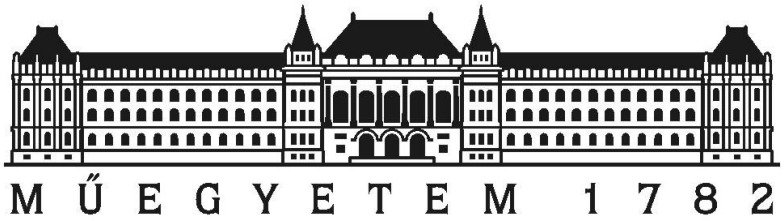

Budapest University for Technology and Economics
Faculty of Electrical Engineering and informatics
Department of Telecommunications and Media Informatics

Modelling Various Extensions of the Time Dependent Traveling Salesman Problem Under Uncertain Conditions

PhD Dissertation of

Ruba AlMahasneh

Research Supervisor:
Prof. László T. Kóczy

March – 2022

Declaration

I, undersigned Ruba AlMahasneh hereby declare that this Ph.D. dissertation was made by myself, and I only used the sources given at the end. Every part that was quoted word-for-word, or was taken over with the same content, I noted explicitly by giving the reference to the source.

Ruba AlMahasneh

(signature) March , 2022

Abstract

The Traveling Salesman Problem (TSP) is one of the most often studied NP-hard graph search problems. There have been numerous publications in the literature that applied various approaches to find the optimal or quasi optimal solution, even though the problem is computationally intractable. The Time Dependent Traveling Salesman Problem (TD TSP) is one of the most practical extensions of the original TSP problem. In the TD TSP, SA (simulated annealing) is used as part of an optimization problem, where the objective function is equated with a Hamiltonian distance (H) between nodes. In the TD TSP, SA usually starts with a random configuration, then series of moves ($\sigma_i \rightarrow \sigma_{i+1}$) are performed at each temperature (T) step. At each step the move is either accepted or rejected with the probability $p(\sigma_i \rightarrow \sigma_{i+1}) = \min\{1, \exp(-\Delta H/T)\}$, T is the temperature, and $\Delta H = H(\sigma_{i+1}) - H(\sigma_i)$ is the energy difference between the two participating moves (configuration steps). In case of rejection, σ_{i+1} is set to σ_i. By lowering the temperature (T) the probability for accepting a deterioration is reduced, ultimately, if the system is cooled down, and loses energy till it freezes, then it is in a local optimum. On the other hand, to represent the jam factors effect in the TD TSP, there is a predefined area that represents the traffic jam region in the city center (Augsburg), in a time dependent form; in which the jam function (f) is assumed to have different values according to its closeness to the city center, and thus, the distances vary in time due to the modified traffic jam cost. The time it takes to drive from one end point to the other one is multiplied in the case of traffic jam factor value when $f > 1$ ($L0$ equals the ground state energy in the cases $f = 1$). Thus, ultimately, the edges are assigned higher costs (longer time) if they were travelled within the predefined oblong shaped area in the city center and lower (shorter time) when they are not traveled within the jam area.

However, in reality, there is no such rigid area for the jam region; on the contrary, those circumstances are rather uncertain, imprecise and can form at any place or time unexpectedly and gradually. Realizing the limitation of representing the jam regions and the rush hour periods effects, and in pursuit of introducing a new perspective on the jam factors costs calculation in the TD TSP, this dissertation's aim is to illustrate thoroughly different novel fuzzy-based models (formulations) to tangibly calculate the effects of simultaneous jam factors (the jam regions and the rush hour periods during the day) on any tour more accurately than it was originally presented in the TD TSP.

This dissertation uses innovative fuzzy-based models to represent the modified costs by the rush hour periods and the jam regions in the TD TSP. Indeed, by applying fuzzy sets (to calculate the jam regions and the rush hour periods during the day) the results show more realistic quantifications for such uncertain factors. Thus, I have built up my work gradually from both aspects, complexity, and accuracy. I started with incorporating type-1fuzzy sets (in the 3FTD TSP) and ended up with my fifth most complex model using type-2 fuzzy sets; the Interval-Valued Intuitionistic Fuzzy sets model for the TD TSP (IVIFTD TSP). Throughout my research work, my models received attention within the fuzzy society, which gave me assurance to dedicate my efforts and focus to this overlooked aspect in the literature.

First, I introduced the novel 3FTD TSP (Triple Fuzzy Time Dependent Traveling Salesman Problem); a further extension towards including costs uncertainties using fuzzy notions, the first ever fully fuzzified model of the original TD TSP. The 3FTD TSP utilizes fuzzy values for determining the costs between any two nodes (cities, locations, shops. etc) within the traffic jam regions and/or during the rush hours periods more practically. It is called triple fuzzy as it has three aspects of uncertainty (the time itself between nodes,

the jam regions costs and the rush hours costs), all factors were represented using type-1 (classic) fuzzy sets. Since this model was characterized using two-dimensional membership functions, the rush hours times can be represented using piecewise linear normal fuzzy sets. On the other hand, the jam regions can be represented by various fuzzy membership functions depending on the area's topography (in the 3FTD TSP I used trapezoidal "oblong shaped" membership function for the city centres). Indeed, this representation simulates the modified costs in case of crossing a specific jam region (jam regions here can be city centres, accidents, or road conjunction).

The 3FTD TSP approach led to fully fuzzy-based costs formulation for the classic TD TSP, and thus, expressing the uncertain costs added to the overall tour length (costs) more efficiently. After achieving good results on small representative instances; I extended the 3FTD TSP (by developing a C++ program that handles different type of fuzzy numbers for the TD TSP) and applied it on the largest universal instances in the literature in pursuit of testing the generality and applicability of the model on real-life scenarios. For optimizing the novel extended 3FTD TSP, the DBMEA (Discrete Bacterial Memetic Evolutionary Algorithm) was proposed, as a meta-heuristic. The DBMEA has been successfully applied on various similar (although crisp) problems for finding the (quasi-) optimal solution. In order to support the claim about the efficiency of this selection, the classic GA (Genetic Algorithm) has been also applied on all universal instances (Eil51, Bier127, Eil76, Eil101 s250_1 and s250_2), enabling the reader to compare the accuracy and the speed.

However, realizing the possible enhancements on the 3FTD TSP representation, and the higher degree of accuracy that can be achieved, led me to introducing a more adequate and realistic formulation, the IFTD TSP (Intuitionistic Fuzzy Time Dependent Traveling Salesman Problem). In this novel model I employed type-2 intuitionistic fuzzy sets which is characterized by three-dimensional membership functions, it involves the so-called hesitation part between the membership functions and the non-membership functions representing the same uncertain components in the previous model (costs, areas of traffic jams, rush hour times) allowing a higher level of vagueness representation. In fact, adding the hesitation part to either the membership values or to the non-membership values (or to both) assigned to the local and temporal embedding of the costs (added from the jam factors) allowed a higher-level involvement of the complex nature of uncertainty, say, the secondary uncertainty originating from the random sampling of costs influencing factors. This additional demonstration using the intuitionistic fuzzy elements to quantify the non-deterministic jam factors and the rush hours periods, resulted in creating an inference system that approximates any tour's costs in an even more adequate way. This model was tested on Bier127 instance and optimized using the DBMEA (Discrete Bacterial Memetic Evolutionary Algorithm) and then a complete comparison between the novel extended IFTD TSP and the classic TD TSP solution (on the same instance under the influence of jam regions and rush hours) was conducted.

Then, I moved forward by introducing another novel and even more accurate approach with the additional notion of interval-valued intuitionistic fuzzy sets; the IVIFTD TSP (Interval-valued Intuitionistic Fuzzy Time Dependent Traveling Salesman Problem). Type-2 fuzzy sets have the potential to provide better performance in modelling problems with higher uncertainties than the traditional type-1 (classic) fuzzy sets, thus, employing interval-valued intuitionistic fuzzy sets for quantifying the traffic jam regions, and the rush hours periods modified costs more precisely was achieved. In addition, by employing the IIFWAA operator (in the IVIFTD TSP model) to aggregate the accumulative interval-valued fuzzy information from all the jam factors simultaneously decreased the information distortion (loss). Indeed, the IVIFTD TSP is the most complicated (yet practical) model that was capable of catering for all the external traffic factors affecting

any edge collectively at any given time while least information loss was guaranteed. For applicability, I optimized the IVIFTD TSP model and the classic TD TSP using the DBMEA on Bier127 instance (with the help of my colleague), and conducted a complete comparison.

Afterwards, I introduced the novel IVFSTD TSP (Interval-Valued Fuzzy Soft Time Dependent Traveling Salesman Problem Model). In the IVFSTD TSP, I have applied the notion of interval-valued fuzzy soft sets to address the overlapping jam factors for any edge in a tour, and thus, calculated the overall modified costs efficiently. The case study I simulated has been taken to exhibit the simplicity of the technique and applicability.

Lastly, in pursuit of proving the applicability and the generality of the novel models, and the fact that they can be applied on different real-world transportation and logistics problems. I used the extended TOPSIS (The Technique for Ordered Preference by Similarity to the Ideal Solution) method using interval-valued intuitionistic fuzzy to solve the Dial-a-ride Problem with Time Window (a prime example of the TD TSP) as a multiple attribute group decision making (MAGDM) problem. This model can determine efficiently both the collective interval-valued intuitionistic positive-ideal solution (PIS) and the interval-valued intuitionistic negative-ideal solution (NIS) by constructing the weighted collective interval-valued intuitionistic fuzzy decision matrix. Based on different distance definitions, I calculated the relative closeness of each alternative to the PIS then ranked the alternatives accordingly (using scoring matrix), and ultimately selected the collective most desirable alternative(s).

Despite of the different novel, fuzzy-based formulations I proposed, each model offers slightly different optimum solution, but it is hard to choose the best model, as each one has its own unique advantages.

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Chapter 1

1.1 Introduction

The Traveling Salesman Problem (TSP) is a classical problem in combinatorial optimization and has been studied extensively in many forms due to its applicability in different domains. It is the problem of a traveling salesman who has *n* locations to visit, returning eventually to the starting point without visiting any location more than once [1]. The goal is to minimize the total cost for any distance traveled (in terms of length units, time, money or other). In the language of graph theory, it means finding the shortest Hamiltonian cycle. Finding the optimal solution of a given instance of the TSP is an NP-hard problem, as obtained by a simple reduction from the Hamilton-Cycle problem [2]. Even in the special case where distances between nodes are represented in the Euclidean plane, it is also proved to be an NP-hard problem. TSP has a polynomial time approximation scheme (PTAS) that means, for any $\varepsilon > 0$, there exists a polynomial time algorithm which guarantees an approximation of size at most $1 + \varepsilon$ times the optimal solution [3]. This explains the added complexity of the TSP when non-geometric (non-deterministic) variants taken in consideration (such as the rush hours periods and the jam regions effects on the tour's overall costs). In the literature, there have been many different extensions and variations of the original TSP [1]. Those approaches main goal was to present a more accurate and realistic costs estimate of any tour, however, those models assigned costs to the edges connecting the nodes (cities) based on Euclidean Distances as constant, which reveals the obvious limitations when simulating real-life cases. Indeed, any trip on the ground is subject not only to the physical distances between cities, but also to the non-deterministic and unexpected natural circumstances (such as the jam regions and the rush hours' periods during the day) that adds to the total cost and the complexity of the problem.

Moving towards a more realistic model, the TD TSP introduced a viable refinement for costs calculation, but still not efficient enough; since it is still a deterministic solution [4]. In the TD TSP, simulated annealing (SA) was used and the problem was handled as an optimization problem with an objective function equated with a Hamiltonian distance. The physical system is successively cooled down, such that it loses energy to a large extent, until it freezes in a (local) optimum configuration. If the cooling process is performed slowly enough, one can trust in ending up in a configuration energetically not far away from the ground state (L_0 in Fig.1.1). With SA, the configuration starts with a random configuration, then series of moves ($\sigma_i \rightarrow \sigma_{i+1}$) are performed at each temperature (T) step. At each step the move is either accepted or rejected with the probability $p(\sigma_i \rightarrow \sigma_{i+1}) = \min\{1, \exp(-\Delta H/T)\}$, T is the temperature, and $\Delta H = H(\sigma_{i+1}) - H(\sigma_i)$ is the energy difference between the two participating moves (configuration steps). In case of rejection, σ_{i+1} is set to σ_i. By lowering the temperature (T) the probability for accepting a deterioration is reduced. When the system is cooled down, and loses energy till it freezes in a local optimum. Thus, applying SA to the given optimization problem, a proper energy function and suitable moves have to be found.

Moreover, the TD TSP assigns a fixed part of the graph to represent the "traffic jam region" in a time dependent form, namely, each graph's edge is assigned given costs in the non-rush hour periods, and higher costs during the rush hour times [4]. In the TD TSP, starting at a fixed point in time at noon for example, the traffic jams affecting specific edges of which both end points are inside a defined region in the city center. The time it takes to drive from one end point to the other one is multiplied in this case by a traffic jam factor

$f > 1$ (traffic jam factor = f). Fig.1.1 shows optimum configurations for different strengths of the traffic jam f (L_0 equals the ground state energy). The node at which the traveling salesman starts his tour is marked by a filled circle. The traveling salesman is supposed to start his tour at one specific node inside this traffic jam area at 9 am. in the morning and return to it at 3 pm. in the afternoon. In case $f = 1$, then he moves along the optimum tour of the original Bier127 problem. Those crisp numbers are multiplied with the length of the corresponding edge between the nodes in the given rush hour times, to quantify the jam factor's effects on the respective costs [4]. In the original TD TSP paper there are optimum configurations for different strengths of the traffic jam function f are shown where it suggest the best possible route for the salesman to take during his trip.

Despite the TD TSP's ability to achieve good results in determining the overall modified cost from the jam factors, one main drawback is the use of concrete numbers (and fixed rigid area in the graph) for representing the proportional traffic jam region's effect on a tour (the city center).

After the classical TD TSP, there have been many other attempts by researchers to quantify the rush hours and the jam regions' effects on the trip's overall cost. Some models used Ant Colony optimization to predict traffic for the TD TSP [5, 6]. Others used mixed approaches, where they incorporated integer programming model to calculate the traveled time, then integrated both Ant Colony System (ACS) and Tabu Search (TS) algorithms [6] to calculate the overall cost. Other authors used linear constrains formulation techniques to present the jam factors' effect on any trip [7]. Also, dynamic programming algorithm was proposed to solve the TD TSP [8]. In fact, there is one common aspect in all the previously proposed approaches in the literature, that is, using crisp numbers to simulate those uncertain and imprecise road conditions, indeed, this is an insufficient formulation.

Although those models proposed good solutions, but without a doubt they all have serious limitation of simulating real-life situations due to using crisp and deterministic approaches in modeling non-deterministic factors (intangible). In fact, it would be closer to reality to use fuzzy sets to capture the uncertainties associated with those imprecise road conditions. In particular, using fuzzy sets [9] in representing the effects of those jam factors as cost coefficients. Thus, I have introduced, for the first time in the literature, five novel fuzzy-based approaches in pursuit of adequately modeling the jam regions and the rush hours periods in realistic environmental circumstances for the classic TD TSP. Each one of those fuzzy based models achieved one step closer to simulating more accurately the non-deterministic costs associated with roads uncertainties in a tour [10,11,12,13,14,15,16,17,18]. Those models ultimately offered practical and realistic formulations for the very first time by overcoming the rigidity presented in all the previously available approaches in the literature.

Moreover, since each fuzzy sets type (and extension) has different characteristics and different dimensions for their membership functions (which will offer flexibilities or limitations), I was eager to build various TD TSP extended models utilizing each type; starting from the classic type-1 fuzzy sets to type-2 fuzzy sets with their different extensions (intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy sets and interval-valued fuzzy soft sets). First, testing each mathematical fuzzy-based model on a small but representative instance, then applying each model on the largest available instances in the literature to compare the results with the state-of-the art solutions after optimizing the models with two different algorithms (the DBMEA and the GA). Afterwards, I tested the applicability of the novel models on different variation of the TD TSP (Dial-a-ride problem with time window), which only validated my results and efforts in this research niche.

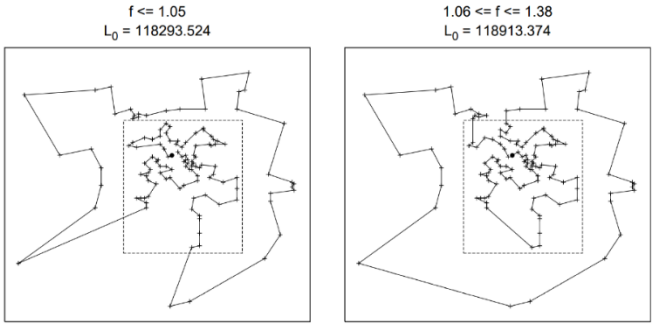

Figure 1.1 TD TSP city center traffic area of Augsburg

1.2 Research Objectives

Historically, effective representation for the TD TSP with the traffic regions and rush hours periods have been difficult to achieve. The reason for this insufficiency, was the fact that all previous models used concrete numbers (except [19], where the first idea of applying fuzzy triangular numbers was raised) to model those uncertain and vague factors. Overlooking the fact that they are non-deterministic, thus, they cannot be accurately represented using crisp numbers. In fact, the most promising and realistic approach is in fact modelling the jam regions and rush hours' effects by using fuzzy sets.

My goal was to introduce and compare various novel fuzzy-based models to accurately quantify the simultaneous effects (modified costs) of the rush hours and the jam factors (the unpredictable road conditions) in the TD TSP with the least amount of information loss. My work has six main tightly connected goals:

First, incorporating classic type-1 fuzzy sets (characterized by a two-dimensional member ship functions) in the first ever fully fuzzy-based version for the TD TSP with the rush hours and the jam regions added costs [10].

Second, applying the intuitionistic fuzzy sets to incorporate the hesitation part between the membership functions and the non-membership functions, which gave higher level of accuracy representation for the road uncertainties [11,12].

Third, formulating a model that calculates the jam factors affecting any edge simultaneously in the TD TSP [13] by using the interval-valued intuitionistic fuzzy sets. This model adequately considered multiple jam factors simultaneously by utilizing the IIFWAA (interval-valued intuitionistic fuzzy weighted arithmetic average aggregation) operator. Moreover, by facilitating different distance measures I was able to find the collective optimal solution and eventually recommend the most desirable route(s) formation (with the least possible collective costs) [13].

Fourth, incorporating different variations (extensions) of type-2 fuzzy sets (which is characterized by a three-dimensional membership functions) such as interval-valued fuzzy soft sets that allowed road uncertainties representation to become even more flexible, yet accurate [18].

Fifth: proving the novel models' applicability by applying them on other transportation problem, such as the Dial-a-ride problem with time window. This in particular proves the applicability and scalability of the IVIFTD TSP (Interval-Valued Intuitionistic Fuzzy Time Dependent Traveling Salesman Problem) [14].

Sixth: developing and publishing a C++ tool box [15,16] to simulate the novel models and integrate them with the Discrete Bacterial Memetic Evolutionary Algorithm and the Genetic Algorithm, to calculate the (quasi-)optimum or semi optimum solution. Then, conducting a thorough comparison between all the novel models and the classical TD TSP on the largest instances available in the literature (Eil51, Bier127, Eil76, Eil101 s250_1 and s250_2) to prove my models' efficiency and generality compared with the state of the art solutions.

1.3 Thesis Structure

The dissertation is organized as follows:

Chapter 2 describes the background of the TSP and gives a thorough overview of the classic TD TSP. Then, this chapter presents some non-fuzzy based extensions of the TD TSP. Also, it present overview of the available fuzzy models, for the TSP and the TD TSP. Then the models' drawbacks are briefly mentioned. Afterwards, it explains how the need emerged for my own models formulations.

Chapter 3 introduces a thorough background of fuzzy sets theory (type-1 and type-2 and their operations), that will be built-on and incorporated throughout the later chapters. Then, it gives an overview of the evolutionary algorithms; the Genetic Algorithms (GA) and the Discrete Bacterial Memetic Evolutionary Algorithm (DBMEA). Lastly, it explains the TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) approach and how it is used in decision making.

Chapter 4 presents my two novel dependent type-1 fuzzy based models. The Triple Fuzzy Time Dependent Travelling Salesman Problem (3FTD TSP) model for small representative instances. Then an optimized and enhanced extension of the 3FTD TSP for the largest universal instances in the literature. In this chapter, I show the run results (comparisons) for the extended fuzzy 3FTD TSP with two optimization algorithms; the DBMEA and the GA.

Chapter 5 presents the Intuitionistic Fuzzy Traveling Salesman Problem Model (IFTD TSP) for the jam regions and the rush hours periods in the TD TSP. This chapter shows the run results and comparison on Bier127 instance, optimized using the DBMEA for both the novel IFTD TSP and the classic TD TSP.

Chapter 6 represents the Extension of the Time Dependent Travelling Salesman Problem with Interval-Valued Intuitionistic Fuzzy model (IVIFTD TSP) by applying Memetic Optimization Algorithm. There is

a complete walk through of the model on a sample case, afterwards the results of the optimized extended IVIFTD TSP and the TD TSP on multiple Bier127 instances are discussed offering full comparison.

Chapter 7 presents my resent work, the Interval-Valued Fuzzy Soft Time Dependent Traveling Salesman Problem (IVFSTD TSP) model, that applies the interval-valued fuzzy soft sets on the TD TSP to find the accumulative overlapping jam factors on a tour.

Chapter 8 presents the extended TOPSIS method for the Dial-a-ride problem with time window using interval-valued intuitionistic fuzzy sets. In this chapter I apply my most complicated novel fuzzy based model; the IVIFTD TSP with further extension of the TOPSIS (The technique for ordered preference by similarity to the ideal solution) method on Dial-a-ride problem as a variation of the TD TSP. This chapter emphasises the applicability of the novel fuzzy based models on real-life problems.

Chapter 9 provides an overview of my future research work plans.

Chapter 10 provides a short overview of my thesis, emphasizing the most important conclusions.

Chapter 11 publications, grants and awards.

