the mixed material. The shape of the particles also affects the mixing power requirement. Grains with a spherical shape require lower mixing power, while grains with a more distorted, irregular shape require higher mixing power [15]. Mixing smaller grains requires lower mixing power, while mixing larger grains requires higher mixing power for the same material properties [7,16]. The moisture content of the mixed materials cannot be neglected. The internal structure of absorbent materials may undergo alterations due to the presence of moisture, which can result in changes to particle size and the composition of the material. The formation of cohesive forces between particles with surface moisture can further increase the mixing power requirement compared to materials without surface moisture [17].

In conclusion, the literature on measurement indicates that an increase in drum loading factor and/or rotational speed leads to an increase in the mixing power requirement. Furthermore, the size of the mixed material and its physical and material properties must be considered, as they influence the requirement to different extents.

A summary of the literature on simulations indicates that the discrete element method is suitable for modelling the mixing power requirement of granular materials. However, it is important to choose the most appropriate contact model for the mixing process.

A summary of the literature on theoretical derivations and empirical equation generation reveals that there is currently no relationship that can be applied to determine the mixing power requirement of the batch-type horizontal paddle mixers.

THE PROGRESS OF THE RESEARCH

In my doctoral thesis, I have developed methods for determining the mixing power requirement of a batch-type horizontal paddle mixer.

The research work consisted of the following main steps:

1. A comprehensive review of the relevant literature was conducted to identify the sources dealing with the measurement, simulation, or theoretical description of mixing power requirement.

2. Laboratory measurements have been carried out to determine the mixing power requirement of granular materials with different moisture contents at different operating settings. The measurement results were used to validate simulation models based on the discrete element method (DEM) and to determine the unknown parameters of the equation created by dimensional analysis.

3. In order to obtain the required setup parameters for the DEM simulations, laboratory measurements were carried out using an air pycnometer and a static angle of repose and a direct shear box apparatus. Furthermore, simulation calibrations were performed with the latter two devices in order to determine additional setup parameters.

4. In the DEM simulations, two different contact models were investigated, both of which contain equations for the forces and moments in the event of particle collisions. One model took into account the cohesive forces due to the surface moisture of the particles, while the other was only suitable for mixing granular materials without surface moisture.

5. The DEM simulations of the direct shear box and the paddle mixer were a time-consuming task. Consequently, the potential for reducing their simulation time was also investigated. For the direct shear box, a shear velocity sensitivity study was performed. For the paddle mixer, 3D model mesh element number, time step and parameter sensitivity studies were carried out.

6. The dimensional analysis approach was employed to derive the generic form of the dimensionless equation for paddle mixers. From this, a material and mixer specific equation was derived due to the limited number of values for some parameters. The resulting dimensionless equation can be used to estimate the mixing power requirement of the investigated paddle mixer.

MEASUREMENTS AND METHODS

The objective of this work was to develop a method for estimating the mixing power requirement in a batch-type horizontal paddle mixer. At the Stokes Laboratory of the Department of Building Services and Process Engineering, Budapest University of Technology and Economics, measurements were carried out using the mixer illustrated in *Figure 2*. The investigated materials are illustrated in Figure 2: hulled millet (*Panicum miliaceum L.*), fodder corn (*Zea mays L.*) and hulled barley (*Hordeum vulgare*).

Figure 2. The investigated granular materials:
a. hulled millet, b. fodder corn, c. hulled barley

Mixing power requirement measurements

Prior to the measurements, the granualr materials were prepared. This involved the use of an air classifier to remove any broken particles or other impurities, thus ensuring that the particle size was as close to the desired value as possible. Subsequently, the addition of water resulted in the creation of bulks with different moisture contents, which was achieved using a rotary drum. Finally, the moisture content of the granular materials was determined using a drying chamber and mass measurement.

The quantity of material loaded into the drum of the paddle mixer was quantified by the drum loading factor, which represents the volume of the bulk relative to the volume of the empty drum. During the mixing power requirement measurements, the rotational speed was varied over the range $n_k = 0.48 - 1.58\ 1/s$ using a frequency inverter, and the drum loading factor was investigated over the range of $l = 10 - 25\%$. The power requirement of the motor was recorded using a three-phase power transmitter.