Chapter 2 The Time Dependent Traveling Salesman Problem (TD TSP)

2.1 Introduction

The traveling salesman problem (TSP), in which a traveling salesperson must find the shortest closed tour between a given set of cities, visiting each city exactly once [1]. In the TSP, the distances between the single nodes are pre-known. However, looking closely to this problem it is obvious that the distances are time dependent. The goal in the classic TSP is to find the shortest closed tour through *N* nodes, touching each node exactly once. The distances $D(i;\ j)$ between nodes (cities, stops or shops) i and j are usually given in units of either length (or time). As this problem is NP-complete, many heuristics and meta-heuristic algorithms have been developed in the literature to solve this problem approximately. The meta-heuristics (such as the Simulated Annealing) are designed to search for global optima; but they can't guarantee finding one after termination criteria are satisfied. However, those methods are effective because they have been tested on many NP-hard problems and shown efficiency in finding high quality solutions in timely fashion.

The time-dependent traveling salesman problem (TD TSP) was the first extension of the TSP; where distances vary in time. It was first introduced by Schneider [4], where he optimized the TD TSP using simulated annealing (SA) on Bier127 instance. Indeed, the SA (Simulated Annealing) was one of the most successful applications and refinements for the original TSP. In the TD TSP, the objective function is equated with a Hamiltonian distance H. When the physical system is cooled down (it loses energy to a large extent) until it freezes in a (local) optimum configuration. If the cooling process is performed slowly enough, it can lead to a configuration energetically not far away from the ground state.

The SA usually starts with a random configuration, then series of moves ($\sigma_i \rightarrow \sigma_{i+1}$) is performed at each temperature step, where each move is either accepted or rejected with the probability $p(\sigma_i \rightarrow \sigma_{(i+1)}) = min\{1;\ exp(-\Delta H/T)\}$ [4] with $\Delta H = H(\sigma_{i+1}) - H(\sigma_i)$ being the energy difference between the two participating configurations. In case of rejection, σ_{i+1} is set to σ_i. By lowering the temperature T, the probability for accepting a deterioration is reduced [4]. A useful representation of a configuration of a TSP is a permutation σ of the numbers $1;\ 2 \ldots \ldots N$, such that the Hamiltonian H can be written as $H(\sigma) = \sum_{i=1}^{N} D\big(\sigma(i), \sigma(i+1)\big)$ $(\sigma(N+1) \equiv \sigma(1)$. The Lin-2-Opt [4], which reverses the direction of a part of the tour, and Lin-3-Opt that exchanges two successive parts of the tour without altering their directions lead to rather good results. Fig. 2.1 shows the results for the mean value $\langle H \rangle$ of the energy versus decreasing temperature T and its corresponding specific heat C, which exhibits a peak at a freezing temperature $T_f \approx 900$. The common TSP is rather looked at as abstract problem which is far from reality. The distances are not constant as they are being modeled in most of the solutions proposed in the literature. This is where Schneider got his motivation to extend the classic TSP into the time dependent traveling salesman problem (TD TSP), in which the distances vary in time due to traffic jam factors. In the TD TSP, the time it takes to drive from one end point to the other one is multiplied in this case by a traffic jam factor $f > 1$. The traveling salesman is supposed to start his tour at one

specific node inside this traffic jam area at 9 am. in the morning and return to it at 3 pm. in the afternoon. In the case of $f = 1$, the courier moves along the optimum tour of the original bier127 problem.

Of course, it is favorable for him first to visit all nodes inside the traffic jam region if $f \gg 1$ Fig. 2.2 shows optimum configurations for various f (L_0 equals the ground state energy). The node at which the traveling salesman starts his tour is marked by a filled circle. According to Schneider's model, the traveling salesman must compromise between driving a detour in order to avoid the traffic jam region in the afternoon and driving through the traffic jam region in order not to extend the tour length too much. Therefore, the Hamiltonian distance can be re written as $H = H_{opt} + H_{det} + H_{time}$ with the tour length $H_{opt} + H_{det}$, consisting of the length of the optimum tour for the original problem and the additional detour the traveling salesman drives, and H_{time}, the amount of time the traveling salesman is supposed to wait due to the traffic jam factors.

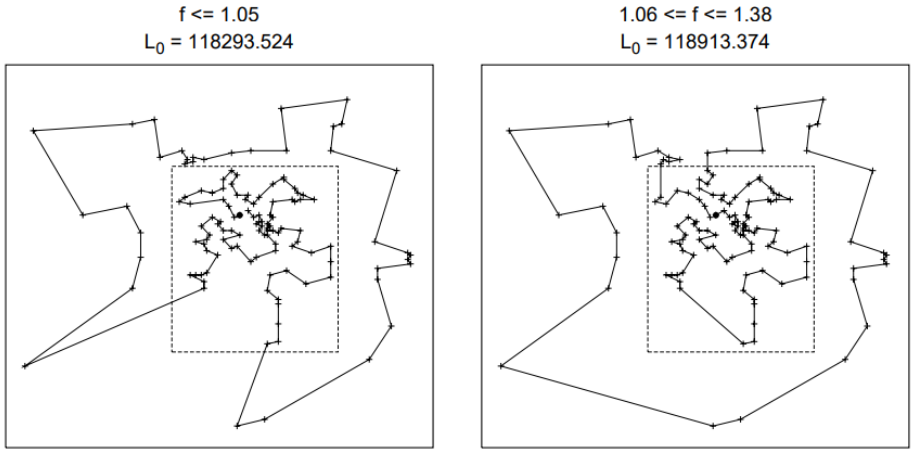

Figure 2.1 TD TSP city center traffic area of Augsburg

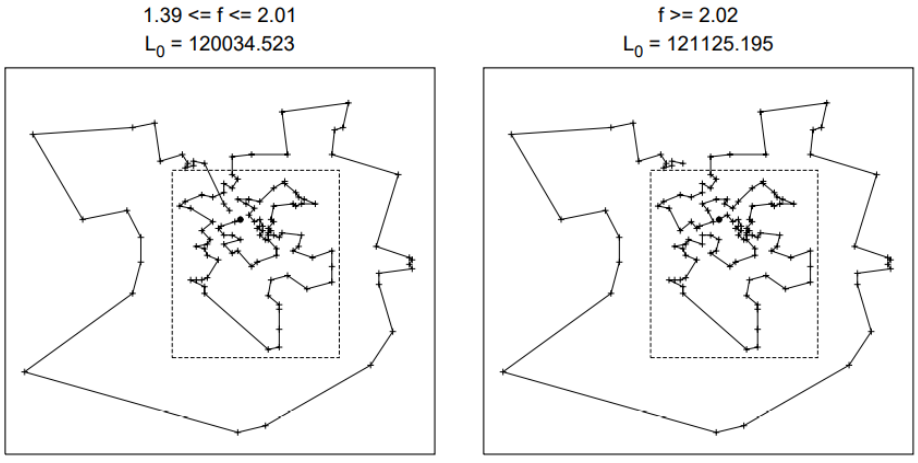

Figure 2.2 TD TSP city center traffic area of Augsburg

The TD TSP introduced an important refinement for costs calculation in the original TSP. Having such an NP-hard problem, calculating the modified tour's costs is more time consuming and requires even more operations than in the original TSP, because the edges exchange can also affect the length of the edges that did not participate in it [4]. Moreover, the classic TD TSP predefined the area of the traffic region in the city center, where the distances between the nodes within the oblong shaped jam region were assigned higher cost than those edges that did not cross that area during travel. Then, those additional costs were multiplied with the time (cost) taken between nodes (cities) and finally added to the overall trip cost in this increased form.

The classic TD TSP was the first viable extension of the TSP, as it transformed the way TSP was perceived, particularly, the edges between nodes became time dependent rather than only independent physical distances. However, looking closer to the TD TSP model, we can realize the limitation in the formulation. The predefined rigid jam area that was the base of the TD TSP formulation for the extra added costs, is not accurate depiction of real-life. In reality, there is no such rigid area for jam region, on the contrary, those circumstances are rather uncertain, they can happen at any time during the day or can form at any place unexpectedly and gradually. Having those two factors (the rush hours and the jam regions) looked at in an uncertain (fuzzy) formulation, changes the whole perspective about the TD TSP costs calculation, which forms the core of the models I propose throughout my work.

2.2 Non-Fuzzy-Based TD TSP Extensions and New Algorithms

In 2016, Taş et al. proposed a variant of the Traveling Salesman Problem with time-dependent service times [20]. The mathematical formulation of the model was extended with sub-tour elimination constraints and was solved using IBM ILOG CPLEX 12.5. The tests were carried out only on smaller instances up to 45 nodes. Even for these relatively small graphs, in quite many cases the optimal solutions were not found within the time limit set by the authors (7200 s).

In 2016, Hurkała proposed a new algorithm for the TD TSP with multiple time windows and compared it with three well-known and often efficient heuristic methods (Simulated Annealing, list-based threshold accepting algorithm, and Variable Neighborhood Search) [21]. The main drawback of these results is that the methods mentioned were tested on even smaller instances, up to 23 nodes only, although these heuristic approaches were able to handle larger instances, as it was shown in a multitude of other papers reporting on meta-heuristic solutions of various other TSP-related graph search tasks.

In 2018, Vu et al. presented an integer programming formulation for solving the TD TSP with time windows. However, this approach was again only able to solve small instances, up to 40 nodes [22].

In 2018, our team also published a novel (non-fuzzy) approach, where a parameter-adapted version of the rather generally applicable (say, "universal"), efficient, and efficacious method called discrete bacterial memetic evolutionary algorithm (DBMEA) was applied for solving the problem, first, for the reason of possible comparison, on Schneider's instances. The proposed meta-heuristics [23] clearly outperformed previous attempts for the solution of this problem, and, by adding a number of new, larger instances, it was also shown that DBMEA did not have restrictions concerning the size of the graph (rather, when knowing the size, a good prediction of obtaining a good approximate solution could be obtained).

In 2019, Ban proposed a two-phase meta-heuristic algorithm [24] for Schneider's version of the Time-Dependent Traveling Salesman Problem, but it did not outperform the earlier published DBMEA approach.

Also in 2019, Vu et al. proposed an integer programming approach which was based on an integer programming formulation of the problem on a time-expanded network. This novel technique outperformed the best-known methods on the benchmark instances [25] and, thus, this may be considered now as “best practice” in a certain sense. However, while Vu’s algorithm is a clear competitor of the DBMEA concerning speed and accuracy, it is still a tailor-made technique with a very narrow applicability scope that cannot easily be adapted to any extension or generalization of the original problem and, thus, it may not be considered as universal (generally applicable). Also, the (rather important) predictability of the expected run time for optimizing a given size problem is not good. In the attempt of extending the model towards more complex, fuzzy, etc. graphs and searching for an efficient algorithm for acceptably good approximate solutions of instances of this extended model, it was obviously important to have some generally applicable approach that promised good results even for new, more general problem classes with unknown, potentially unexpected behavior. (For the investigations concerning the general applicability, called universality, of the DBMEA algorithm family [26,27])
The extensions mentioned above are more general and more realistically applicable compared to the original TSP. However, these models do not consider the inherited uncertainty involved in real life situations, such as the uncertainty of the costs between the pairs of nodes, the uncertainty of the time-dependence periods, the uncertainty of the geographic extensions of the traffic jam areas, the rush hour periods, or the uncertainty of the time windows (TW) in the further extended TD TSP TW.

Finally, it should be mentioned that several further meta-heuristic alternatives for solving the TD TSP have been proposed which yield generally efficient approaches. The most promising one was proposed by Roy et al. [28], but it has a serious disadvantage than applying the original genetic algorithm for the base of the memetic approach, which has been shown in numerous investigations to be inferior to the bacterial evolutionary algorithm, being considerably slower and less accurate [26,27].

2.3 Fuzzy-Based TSP Extensions in the Literature

The TriIFS (Triangular intuitionistic fuzzy number) approach employed the Intuitionistic fuzzy weighted hyperpath [29] with the lower accuracy using scoring between IFS to find the shortest path. Although, it is a simple and applicable approach but still the author did not address the jam factors effects on a tour or an edge in the graph and treated the cost purely from the length of the edges only.
Another interesting approach was introduced by Anitha [30] where the author discussed the Symmetric Traveling Salesman Problem under Intuitionistic Fuzzy environment. The author considered the grade of uncertainty while scheming the Euclidean interval for Intuitionistic Fuzzy sets to minimize the uncertainty in the selected routes. This models adds targets and limitations in Optimization problem (as additional constrains). Thus, making the most of the grade of acknowledgement of Intuitionistic Fuzzy objectives along with limitations and to reduce the grade of rejection of Intuitionistic Fuzzy objectives and to reduce the grade of uncertainty of Intuitionistic Fuzzy objectives along with limitations. However, this model was never tested on universal instances; it was only tested on example with three cities.

2.4 Fuzzy-Based TD TSP Extensions in the Literature

There was one attempt to apply fuzzy sets on the TSP in 2017; the fuzzy based solution to the travelling salesman problem [19], this model was applied on 5 nodes (cities) and used triangular membership functions to represent all traffic conditions as added costs to the overall edges lengths [10]. The drawback of this representation is obvious, the model's efficiency, practicality or applicability was never proven or tested on large scale instances.

Chapter 3 Fuzzy Sets Methodological Bases

Fuzzy Sets and Extensions

3.1 Type-1 (Classic) Fuzzy Sets

In 1965, fuzzy sets as an extension of the classical notion of sets were introduced by L. A. Zadeh [9]. The classical sets can be seen as special cases of the fuzzy sets membership functions if they take values from {0,1}, while elements of fuzzy sets may assume any value in [0,1]. In classical set theory, according to a bivalent condition, the elements membership in a set is assessed in binary terms. A membership function values in the real unit interval [0, 1] can help to describe any phenomenon more efficiently, wherever the notion of "belonging" is vague, gradual (this graduality expressed by degree from the unit interval has nothing to do with uncertainty of randomness type) or uncertain [9, 35].

let X denote a universal set $X(\neq \emptyset)$ with a generic element of X denoted by x, that is, $X = \{x\}$. Then the membership function by which a fuzzy set A is defined has the form :

$$\mu_A: X \to [0,1] \quad (3.1)$$

where $[0,1]$ denotes the interval of real numbers from 0 to 1, inclusive. It is the degree of membership of the element x to the set A.

Then a fuzzy set A has the form by which μ_A is the membership function and x is a particular element of X:

$$\boldsymbol{A = \{\langle x, \mu_A(x) \rangle | x \epsilon X\}} \quad (3.2)$$

3.1.1 Operations on Type-1 Fuzzy Sets

The original theory of Fuzzy sets was formulated in terms of the following specific operators of set; complement, union and intersection. When the range of membership functions perform precisely as the corresponding operator for crisp sets, thus establishing them as clear generalization of the latter. For two fuzzy sets A and B, μ_A and μ_B are the membership functions respectively, where x is an arbitrary element of X. It is possible to define new sets from them or to operate with them. The basic operations between sets are described as follows (where $\vee$= max; $\wedge$ = min).

1) Intersection operator (usually called T-norm): the membership function of the Intersection of two fuzzy sets A and B with membership functions and respectively is defined as the minimum of the two individual membership functions. This is called the minimum criterion. The Intersection operation in Fuzzy set theory is the equivalent of the AND operation in Boolean algebra. It is denoted by $A \cap B$ and is defined as the set formed by those elements of X belonging to A and B simultaneously:

$$A \cap B \Leftrightarrow \mu_{A\cap B} = \{\langle x, \mu_A(x) \wedge \mu_B(x)\rangle | x \in X\} \quad (3.3)$$

2) Union operator (usually called T-conorm): the membership function of the Union of two fuzzy sets A and B with membership functions and respectively is defined as the maximum of the two individual membership functions. This is called the maximum criterion. The Union operation in Fuzzy set theory is the equivalent of the OR operation in Boolean algebra. It is the set formed by those elements that belong to A or B, or both simultaneously. It is denoted by $A \cup B$.

$$A \cup B \Leftrightarrow \mu_{A\cup B} = \{\langle x, \mu_A(x) \vee \mu_B(x)\rangle | x \in X\} \quad (3.4)$$

3) Complement : The membership function of the Complement of a Fuzzy set A with membership function is defined as the negation of the specified membership function. This is called the negation criterion. The Complement operation in Fuzzy set theory is the equivalent of the NOT operation in Boolean algebra. The following rules which are common in classical set theory also apply to Fuzzy set theory. The complement of A is denoted by $\bar{A}$, and consists of all elements of X that do not belong to A.

$$\bar{A} \Leftrightarrow \mu_{\bar{A}} = 1 - \mu_A(x) \quad (3.5)$$

3.2 Type-2 Fuzzy Sets

3.2.2 General Type-2 Fuzzy Sets

Since Zadeh [9] introduced fuzzy sets, there have been many approaches and theories introduced to treat imprecision and uncertainty even more adequately. Generally, classical "type-1" fuzzy sets are not able to directly model complex uncertainties adequately, because their membership functions assign crisp values to the membership degrees and, thus, the measure of uncertainty is "rigid".

It is not always possible for a membership function of the type $X \rightarrow [0,1]$ to assign precisely one point from the interval $[0,1]$ to each element in $x \in X$ without the loss of at least a part of information (which is the basis of type 1 fuzzy sets). This is where type-2 fuzzy sets came more sufficient. Type-2 fuzzy sets have $(n+2)$ −dimensional membership functions representation, while the representation of type-1 fuzzy sets membership functions are only $(n+1)$ −dimensional (assuming that the universe of discourse has *n* dimensions). Thus, type-2 fuzzy sets model uncertainties more efficiently, because their membership functions assign degrees which are fuzzy themselves [36,37,38]. Indeed, type-2 fuzzy sets allow more degrees of freedom in representing uncertain phenomena.

To illustrate, as Mendel explained in his work [36,37], if we blur a type-1 membership function (Fig. 3.1a) by shifting the points on the triangle left or right we get the membership function in Fig. 3.1b. Then, to a

specific element of the universe, no longer is a single value assigned as membership degree; instead, the membership function assumes all values where the "vertical" line intersects the blur (in the figure, those values were given equal weight, but in general their might be a membership degree assigned to each value). This additional degree of freedom conveys new information (Fig. 3.2), which offers more control on the effects of uncertainties and vagueness in rule-based fuzzy systems and thus, makes it possible to directly model uncertainties [37,38], which in turn, will offer the possibility of more control in modeling and simulating any phenomenon that contains a considerable amount of vagueness and/or inconsistency. This is very useful when uncertainties affect the nature of specific problems and provides better performance, especially, in modeling real-life situations. A prime example may be modeling the traffic conditions in urban areas for the TD TSP.

A type-2 fuzzy set $\tilde{A}$ is characterized by type-2 membership function $\mu_{\tilde{A}}(x,u)$, where $x\epsilon\ X$ and $u \in J_x \subseteq [0,1]$, and J_x is connected. $\tilde{A}$ is given as follows:

$$\tilde{A} = \{\langle (x,u), \mu_{\tilde{A}}(x,u) \rangle | \forall x\epsilon\ X, \forall u \in J_x \subseteq [0,1]\} \quad (3.6)$$

in which $0 \leq \mu_{\tilde{A}}(x,u) \leq 1$.

When all $\mu_{\tilde{A}}(x,u) = 1$ (on the positive points), then $\tilde{A}$ is an interval type-2 fuzzy set. The uncertainty of the primary memberships of a type-2 fuzzy set $\tilde{A}$ consists of a bounded region called "footprint of uncertainty" (FOU), the union of all primary memberships given in Eq. (3.7):

$$\mathrm{FOU}(\tilde{A}) = \cup_{x\in X} J_x \quad (3.7)$$

FOUs emphasize the distribution that sits on top of the primary membership function of the type-2 fuzzy set.

However, due to the increased computational complexity of applying general type-2 fuzzy sets, a compromise is often accepted to reduce computational needs, namely, the application of *interval type-2 fuzzy sets*. This simplification of the third dimension of the membership function representation makes mathematical operations more manageable.
Since the TD TSP is known as an NP-hard problem itself [4], modeling it by general type-2 fuzzy sets will just increase the complexity further. Thus, from complexity point of view to apply interval type-2 fuzzy sets more is manageable (See section 3.2.3). In fact, interval type-2 fuzzy sets offer a quite practical and tractable model while the double uncertainty feature is still preserved [36,37].

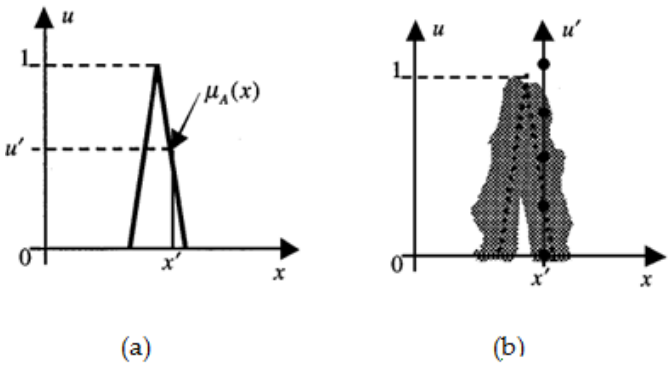

Figure 3.1 (a) Type-1 membership function (b) Blurred type-1 membership function.

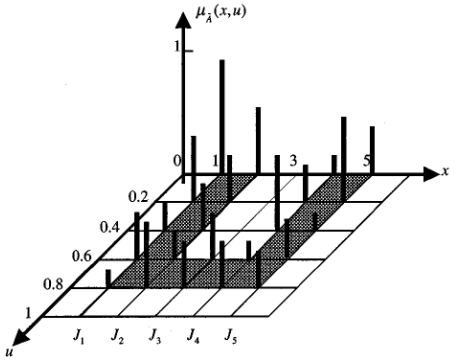

Figure 3.2 General Type-2 fuzzy set.

3.2.3 Interval-Valued Fuzzy Sets

An interval-valued fuzzy set (IVFS) A defined on X is given by :

$$\tilde{A} = \{\langle x, [\mu_A{}^{\mathrm{L}}(x), \mu_A{}^{\mathrm{U}}(x)]\rangle \big| \forall x \epsilon\, X\} \tag{3.8}$$

$$\mu_A{}^{\mathrm{L}}, \mu_A{}^{\mathrm{U}} : X \rightarrow [0,1] \mid \forall x \epsilon\, X, \mu_A{}^{\mathrm{L}}(x) \leq \mu_A{}^{\mathrm{U}}(x) \tag{3.9}$$

Let

$$\tilde{\mu}_A(x) = [\mu_A{}^{\mathrm{L}}(x), \mu_A{}^{\mathrm{U}}(x)], \forall x \epsilon\, X$$

and let $D[0,1]$ be the set of all closed subintervals of the interval $[0,1]$. Therefore, for $x \epsilon\, X$, $\tilde{\mu}_A(x) \in D[0,1]$. Thus,

$$\tilde{A} = \{(x, \tilde{\mu}_A(x))\}, \tilde{\mu}_A : X \rightarrow D[0,1], x \epsilon\, X \tag{3.10}$$

Then, interval $\tilde{\mu}_A(x) = [{\mu_A}^{\mathrm{L}}(x), {\mu_A}^{\mathrm{U}}(x)]$, is assigned for each element $x \epsilon\, X$, where ${\mu_A}^{\mathrm{L}}$ and ${\mu_A}^{\mathrm{U}}$ are the lower extreme and the upper extreme, respectively, of the generalized membership function $\tilde{\mu}_A$.

3.2.4 Interval-Valued Fuzzy Sets Operations

An interval-valued fuzzy set $\tilde{X}$ on the universe U is a mapping such that $\tilde{X}$: $U \rightarrow Int\,([0,1])$, where $Int\,([0,1])$ stands for all closed subintervals of $[0,1]$. The set of all interval-valued fuzzy sets on U is denoted by $\tilde{P}(U)$ [55,56].

$$\tilde{F} : U \rightarrow Int([0,1]),$$

Suppose that $\tilde{X} \in \tilde{P}(U)\ \forall x \in U, \mu_{\tilde{x}}(x) = [{\mu_{\tilde{x}}}^{\mathrm{L}}(x), {\mu_{\tilde{x}}}^{\mathrm{U}}(x)]$ is called the degree of membership of an element x to $\tilde{X}$ and ${\mu_{\tilde{x}}}^{\mathrm{L}}(x)$ and ${\mu_{\tilde{x}}}^{\mathrm{U}}(x)$ are referred to as the lower and upper degrees of membership x to $\tilde{X}$ respectively, where $0 \leq {\mu_{\tilde{x}}}^{\mathrm{L}}(x) \leq {\mu_{\tilde{x}}}^{\mathrm{U}}(x) \leq 1$.

Let $\tilde{X}, \tilde{Y}$ be two IVFS. Such that $\tilde{X}, \tilde{Y} \in \tilde{P}(U)$, then :

1. **The union of $\tilde{X}$ and $\tilde{Y}$ denoted by $\tilde{X} \cup \tilde{Y}$, is given by:**

$$\boldsymbol{\mu_{\tilde{X} \cup \tilde{Y}}(x)} =$$

$$\mathbf{sup}[\,\boldsymbol{\mu_{\tilde{X}}}(\boldsymbol{x}), (\,\boldsymbol{\mu_{\tilde{Y}}}(\boldsymbol{x}),)] = [\sup(\boldsymbol{\mu^{\mathrm{L}}}_{\tilde{X}}(\boldsymbol{x}),\ \boldsymbol{\mu^{\mathrm{L}}}_{\tilde{Y}}(\boldsymbol{x})), \mathbf{sup}(\ \boldsymbol{\mu^{\mathrm{U}}}_{\tilde{X}}(\boldsymbol{x}),\ \boldsymbol{\mu^{\mathrm{U}}}_{\tilde{Y}}(\boldsymbol{x}))] \quad (3.11)$$

2. **The intersection of $\tilde{X}$ and $\tilde{Y}$ denoted by $\tilde{X} \cap \tilde{Y}$ is given by:**

$$\mu_{\tilde{X} \cap \tilde{Y}}(x) =$$

$$\inf[\,\mu_{\tilde{X}}(x),\ \mu_{\tilde{Y}}(x)] = [\inf(\ {\mu^{\mathrm{L}}}_{\tilde{X}}(x),\ {\mu^{\mathrm{L}}}_{\tilde{Y}}(x)), \inf(\ {\mu^{\mathrm{U}}}_{\tilde{X}}(x),\ {\mu^{\mathrm{U}}}_{\tilde{Y}}(x))] \quad (3.12)$$

3. **The complement of $\tilde{X}$, denoted by $\tilde{X}^{c}$, is given by:**

$$\mu_{\tilde{X}^{c}}(x) = 1 - \mu_{\tilde{X}}(x) = \left[1 - {\mu^{\mathrm{U}}}_{\tilde{X}}(x), 1 - {\mu^{\mathrm{L}}}_{\tilde{X}}(x)\right] \quad (3.13)$$

3.2.5 Intuitionistic Fuzzy Sets

The concept of intuitionistic fuzzy sets (IFS) was introduced by Atanassov [39] as a new essential generalization of the original concept of fuzzy sets. The basic idea is that it is not always true that the degree of non-membership of an element in a fuzzy set is equal to the complement of the membership degree (in Zadeh's definition "one minus the membership"). Indeed, due to the subjectivity and uncertainty of the human assessment of the degree of belonging and not belonging there may be a degree of *hesitation* when assigning such conceptually complementary measures, which expresses the difference of the two aspects. This concept is well addressed in IFSs that formally incorporate a degree of hesitation, which is defined as "one minus the sum of membership and non-membership degrees". Adding a hesitation part to either the membership value or to the non-membership value or to both, allows a higher-level representation of the complex nature of the problems on hand, including the modeling of higher-level uncertainties [40,41,42].

Let X be a set (space), with a generic element of X denoted by x, that is $X = \{x\}$. For a set X, an IFS A is given by :

$$\tilde{A} = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \epsilon X\} \quad (3.14)$$

where the functions $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$, with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$.
The numbers $\mu_A(x) \in [0,1]$ and $\nu_A(x) \in [0,1]$, denote the degree of membership and the degree of non-membership of the element x to the set A, respectively. For each IFS, A in X, the amount $\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))$ is called the degree of indeterminacy (hesitation part), which may cater to membership value, non-membership value or both.

3.2.5.1 Operations on Intuitionistic Fuzzy Sets

A crisp relation represents the presence or absence of association, interaction, or interconnectedness between the elements of two or more sets. This concept can be generalized to allow for various degrees or strengths of relation or interaction between elements. Degrees of association can be represented by membership grades in a fuzzy relation in the same way as degrees of set membership are represented in the fuzzy set. In fact, just as the crisp set can be viewed as a restricted case of the more general fuzzy set concept, the crisp relation can be considered to be a restricted case of the fuzzy relations. Indeed, the dual notion to a triangular norm is a triangular conorm (abbreviation t-conorm, also s-norm). The t-norm is sometimes called a *fuzzy intersection*. Depending on the choice of a t-norm, we obtain different fuzzy intersections. Dually, a t-conorm corresponds to a *fuzzy union*.
The use of intuitionistic fuzzy sets, instead of traditional fuzzy sets, provide an even more sensitive and realistic estimate for the non-deterministic factors present in any problem.

Let X and Y be two sets, then a relation between X and Y (over $X \times Y$) is known as a binary relation: xRy means that, x is related to y under the given relation R. Thus, an intuitionistic fuzzy relation (IFR) R is an IFS of $X \times Y$, characterized by the membership function μ_R: $X \times Y \rightarrow [0,1]$ and the non-membership function $v_R : X \times Y \rightarrow [0,1]$. It will be denoted by $R(X \rightarrow Y)$. Where for any $(x, y) \in X \times Y$ the value $\mu_R(x, y)$ shows the degree to which x and y are in relation with R, For any $x \in X$ and $y \in Y$.

A is an IFS of X, the max-min-max composition of the IFR R ($X \rightarrow Y$) with A gives an IFS B of Y, and it is denoted by $(B = R \circ A)$ and is defined by the membership function:

$$\mu_{R \circ A}(y) = \bigvee_x [\mu_A(x) \wedge \mu_R(x, y)] \quad (3.15)$$

and the non-membership function

$$v_{R \circ A}(y) = \bigwedge_x [v_A(x) \vee v_R(x, y)] \quad (3.16)$$

for all $y \in Y$ (where $\bigvee$= max; $\bigwedge$ = min).

3.2.6 Interval-Valued Intuitionistic Fuzzy Sets

Atanassov and Gargov [41] defined the concept of interval-valued intuitionistic fuzzy sets (IVIFS) in 1989, which is a further generalization of the definition of the IFS (Intuitionistic Fuzzy Sets). Each IVIFS is characterized by a pair of a membership function and a non-membership function whose values are intervals rather than real numbers. The IVIFS (Fig. 3.3) is even more powerful in dealing with multiple and complex vagueness and uncertainty than the plain IFS, because it incorporates the hesitation degree (hesitation margin) in calculating the membership function by using the new, third dimension of type-2 fuzzy sets, thus combining the two powerful features of IFS and IVFS [42]. Modeling problems with high uncertainty and hesitation via IVIFS looks more promising in terms of capturing the intangible vagueness factors and their numerical effects. Also, IVIFSs decrease the information loss that comes from using IFSs solely. Indeed, IVIFSs have received much attention from researchers concerning how to deal with the interval-valued intuitionistic fuzzy numbers (IIFNs), which is the basic area of using IVIFS.

We will use the Eqs. (3.1) to (3.10) for IVFS. For a universal set X, an IVIFS A is an object having the form as defined in Eq. (3.11).

$$\tilde{A} = \{\langle x, \tilde{\mu}_A(x), \tilde{v}_A(x)\rangle | x \epsilon X\} \quad (3.17)$$

where $\tilde{\mu}_A(x) \subset [0,1]$ and $\tilde{v}_A(x) \subset [0,1]$ are intervals such that for all $x \epsilon X$ with the condition :

$$0 \leq \sup(\tilde{\mu}_A(x)) + \sup(\tilde{v}_A(x)) \leq 1, \forall x \in X \quad (3.18)$$

The intervals $\tilde{\mu}_A(x)$ and $\tilde{v}_A(x)$ denote the degrees of membership and non-membership of x to $\tilde{A}$, respectively. Clearly, if $\inf(\tilde{\mu}_A(x)) = \sup(\tilde{\mu}_A(x))$ and $\inf(\tilde{v}_A(x)) = \sup(\tilde{v}_A(x))$, then the IVIFS $\tilde{A}$ is reduced to traditional IFS.

In the literature, many ideas were presented to address the definition of some proper interval-valued intuitionistic fuzzy aggregation operators. Xu and Chen [43,44] proposed several arithmetic aggregation operators for IVIFSs, such as the interval-valued intuitionistic fuzzy weighted arithmetic mean aggregation (IIFWA), the interval-valued intuitionistic fuzzy ordered weighted mean aggregation (IIFOWA), and the interval-valued intuitionistic fuzzy hybrid aggregation (IIFHA). Xu and Chen [43,44,45] also introduced the interval-valued intuitionistic fuzzy weighted geometric mean (IIFWG), the interval-valued intuitionistic fuzzy ordered weighted geometric mean (IIFOWG), and the interval-valued intuitionistic fuzzy hybrid geometric mean (IIFHG) operators.

Next, I will introduce the interval-valued intuitionistic fuzzy sets weighted arithmetic average (IIFWAA) operator (see in Eq. (3.19)).

$$\text{IIFWAA}(A) = \left(\left[1-\prod_{i=1}^{n}(1-\mu_{AL}(x_i))^{\omega_i}, 1-\prod_{i=1}^{n}(1-\mu_{AU}(x_i))^{\omega_i}\right], \left[\prod_{i=1}^{n}(v_{AL}(x_i))^{\omega_i}, \prod_{i=1}^{n}(v_{AU}(x_i))^{\omega_i}\right]\right) \quad (3.19)$$

where $\omega = (\omega_1, \omega_2 \dots \dots . \omega_i)^T$ are the weight vectors of *A*. Further, $\omega_i > 0$ and $\sum_{i=1}^{n} \omega_i = 1$.
$\mu_A(x) = [\mu_{AL}(x), \mu_{AU}(x)\ ]$ and $v_A(x) = [v_{AL}(x), v_{AU}(x)\ ]$.
In our simple model, we gave equal weights to all the jam factors by $\omega_i = 1/n$, but the weight can be assigned according to the factor importance.

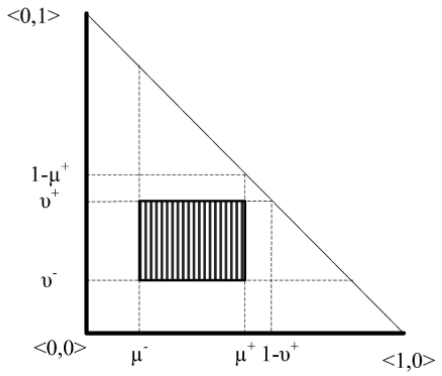

Figure 3.3 Interval-valued intuitionistic fuzzy set

For finding the final jam factors modified costs, I used a measure based on Park's distance between IVIFS. In fact, finding the distance between IFS has attracted many researchers as an alternative to the max-min-max composition. Park et al. [46,47] proposed new distance measures between IVFSs based on the Hamming distance.

In my model I adopted the normalized Hamming distance considering the hesitation part as in Eq. (3.20). I proposed here to give normalized weights to the factors (it can be assigned according to their importance). $w_i = 1/n$. Park's measure [46,47,48] for the normalized Hamming distance considering the hesitation part is defined as follows:

For any two IVIFSs A and B, where $A = \{\langle x_i, \mu_A(x_i), v_A(x_i)\rangle \,|x\epsilon\, X\}, (i = 1,2 \dots \dots n)$ and $B = \{\langle x_i, \mu_B(x_i), v_B(x_i)\rangle \,|x\epsilon\, X\}, (i = 1,2 \dots \dots n)$, the normalized Hamming distance considering the hesitation part is defined as in [46,47,48]:

$$l_h(A,B) = (1/4n)\sum_{i=1}^{n}[|\mu_{AL}(x_i) - \mu_{BL}(x_i)| + |\mu_{AU}(x_i) - \mu_{BU}(x_i)| + |v_{AL}(x_i) - v_{BL}(x_i)| + |v_{AU}(x_i) - v_{BU}(x_i)| + |H_{AL}(x_i) - H_{BL}(x_i)| + |H_{AU}(x_i) - H_{BU}(x_i)|] \quad (3.20)$$

where H is the hesitation part.

3.2.7 Soft Sets

Soft set theory was introduced by Molodtsov [54] in 1999 as a new mathematical theory for dealing with uncertainty, it has been applied in several domains, such as smoothness of functions, game theory, operations research, Riemann integration and probability. In fact, Molodtsov applied this theory in many areas [56,57,58], and then formulated the notions of soft number, soft derivative, soft integral, etc. in [59]. On the other hand, Maji et al. [55] worked on the application of soft set in the decision making problem using the reduction of rough sets [60]. Chen et al. [61] proposed parametrization reduction of soft sets, and then Kong et al. [62] presented the normal parametrization reduction of soft sets.
This growing interest led many researchers to study the properties and applications on the soft set theory [63,64,65,66]. In contrast with the axiomatic approach in the classical probability theory, the soft probability is directly based on measurements [67,68]. The soft probability may be applied to any event, in contrast to the classical probability which may be applied only to stochastically stable events. If the event is stochastically stable, then the soft probability will be a sufficiently narrow interval with a large trust degree if n is large enough. If the event is not stochastically stable, then the soft probability may be a large interval. The proposed approach can also be applied to construct a theory of soft clusters [54].

In this section, we present the basic definitions of soft set theory [56] and fuzzy set theory [9] that are useful for subsequent discussions.

Throughout this work, U to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A \subseteq E$.

A soft set F_A over U is a set defined by a function f_A representing a mapping

$$f_A : E \to P(U) \text{ such that } f_A(x) = \emptyset \; if \; x \notin A \quad (3.21)$$

Here, f_A is called approximate function of the soft set F_A, and the value $f_A(x)$ is a set called $x - element$ of the soft set for all $x \in E$. It is worth noting that the sets $f_A(x)$ may be arbitrary, empty, or have nonempty intersection. Thus a soft set over U can be represented by the set of ordered pairs. Also, the set of all soft sets over U will be denoted by $S(U)$.

$$F_A = \{x, f_A(x) : x \in E, f_A(x) \in P(U)\} \quad (3.22)$$

3.2.8 Fuzzy Soft Sets

Type-2 fuzzy sets allowed more degrees of freedom in representing uncertainty while simulating any phenomenon that contains a considerable amount of vagueness or inconsistency [37]. This in turn gave better calculation when applied on real-life situations as many researchers showed [48 ,49,50,51,52]. After Molodtsov [54] introduced the Soft set, Maji et al. [70] presented the concept of the fuzzy soft sets, he also defined many operations for soft sets to make a detailed theoretical study on them. Also, Maji et al. successfully applied the fuzzy soft set in decision-making problems [62,65]. Mukherjee and Chakraborty [71] worked on intuitionistic fuzzy soft relations. Yang et al. [73] defined the operations on fuzzy soft sets which are based on three fuzzy logic operators: negation, triangular norm and triangular conorm. Zou and Xiao [74] introduced the soft set and fuzzy soft set into the incomplete environment. Also, Cagman et al [75,76] defined the concept of fuzzy parameterized fuzzy soft set (FPFS-set).

Let U be a universal set, E a set of parameters and $A \subset E$. Let $F(U)$ denotes the set of all fuzzy subsets of U. Then a pair (F, A) is called a fuzzy soft set over U, where F is a mapping from A to $F(U)$.

A fuzzy soft set $\tilde{F}_A$ over U is a set defined by a function $\tilde{\mu}_A$ that represents the mapping

$$\tilde{\mu}_A(x) : E \to F(U) \text{ such that } \tilde{\mu}_A(x) = \emptyset \; if \; x \notin A \quad (3.23)$$

Here, $\tilde{\mu}_A$ is called fuzzy approximate function of the fuzzy set $\tilde{\mathrm{F}}_A$, and the value $\tilde{\mu}_A(x)$ is a set called *x-element* of the fuzzy soft sets for all $x \in E$. Thus, $\tilde{\mathrm{F}}_A$ over U can be represented by the set of ordered pairs

$$\tilde{\mathrm{F}}_A = \{(x, \tilde{\mu}_A(x)) : x \in E, \tilde{\mu}_A(x) \in F(U)\} \quad (3.24)$$

all the fuzzy sets over U will be denoted by $F(U)$ and the set of all fuzzy soft-sets over U will be denoted by $FS(U)$

3.2.9 Interval-Valued Fuzzy Soft Sets and Operations.

Yang et al. [77] presented the combination of interval-valued fuzzy set and soft set. Kong et al. [78] defined the normal parameter reduction in the fuzzy sets, and showed that Roy and Maji's [80,81] algorithm is not convenient in general cases. The combined result of fuzzy set and soft set theory is a fuzzy soft set. However, it should be noticed that in many real applications, the membership degree in a fuzzy set cannot be lightly confirmed. It is more reasonable to give an interval-valued data to describe membership degree. From such point of view, Zadeh further proposed the concept of an interval-valued fuzzy set. By combining the interval-valued fuzzy set and soft set, it is natural to define the interval-valued fuzzy soft set model [82].

(IVFS fully explained in sections 3.2.3 and 3.2.4 which will be built on here), let U be an initial universal set, and E be a set of parameters. A pair $(\widetilde{F}, E)$ is called an interval-valued fuzzy soft set over $\tilde{P}(U)$, where $\tilde{F}$ is a mapping given by:

$$\tilde{F}: E \rightarrow \tilde{P}(U) \qquad (3.25)$$

An interval-valued fuzzy soft set is a parameterized family of interval-valued fuzzy subsets of U, thus, its universe is the set of all interval-valued fuzzy sets of U (ie. $(\tilde{P}(U))$. An interval-valued fuzzy soft set is also a special case of a soft set because it is still a mapping from parameters to $\tilde{P}(U)$.

$\forall\, e \in E$, $\tilde{F}(e)$ is referred as the interval fuzzy value set of parameter e, it is actually an interval-valued fuzzy set of U where $x \in U$ and $e \in E$, it can be written as:

$$\tilde{F}(e) = \{\langle x, \mu_{\tilde{F}(e)}(x)\rangle : x \in U : E \rightarrow \tilde{P}(U))\} \qquad (3.26)$$

$\tilde{F}(e)$ is the interval-valued fuzzy membership degree that object x holds on parameter e. If $\forall\, e \in E$ and $\forall\, x \in U$, $\mu^{L}{}_{\tilde{F}(e)}(x) = \mu^{U}{}_{\tilde{F}(e)}(x)$, then $\tilde{F}(e)$ will degenerated to be a standard fuzzy set and then $(\tilde{F}, E)$ will be degenerated to be a traditional fuzzy soft set.

3.3 Evolutionary algorithms

3.3.1 The Genetic Algorithms (GA)

Genetic Algorithms (GA) are the archetypical evolutionary search and optimization algorithms that are based on concepts of natural selection and natural genetics that belong to the larger class of evolutionary algorithms (EA). The original GA algorithm was developed to simulate some of the processes observed in natural evolution, a process that operates on chromosomes (organic devices for encoding the structure of living being). They are commonly used to generate high-quality solutions to optimization and search problems by relying on biologically inspired operators such as mutation, crossover and selection. A genetic

algorithm (GA) searches among a population (of points) and uses objective function information without any gradient information [31]. The transition scheme of the GAs are used as general purpose optimization algorithm. They are also efficient to search in irregular spaces, and thus, are applied to a variety of function optimizations and parameters estimation problems [32,33,34].