Prior to loading the granular material into the drum, measurements were taken to determine the average idling power requirement due to the shaft and agitator rotation, bearing friction and the drag force created by the air resistance. The power values measured as a function of time and were averaged for the set drum loading factor and rotational speed. By subtracting the average idling power requirement from the average power requirement measured with the material inside the drum, the average mixing power requirement of the granular material was determined.

Discrete element simulations

The discrete element simulations were performed using the open-source Yade-DEM software. This afforded the opportunity to make modifications or implement new contact models into the software. Two contact models were investigated in the doctoral thesis.

The first contact model was the '*CohFrictPhys*' model, which was already included in the software. It is illustrated schematically in *Figure 3*. This model considers the cohesive forces due to moisture on the surface of the particles. The equations in the model are derived from those in the classical elastic and Bond contact models. In the simulations, it was necessary to apply a velocity-dependent parameter to simulate the mixing power requirement with the appropriate order of magnitude. This led to the conclusion that the introduction of damping was required in order to ensure that the dissipation of energy was not solely attributed to friction.

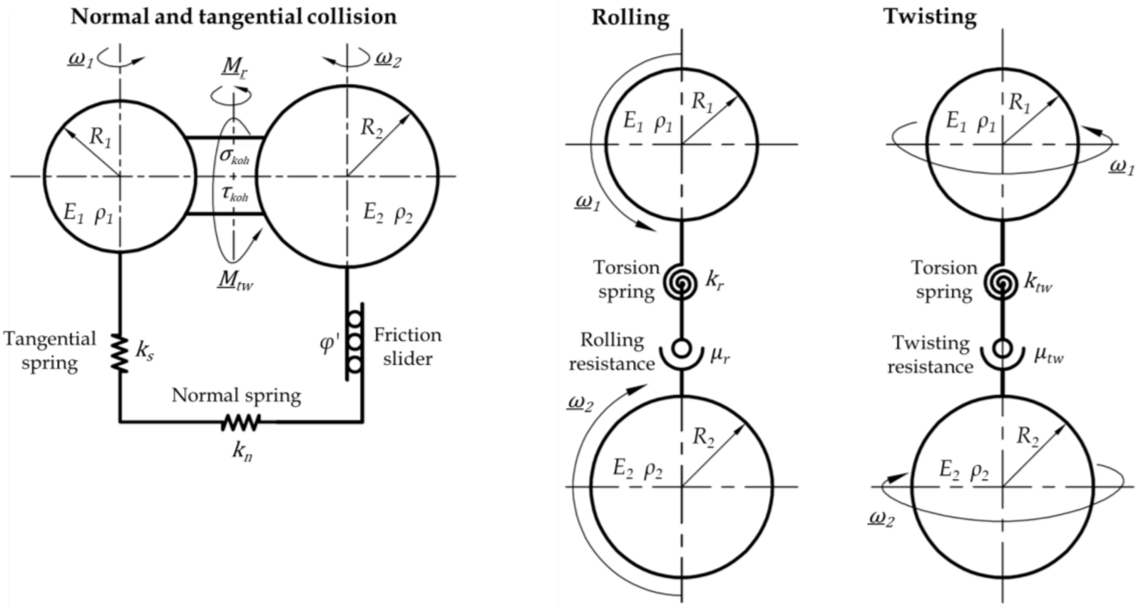


Figure 3. Schematics of the 'CohFrictPhys' contact model

The second contact model was a developed viscoelastic model (named: '*ViElTo*'), which I implemented into the Yade-DEM software using the C++ programming language. *Figure 4* illustrates the schematic structure of the *ViElTo* contact model. Viscous damping was implemented into the model for normal and tangential collision directions, as well as for modelling particle rolling and twisting. The model did not consider cohesive forces, so the results of the simulations were validated only by measurements with air-dry granular materials. Consequently, it was unnecessary to calibrate the micromechanical parameters modelling the cohesive relationships, as in the *CohFrictPhys* contact model for cohesive normal and shear strengths.

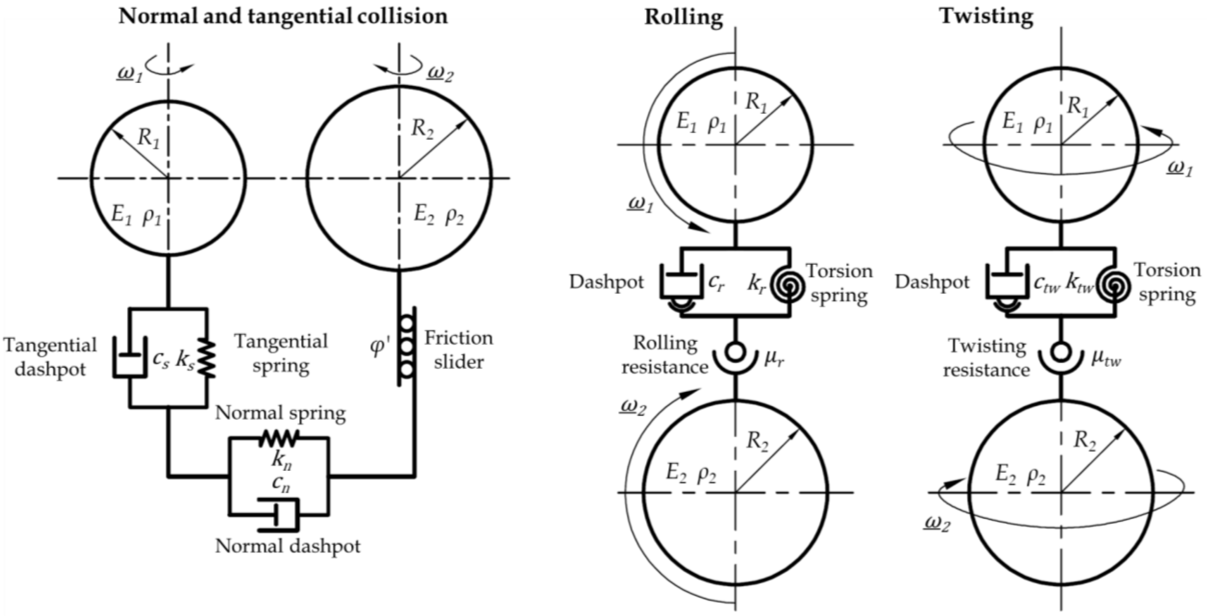


Figure 4. Schematics of the 'ViElTo' contact model

In order to determine the setup parameters for the DEM simulations, air pycnometer, static angle of repose and direct shear box measurements were performed. Furthermore, the latter two were also used for calibration simulations, which enabled the determination of additional setup parameters.

The direct shear box and the mixing simulations were time-consuming tasks. In order to reduce the simulation time, it was proposed to increase the shear velocity for the direct shear box test. The initial shear velocity of $0.02\ mm/s$ could be increased up to $50\ mm/s$ without significant change in the simulated shear force. When a shear velocity exceeding $50\ mm/s$ was employed, the force on the load plate exhibited a notable change in comparison to the nearly constant force associated with the initial shear velocity. With regard to the mixing simulations, a reduction in the number of mesh elements (the number of triangles representing the surfaces) of the mixer's 3D model was proposed. The results indicated that the minimum number of mesh elements required for the collisions in the simulations was sufficient, resulting in a reduction in the simulation time requirement.

Dimensional analysis

In order to apply dimensional analysis, I have collected the quantities that influence the mixing power requirement from both the literature and my own measurement and simulation experience. My objective was to create a dimensionless equation to estimate the mixing power requirement in the paddle mixer:

$$N_P = \left(\frac{l}{N_{kl}}\right)^{B_{k,1}} {N_C}^{B_{k,2}} {Fr_M}^{B_{k,3}} \left(\frac{L_d}{W_{kl}}\right)^{B_{k,4}} \left(\frac{D_d}{d_{sz}}\right)^{B_{k,5}} \left(\frac{H_d}{H_{kl}}\right)^{B_{k,6}} \left(\frac{\alpha_{kl}}{\alpha_{AoR}}\right)^{B_{k,7}} \left(\frac{\mu_d}{\mu_{sz}}\right)^{B_{k,8}}. \qquad (1)$$

In Equation (1), N_P represents the power number, l denotes the drum loading factor, N_{kl} is the number of mixing paddles, N_C is the cohesion number, Fr_M is the mixing Froude number, d_{sz} is the typical diameter of the grains, α_{AoR} is the bulk's static angle of repose, μ_{sz} and μ_d is the friction coefficient of the bulk and the mixer. The remaining geometric dimensions of the mixer are illustrated in *Figure 5*.