3.3.2 The Discrete Bacterial Memetic Evolutionary Algorithm (DBMEA)

The DBMEA is a memetic algorithm, a combination of the bacterial evolutionary algorithm and 2-opt, 3-opt local search [26,27]. Memetic algorithms combine the global search evolutionary algorithms with local search methods; thus, they can eliminate the disadvantages of both methods. Also, in many cases the addition of local search approaches usually can significantly improve the performance of the classical evolutionary algorithms, so the Memetic algorithms can be efficient tools for solving TSP and other NP-hard optimization problems. Hence, significant improvement in the performance of the classical evolutionary algorithms was achieved. Since its run time is more predictable than in the case of Concorde algorithm, then DBMEA is considered efficient, especially for large-sized problems with a complicated structure. Moreover, testing DBMEA for the biggest tested instance, it could give even a better solution than the best-known value and the average runtime was smaller than in the case of the state-of-the-art methods for the problem [15,16,17,18].

Below is a brief description on how the DBMEA performs the optimization [23,26,27]:

Creating the Initial Population: Population is a group of individuals which mean solutions for the problem. In the DBMEA algorithm the individuals represent possible tours for the Minimum Latency Problem. In the DBMEA for Minimum Latency Problem the individuals in the initial Population are generated randomly

Bacterial mutation: bacterial mutation optimizes the bacteria individually. It tries to improve the bacterium by modifying randomly the segments in the clones. As a result of the bacterial mutation the bacteria are more or equally fit than the original bacteria.

Gene transfer: the gene transfer operation allows the transfer of information between the bacteria in the population in the hope that the bacteria become better and better. In the gene transfer operation a “good” bacterium transfers a part of its solution to a “bad” bacterium.

Local search: Local search techniques are crucial parts of every memetic algorithm because they are responsible for the improvement of the candidate solutions in the population. In the DBMEA for solving the Minimum Latency Problem the local searches are the 2-opt and 3-opt techniques. The execution time of the local search was reduced by examining only the close vertices to every vertex.

*2-opt local search***:** 2-opt local search replaces two edge pairs in the original graph to reduce the length of the tour. Itis stopped when no further improvement is possible.

*3-opt local search***:** the 3-opt local search replaces edge triples. The deleting of three edges results three sub-tours. There are four possible ways to reconnect these sub-tours. The output of the 3-opt step is always the less costly tour.

3.4 The TOPSIS Approach

Among the many approaches of MADM, it is possible to consider a subgroup of methods that involves cost and benefit calculations. One of the most widely used approach among them is the Technique for Order Performance by Similarity to Ideal Solution (TOPSIS). TOPSIS was first proposed in 1981 by Hwang and Yoon [83]. This approach is employed mainly for four reasons; it is logically rational and understandable, the computation processes are straightforward, the concept permits the pursuit of the best alternatives for each criterion depicted in a simple mathematical form, and lastly, the importance weights are incorporated in the comparison procedures. Also, the number of steps in the TOPSIS method remains the same, regardless of the number of attributes [84,85]. The TOPSIS method adopts the concepts of “ideal” and “anti-ideal” solutions as suggested by Hwang and Yoon and computes the relative distances from the ideal and anti-ideal solutions for each alternative. The best alternative should be as close as possible to the ideal solution and as far as possible from the anti-ideal solution in a multi-dimensional computing space [85,86].

Chapter 4 The Triple Fuzzy Time Dependent Travelling Salesman Problem (3FTD TSP)

4.1 Introduction

The Traveling Salesman Problem (TSP) is one of the most often studied NP-hard graph search problems. There have been numerous publications in the literature that applied various approaches to find the optimal or semi-optimal solution. Although the problem is computationally intractable, the Time Dependent Traveling Salesman Problem (TD TSP) is one of the most realistic extensions of the original TSP problem. In the TD TSP, the added costs (to the actual distances) between nodes vary, namely, the edges are assigned higher costs if they were travelled within a predefined oblong shaped area (the jam region in the city center). Realizing that the jam regions and the rush hours costs on a tour are uncertain and can never be accurately represented by concrete numbers, firstly, I introduced the novel 3FTD TSP (Triple Fuzzy Time Dependent Traveling Salesman Problem); the first fully fuzzy-based model of the original TD TSP. The 3FTD TSP utilizes fuzzy values for determining the costs between any two vertices within the traffic jam regions and during the rush hours periods more precisely. Secondly, I extended the 3FTD TSP and applied it on the biggest universal instances in the literature. I also proposed the application of the DBMEA (Discrete Bacterial Memetic Evolutionary Algorithm), as a meta-heuristic and the classic GA (Genetic Algorithm) enabling the reader to compare the accuracy and the speed of the (quasi-) optimum solutions convergence.

4.2 Related Work

In the literature, there have been numerous proposed extensions and variations of the original TSP to simulate the added costs by the traffic conditions (such as traffic regions and rush hours' delays that occur during the day). Those models assigned costs to the edges connecting the nodes (cities) based on Euclidean Distances. However, those costs are constant, which is an obvious limitation when applied to actual cases. Because the traffic circumstances are intangible factors. Moving one step towards a more realistic model, the TD TSP introduced a viable refinement for the costs calculation, yet not efficient enough, since it is still a deterministic solution [4]. In fact, the TD TSP assigns a fixed part of the graph to represent the "traffic jam region" in a time dependent form, namely, each graph edge is assigned given costs in the non-rush hour periods, and higher costs during the rush hour times [4]. Afterwards, those crisp numbers are multiplied with the length of the corresponding edge between the nodes in the given rush hour times, in order to quantify the jam factor's effects. Despite the TD TSP's ability to achieve good results in determining the overall cost, one main drawback is the use of concrete numbers for representing the proportional jam factors and the simplified representation of the traffic jam region (city centre). After the classical TD TSP, there have been many other attempts by researchers to quantify the rush hours and the jam regions effects on the trip's overall cost. Some models used Ant Colony optimization to predict traffic for the TD TSP [5]. Others used a mixed approach, where they incorporated integer programming model to calculate the travelled time, then integrated both Ant Colony System (ACS) and Tabu Search (TS) algorithms [6] to calculate the overall

cost. Other authors used linear constrains formulation techniques to present the jam factors' effect on any trip [7]. Also, dynamic programming algorithm was proposed to solve the TD TSP [8]. Looking closely, there is one common aspect in all the previously proposed approaches, that is, using crisp numbers to simulate those uncertain and imprecise road conditions, indeed, this is an insufficient formulation. In fact, it would be closer to reality to use fuzzy numbers to capture the uncertainties associated with road conditions. There was one attempt to apply fuzzy sets on the TSP in 2017; the fuzzy based solution to the travelling salesman problem introduced [19], this model was applied on 5 nodes (cities) and used triangular membership functions to represent all traffic conditions as added costs to the overall edges length. The drawback of this representation is obvious, the model's efficiency, practicality or applicability was never proven or tested on large scale instances.

4.3 Methodology

This part of my work was divided into two main approaches:

1. Introducing a novel fully fuzzy-based mathematical model for the TD TSP (the 3FTD TSP) and testing it on small size (but representative) instances problem containing 12 nodes (cities) to prove its preliminary functionality.

2. Optimizing the Extended Triple Fuzzy Time Dependent Travelling Salesman Problem (3FTD TSP) on the largest universal benchmarks with DBMEA (Discrete Bacterial Memetic Evolutionary Algorithm) and GA (Genetic Algorithm).

The original TSP was first formulated in 1930 [1], it involves a salesman who starts the journey from the company headquarters, visits all planned destination (cities or shops) exactly once, and then returns to the original starting point with the minimum overall travelled distance. TSP can be defined as a graph search problem with edge weights (costs) as in Eq. (4.1). To formulate the symmetric case with *n* nodes (cities) where $c_{ij} = c_{ji}$ represents the cost of going from city (vertex) i to city j. G_{TSP} in the graph on hand. Then, C is called cost (time) matrix, $C = [c_{ij}]$.

Let $G_{TSP} = \langle V_{cities}, E_{conn} \rangle$

where

$$V_{cities} = v_1 \cup \{v_2, \dots v_i \dots, v_n\}, E_{conn} \subseteq \{(v_i, v_j) | i \neq j\}, i = (2 \dots, n) \qquad (4.1)$$

v_1 is the starting vertex. E_{conn} is the set of all edges. The cost matrix C is the mapping all subscripts of the vertices, thus:

$$C: V_{cities} \times V_{cities} \rightarrow R^+ \qquad (4.2)$$

R^+ is the range of the costs from the set of edges to the positive real lines.

The initial set of vertices in any tour is $\{1, k_2, \dots k_i \dots, k_n\}$ where $k_1 = 1$ the starting subscript, $i = (2 \dots, n)$. The goal is to find the path of edges that minimizes the sum of the costs (C_{sum}) by traveling through the vertices exactly once.
C_{sum} calculated in Eq. (4.3) and should be minimized as in Eq. (4.4). Here c_{1,k_2} is the cost of traveling the starting edge, and $c_{k_{n-1},1}$ is the cost of returning to the first vertex (traveling the last edge).

Thus, C_{sum} can be written as:

$$C_{sum} = c_{1,k_2} + \sum_{j=2}^{n} c_{k_j,k_{j+1}} \, , (j = 2, \dots, n-1) \, , c_{k_j,k_{j+1}} \epsilon \left[c_{ij}\right], k_2, k_n \epsilon \{2, \dots, n\} \qquad (4.3)$$

Then to minimizes the sum of the costs:

$$\text{Minimize } C_{sum} \qquad (4.4)$$

A valid permutation for vertices (the subscripts) is:

$$\{v_1, v_{k_2}, \dots v_{k_i} \dots, v_{k_n}\} \qquad (4.5)$$

where $i = (2 \dots, n)$. v_1 is the starting vertex.

Let $\{k_1, \dots . k_n\}$ be a permutation of $\{1 \dots, n\}$, then a valid sequence of edges that forms a path that goes through all vertices (exactly once and goes back to the first one) can be expressed in the form:

$$e_{1,k_2}, e_{k_2,k_3}, \dots . e_{k_i,k_j} \dots \dots e_{k_{n-1},1} \, , i, j \, \epsilon \{2 \dots, n\} \qquad (4.6)$$

where first edge is e_{1k_2} , $k_i \neq k_j$ so that $i \neq j$.

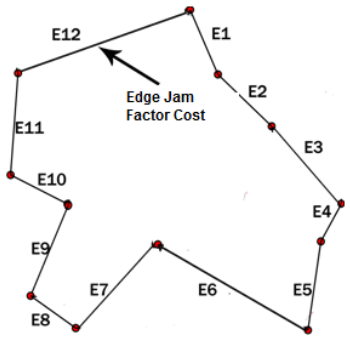

Figure 4.1 Tour for a Simple Example

Considering an actual trip (for simplicity we will consider small sized tour Fig. 4.1), clearly, it is subjected not only to the topography of the given route, but also to the unexpected circumstances and road conditions, which affect the overall timing (cost) of the tour. Naturally, in the city centers and during the rush hours periods, trips between nodes (vertices) take longer. Thus, those two factors must be looked at as vague or uncertain (non-deterministic but not in a random sense) values, and the relevant estimated costs for any tour traveled between two nodes are not constant themselves. Hence, it is more appropriate to represent those imprecise (uncertain) costs using fuzzy numbers. This was my main motivation for introducing the novel 3FTD TSP model, which is a more adequate and realistic version of the classic TD TSP (the crisp version fuzzy-based from multiple aspects).

4.3.1 The Triangular Fuzzy Costs of the Time Dependent Traveling Salesman Problem (3FTD TSP)

The 3FTD TSP is so far the only fully extended and published fuzzy version of the classic TD TSP. In the 3FTD TSP model, three "modifying parameters" of the edge costs were represented using fuzzy sets. Each cost in the trip has its own membership function, namely, the fuzzy time (cost) needed for crossing edge (v_i , v_j) which is denoted by $\tilde{c}_{ij}$. The fuzzy jam region influenced cost with membership function $\mu_J(v_i, v_j)$ and the fuzzy rush hours cost with membership function $\mu_{RH}(h)$. Hence, according to the degree of belonging to the specified jam region or if the edge was being crossed during the fuzzy rush hours; the original fuzzified time of the related edge(s) will be influenced accordingly. It is worth mentioning that, in our fuzzy formulation, the cost of crossing the edges is defined by the time needed to cross them. The cost between edges is expressed by time, thus, from here on we will refer to time as cost. Moreover, cost here is a fuzzy set of the universe of the potential costs between two vertices (the universe of potential travelling time).

Let c_{ij} be the cost of traveling between vertices v_i and v_j, $\tilde{C}$ is the fuzzy cost matrix, which can be defined as:

$$\tilde{C} = [\tilde{c}_{ij}] \tag{4.7}$$

where $\tilde{c}_{ij} : T \rightarrow [0, 1]$. $i \neq j$, and $i, j \in \{1, \dots . n\}$.
T is the universe of the costs $[0, \max\{T\}]$, such that $(\max\{T\})$ is the highest cost in the trip under study.

Thus,

$$\tilde{c}(v_i, v_j) = \tilde{c}_{ij} \tag{4.8}$$

$$\tilde{c}_{ij} = \langle T, \mu_{ij}(t): T \rightarrow [0, 1] \rangle \text{ , such that } t \in T \tag{4.9}$$

In our illustrative example, $\mu_{ij}(t)$ defines $\tilde{c}_{ij}$ over T (the traveling time between vertices v_i and v_j). $\mu_{ij}(t)$ is a triangular membership function between vertices v_i and v_j. x, y, and z represent the break points of $\langle \mu_{ij}(x, y, z) \rangle$ in Eq. (4.11):

$$\tilde{c}(v_i, v_j) = \langle T, \mu_{ij}(x, y, z) \rangle \tag{4.10}$$

$$\mu_{ij}(x,y,z) = \begin{cases} 0 & (v_i, v_j) \leq x \\ \frac{(v_i,v_j)-x}{y-x} & x \leq (v_i, v_j) \leq y \\ \frac{z-(v_i,v_j)}{z-y} & y \leq (v_i, v_j) \leq z \\ 0 & z \leq (v_i, v_j) \end{cases} = \max\left\{\min\left\{\frac{(v_i,v_j)-x}{y-x}, \frac{z-(v_i,v_j)}{z-y}\right\}, 0\right\} \tag{4.11}$$

To calculate the modified cost due to crossing the jam region, we assume (in the presented example) there is a center point at the city center, and the jam region has a truncated conic shaped membership function $\mu_J(v)$ (see Fig.4.2.). Where the membership degree depends on the distance measured from the center point. Since we assume a truncated conic type membership function for the jam region $\langle x, y; r1, r2 \rangle$, there are two radiuses ($r1$ and $r2$); one that of the conical kernel ($r1$) and the other is that of the support ($r2$).

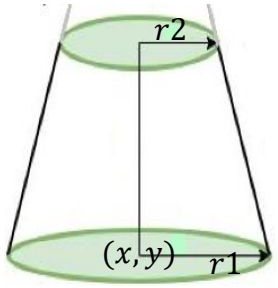

Figure 4.2 Truncated conic jam region representation

To calculate the modified cost that affects edge (v_i, v_j) due to crossing the jam region, we must first find the membership degree μ_J of each vertex.

Let V be set of vertices that is $V = v_1 \cup \{v_2, \dots v_i \dots, v_n\}, v_1$ is the starting vertex. $i = (2, \dots n)$. J denotes the fuzzy set of the jam region.

$$J = \langle V, \mu_J(v): V \rightarrow [0,1] \rangle \quad , v \in V \tag{4.12}$$

Then, to calculate the cost modified by being (partially) in the jam region for edge (v_i, v_j) we apply Eq.(4.13):

$$\mu_{CJ}(v_i, v_j) = \frac{\mu_J(v_i)+\mu_J(v_j)}{2} \tag{4.13}$$

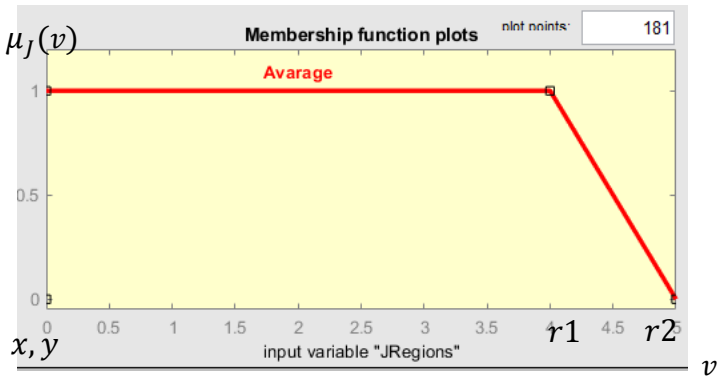

Figure 4.3 One dimensional cut of membership function (μ_J)

Fig. 4.3 represent the cut of the jam region membership function in two dimensions (for our illustrative example), that has trapezoidal shape with break point values [0,0, 4000, 5000].

We use Eq. (4.14) which defines a trapezoidal membership function$(v; \dot{a}, \dot{b}, \dot{c}, \dot{d})$, where $\dot{a}, \dot{b}, \dot{c}$ and $\dot{d}$ represent the break points.

$$\mu_J(v; \dot{a}, \dot{b}, \dot{c}) = \begin{cases} 1 & v \leq \dot{c} \\ \frac{\dot{d}-v}{\dot{d}-\dot{c}} & \dot{c} \leq v \leq \dot{d} \\ 0 & \dot{d} \leq v \end{cases} \tag{4.14}$$

where $\dot{a} = \dot{b} = \dot{c}$ (in our example).

This formulation eliminates (to some extent) one of Schneider's model inaccuracies, which ignores the intersections of the jam regions and the fact that such modified cost builds up gradually (which is better captured using fuzzy sets as we have shown).

To show the effect of the modified fuzzy cost that might affect a trip is the rush hour periods. We propose a double trapezoidal membership function for the traffic rush hour periods during the day (in our simple illustrative example, see Fig. 4.4). It has the break point values {5 ,7.5, 10, 12.5, 12.5, 15, 17.5, 20}, and its membership values are: {0, 1, 1, 0, 0, 1, 1, 0, 0}.
$\mu_{RH}(h)$ was represented with two main rush hour periods during the day (with membership values equal to 1) the peaks are from 7.5 a.m. to 10 a.m. and from 3 p.m. to 5.5 p.m. Between the two periods the traffic intensity is lower but not zero.

Let H be the hours' time line (24 hours of the day), h the elements of the time line such as $h \in H$. The membership function for the rush hours is:

$$RH: \mu_{RH}(h): H \rightarrow [0,1], h \in H \tag{4.15}$$

$\mu_{RH}(h)$ is the degree of membership of belonging to the rush hour periods (whether the edge was crossed during rush hours periods).

Here a, b, c and d represent the break points, then the a trapezoidal $(h;\ a, b, c, d)$ membership function for the rush hours periods (in the morning or the evening in our example) can be written as in Eq. (4.16):

$$\mu_{RH}(h) = \begin{cases} 0 & h \leq a \\ \frac{h-a}{b-a} & a \leq h \leq b \\ 1 & b \leq h \leq c \\ \frac{d-h}{d-c} & c \leq h \leq d \\ 0 & d \leq h \end{cases} \tag{4.16}$$

$\mu_{RH}(h) = \max\left\{\min\left\{\frac{h-a}{b-a}, 1, \frac{d-h}{d-c}\right\}, 0\right\}$.

Finally, to calculate the resulting fuzzy cost that can possibility affect a trip (all edges in a trip), we must first sum the resulting fuzzy costs on each edge (v_i, v_j), then on the whole trip.

Hence, for calculating the resulting fuzzy cost $\tilde{C}_{sum}$ between vertex v_i and v_j we apply Eq.(4.17).

$$\tilde{C}_{sum}(v_i, v_j) = \left[\tilde{c}(v_i, v_j)\right] * [1 + \mu_{RH}(h)] * \left[1 + \frac{\mu_J(v_i) + \mu_J(v_j)}{2}\right] \tag{4.17}$$

Let $\{k_1, \ldots . k_n\}$ be a permutation of $\{1 \ldots, n\}$, then a valid sequence of edges that forms a path that goes through all vertices (exactly once and goes back to the first one) can be expressed in the form:

$$p = e_1, e_{k_2,k_3}, \ldots . e_{k_i,k_j} \ldots \ldots, e_1, i \neq j \text{ so that } k_i \neq k_j \text{ and } i, j \in \{1 \ldots, n\} \tag{4.18}$$

where the first edge is e_1.

To calculate the cumulative modified fuzzy cost for the first two edges e_1(with membership break points a, b, c) and e_{k_2,k_3} (with membership break points p, q, r) whose triangular membership functions are shown in Eq (4.11), we apply Eq.4.19 [87]:

$$\bar{\bar{C}}_{sum}(e_1 + e_{k_2,k_3}) = \mu_{e_1+e_{k_2,k_3}}(t) = \begin{cases} \frac{t-(a+p)}{(b+q)-(a+p)}, & (a+p) \le t \le (b+q) \\ \frac{(c+r)-t}{(c+r)-(b+q)}, & (b+q) \le t \le (c+r) \end{cases} \quad (4.19)$$

We apply this addition to all edges of the path (p) to find $\bar{\bar{C}}_{sum}(p)$:

$$\bar{\bar{C}}_{sum}(p) = \begin{cases} \frac{t-(\sum_{ij\in n} a_{ij})}{(\sum_{ij\in n} b_{ij})-(\sum_{ij\in n} a_{ij})}, & (\sum_{ij\in n} a_{ij}) \le t \le (\sum_{ij\in n} b_{ij}) \\ \frac{(\sum_{ij\in n} c_{ij})-t}{(\sum_{ij\in n} c_{ij})-(\sum_{ij\in n} b_{ij})}, & (\sum_{ij\in n} b_{ij}) \le t \le (\sum_{ij\in n} c_{ij}) \end{cases} \quad (4.20)$$

In Fig. 4.3 And Fig. 4.4, the following three elements are described by fuzzy membership functions [zero values here mean the normal traffic, away from the city centre and not during rush hours]:

- Triangular fuzzy costs (time) between the edges; $\mu_{ij}(t)$.
- Membership function of the fuzzy jam region; $\mu_{CJ}(v_i, v_j)$.
- Membership function(s) of the traffic rush hour time period(s) $\mu_{RH}(h)$.