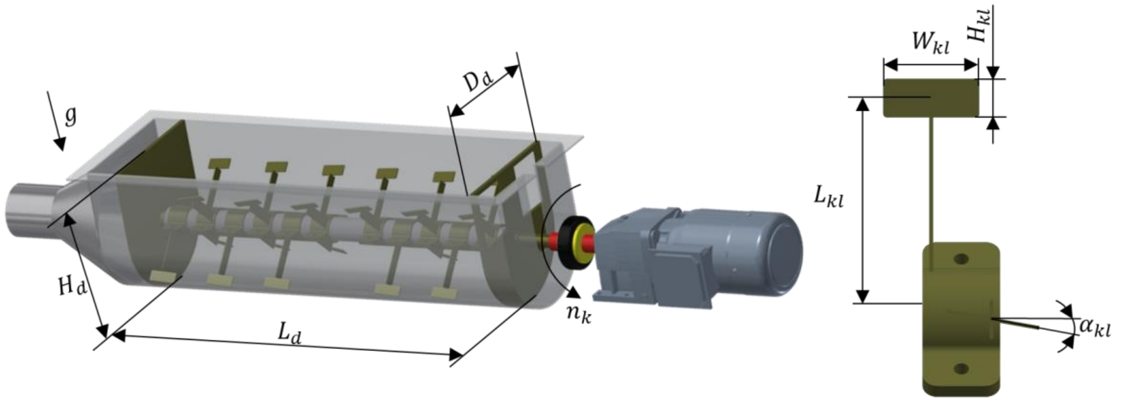


Figure 5. 3D model of the paddle mixer and a mixing paddle
(L_d: drum length; D_d: drum width; H_d: drum height; W_{kl}: mixing paddle width; H_{kl}: mixing paddle height; α_{kl}: inclination angle of the mixing paddle; L_{kl}: power lever of the mixing paddle; n_k: rotational speed; g: gravitational acceleration)

The parameters that remained constant throughout the measurements were merged into a single parameter $(B_{k,0})$, given that their values remained constant across the investigated parameter ranges. Furthermore, a novel dimensionless number, the cohesive power number, was introduced:

$$N_{PC} = \frac{\overline{P}_k}{C_h {L_{kl}}^3 n_k}, \tag{2}$$

where $\overline{P}_k$ denotes the average mixing power requirement, C_h the bulk cohesion, L_{kl} is the mixing paddle force lever and n_k is the rotational speed. Finally, regression analysis and a nonlinear solver were used to determine the unknown $B_{k,i}$ parameters of the material and mixer-specific equation, based on the measurement results of air-dry hulled millet:

$$N_{PC} = B_{k,0} l^{B_{k,1}} {N_C}^{B_{k,2}} {Fr_M}^{B_{k,3}}, \tag{3}$$

In comparison to the literature, the dimensionless equation is notable for two reasons. Firstly, the cohesive force was not considered, but rather the direct shear box independent bulk cohesion was taken into account. Secondly, the mixing Froude number was included. In the case of liquid mixing in horizontal mixers, the mixing Froude number is negligible because the residence time of the liquid on the surface of the mixing element is short, and the effect of gravity is minimal. In contrast, when mixing solid granular materials, the particles may remain on the surface of the paddles for a longer period of time, thereby exerting additional torque on the agitator shaft during mixing. With the introduction of the cohesive power number, the dependence of the mixing power requirement of granular materials on the moisture content is characterised by bulk cohesion. In the classical power number, which is also used for liquid mixing, the bulk density fulfils this role. It can be observed that changes in the moisture content of granular materials have a greater effect on the bulk cohesion than on the bulk density. Consequently, it is recommended that the cohesive power number be employed in such cases.

RESULTS

A series of experiments were conducted in a batch-type horizontal paddle mixer, located in the department's laboratory, in order to validate the results of the simulations and to obtain the values of the unknown parameters within the dimensionless equation.

The DEM simulations demonstrated that the developed '$ViElTo$' contact model was capable of accurately simulating the mixing power requirement. Increasing the shear velocity in direct shear box simulations can reduce the time requirement, but it is essential to monitor the variation of the force acting on the preload plate. If the force acting on the load plate, compared to the weight force from the load plate mass, significantly changes (in my case, $\pm 14.5\%$), the shear force will also vary and become a shear velocity-dependent value. Consequently, it is necessary to reduce the shear velocity, otherwise a simulation result different from the shear force measured on the laboratory shear box obtained. A reduction in the time required for mixing power simulations can be achieved by reducing the number of mesh elements in the 3D model of the mixer. It is sufficient to use the minimum number of mesh elements required for the collision during the DEM simulations.

The unknown parameters of the new dimensionless equation were determined by means of regression analysis using a nonlinear solver. Once the values of each parameter had been fixed, their disorting effect was taken into account by means of statistical indicators.

New scientific results

Thesis 1

The shear velocity of the direct shear box DEM simulations can be increased to $50\ mm/s$ with 3,3% relative error of the shear force and 14,5% maximum relative fluctuation of the force acting on the load plate, under the following conditions:

- simulation of a standard $60\ mm\ \times 60\ mm\ \times 30\ mm$ direct shear box;
- spherical particles with a typical diameter of $1.8 \pm 0.1\ mm$ and normal distribution;
- application of the $ViElTo$ viscoelastic contact model and;
- preload of $19.61\ kPa$.

The results of the thesis are discussed in Section 7.2.2.1 in the dissertation.
Related publications: [S2], [S12], [S13], [S14]

Thesis 2

The mixing power requirement of the batch-type horizontal paddle mixer can be determined with 14.5% maximum relative error by using the viscoelastic contact model in the DEM simulations.

Equation of motion in the model:

$$m\frac{\partial \underline{v}}{\partial t}=\underline{F_n}+\underline{F_s}+\underline{F_g}=$$

$$=4k_n u_n \underline{n}+\left(\begin{cases}2\sqrt{k_n\dfrac{m_1 m_2}{m_1+m_2}} & sphere-sphere\ collision\\ 2\sqrt{k_n m_1} & sphere-structural\ wall\ element\ collision\end{cases}\right)\beta_n \underline{v_n}$$

$$+\left(\begin{cases}\underline{F_s}_{|t-\Delta t}+k_s\Delta t\underline{v_s}\ ,if\ \left|\underline{F_s}\right|<\left|\underline{F_n}\right|tan(\varphi')\\ \left|\underline{F_n}\right|tan(\varphi')\dfrac{\underline{F_s}}{\left|\underline{F_s}\right|},if\ \left|\underline{F_s}\right|\geq\left|\underline{F_n}\right|tan(\varphi')\end{cases}\right)$$

$$+\left(\begin{cases}2\sqrt{k_s\dfrac{m_1 m_2}{m_1+m_2}} & sphere-sphere\ collision\\ 2\sqrt{k_s m_1} & sphere-structural\ wall\ element\ collision\end{cases}\right)\frac{\beta_{s1}+\beta_{s2}}{2}\underline{v_s}+m_i\underline{g};$$

Equation of momentum in the model:

$$\boldsymbol{I}\frac{\partial \underline{\omega}}{\partial t}=\underline{M_r}+\underline{M_{tw}}+\underline{M_{kon,i}}=$$

$$=\left(\begin{cases}\underline{M_r}_{|t-\Delta t}-k_r\underline{\omega_r}\Delta t\ \ ,if\ \left|\underline{M_r}\right|<\mu_r\left|\underline{F_n}\right|\dfrac{2\mathrm{R}_1\mathrm{R}_2}{\mathrm{R}_1+\mathrm{R}_2}\\ \mu_r\left|\underline{F_n}\right|\dfrac{2\mathrm{R}_1\mathrm{R}_2}{\mathrm{R}_1+\mathrm{R}_2}\dfrac{\underline{M_r}}{\left|\underline{M_r}\right|}\ ,if\ \left|\underline{M_r}\right|\geq\mu_r\left|\underline{F_n}\right|\dfrac{2\mathrm{R}_1\mathrm{R}_2}{\mathrm{R}_1+\mathrm{R}_2}\end{cases}\right)$$