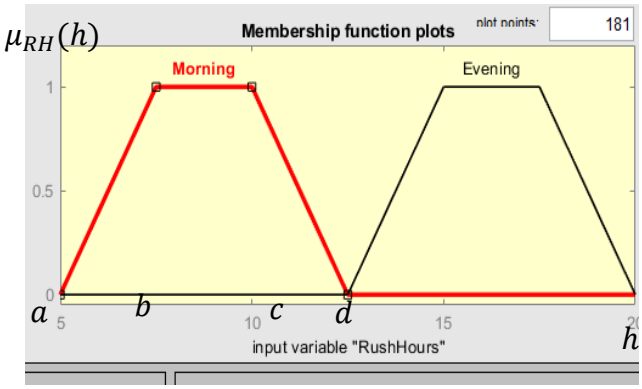

Figure 4.4 Traffic rush hours' membership function (μ_{RH})

4.3.2 Defuzzification Operation in the Extended 3FTD TSP

Defuzzification is the process of obtaining a single number from the output of the aggregated fuzzy set. It is used to transfer fuzzy inference results into a crisp output [87,88]. When I extended the 3FTD TSP, I took in consideration that an edge can be affected by different jam factors simultaneously. For example, it can cross a jam region during the rush hours. Thus, I used the weighted average method [88,89] for finding the crisp output value and add it to the overall cost of each edge respectively. As in Eq. (4.21) x_j^* computed by weighting each membership function in the output by its respective maximum membership values and dividing it by the sum of the weighting functions. This representation was chosen when I developed the extended 3FTD TSP C++ modules [15] due to its computational efficiency (by conducting many experiments, the calculation of COA and weighted average method showed very close results). This concept is presented in Eq. (4.21) and in Fig.4.5., where $\mu_{\bar{C}_i}$ the centroid of each symmetric membership function and x_j^* the total defuzzified number. $i, j \in \{1,2 \dots, n\}$:

$$x_j^* = \frac{\sum_{i=1}^{n}\left[x_{ij} * \mu_{\bar{C}_{ij}}(x_{ij})\right]}{\sum_{i=1}^{n} \mu_{\bar{C}_{ij}}(x_{ij})} \tag{4.21}$$

Finally, these crisp numbers x_j^* are summed up providing the total added costs of the tour under study.

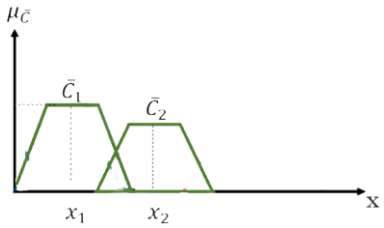

Figure 4.5 Weighted average method of defuzzification

4.3.3 Mamdani-type Fuzzy Rule-based Modelling Inference System.

Zadeh in 1965 introduced fuzzy concept to develop a mathematical framework for imprecisely presented data. It is a generalized form of interval analysis, that the parameters are defined with lower and upper values. In a traditional set theory, an element is defined as a member of a set. If the element is in the set, the membership grade is unity, otherwise it is zero. A fuzzy set is an extension of the traditional set theory in which elements have certain degree of membership between zero and one $\mu(0-1)$ in the set. Fuzzy set establishes the relationship between uncertain data and the membership function μ, which ranges from 0 to 1 [9]. Thus, a fuzzy model expresses a complex system in the form of fuzzy implications.

In 1975, Mamdani represented one of the first fuzzy systems which applied a set of fuzzy rules supplied by experienced human operators to control a steam engine and boiler combination. This approach has been successfully used in a variety of industrial problems. Fig 4.6. shows the main components of a fuzzy inference system [9,49].
Moreover, the Mamdani model can be built by using linguistic relationships (implications) and observed data. The Mamdani-based fuzzy models use excessive number of rules for system modelling (Fig 4.6.) as below:
Let X be input (regression) matrix and g an output vector defined as follows:

$$X = [x_1, \ldots\ldots, x_2]^T = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ \cdot & \cdot \\ \cdot\cdot & \cdot\cdot \\ x_{n1} & x_{n2} \end{bmatrix}$$

$$g = [g_1, \ldots\ldots, g_n]$$

A fuzzifier establishes the mapping from crisp input values to fuzzy sets defined in the universe of discourse of those inputs in a FIS (fuzzy inference system). The membership function of the fuzzy set is defined over the universe of discourse U associated to a crisp input value. $U = U_1 \times U_{2\ldots} \times U_n \subset R^n$ is the input space and $V \subset R$ is the output space. A fuzzy rule base contains a set of fuzzy IF-THEN rules. The fuzzy IF-THEN rules are recognized as the core of the FIS, and all other components such as membership functions have been used to implement these rules in a reasonable, realistic and efficient manner. These fuzzy IF-THEN rules are employed by the FIS to define a mapping from fuzzy sets in the input universe of discourse $U \subset R^n$ to fuzzy sets in the output universe of discourse $V \subset R$, based on fuzzy logic principles. The practical experience discloses that these fuzzy IF-THEN rules make a convenient framework to incorporate human expert knowledge. The rules also allow quantitative, qualitative and judgmental data to be integrated in a uniform manner [9]. The fuzzy IF-THEN rules can be defined as:

$$R^{(l)}: \text{IF } x_1 \text{ is } F_1^l \text{ and } x_2 \text{ is } F_2^l \text{ ....and } x_n \text{ is } F_n^l \text{ then } y \text{ is } G^l \qquad (4.22)$$

where F_i^l ; and G^l are fuzzy sets in $U_n \subset R^n$ and $x = (x_1, x_2, \ldots x_n)^T \in U$ and $y \in V$ are input and output linguistic variables of the FIS which belong to the input and output universes, respectively.

Let M be the number of rules in the fuzzy rule base ($l = 1,2, \ldots . M$). The practical experience discloses that these fuzzy IF-THEN rule represents fuzzy set F_1^l, F_2^l .....$F_n^l \rightarrow G^l$ for $l = 1,2, \ldots . M$; in the product space $U \times V$. The processing structure (which is presented by these rules) puts into effect the inference process on the system inputs. The fulfilment of rule antecedent gives rise to the execution of its consequent output.

On the other hand, the defuzzifier performs the defuzzification, where the output fuzzy numbers are changed to a crisp number. The defuzzifier maps fuzzy sets in V to crisp points in V, it performs the opposite

operation of the fuzzifier by defining a mapping between fuzzy sets defined in the output domain V and crisp values defined in the same universe. Moreover, in the defuzzification interface the aggregation of the information provided by the individual outputs must be performed to obtain a crisp output value (Fig 4.7 visualize the output membership functions of the FIS in hand). It should be mentioned that aggregation could be done by utilizing different operators, such as arithmetic, geometric, or harmonic means, Min and Max [9,63] and center of area (COA) [87,88].

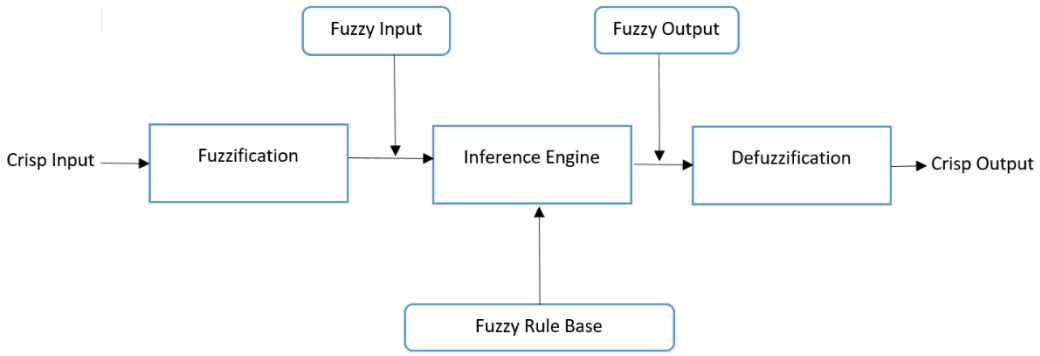

Figure 4.6 Mamdani fuzzy inference system (FIS)

In the Mamdani inference method, the consequents in the base of rules are calculated by the specialist idea [90,91,92]. The t-norm ∧ (minimum) is usually adopted for the logic connective ‘‘and’’ as expressed by Eq. (4.23):

$$\mu_A(x) \text{ AND } \mu_B(x) = \text{MIN}\{\mu_A(x), \mu_B(x)\} \quad (4.23)$$

for the logic connective ‘‘or’’ s-norm, ∨ (maximum) is usually represented.

$$\mu_A(x) \text{ OR } \mu_B(x) = \text{MAX}\{\mu_A(x), \mu_B(x)\} \quad (4.24)$$

Alternative operators are the Max-Min (Zadeh) and the multiplication (Larsen), respectively, in Eqs. (4.25) and (4.26):

$$\mu_R(x,y) = \text{MAX}\{1 - \mu_A(x), \text{MIN}\{\mu_A(x), \mu_B(x)\}\} \quad (4.25)$$

$$\mu_R(x,y) = \{\mu_A(x) * \mu_B(x)\} \quad (4.26)$$

The composition operators of fuzzy relations commonly used are Max-Min, Max-prod and Max-Media, respectively, in Eqs. (4.27), (4.28) and (4.29) [91]. Suppose S is a fuzzy relation on the Cartesian space $X \times Y$, R is a fuzzy relation on Cartesian space $Y \times Z$, then:

$$S \circ R = \mu_{S_{(x,y)}} \circ \mu_{R_{(y,z)}} = \mathrm{MAX}_y\{\mathrm{MIN}\{\mu_s(x,y), \mu_R(y,z)\}\} \quad (4.27)$$

$$S.R = \mu_{S_{(x,y)}}.\mu_{R_{(y,z)}} = \mathrm{MAX}_y\{(\mu_s(x,y) \times \mu_R(y,z))\} \quad (4.28)$$

$$S \oplus R = \mu_{S_{(x,y)}} \oplus \mu_{R_{(y,z)}} = \mathrm{MAX}_y\{1/2\,(\mu_s(x,y) + \mu_R(y,z))\} \quad (4.29)$$

The advantages of the Mamdani fuzzy inference system:

1. It's intuitive.
2. It has widespread acceptance.
3. It's well suited to human cognition.

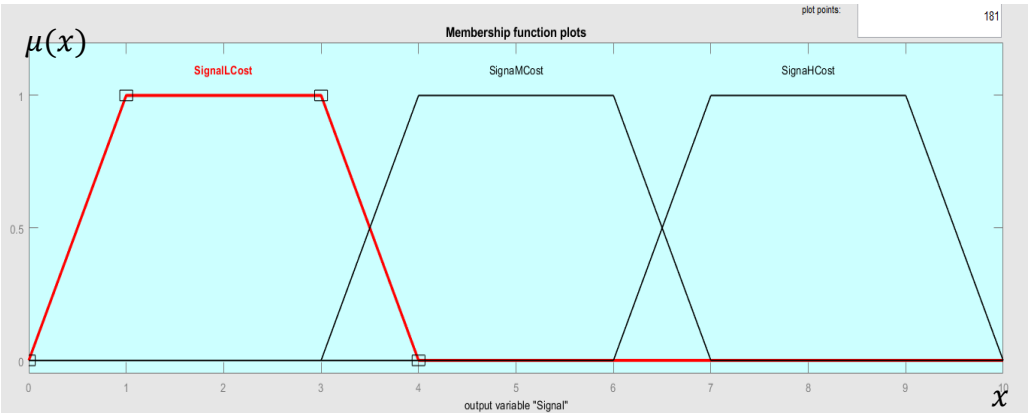

Figure 4.7 Output membership functions

4.3.4 The Fuzzy C++ Program Modules

Due to lack of available free recourse for the Mamdani model inference system in C++ language; I had to develop a fuzzy based C++ program to accommodate for the proposed 3FTD TSP on the one hand, and to perform the optimization and integration with the different algorithms on the other hand (such as the Discrete Bacterial Memetic Evolutionary Algorithm and the Genetic Algorithm). The C++ program I developed can simulate the inference system of any problem. It has two parts: the linguistic part (the fuzzy rules) and the numerical part. It can handle type-1 and type-2 fuzzy sets (*full code and ready-to-run test case will be included in the dissertation's appendix*).

To completely specify the operation of a Mamdani fuzzy inference system, a function for each of the following operators was assigned [16]:

1) AND operator (usually T-norm) for the rule firing strength computation with AND'ed antecedents.
2) OR operator (usually T-conorm) for calculating the firing strength of a rule with OR'ed antecedents.
3) Implication operator (usually T-norm) for calculating qualified consequent MFs based on given firing strength.
4) Aggregate operator (usually T-conorm) for aggregating qualified consequent membership functions to generate an overall output membership function.
5) Defuzzification operator for transforming an output membership function to a crisp single output value.

The C++ program built is mainly divided into the following sub modules [16]:

- *Fuzzy Rule Module:* this module consists of the linguistic variables and fuzzy rules. This class main purpose to translate and sort the fuzzy rules provided for the case under study.
- *Fuzzy Input Module*: this class is responsible for handling the input variables for any case under study.
- *Fuzzy Reasoner Module:* basically, it takes the inputs and the fuzzy rules provided, then each fuzzy rule is mapped to output variable and lastly it performs the defuzzification process.

It is worth mentioning due to complexity, I have used the weighted average method [88] in my C++ modules for defuzzification (as explained in section 4.3.2).

4.3.5 The Fuzzy C++ Program's Inference Rules

As I explained in the previous sections, the inference engine (in Fig.4.6.) is the component that derives the fuzzy outputs from the input fuzzy sets according to the relations defined through the fuzzy rules [9]. Generally, the rule base expression are formulated such as: IF X is A THEN Y is B. It describes a conditional statement that in this case is a fuzzy conditional statement, since A and B are fuzzy sets, and X and Y are linguistic variables. A fuzzy conditional statement like this one represents a fuzzy relation between A and B defined in $U \times V$. This fuzzy relation is expressed again by a fuzzy set (R) whose membership function $\mu_R(x, y)$ is given by

$$\mu_R(x, y) = I(\mu_A(x), \mu_B(y)), \forall x \in U, y \in V \quad (4.30)$$

in which $\mu_A(x)$ and $\mu_B(y)$ are the membership functions of the fuzzy sets A and B, and I is a fuzzy implication operator that models the existing fuzzy relation.

This approach corresponds to the original Mamdani Fuzzy Rules Based System (FRBS) [92], being the main tool to develop linguistic models, and is the approach that has been mainly considered in my work

(while developing the program). A Mamdani FRBS provides a natural framework to include expert knowledge in the form of linguistic rules. This knowledge can be easily combined with rules which are automatically generated from data sets that describe the relation between system input and output. In addition, this knowledge is highly interpretable. Therefore, each rule is a description of a condition-action statement that exhibits a clear interpretation to a human – for this reason, these kinds of systems are usually called linguistic or descriptive Mamdani FRBSs.

In the model I built, the linguistic fuzzy rules for the 3FTD TSP were set as per below (those rules are configurable in the dataset as the program is completely dynamic):

1. *IF* JamRegion is Average AND Rush is Morning *THEN* Signal is HCost
2. *IF* JamRegion is Average AND Rush is Evening *THEN* Signal is MCost
3. *IF* JamRegion NOT Average AND Rush is Morning *THEN* Signal is LCost
4. *IF* JamRegion NOT Average AND Rush is Evening *THEN* Signal is LCost
5. *IF* JamRegion is Average AND Rush NOT Morning *THEN* Signal is MCost
6. *IF* JamRegion is Average AND Rush NOT Evening *THEN* Signal is MCost

The rules will be incorporated in the defuzzification stage (as explained above). Here I defined three trapezoidal membership functions of the Output Signals that are shown in Fig.4.6., HCost (High cost), MCost (Meduim cost), and LCost (Low cost), that have the below break points (which are also configurable in the settings file of the C++ program, to accommodate for the case under study):

- LCost (0,1,3,4)
- MCost (3,4,6,7)
- HCost (6,7,9,10)

This part of my program was used to test the 3FTD TSP on bigger instances and compare it with the TD TSP results. However, integrating my program with the DBMEA was necessary. Thus, (with the help of my colleague) my fuzzy based C++ software was integrated with the DBMEA (Discrete Bacterial Memetic Evolutionary Algorithm) and GA (Genetic Algorithm) for optimization, validation, and scalability purposes. Lastly, the integrated optimized program was run on the largest TD TSP instances in the literature (bier127, eil51, eil76, il101, s250) to compare my models with the classic TD TSP results on the one hand, and to prove my fuzzy-based models' generality, applicability, scalability, and efficiency on the other hand.

Following this methodology to simulate the IFTD TSP model on larger instances, I had to extend my C++ program to accommodate for Type-2 fuzzy sets. Indeed, Type-2 fuzzy sets module that I built, is complementary and extension of the type-1 module mentioned previously, where additional calculation must be performed to conduct the intuitionistic fuzzy part as follows:

Depending on Eqs. (3.1 to 3.10), let A be an IFS of X , the max-min-max composition of the IFR R ($X \rightarrow Y$) with A is an IFS B of Y denoted by ($B = R \circ A$) and is defined by the membership function:

$$\boldsymbol{\mu_{R \circ A}(y) = \bigvee_x [\mu_A(x) \wedge \mu_R(x, y)]} \quad (4.31)$$

and the non-membership function:

$$v_{R\circ A}(y) = \wedge_x [v_A(x) \vee v_R(x,y)] \quad (4.32)$$

for all $y \in Y$ (where $\vee$= max; $\wedge$ = min).

4.4 Results and Discussion for Optimizing the Extended Triple Fuzzy Time Dependent Travelling Salesman Problem (3FTD TSP) on the Largest Universal Benchmarks with DBMEA

The first results I have achieved were the ones in Table 4.1 [10]. The run results for the Bier127 benchmark problem applied on the extended 3FTD TSP, with different jam factors. This original TSP benchmark is modelling the locations of the Bier127 gardens in the area of Augsburg (Germany). The traffic jam region was defined with a circle of radius 5000m and with centre (10540, 11945). Velocity s is 6000 m/h in each test. The middle point of the fuzzy triangular cost for each edge is the Euclidean distance between the endpoints, namely, the left-side and right-side points were determined randomly (0-50% lower and higher than the middle point) in this test.
Our algorithm was tested on an Intel Core i7-7500U 2.7 GHz, 8GB of RAM memory workstation under Linux Mint 18. The loss aversion approach was applied and the supports rather than the cores or the whole membership functions of the fuzzy cost factors were taken into account (e.g. with jam factor = 5.0) the average elapsed time was 22.286 hours, the average support value of the output triangular fuzzy numbers was 10.76 hours when the loss aversion was taken into account (without loss aversion, these values were 22.196 hours and 10.92 hours).

In Table 4.2. and Table 4.3., the 3TD TSP was compared with the TD TSP on Bier127 instance and our own instance that contains 250 nodes (s250_1 and s250_2), the results were calculated by averaging five test runs. The total time in hours required to visit each location with different jam regions and rush hours costs (Fig.4.8.).

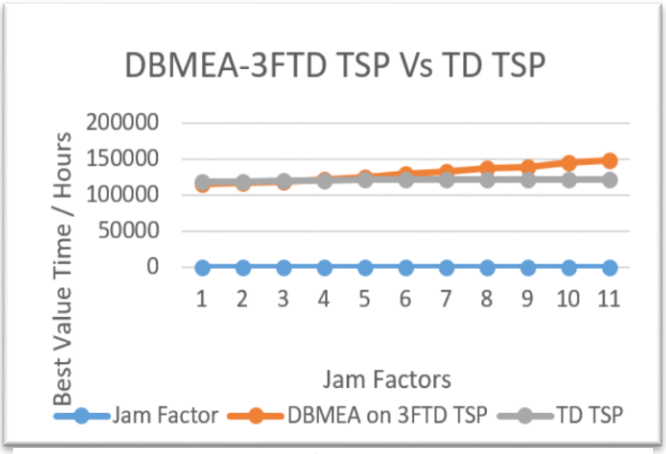

Figure 4.8 Bier127 for DBMEA and TD TSP

After achieving good results, I was eager to test it on the largest universal TD TSP benchmarks, to prove my model's efficiency and compare it with state-of-the-art solutions in the literature. Thus, I integrated my 3FTD TSP C++ software with various optimizations algorithms such as the DBMEA and the GA algorithm. The benchmarks we used are Eil51, Bier127, Eil76, Eil101 s250_1 and s250_2. To illustrate, each instance's name reflects the number of cities (nodes) under study; Eil51has a total of 51 cities that a salesman must visit. The coordinates of all the cities have been downloaded from the library TSP-Lib12.

Table 4.4. shows different run results for the Eil51 benchmark problem applied using the fuzzified version (3FTD TSP), with different jam regions and rush hours' values. Table 4.5. shows different run results for the Eil76 benchmark problem applied using the 3FTD TSP with different jam regions and rush hours' values. Table 4.6. shows different run results for the Eil101 benchmark problem applied using 3FTD TSP with different jam regions and rush hours' values. Table 4.7. and Table 4.8. show different run results for the s250 benchmark problem applied using the extended 3FTD TSP with different jam regions and rush hours' values (our own instances s250_1 and s250_2 which consist of 250 cities each).

Testing our algorithm on the standard TD TSP benchmarks and achieving promising results (Tables 4.1 to 4.9, are summarized in Figs. 4.9 and 4.10) only proves that our new model is general enough to be applied for the extended problem, and we also want to show that it converges with a (quasi-)optimal solution in all cases. Indeed, the DBMEA was able to find high quality solutions for the 3FTD TSP problem, with different membership functions $\mu_J(x)$ and $\mu_{RH}(t)$, even with large rush hours and jam regions values. The obtained results are validated in the context of the known optimum solution for the original TSP, and Schneider's [2] solution for the TD TSP.

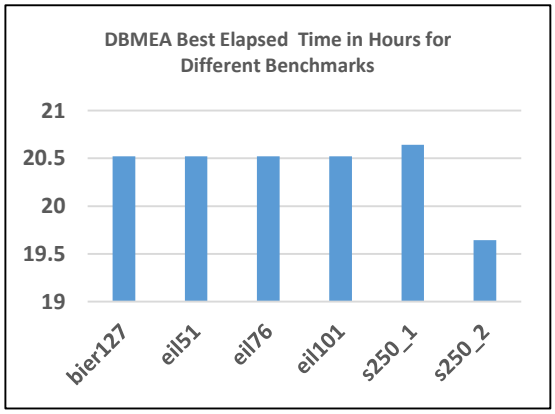

Figure 4.10 Best elapsed time for different benchmarks with DBMEA

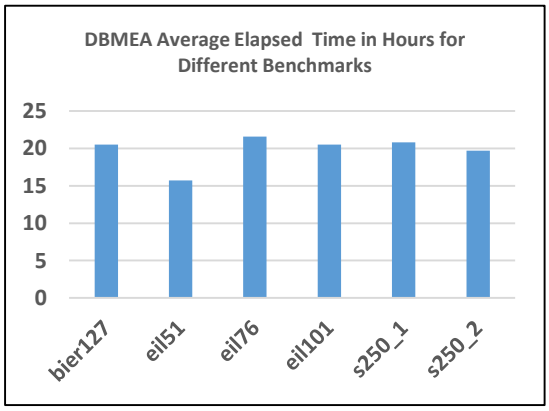

Figure 4.9 Average elapsed time in hours for different benchmarks with

Table 4.1 Bier127 run results for 3FTD TSP vs TD TSP

Jam factor	3FTD TSP Best value	TD TSP Best value
1.00	115929.0	118293.5
1.05	117498.6	119053.2
1.20	117997.2	119714.1
1.50	121076.4	120571.7
2.00	125083.8	121125.2
3.00	129369.0	121125.2
5.00	132220.2	121125.2
10.00	137813.4	121125.2
20.00	139286.4	121125.2
50.00	144363.1	121125.2
100.00	147520.2	121125.2

Table 4.2 Bier127 computational results for 3FTD TSP with DBMEA

Testing Instance	Best elapsed time			Best elapsed time	Average elapsed time	Average runtime [s]
bier127	20.5317273	20.58075	20.52149	20.52149	20.54466	5.698
s250_1	21.08152503	20.71037	20.64352	20.64352	20.8118	27.493
s250_2	19.75256584	19.67563	19.64307	19.64307	19.69042	27.383

In fact, the results only prove that the extended 3FTD TSP gives realistic run times and a timely conversion speed which proves the applicability of the model. In addition, it indicates that it is a more general and robust solution to simulating real life scenarios than the original TD TSP is, the former offering more realistic representation of the rush hours and jam regions with high adequacy.

Table 4.3 Bier127 computational results for 3FTD TSP with DBMEA

	Elapsed time			Best	Average elapsed time
bier127	20.5317273	20.5807	20.5215	20.5215	20.5215
bier127	20.0984	20.3999		20.5215	20.2492
s250_1	21.08152503	20.71037	20.64352	20.64352184	20.81180495
s250_2	19.75256584	19.67563	19.64307	19.64307403	19.69042

Table 4.4 Eil51 computational results for 3FTD TSP with DBMEA

Eil51 -hours	**Elapsed time**			**Best elapsed time**	**Average elapsed time**
	15.7147	15.7573	15.7147	20.5215	15.7289

Table 4.5 eil76 computational results for 3FTDTSP with DBMEA

Eil76-hours	**Elapsed time**			**Best elapsed time**	**Average elapsed times**
	21.4841	21.5433	21.7321	20.5215	21.5865

Table 4.6 Eil101 computational results for 3FTDTSP with DBMEA

Eil101 -hours	**Elapsed time**			**Best elapsed time**	**Average elapsed time**
	23.6234	23.5858	23.5537	20.5215	23.5876

Table 4.7 s250_1 computational results for 3FTDTSP with DBMEA

s250_1 -hours	**Elapsed time**			**Best elapsed time**	**Average elapsed time**
	21.08152503	20.7104	20.6435	20.6435	20.8118

Table 4.8 s250_2 computational results for 3FTDTSP with DBMEA

s250_02 -hours	**Elapsed time**			**Best elapsed time**	**Average elapsed time**
	19.75256584	19.6756	19.6431	19.6431	19.6904

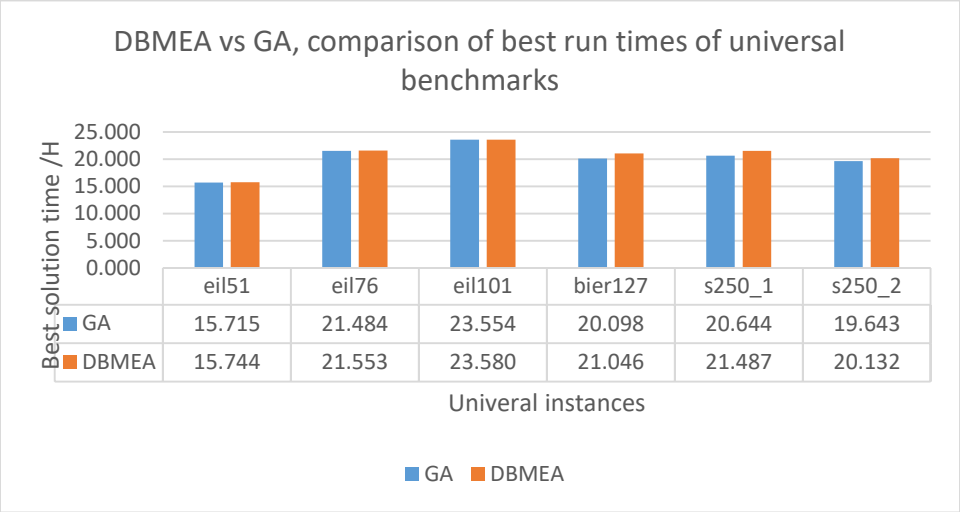

Figure 4.11 DBMEA vs GA, comparison of best run times of universal benchmarks

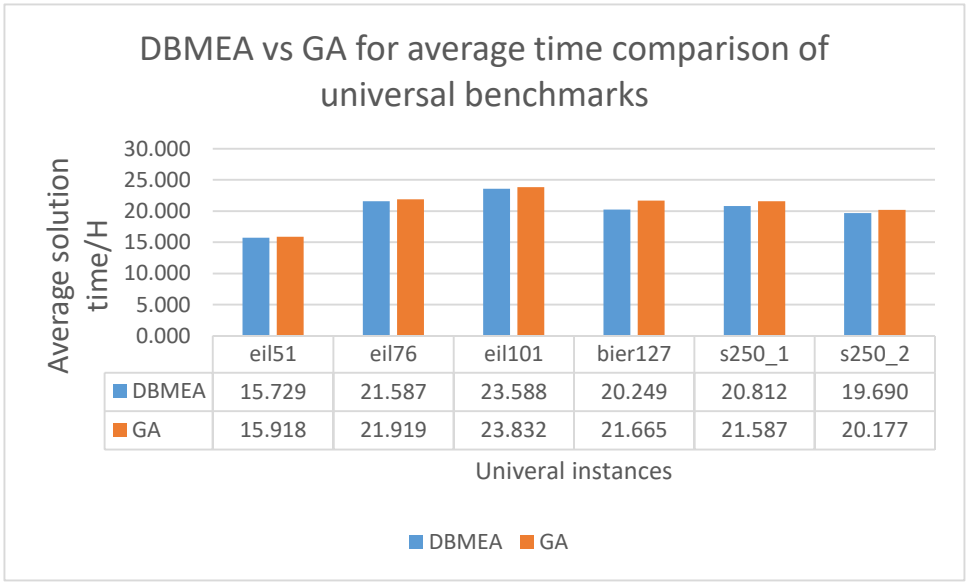

Figure 4.12 DBMEA vs GA for average time comparison of universal benchmarks

In Fig. 4.11, the results show the comparison of the best elapsed times for both GA and the DBMEA using the 3FTD in TSP extension. It is clear that most of the runs, the DBMEA outperformed the GA. In Fig. 4.12, the graph shows the comparison of the average elapsed time for both algorithms (the GA and the DBMEA).

In Tables 4.9. and 4.10, we have tested both the DBMEA and the GA using the 3FTD TSP model. The results show that the DBMEA outperformed the GA (in bold), although both approaches gave feasible solutions. We have run both algorithms on 300 generations. The Genetic Algorithm was faster than DBMEA after 100 generations. However, what makes the DBMEA generally outperforming the GA is the fact that – to some

surprise - the GA gets stuck in a local optimum and seems to be unable to significantly improve the solution between 100 and 300 generations. Of course, there may be some changes after a larger, maybe significantly larger number of generations, however, this could happen in a definitely not feasible run time.

Table 4.9 100 generation comparison of results for the 3 FTD TSP and the TD TSP

	DBMEA	
Benchmark	**Best value**	**Average value**
eil51	**15.715**	**15.729**
eil76	**21.484**	**21.587**
eil101	**23.554**	**23.588**
bier127	**20.098**	**20.249**
s250_1	**20.644**	**20.812**
s250_2	**19.643**	**19.690**

Table 4.10 . 100 generation comparison of results for the 3 FTD TSP and the TD TSP

	Genetic algorithm	
Benchmark	**Best value**	**Average value**
eil51	15.744	15.918
eil76	21.553	21.919
eil101	23.580	23.832
bier127	21.046	21.665
s250_1	21.487	21.587
s250_2	20.132	20.177

4.5 Summary

The results of this chapter are summarized in the booklet sections (4.1) and (4.2).

In this chapter, I proposed, a novel fuzzy based approach for the TD TSP (the Triple Fuzzy Time Dependent Traveling Salesman Problem). Then I extended the 3FTD TSP approach. The two models were presented and tested on several TD TSP benchmark instances in the literature (Eil51, Bier127, Eil76, Eil101 s250_1 and s250_2). Originally, in the TD TSP model, the costs between the nodes are time dependent and this dependency is connected with a predefined oblong shaped area. On the other hand, in the 3FTD TSP model, the road conditions are represented by type-1 fuzzy sets. The uncertainty associated with the traffic jam areas and the fuzzy occurrence of the rush hours' phenomenon in time, depends on several unknown or non-deterministic factors. Thus, the uncertainty was represented by fuzzy sets of the respective universes, this way determining an uncertain overall tour length (cost) was more efficient and practical.

Two meta-heuristic approaches, Genetic Algorithm and Discrete Bacterial Memetic Evolutionary Algorithm were chosen to optimize the model due to their former success in similar (although non-fuzzy) problems. The simulation results confirm the efficiency (advantageous time and space complexity) and the predictability of the Discrete Bacterial Memetic Evolutionary Algorithm. Furthermore, it was found that the

Discrete Bacterial Memetic Evolutionary Algorithm was able to converge in reasonable time for large instances with the maximum of 250 cities, without being stuck in a local optimum. Our novel extended model was able to adequately simulate complicated tour graphs which proves the universality of the Triple Fuzzy Traveling Salesman problem model, its applicability, generality, and scalability on real-life problems.

- *Using type-1 fuzzy sets, I introduced a novel mathematical formulation for the jam regions effects on a tour's costs simulating real-life circumstances.*
- *Using type-1 fuzzy sets I introduced a novel mathematical formulation to quantify effectively the rush hours effects on a tour's costs simulating real-life circumstances.*

- *I introduced for the first time, a fully fuzzified version of the classic TD TSP with jam regions, rush hours, and overall trip's costs; the 3FTD TSP model. I have shown that the costs between the nodes (cities) may depend on time (and location), due to the inherent uncertainty involved, they can be better translated as imprecise (fuzzy) values. Building on this concept, I mathematically validated the 3FTD TSP model on a representative size tour by modelling the uncertainties caused by road unpredicted conditions using fuzzy sets (convex and normal fuzzy sets of the respective universes). Using typ-1 fuzzy sets I expressed the costs influenced by the rush hours times and the jam situations, and ultimately, calculated the overall tour length (costs) quantitatively.*

- *I built a dynamic C++ program that can simulate numerous variations of the jam regions and rush hour periods for the extended 3FTD TSP in fuzzy environment. My C++ modules simulated my previously proven thesis group (4.1) on the largest instances (Eil51, bier127, eil76, eil101 and our own s250 cities instances) available in the literature, after it was integrated with two different optimization algorithms (DBMEA and GA). Leading to valuable comparative statistics to assess the novel 3FTD TSP's applicability, generality, and scalability.*

Chapter 5 The Intuitionistic Fuzzy Based Model of the Jam Regions and the Rush Hours for the Time Dependent Traveling Salesman Problem

5.1 Introduction

In the literature, there have been many algorithms to quantify the effect of the jam regions and the rush hours in the TD TSP. Since those factors are intangible, such task considered hard combinatorial optimization problem, with real life motivation [2,3,4]. The original abstract problem assumes that a fixed part of the graph (the traffic jam region) has time dependent costs, namely, each graph edge is assigned a cost (weight) in the non-rush hour periods, and another (higher) one during the rush hours times. The aim is to find the tour with the lowest costs in the same way as in the case of the classic TSP. Despite the TD TSP's ability to achieve good results in determining the overall cost, a major pitfall is that a single concrete number is used to represent the proportional jam factors. This is rather unrealistic representation, as usually, the traffic jam regions build up gradually, and their respective effects are not uniformly affecting the area – in some border regions there might be a much smaller increase of the costs, compared to the real core. This work was motivated by realizing the limitation and impreciseness of jam factors representation in TD TSP, and continuing on our previous work (the 3FTD TSP). indeed, the unrealistic simplification of the rush hours and the jam regions calculation were overcome (at least partly) in this IFTD TSP model (Intuitionistic Fuzzy Traveling Salesman Problem Model).

There are several generalizations of fuzzy sets theory satisfying various objectives; among them, the notion of intuitionistic fuzzy sets (IFSs) introduced by Atanassov [37] is an interesting and practically useful one. The IFS theory has been utilized in various areas that have to do with decision making under vagueness and with unstructured problems and indeed proved being successful [37,41,42,44,47]. In the literature, there were some attempts to apply IFS on the TSP and the TD TSP, section 2.3 explains these models briefly with their drawbacks. These models' shortages were (at least partly) overcome in my proposed model, the IFTD TSP. I applied the intuitionistic fuzzy sets to model the jam regions effects (with the assumption some edges cross city centers) on the one hand, and on the rush hours effects on the other hand (assuming some edges will be traveled during rush hours).

5.2 Related Work

Since I have proven in the previous chapters that the estimated tour distances (between nodes) are not constant, it is appropriate to eliminate this imprecision by using fuzzy numbers; such as intuitionistic fuzzy sets, afterwards these fuzzy numbers may be summed up in a tour in order to calculate the total distance. Fuzzification of the TD TSP was presented in one of my previous research papers [10], where I introduced a fuzzy model for the Time Dependent Traveling Salesman Problem (3FTD TSP). That model redefined

the TSP entirely and transformed it to the Triple Fuzzy extension (3FTD TSP) with fuzzy costs, fuzzy jam regions and fuzzy rush hours. The model was able to express the costs influenced by the jam regions and the rush hours and eventually quantify the overall tour length of the salesman trip more accurately corresponding to the real-life situation [10]. Afterwards, optimization the 3FTD TSP model using the heuristic DBMEA confirmed the effectiveness and the predictability of the proposed technique. Thus, I was more motivated to enhance my previous model, and present another extension of the TD TSP [11], then validating, and comparing the newly extended fuzzy based model (IFTD TSP) with my previous models [12,14]. Since intuitionistic fuzzy sets are type-2 fuzzy sets, they have three-dimensional membership functions, while type-1 sets have only two; the intuitionistic fuzzy sets are more flexible to fuzzify the TD TSP under the jam regions and the rush hours periods influence.

5.3 Methodology

Below is a short review of the basic concepts, operations and definitions related to intuitionistic fuzzy sets which are used in the upcoming sections in a more analytical and formal model (main definitions were introduced in chapter 3).

Using on Eqs. (3.1) to (3.16), Let A and B be two IFSs of the set X, Then

$$A \subset B \text{ IFF } \forall x \in X, [\mu_A(x) \leq \mu_B(x) \text{ and } v_A(x) \geq v_B(x)] \tag{5.1}$$

$$A \supset B \text{ IFF } B \subset A \tag{5.2}$$

$$A = B \text{ IFF } \forall x \in X, [\mu_A(x) = \mu_B(x) \text{ and } v_A(x) = v_B(x)] \tag{5.3}$$

$$A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(v_A(x), v_B(x))\rangle | x \in X\} \tag{5.4}$$

$$A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(v(x), v_B(x))\rangle | x \in X\} \tag{5.5}$$

The max-min-max composition will be used in this chapter which was explained in chapter 3 (Eq. 3.15 and Eq. 3.16).

To explain the IFTD TSP model [11,12], let, there be n number of edges E that form a whole tour for a salesman. $e_i \in E$. Let R be an IFR $(J \times C)$ and construct an IFR Q of $(E \times J)$. Clearly, the composition T of IFRs R and Q $(T = R \circ Q)$ give the costs of any edge e_i in terms of E to C given by the membership functions in Equations

$$\boldsymbol{\mu_T(e_i, c) = \vee_{j\epsilon J}\left[\mu_Q(e_i j) \wedge \mu_R(j, c)\right]} \quad (5.6)$$

and the non-membership function given by

$$\boldsymbol{v_T(e_i, c) = \wedge_{j\epsilon J}\left[v_Q(e_i j) \vee v_R(j, c)\right]} \quad (5.7)$$

$$\forall e_i \in E \text{ and } c \in C$$

For given R and Q, the relation $(T = R \circ Q)$ can be computed. From the knowledge of Q and T, an improved version of the IFR R can be computed, for which the following holds valid:

(i) $J_R = \mu_R - v_R \cdot \pi_R$ is greatest. Thus, calculating J_R will lead to find the additional cost that must be added from the jam regions effect on any edge, and ultimately, quantify the intangible cost of the such uncertain factor with higher degree of accuracy. Lastly, it will be added to the overall cost for the edge that crosses that specific jam region.

(ii) The equality $T = R \circ Q$ is retained

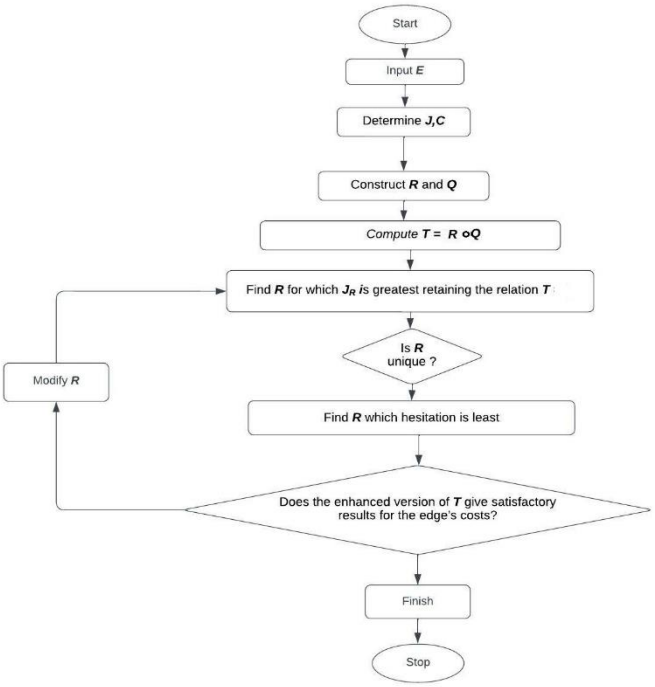

Figure 5.1 IFTD TSP flow chart

As seen in Fig.5.1, from the flow chart, it is clear that this proposed concept improved version of R will be a more significant IFR translating the higher degrees of association and lower degrees of non-association of J as well as lower degrees of hesitation to the cost evaluation C. If almost equal values for different C in T

are obtained, then we consider the case for which hesitation is least. From a refined version of R one may infer the costs from the jam factors in the sense of a paired value, one being the degree of association and other the degree of non-association.

5.3.1 Quasi-optimization of the Time Dependent Travelling Salesman Problem by Intuitionistic Fuzzy Model and Memetic Algorithm

In this section, I have applied the IFTD TSP on simple tour with 12 nodes (Table 5.1 to Table 5.4), to test the methodology first [11]. Then we used the DBMEA on both the classic TD TSP, and the extended IFTD TSP on Bier127 instance for comparison [12]. Let there be a simple tour that has only 12 edges ($E\ 1 \ldots E12$) as shown in Fig. 4.1. Each edge connects two nodes. Each edge eventually will have an added jam factors costs, depending on the jam area(s) the edge crosses (where they edges were travelled) or if it was travelled during the rush hours periods. Table 5.1 shows each edge and the jam factors associated with it. The ultimate goal is to be able to calculate the total tour jam factors added costs, which will be multiplied by the physical distance between the corresponding nodes, resulting in quantifying the total jam factors added costs of that tour.
The IFR $Q(E \to J)$ is given as shown in Table 5.1. The set of jam factors costs be C is given in Table 5.2. The IFR $R(J \to C)$ is given in Table 5.2. Therefore, the composition ($T = R \circ Q$) is given in Table 5.3.
Also, I calculated the jam region costs factors on an edge e (c_{jam_e} is the total jam cost affecting an edge e) given in Table 5.4, where the four cost factors are ($c_1 = 1.2, c_2 = 1.5, c_3 = 2, c_4 = 5$) with weighted average calculations :

$$\boldsymbol{c_{jam_e}} = \frac{\sum_{i=1}^{n}[\boldsymbol{c_{ij}} * \boldsymbol{j_{ij}}]}{\sum_{i=1}^{n} \boldsymbol{j_{ij}}} \quad (5.8)$$

The rush hour factors costs of each tour edge (c_{rush}) are determined in similar intuitionistic model. The relations between the time and the rush hour periods ($\grave{Q}$) are described with intuitionistic fuzzy functions Fig. 5.2. An intuitionistic fuzzy relation ($\grave{R}$) is given between the rush hour periods and the jam factors costs, similarly, as done in Table 5.2 (for jam regions). Then the composition ($\grave{T} = \grave{Q} \circ \grave{R}$) is calculated Table 5.3. Finally rush hour cost factors were calculated with weighted averaging Table 5.4.