$$+\left(\begin{cases}\underline{M_{tw}}_{|t-\Delta t}-k_{tw}\underline{\omega_{tw}}\Delta t\ \ ,if\ \left|\underline{M_{tw}}\right|<\mu_{tw}\,tan(\varphi')\left|\underline{F_n}\right|\dfrac{2\mathrm{R}_1\mathrm{R}_2}{\mathrm{R}_1+\mathrm{R}_2}\\ \mu_{tw}\,tan(\varphi')\left|\underline{F_n}\right|\dfrac{2\mathrm{R}_1\mathrm{R}_2}{\mathrm{R}_1+\mathrm{R}_2}\dfrac{\underline{M_{tw}}}{\left|\underline{M_{tw}}\right|},if\ \left|\underline{M_{tw}}\right|\geq\mu_{tw}\,tan(\varphi')\left|\underline{F_n}\right|\dfrac{2\mathrm{R}_1\mathrm{R}_2}{\mathrm{R}_1+\mathrm{R}_2}\end{cases}\right)$$

$$-R_i\underline{n}\times\left(\underline{F_n}+\underline{F_s}\right).$$

The results of the thesis are discussed in Section 7.2.2.3 in the dissertation.
Related publications: [S1], [S2]

Notation	***Name***	***Unit***
E	elasticity modulus of colliding elements	Pa
F_g	weight force	N
F_s	resultant force in tangential direction	N
$F_{s_{\|t-\Delta t}}$	tangential force one time step earlier	N
F_n	resultant force in normal direction	N
g	gravity acceleration	m/s^2
i	running index, $i=1,2$	1
$\boldsymbol{I}$	moment of inertia matrix	kgm^2

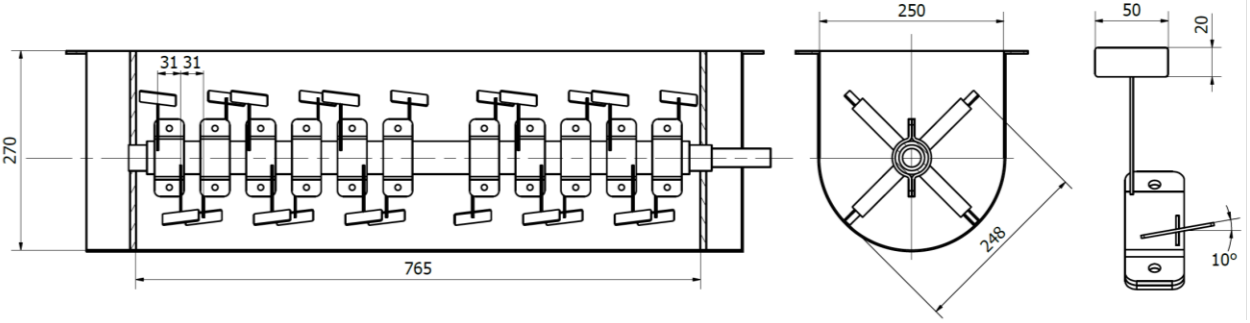
k_n	resultant spring stiffness in normal direction, $k_n = 4\frac{E_1R_1\,E_2R_2}{E_1R_1\,+E_2R_2}$;	$\frac{N}{m}$
k_r	resultant rolling spring stiffness, $k_r = \beta_{kr}k_nR_1R_2$	Nm
k_s	resultant spring stiffness in tangential direction, $k_s = 2\frac{k_{n1}v_1\;\,k_{n2}v_2}{k_{n1}v_1\;\,+k_{n2}v_2}$	$\frac{N}{m}$
k_{tw}	resultant twisting spring stiffness, $k_{tw} = \beta_{ktw}k_sR_1R_2$	Nm
m	mass of colliding element	kg
M_{kon}	contact moment	Nm
M_r	rolling moment	Nm
$M_{r\vert t-\Delta t}$	rolling moment one time step earlier	Nm
M_{tw}	twisting moment	Nm
$M_{tw\vert t-\Delta t}$	twisting moment one time step earlier	Nm
$\underline{n}$	normal vector	1
R	radius of colliding element	1
t	time	s
u_n	normal direction overlap of colliding elements	m
v	velocity of colliding element	m/s
v_n	normal direction velocity, $\underline{v}_n = \left(\underline{v}_1 - \underline{v}_2\right)\cdot\underline{n}$	m/s
v_s	tangential direction velocity, $\underline{v}_s = \underline{v}_1 - \underline{v}_2 - \left(\left(\underline{v}_1 - \underline{v}_2\right)\cdot\underline{n}\right)\underline{n}$	m/s
β_{kr}	rolling spring proportionality factor $\beta_{kr} = \begin{cases} \frac{2\beta_{kr1}\beta_{kr2}}{\beta_{kr1}+\beta_{kr2}} & ,ha\ \beta_{kr1} > 0 \text{ és } \beta_{kr2} > 0 \\ 0 & ,ha\ \beta_{kr1} \leq 0\ vagy\ \beta_{kr2} \leq 0 \end{cases}$	1
β_{ktw}	twisting spring proportionality factor $\beta_{kr} = \begin{cases} \frac{2\beta_{ktw1}\beta_{ktw2}}{\beta_{ktw1}+\beta_{ktw2}} & ,ha\ \beta_{ktw1} > 0 \text{ és } \beta_{ktw2} > 0 \\ 0 & ,ha\ \beta_{ktw1} \leq 0\ vagy\ \beta_{ktw2} \leq 0 \end{cases}$	1
β_n	normal viscous damping coefficient, $\beta_n = \frac{\beta_{n1}+\beta_{n2}}{2}$	1
β_s	tangential viscous damping coefficient, $\beta_s = \frac{\beta_{s1}+\beta_{s2}}{2}$	1
Δt	time step	s
μ_r	resultant rolling resistance coefficient, $\mu_r = \min(\mu_{r1}, \mu_{r2})$	1
μ_{tw}	resultant twisting resistance coefficient, $\mu_{tw} = \min(\mu_{tw1}, \mu_{tw2})$	1
ν	the proportionality factor between tangential and normal direction spring stiffness (Poisson's ratio)	1
φ'	resultant friction angle, $\varphi' = \frac{\varphi'_1+\varphi'_2}{2}$	°
ω	angular velocity of colliding elements	$1/s$
ω_r	rolling angular velocity, $\underline{\omega}_r = \underline{\omega}_1 - \underline{\omega}_2 - \left(\left(\underline{\omega}_1 - \underline{\omega}_2\right)\cdot\underline{n}\right)\underline{n}$	$1/s$
ω_{tw}	twisting angular velocity, $\underline{\omega}_{tw} = \left(\left(\underline{\omega}_1 - \underline{\omega}_2\right)\cdot\underline{n}\right)\underline{n}$	$1/s$