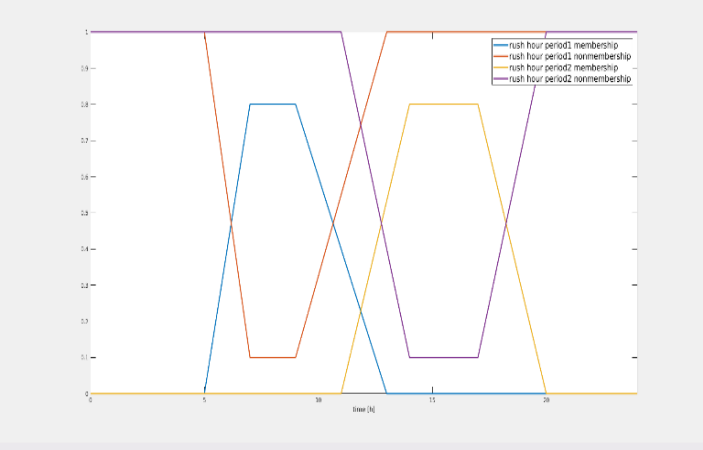

Figure 5.2 Fuzzy membership and non-membership functions of the rush hour periods

The added costs of the edges are calculated considering based on Eqs. (4.1 to 4.20) in chapter 4.

Table 5.1 Route1= (edge 1…edge12)

(Q)	**Jam Region1**	**Jam Region2**	**Jam Region3**	**Jam Region4**
E1	(0.8, 0.1)	(0.6, 0.1)	(0,1)	(0,1)
E2	(0,1)	(0,1)	(0.2, 0.8)	(0.6, 0.1)
E3	(0.8, 0.1)	(0.8, 0.1)	(0,1)	(0,1)
E4	(0,1)	(0,1)	(0, 0.6)	(0.2, 0.7)
E5	(0.8, 0.1)	(0.8, 0.1)	(0, 0.6)	(0.2, 0.7)
E6	(0, 0.8)	(0.4, 0.4)	(0,1)	(0,1)
E7	(0,1)	(0,1)	(0.6, 0.1)	(0.1, 0.7)
E8	(0, 0.8)	(0.4, 0.4)	(0.6, 0.1)	(0.1, 0.7)
E9	(0.6, 0.1)	(0.5, 0.4)	(0,1)	(0,1)
E10	(0,1)	(0,1)	(0.3, 0.4)	(0.7, 0.2)
E11	(0, 0.8)	(0.4, 0.4)	(0.6, 0.1)	(0.1, 0.7)
E12	(0.4, 0.4)	(0.6, 0.1)	(0,1)	(0.2, 0.8)

Table 5.2 Jam costs

Jam area (R)	**Cost factor 1 (c1)**	**Cost factor 2 (c2)**	**Cost factor 3 (c3)**	**Cost factor4 (c4)**
Jam Region1	(0.4, 0)	(0.7, 0)	(0.3, 0.3)	(0.1, 0.7)
Jam Region2	(0.3, 0.5)	(0.2, 0.6)	(0.6, 0.1)	(0.2, 0.4)
Jam Region3	(0.1, 0.7)	(0, 0.9)	(0.2, 0.7)	(0.8, 0)
Jam Region4	(0.4, 0.3)	(0.4, 0.3)	(0.2, 0.6)	(0.2, 0.7)

Table 5.3 T=R ∘Q

Jam Cost (T)	**Cost factor1**	**Cost factor2**	**Cost factor3**	**Cost factor4**
E1	(0.4, 0.1)	(0.7, 0.1)	(0.6,0.1)	(0.2, 0.4)
E2	(0.4,0.3)	(0.4,0.3)	(0.2, 0.6)	(0.2, 0.2)

E3	(0.4, 0.1)	(0.7, 0.1)	(0.6,0.1)	(0.2, 0.4)
E4	(0.2,0.7)	(0.2,0.7)	(0.2, 0.7)	(0.2, 0.6)
E5	(0.3, 0.1)	(0.7, 0.1)	(0.6, 0.1)	(0.2, 0)
E6	(0.3, 0.5)	(0.2, 0.6)	(0.4, 0.4)	(0.2,0.4)
E7	(0.1,0.7)	(0.1,0.7)	(0.2, 0.7)	(0.6, 0.1)
E8	(0.3, 0.5)	(0.2, 0.6)	(0.4, 0.4)	(0.6, 0.1)
E9	(0.4, 0.1)	(0.6, 0.1)	(0.5, 0.3)	(0.2, 0.4)
E10	(0.4,0.3)	(0.2,0.6)	(0.2, 0.6)	(0.3, 0.4)
E11	(0.3, 0.5)	(0.2, 0.6)	(0.4, 0.4)	(0.6, 0.1)
E12	(0.4,0.4)	(0.4,0.4)	(0.6, 0.1)	(0.2, 0.4)

Table 5.4 intuitionistic jam region cost factors for edges

J_R	(J_R1)	(C 1)	(J_R2)	(C 2)	(J_R3)	(C 3)	(J_R4)	(C4)	*Total Jam region cost factors* $[c_{jam_e}]$
E1	0.35	1.2	0.68	1.5	0.57	2	0.04	5	1.695
E2	0.31	1.2	0.31	1.5	0.08	2	0.08	5	1.791
E3	0.35	1.2	0.68	1.5	0.57	2	0.04	5	1.695
E4	0.13	1.2	0.13	1.5	0.13	2	0.08	5	2.151
E5	0.24	1.2	0.68	1.5	0.57	2	0.2	5	2.04
E6	0.2	1.2	0.08	1.5	0.32	2	0.36	5	2.917
E7	0	1.2	0	1.5	0.13	2	0	5	2
E8	0.2	1.2	0.08	1.5	0.32	2	0.57	5	3.291
E9	0.35	1.2	0.57	1.5	0.44	2	0.04	5	1.682
E10	0.31	1.2	0.08	1.5	0.08	2	0.18	5	2.388
E11	0.2	1.2	0.08	1.5	0.32	2	0.57	5	3.291
E12	0.32	1.2	0.32	1.5	0.57	2	0.04	5	1.763

5.4 Results and Discussion

I proposed a novel intuitionistic fuzzy sets based model for the realistic extension of the original TD TSP, namely, the Intuitionistic Fuzzy Traveling Salesman Problem Model (IFTD TSP), for considering jam factors and rush hours in the original TDT SP problem by applying the IFS theory [11]. Obviously, this improved version is a promising approach in translating the higher degrees of association and lower degrees of non-association of the jam factors and rush hours periods, as well as simulating lower degrees of hesitation to any edge costs. The IFTD TSP ultimately achieved a more realistic calculation for the traveled routes under uncertain road circumstances. The DBMEA algorithm was modified (with the help of my colleague), and tested for the new IFTD TSP problem on Bier127 instance [12]. Bier127 instance was selected for the

test because our team had some former results for TD TSP with this instance to validate the obtained results. These TSP benchmark instances were first examined by Schneider for the Time Dependent TSP [4]. He defined a traffic jam region (coordinate of the left corner point (7080, 7200), width is 6920, and the height is 9490). In our test this jam region was divided into four equal sized subdivisions. Velocity *v* is 6000 m/h in each test. Two rush hour periods were defined as in Fig. 5.3, Fig 5.4.

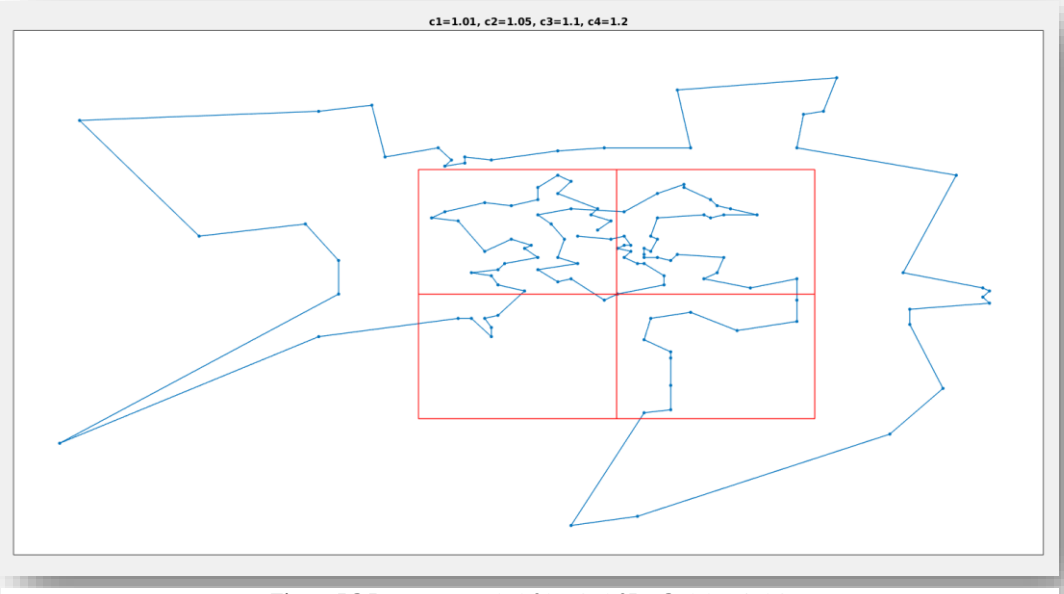

Figure 5.3 Best tour c_1=1.01 c_2=1.05 c_3=1.1 c_4=1.2

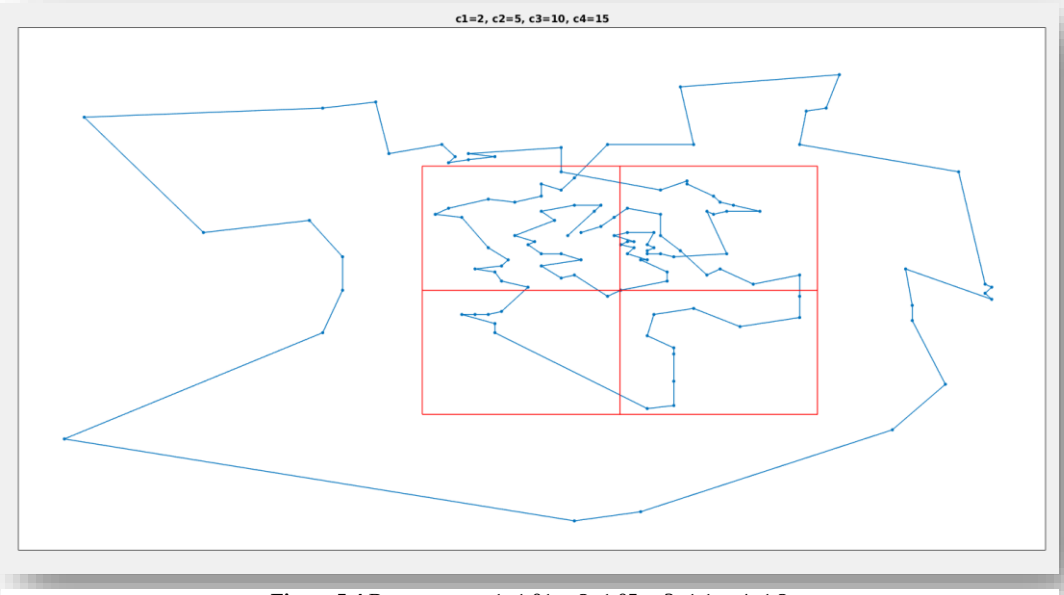

Figure 5.4 Best tour c_1=1.01 c_2=1.05 c_3=1.1 c_4=1.2

Table 5.5. contains the total time in hours required to visit each location with different cost factors. This new problem is a generalization of the original TSP (with $c_1 = 1.2, c_2 = 1.5, c_3 = 2, c_4 = 5$ the edge costs are the Euclidean distances, so these parameters result in the TSP problem). As it can be seen in Table 5.5 and Fig 5.5., the DBMEA also found high quality solutions with big jam factors values.

Table 5.5 computational results for IFTD TSP

	DBMEA		
cost factors	**best elapsed time**	**average elapsed time**	**average runtime [s]**
$c_1 = 1.01$ $c_2 = 1.05$ $c_3 = 1.1$ $c_4 = 1.2$	20.605	20.712	170.535
$c_1 = 1.05$ $c_2 = 11$ $c_3 = 1.2$ $c_4 = 1.5$	21.166	21.349	176.691
$c_1 = 1.1$ $c_2 = 1.3$ $c_3 = 1.5$ $c_4 = 2$	22.061	22.186	192.313
$c_1 = 1.2$ $c_2 = 1.5$ $c_3 = 2$ $c_4 = 5$	22.885	23.124	207.919
$c_1 = 1.5$ $c_2 = 2$ $c_3 = 5$ $c_4 = 10$	23.867	24.185	195.417
$c_1 = 2$ $c_2 = 5$ $c_3 = 10$ $c_4 = 15$	23.909	24.302	207.109
$c_1 = 5$ $c_2 = 10$ $c_3 = 20$ $c_4 = 50$	24.058	24.629	124.910

Lastly, Fig.5.5 shows comparison for averaged 5 runs and the best elapsed time (in hours) for the DBMEA applied on the extended IFTD TSP on Bier127 instance [11,12,15].

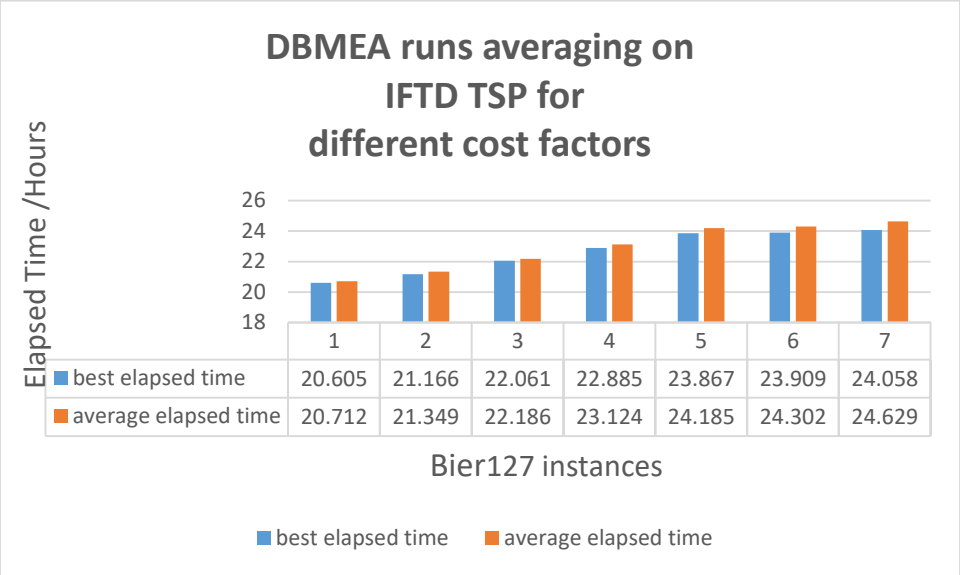

Figure 5.5 DBMEA Comparison results for best/average elapsed time for the IFTD TSP on Bier127

5.5 Summary

The results of this chapter are summarized in the booklet sections (4.3) and (4.4).

I proposed a novel intuitionistic fuzzy sets based model for the realistic extension of the TD TSP, namely, the IFTD TSP model, to calculate more accurately jam factors and rush hours in the real life extension of the TDT SP problem by applying the IFS theory. Clearly, this enhanced model offered more promising modelling to translating the higher degrees of association and lower degrees of non-association of the jam regions and the rush hours as well as lower degrees of hesitation to any edge's costs and, ideally, a more realistic calculation for the travelled routes under uncertain road circumstances.

- *I formulated a novel mathematical model for the jam regions effects on any tour using the intuitionistic fuzzy sets.*

- *I formulated a novel mathematical model for the rush hours effects on any tour using the intuitionistic fuzzy sets.*

- *I introduced a complete novel intuitionistic fuzzy sets based model, The Fuzzy Time Dependent Traveling Salesman Problem model (IFTD TSP), to translate the higher degrees of association and the lower degrees of non-association for trip's costs as well as the lower degrees of hesitation to edges costs and ultimately, I introduced a more realistic calculation for the travelled routes under vague and unpredicted road conditions.*

- *Quasi-optimization of the Time Dependent Travelling Salesman Problem by Intuitionistic Fuzzy Model and Memetic Algorithm.*

- *My novel extended IFTD TSP model was optimized using the DBMEA on Bier127 instance. High quality solutions were found for different jam factor values affecting any edge in a tour using the weighted average operator.*

Chapter 6 Optimizing The Extended Interval-Valued Intuitionistic Fuzzy Time Dependent Travelling Salesman Problem (IVIFTD TSP) Using the DBMEA

6.1 Introduction

In the classic TD TSP, the edges are assigned different weights, depending on whether they are traveled in the traffic jam regions (such as busy city centers) and during the rush hour periods, or not. In such circumstances, edges are assigned higher costs, expressed by a multiplying factor. Here, I introduced a novel and even more realistic approach, the Interval-Valued Intuitionistic Fuzzy Time Dependent Traveling Salesman Problem (IVIFTD TSP). It is a further extension of the TD TSP, with the additional notion of interval-valued intuitionistic fuzzy sets for describing uncertainties. The core concept employs interval-valued intuitionistic fuzzy sets for quantifying the traffic jam regions, and the rush hours periods added costs, which are always uncertain in real life. Since type-2 fuzzy sets (such as interval-valued Intuitionistic fuzzy sets) have the potential to provide better performance in modeling problems with higher uncertainties than the traditional fuzzy sets, the new approach is considered as an extended, more applicable and practical version of the original abstract problem and a step forward from my previous fuzzy-based model the IFTD TSP which was built on IFS (Intuitionistic Fuzzy Sets) solely. The optimization of such a complex model is obviously very difficult; it is a computationally intractable problem. However, the IVIFTD TSP proven to be practical and applicable extension of the classic TD TSP even on big instances such as Bier127 when it was optimized using the DBMEA.

6.2 Related Work

Further consideration of the uncertainty factors was done when I first proposed the Intuitionistic Fuzzy Traveling Salesman Problem (IFTD TSP) model [11,12]. I have previously proposed a novel intuitionistic fuzzy set based model as extension of the TD TSP, namely, the IFTD TSP model, to calculate more accurately the jam regions and the rush hours effects. The IFTD TSP was tested on universal instances (such as Bier127) and proved its efficiency in translating higher degrees of association and lower degrees of non-association of the jam regions and the rush hours as well as lower degrees of hesitation to any edge costs. Despite the essential extension of the original abstract model and the inclusion of uncertainties, the IFTD TSP approach has some drawbacks, namely, the properties of the max-min-max composition that was used (chapter 5) in determining the ultimate costs of the rush hours and jam regions of a tour (refer to Table 5.1 to 5.5). The result of this composition operation is only affected by the extreme values and ignores the smaller values of the jam regions and the rush hours periods which might be considered problematic. In fact, the costs calculation using this operation leads to quite conservative results; because it does not cater for all the jam regions and rush hours' costs, on the contrary, it takes into consideration only the largest values and neglects the rest, leading to more information loss.
This gave me the motivation to move forward with formulating another novel fuzzy based approach, using the Interval-Valued Intuitionistic Fuzzy Sets (IVIFS), namely, the with Interval-Valued Intuitionistic Fuzzy Model (IVIFTD TSP) [13].

6.3 Methodology

A mathematically more adequate solution (than the IFTD TSP model) would be aggregating the traffic jam factors by a strictly monotonic operation, and applying a corresponding relational operation as well, rather than executing the max-min-max composition. Such a new approach seems to offer more interpretable results. This last observation was the main turning point in proposing the new IVIFTD TSP (Interval-Valued Intuitionistic Fuzzy Time Dependent Traveling Salesman Problem) model.

In the IVIFTD TSP, the jam factors costs of the edges in a tour were represented as fuzzy relations between the jam factors and the predicted edge assigned costs (delays caused by the traffic jam phenomena).

(Depending on the definitions in section 3.2.6, Eqs. 3.17 and 3.18), let $J = \{J_1, J_2, J_3, \ldots . J_m\}$ denote the universal set of jam factors, $C = \{C_1, C_2, C_3, \ldots . C_n\}$ and $E = \{E_1, E_2, E_3, \ldots . E_q\}$ denotes the universal sets of costs and edges, where $n, m, q \in \{1,2 \ldots, n\}$. We will introduce two fuzzy relations: Q that is defined over set (Cartesian space) $E \times J$, see Eq. (6.1), and R that is defined over set (Cartesian space) $J \times C$ as in Eq. (6.2).

$$\boldsymbol{Q} = \{\langle (\boldsymbol{e},\boldsymbol{j}), \boldsymbol{\mu_Q}(\boldsymbol{e},\boldsymbol{j}), \boldsymbol{v_Q}(\boldsymbol{e},\boldsymbol{j}) \rangle \mid (\boldsymbol{e},\boldsymbol{j}) \in \boldsymbol{E} \times \boldsymbol{J}\} \quad (6.1)$$

$$\boldsymbol{R} = \{\langle (\boldsymbol{j},\boldsymbol{c}), \boldsymbol{\mu_R}(\boldsymbol{j},\boldsymbol{c}), \boldsymbol{v_R}(\boldsymbol{j},\boldsymbol{c}) \rangle \mid (\boldsymbol{j},\boldsymbol{c}) \in \boldsymbol{J} \times \boldsymbol{C}\} \quad (6.2)$$

here $\mu_Q(e,j)$ indicates the degree to which jam factor j affects edge e, while $v_Q(e,j)$ indicates the degree to which jam factor j does *not* affect edge e. Similarly, $\mu_R(j,c)$ and $v_R(j,c)$ are the relationships between jam factors and their respective costs. (This is what we refer to as *confirmability degree* in the coming sections). Thus, $\mu_R(j,c)$ and $v_R(j,c)$ represent the degrees to which jam factor j confirms and does not confirm the presence of cost c, respectively. Since Q is defined over set $E \times J$ and R over set $J \times C$, the composition T of R and Q $(T = R \circ Q)$ for predicting the costs for a specific edge can be represented by FR from E to C given the membership function in Eq. (6.3) and non-membership function in Eq. (6.4)

$$\boldsymbol{\mu_T}(\boldsymbol{e},\boldsymbol{c}) = \mathbf{max}_{\boldsymbol{j}}\{\mathbf{min}[\boldsymbol{\mu_Q}(\boldsymbol{e},\boldsymbol{j}), \boldsymbol{\mu_R}(\boldsymbol{j},\boldsymbol{c})]\} \quad (6.3)$$

$$\boldsymbol{v_T}(\boldsymbol{e},\boldsymbol{c}) = \mathbf{min}_{\boldsymbol{j}}\{\mathbf{max}[\boldsymbol{v_Q}(\boldsymbol{e},\boldsymbol{j}), \boldsymbol{v_R}(\boldsymbol{j},\boldsymbol{c})]\} \quad (6.4)$$

for all $e \in E$ and $c \in C$

Below I will apply my proposed approach on a real-life TD TSP cost problem, taking into consideration the gradual build-up of rush hours and the uncertain geographical extent of the jam regions. The approach consists of four main sequential stages:

Stage 1 Calculate a prediction for the rush hours periods and the jam regions of the edges, in the sense that if the trip between the two cities happens during the rush hours periods and/or within the jam regions, both will be taken into consideration and none of the factors will be neglected. Table 6.1 identifies the cost of each jam factor, which is determined by experts in this domain, according to the rush hours and the jam regions of the city of study.