Thesis 3

The mixing power requirement for the mixing of air-dry hulled millet with no surface moisture content in a batch-type horizontal paddle mixer can be calculated from the following dimensionless equation:

$$N_{PC} = 15.68 l^{1.04} {N_C}^{0.1} {Fr_M}^{-1/4}.$$

The geometry and dimensions of the investigated batch-type horizontal paddle mixer:



The U-shaped static drum was 765 mm in length, 250 mm in width and 270 mm in height, resulting in an empty volume of 47.4 dm^3. The mixing paddles were 50 mm × 20 mm × 2 mm in size and had an inclination angle of 10°. The 22 pcs mixing paddles were fixed to the Ø45 mm shaft by means of flat steel stems of the same cross-section turned on their edges, and by means of removable joints. The spacings between the paddle stems were 31 mm.

The maximum relative error of the equation is 13.7% for the following ranges of validity and measurement conditions:

$$0.1 \le l \le 0.25$$
$$0.0004 \le N_C \le 0.0043$$
$$0.0026 \le Fr_M \le 0.0291$$
$$0.48 \frac{1}{s} \le n_k \le 1.58 \frac{1}{s}$$
$$x = 8.9\%$$
$$g = 9.81\ m/s^2$$

The results of the thesis are discussed in Section 7.3.1 in the dissertation.
Related publications: [S1], [S3], [S4], [S5], [S10], [S11], [S15], [S16], [S17]

Notation	***Name***	***Unit***
C_h	bulk cohesion	Pa
Fr_M	mixing Froude number, $Fr_M = \frac{L_{mp} {n_k}^2}{g}$	1
g	gravity acceleration	m/s^2
l	drum loading factor, the ratio of the volume of the material loaded in to the volume of the drum;	$\frac{m^3_{material}}{m^3_{drum}}$
L_{kl}	force lever of the mixing paddles $\left(L_{mp} = 0{,}114\ m\right)$	m
N_C	cohesion number, $N_C = \frac{{L_{kl}}^2 {n_k}^2 \rho_h}{\mathrm{C_h}}$	1
n_k	rotational speed	$1/s$

N_{PC}	cohesive power number, $N_{PC} = \frac{\overline{P}_k}{C_h {L_{kl}}^3 n_k}$	1
$\overline{P}_k$	average mixing power requirement	W
x	moisture content on wet basis	$\frac{kg_{H2O}}{kg_P}\%$
ρ_h	bulk density	kg/m^3

Thesis 4

The cohesive power number is a dimensionless number that can be used to characterise the mixing power requirement of granular materials, taking into account the bulk cohesion, the characteristic size of the mixer and the rotational speed, with the following relationship:

$$N_{PC} = \frac{\overline{P}_k}{C_h {L_{kl}}^3 n_k}$$

The cohesive power number can also be defined as the product of the cohesion number and the power number which is also used in case of liquid mixing:

$$N_{PC} = N_C N_P$$

The results of the thesis are discussed in Section 7.3.1 and 7.3.2 in the dissertation. Related publications: [S3], [S15]

Notation	***Name***	***Unit***
C_h	bulk cohesion	Pa
L_{kl}	force lever of the mixing paddles $\left(L_{mp} = 0{,}114\ m\right)$	m
N_C	cohesion number, $N_C = \frac{{L_{kl}}^2 {n_k}^2 \rho_h}{C_h}$	1
n_k	rotational speed	$1/s$
N_P	power number, $N_P = \frac{\overline{P}_k}{{L_{kl}}^5 {n_k}^3 \rho_h}$	1
N_{PC}	cohesive power number, $N_{PC} = \frac{\overline{P}_k}{C_h {L_{kl}}^3 n_k}$	1
$\overline{P}_k$	average mixing power requirement	W
ρ_h	bulk cohesion	kg/m^3

PRACTICAL APPLICATION OF THE RESULTS

In the design of mixing equipment, it is essential to consider the resulting power requirement and select a mixing motor that is capable of moving the granular material. In the event of underestimation, there is a risk that the motor may exhibit erratic rotation or fail to start to rotate at all. Consequently, the mixing process is only partially realised or not at all. If the power requirement is overestimated, the investment cost of the motor and the space required for installation will also increase. The power requirement of the motor can be attributed to two factors: the idling power requirement, which occurs when operating the mixer without granular material, and the mixing power requirement, which occurs when there is granular material in the mixer during the operation. By determining the mixing power requirement and the operation time of the mixing process, the energy requirement for mixing granular materials can be calculated. In order to optimise the energy efficiency of mixing equipment, it is crucial to examine this quantity.

Based on the literature, until now there has been no simulation models or theoretical relationships that could be used to estimate the mixing power requirement in a batch-type, horizontal paddle mixer.

A simulation model of the investigated paddle mixer was created using the discrete element method (DEM) and a particle contact model was developed. The two models can be used with the DEM to simulate the mixing power requirement with sufficient accuracy. The applicability of the developed contact model was supported by static angle of repose and direct shear box test simulations. The DEM simulation time required for the direct shear box and the horizontal paddle mixer was considerable. In order to reduce the simulation time requirement of the former equipment, the shear velocity was increased. In contrast, the latter was reduced by reducing the number of 3D model mesh elements and adjusting the time step based on sensitivity studies. The developed methods can be employed to reduce the simulation time required for DEM simulation of other equipment. Furthermore, the simulation model of the mixer can be utilised for energy optimisation, taking into account the additional purpose of the mixing operation (drying).

A new dimensionless equation was created using dimensional analysis. This equation can be used to describe the mixing power requirement of horizontal paddle mixers for mixing granular materials. The unknown parameters of the equation can be determined by knowing the physical and material characteristics of the tested material, as well as the main characteristic size of the mixer. As a result, the mixers can be compared with each other using the dimensionless numbers in the dimensionless equation. This method is already a common practice in the case of liquid mixing, but has not yet been adopted in the case of mixing granular materials.

The DEM simulation environment and dimensionless equation can be employed to estimate the mixing power requirements of the investigated horizontal paddle mixer. Furthermore, these tools can also assist engineers and professionals engaged in the design, operation and development of similar mixers.

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Publications related to the thesis

[S1] Horváth, D., Poós, T., and Tamás, K., 2019, "Modeling the Movement of Hulled Millet in Agitated Drum Dryer with Discrete Element Method," Computers and Electronics in Agriculture, **162**, pp. 254–268.

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