Stage 2 Calculate the interval-valued intuitionistic fuzzy weighted arithmetic average (IIFWAA) of the confirmability and non-confirmability degrees of the edges using aggregation in Eq. (3.19). Normally, the jam regions and the rush hours affect different edges within a tour simultaneously, but with different degrees. In this case, we need to aggregate the interval-valued intuitionistic fuzzy information corresponding to their degrees of confirmability.

Stage 3 Calculate Park's distance between the IFSs using the IIFWAA obtained in Step 2. (Eq. 3.20).

Stage 4 Determine the final jam factors costs (caused by the rush hours and jam regions) based on the measured Park's distance in the third stage (The weights can be assigned according to the topography and importance of the routes; in our case we simply assumed equal weights for all factors).

Here I present an example to illustrate how to apply the proposed new model on a simple TD TSP case study with double uncertain jam regions and rush hours. Afterwards, we run the optimized IVIFTD TSP using DBMEA on Bier127 and compare the run results with the classic TD TSP for the reader.

Let us consider that E_1 is the first edge of a tour as in Fig. 4.1. In the case study, there are four different traffic factors assumed (which may be jam regions and/or rush hour periods) affecting E_1 simultaneously. In the illustration we use $\langle \mu_Q(e,j), v_Q(e,j) \rangle$ estimated to define the degree on how a jam factor j affects edge e as in Eq. (6.1). The jam factors of E_1 affecting that edge in the example are (1.1, 1.2, 1.3, 1.4, 2.1 and 2.2), which are all related to a specific cost (delay) as shown in Table 6.1.

Table 6.1 Knowledge base for rush hour traffic factor costs

Traffic Factors	**Cost1**		**Cost 2**		**Cost 3**	
[*jam regions or rush hours]*	μ_R	v_R	μ_R	v_R	μ_R	v_R
Factor 1.1	[0.6,0.7]	[0.1,0.2]	[0.2,0.3]	[0.5,0.6]	[0.1,0.3]	[0.4,0.6]
Factor 1.2	0.6,0.7]	[0.2,0.3]	[0.2, 0.4]	[0.4,0.6]	[0.4,0.6]	[0.1,0.2]
Factor 1.3	[0.5,0.6]	[0.1,0.2]	[0.1, 0.2]	[0.6,0.7]	[0.3,0.4]	[0.3,0.5]
Factor 1.4	[0.7,0.8]	[0.1,0.2]	[0.1, 0.2]	[0.6,0.8]	[0.1,0.2]	[0.7,0.8]
….	….	…	….	….	….	…..
Factor 2.1	[0.5,0.6]	[0.2,0.3]	[0.2,0.3]	[0.4,0.6]	[0.2,0.3]	[0.5,0.6]
Factor 2.2	[0.7,0.8]	[0.1,0.2]	[0.1,0.2]	[0.6,0.7]	[0.1,0.2]	[0.6,0.7]
Factor 2.3	[0.5,0.6]	[0.2,0.3]	[0.3,0.4]	[0.4,0.6]	[0.1,0.3]	[0.3,0.5]
….	….	…	….	….	….	…..
Factor $n.n$	[0.6,0.8]	[0.1,0.2]	[0.1,0.2]	[0.6,0.7]	[0.2,0.3]	[0.6,0.7]

Table 6.2 shows the jam factors assigned to E_1. Table 6.3 gives the confirmability and non-confirmability degrees with jam factors (1.1, 1.2, 1.3, 1.4, 2.1 and 2.2), according to their degrees of belonging to the rush hour periods and jam regions (referred to from here as "traffic factors", as a generalized term). Table 6.3 gives confirmability degrees for E_1, which is extracted from the general knowledge base represented in Table 6.1.

Table 6.2 E_1 degrees of jam factors. $\mu_Q(E_1, j)$, $v_Q(E_1, j)$

E_1 **Traffic Factors**	**1.1**	**1.2**	**1.3**	**1.4**	**2.1**	**2.2**
$\mu_Q(E_1)$	[0.5, 0.6]	[0.5, 0.6]	[0.4, 0.6]	[0.7, 0.8]	[0.5,0.6]	[0.5,0.7]
$v_Q(E_1)$	[0.2, 0.3]	[0.1, 0.3]	[0.1, 0.2]	[0.1, 0.2]	[0.1,0.2]	[0.2,0.3]

Table 6.3 Confirmability degrees $\mu_R(j, c)$, $v_R(j, c)$

Traffic Factors	**IF degree**					
	Cost1		**Cost2**		**Cost 3**	
Factor 1.1	**[0.6,0.7]**	***[0.1,0.2]***	[0.2,0.3]	[0.5,0.6]	[0.1,0.3]	[0.4,0.6]
Factor 1.2	**[0.6,0.7]**	***[0.2,0.3]***	[0.2, 0.4]	[0.4,0.6]	[0.4,0.6]	[0.1,0.2]
Factor 1.3	**[0.5,0.6]**	***[0.1,0.2]***	[0.1, 0.2]	[0.6,0.7]	[0.3,0.4]	[0.3,0.5]
Factor 1.4	**[0.7,0.8]**	***[0.1,0.2]***	[0.1, 0.2]	[0.6,0.8]	[0.1,0.2]	[0.7,0.8]
Factor 2.1	[0.3,0.4]	*[0.4,0.5]*	[0.6,0.7]	[0.1,0.2]	[0.2,0.3]	[0.5,0.6]
Factor 2.2	[0.2,0.4]	*[0.4,0.6]*	[06,0.7]	[0.1,0.3]	[0.1,0.3]	[0.5,0.6]

Step 1 Based on Table 6.2 and 6.3 we calculate the results in Table 6.4 and 6.5 by applying the IIFWAA operator on μ_R, v_R. For example, [0.61, 0.71], an IIFWAA μ_R of Table 6.5, is calculated as follows (***bolded*** in the first two columns of Table 6.3) :

The confirmability membership degrees of the edge's jam factors (1.1,1.2,1.3 and 1.4) are ([0.6, 0.7], [0.6,0.7], [0.5, 0.6], and [0.7, 0.8]), and here the first edge is affected by four jam factors, $w_i = 1/4$, and so, the distributed weight is calculated as $w = [\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}]$ as follows:

$$0.61 = 1 - \left\{\left(1 - 0.6\right)^{\frac{1}{4}}\right\} * \left\{\left(1 - 0.6\right)^{\frac{1}{4}}\right\} * \left\{\left(1 - 0.5\right)^{\frac{1}{4}}\right\} * \left\{\left(1 - 0.7\right)^{\frac{1}{4}}\right\}$$

$$0.71 = 1 - \left\{(1 - 0.7)^{\frac{1}{4}}\right\} * \left\{(1 - 0.7)^{\frac{1}{4}}\right\} * \left\{(1 - 0.6)^{\frac{1}{4}}\right\} \left\{(1 - 0.8)^{\frac{1}{4}}\right\}$$

μ_R in Table 6.5 is calculated as follows:

Then IIFWAA v_R in Table 6.5 values for the non-membership degrees of the jam factors [0.1, 0.2] [0.2, 0.3][0.1, 0.2][0.1, 0.2] and apply IIFWAA, as below :

$$0.12 = \left\{\left(0.1^{1/4}\right) * \left(0.2^{1/4}\right) * \left(0.1^{1/4}\right) * \left(0.1^{1/4}\right)\right\}$$

$$0.22 = \left\{\left(0.2^{1/4}\right) * \left(0.3^{1/4}\right) * \left(0.2^{1/4}\right) * \left(0.2^{1/4}\right)\right\}$$

Table 6.4 *E1* IVIF degrees (IIFWAA $\boldsymbol{\mu_Q}$,**IIFWAA**$\boldsymbol{v_Q}$)

Q	Factor1	Factor2
Edge 1	([0.54,0.66], [0.12,0.24])	[0.5,0.65],[0.14,0.24]

Table 6.5 *E1* confirmability degrees (IIFWAA $\boldsymbol{\mu_R}$, **IIFWAA** $\boldsymbol{v_R}$)

R	Cost1	Cost 2	Cost3
IIFWAA Factor1	([0.61,0.71],[0.12,0.22])	([0.15,0.28],[0.52,0.67])	[0.24,40],[0.30,0.47]
IIFWAA Factor2	[0.37,0.51],[0.24,0.39]	[0.21,0.31],[0.35,0.46]	[0.55,70],[0.10,0.24]

Step 2 Calculate the distance by applying Eq. (3.20), taking values from Tables 6.4 and 6.5.

Table 6.6 Distance for $\boldsymbol{E_1}$ with traffic factors $\boldsymbol{l_h}$

T	Cost1	Cost 2	Cost3
Edge 1	0.16	0.26	0.24

Step 3 Find the lowest distance, because it points out the jam factor that is causing the highest delay and thus the one worth considering (in our case this is 0.16, as shown in Table 6.6). If we carry on with the same calculations for all the edges, we reach the data presented in Table 6.7; that shows all the jam factor costs for the edges, depending on their respective confirmabilities.

Table 6.7 Distances for $\boldsymbol{E_1, E_{2,\ldots.} E_{12}}$ with traffic factors $\boldsymbol{l_h}$

	Cost1	Cost 2	Cost 3
Edge 1	0.16	0.26	0.24
Edge 2	0.13	0.20	-
Edge 3	0.15	-	0.24
Edge 4	0.16	0.14	0.44
Edge 5	0.3	0.6	0.2
Edge 6	0.16	-	0.24
Edge 7	0.16	0.30	0.20
Edge 8	0.16	0.06	0.44
Edge 9	0.04	0.36	-
Edge 10	0.1	0.23	0.34
Edge 11	0.6	-	0.3
Edge 12	-	0.27	0.23

After observing the results in Table 6.7 from the numerical example, we may notice that applying the IVIFTD TSP with rush hours periods and jam regions was done by aggregating the information of the degrees of

belonging and not belonging of a specific node. Thus, all jam factors were taken in consideration and none of the factors' effects were neglected. In addition, using Park's distance [46,47,93] of IVIFS data to reduce the loss of information lead to more accurate calculations compared to our previous approach with considering less complex uncertainty, namely, the IFTD TSP [11]. The IVIFTD TSP [12] approach can be applied in many related areas with similar time dependent traffic conditions, especially, in logistics and transportation, like single and multiple vehicles routing (VRP), which I will prove later in chapter 7 of this thesis.

6.4 Results and Discussion

The modeling of complex uncertainty, by applying the Interval-Valued Intuitionistic Fuzzy Sets led to a rather realistic and adaptable model that offers several advantages:

1. This model is suitable for real life situations when costs are uncertain and unpredictable due to the influence of unknown and non-deterministic circumstances in many day-to-day transport situations. Also, this approach easily applicable in practice and automated through a computer program module (section 6.4.1).

2. The prediction for the rush hours and the jam regions effects is done by aggregating the information on the degree of belonging (and not belonging) to a specific jam factor; taking in consideration all the jam factors simultaneously according to their weights (importance and severity) which will guarantee less information loss.

3. Using the concept of Park's distance [46,47,72,73] of IVIFS data in order to reduce the loss of information and specify which jam factors accumulatively will cause the most possible delays, and hence, avoid (mitigate) that specific route.

In the next section I will show the results of optimizing the IVIFTD TSP with DBMEA.

6.4.1 The IVIFTD TSP Optimization Results with the DBMEA

To test and compare the new model with the original TD TSP, I developed a small repository with interval valued type-2 fuzzy degrees based on the relations among jam regions according to closeness of city centres and rush hours during the day and their estimated costs in terms of the elapsed time (additional delay) during the passing of those sections (Table 6.8 and Table 6.9). Those costs essentially affect the overall tour length (tour's cost or time) and even the topography of the optimal tour. Then, the DBMEA algorithm was modified (by my colleague) and tested on Bier127 for the new IVIFTD TSP problem. The results were calculated by averaging five runs for different jam costs values. Table 6.10 and Fig. 6.1 clearly indicate that the DBMEA was able to find high quality solutions for the proposed problem even with big jam factors values [13]. The Bier127 problem was selected to continue on our previous work and comparisons [15] (Bier127 instance's setup and our configurations is fully explained in section 4.4.).

Table 6.8 factor1 - jam factor costs membership and non-membership values

Jam factor1	Cost1		Cost 2		Cost 3		Cost 4	
	μ_R	v_R	μ_R	v_R	μ_R	v_R	μ_R	v_R
[0,0.2]	[0.3,0.4]	[0.5,0.6]	[0.2,0.3]	[0.4,0.5]	[0.1,0.2]	[0.6,0.7]	[0,0.1]	[0.7,0.8]
[0.2,0.4]	[0.6,0.8]	[0.1,0.2]	[0.4,0.5]	[0.2,0.3]	[0.2,0.3]	[0.5,0.6]	[0.1,0.2]	[0.6,0.7]
[0.4,0.6]	[0.4,0.5]	[0.3,0.4]	[0.6,0.8]	[0.1,0.2]	[0.4,0.5]	[0.3,0.4]	[0.2,0.3]	[0.5,0.6]
[0.6,0.8]	[0.2,0.3]	[0.5,0.6]	[0.3,0.4]	[0.5,0.6]	[0.6,0.8]	[0.1,0.2]	[0.3,0.4]	[0.5,0.6]
[0,8,0.1]	[0,0.1]	[0.8,0.9]	[0.2,0.3]	[0.5,0.6]	[0.4,0.5]	[0.2,0.3]	[0.8,0.9]	[0,0.1]

Table 6.9 Factor2 jam factor cost membership and non-membership values

Jam factor2	Cost1		Cost2		Cost3		Cost4	
	μ_R	v_R	μ_R	v_R	μ_R	v_R	μ_R	v_R
[0,5]	[0.3,0.4]	[0.5,0.6]	[0.2,0.3]	[0.4,0.5]	[0.1,0.2]	[0.6,0.7]	[0,0.1]	[0.7,0.9]
[5,7]	[0.4,0.5]	[0.3,0.4]	[0.6,0.8]	[0.1,0.2]	[0.4,0.5]	[0.3,0.4]	[0.1,0.3]	[0.4,0.6]
[7,8]	[0.2,0.3]	[0.5,0.6]	[0.3,0.4]	[0.5,0.6]	[0.6,0.7]	[0.2,0.3]	[0.7,0.9]	[0,0.1]
[8,14]	[0.4,0.5]	[0.3,0.4]	[0.6,0.8]	[0.1,0.2]	[0.4,0.5]	[0.3,0.4]	[0.1,0.2]	[0.5,0.7]
[14,17]	[0.2,0.3]	[0.5,0.6]	[0.3,0.4]	[0.5,0.6]	[0.6,0.7]	[0.2,0.3]	[0.8,0.9]	[0,0.1]
[17,22]	[0.4,0.5]	[0.3,0.4]	[0.6,0.8]	[0.1,0.2]	[0.4,0.5]	[0.3,0.4]	[0.1,0.2]	[0.5,0.7]
[22-24]	[0.3,0.4]	[0.5,0.6]	[0.2,0.3]	[0.5,0.6]	[0.1,0.2]	[0.7,0.8]	[0,0.1]	[0.8,0.9]

Table 6.10 DBMEA vs Simulated Annealing results on the Bier127 IVIFTD TSP extension example

Jam factor	DBMEA			Simulated Annealing		
	Best elapsed time	Average elapsed time	average runtime [s]	Best elapsed time	Average elapsed time	Average runtime [s]
c1=1, c2=1.20, c3=1.50, c4=2	19.8315	19.8404	2358.75	20.8090	21.5924	23.89
c1=1, c2=1,5, c3=2, c4=5	19.8352	19.8609	2115.33	21.6720	21.9057	23.13
c1=1, c2=2, c3=5, c4=10	19.8537	19.8741	2249.31	22.0331	22.7257	26.25
c1=1, c2=5, c3=10, c4=25	19.8951	19.9395	2384.19	22.1808	22.8266	27.24

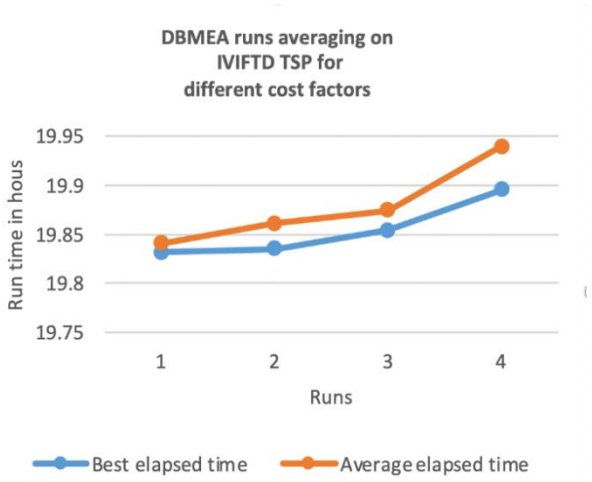

Figure 6.1 DBMEA Averaged runs on IVIFTD TSP

6.5 Summary

The results of this chapter are summarized in the booklet section (4.5).

The interval-valued intuitionistic fuzzy sets extension was applied to construct a more accurate model of The TD TSP. This model was able to calculate multiple uncertain additional costs coming from the jam regions and the rush hours periods. To efficiently combine the factors that are expressed with interval-valued intuitionistic fuzzy values, I utilized the interval-valued intuitionistic fuzzy weighted arithmetic average operator for aggregation. I also proposed a measure based on Park's distance [46,47] between IVIFS for the extended TD TSP costs under unpredicted road circumstances. Optimizing and comparing the IVIFTD TSP runs results on Bier127 with the classic TD TSP can be seen in Table 6.10 and Fig 6.2., they indicate that the new model is effective while it simulates real-life conditions and successfully quantifies the traffic effects without significant information loss. This model will give a more tangible treatment of such uncertain factors, and thus, it offers a better model in terms of accuracy and flexibility than all previous models in the literature [13,15,16].

- *Using the Interval-Valued Intuitionistic Fuzzy sets, I have mathematically fuzzified the jam regions costs in the classic TD TSP. I employed the weighted average operation to take in consideration all the rush hours costs that are affecting any edge simultaneously and thus, I decreased the accumulative information loss.*

- *I formulated a mathematical fuzzified model for the rush hour periods effects in the classic TD TSP using the Interval-Valued Intuitionistic Fuzzy sets. To cater for all simultaneous rush hours costs affecting any edge at any given time; I employed the weighted average operation which decreased the information loss and offered an accurate tangible model.*

- *I have introduced a complete novel fuzzified costs formulation for the classic TD TSP using the Interval-Valued Intuitionistic Fuzzy sets. I utilized the IIFWAA (interval-valued intuitionistic fuzzy weighted arithmetic average) operator to aggregate accumulative information from all the jam factors (depending on their weight/importance). Also, I calculated the hesitation part by*

incorporating the modified Park's distances between IVIFSs. This model was able to consider not only all the jam factors affecting an edge at any time, but also, to cater for the severity/conformability of the edge's belonging and not-belonging for a specific jam region or rush hour period. Finally, ranking the factors according to their closeness (distances between IFSs) lead to less information loss and more accurate tangible calculation for all traffic non-deterministic vague factors.

- *The IVIFTD TSP was optimized on universal instances Bier127 using the DBMEA and compared with the classical TD TSP's average and best elapsed times.*

Chapter 7 Extension of The Time Dependent Travelling Salesman Problem Model with Interval-Valued Fuzzy Soft Sets (IVFSTD TSP)

7.1 Introduction

Since we established the argument of insufficient representation of the jam regions and the rush hours in the original TD TSP, where it assigns higher costs (using concrete numbers) to those edges that cross the predefined jam area in the city centers and lower costs to the edges that do not cross that area. I kept searching for better models to represent those non-deterministic factors (the jam regions and the rush hours) on any tour. My research series went from using type-1 fuzzy sets to type-2 with different formulations, to explore all possible representations and conduct numerous comparisons between all the available models and mine. In 1999, Molodtsov introduced the concept of soft set theory as a general mathematical tool for dealing with uncertainty [54]. Alkhazaleh and Salleh defined the concept of soft expert sets where the user can know the opinion of all experts in one model and give an application of this concept in decision making problems [94]. Thus, in this chapter I introduce another novel extended fuzzy soft set-based model, the IVFSTD TSP (Interval-Valued Fuzzy Soft Time Dependent Traveling Salesman Problem). The core concept employs interval-valued fuzzy soft sets on the costs between nodes to realistically quantify the traffic jam regions, and the rush hours periods effects on any tour. Since the interval-valued fuzzy sets are generalization of the original fuzzy sets (see chapter 3 section 3.2), they have the ability to simulate uncertain road conditions more efficiently than concrete numbers, then this proposed approach (the IVFSTD) is a useful extension and a practical alternative model of the original abstract problem [4].

7.2 Related Work

Previously, I have proposed three novel approaches. First, the Triple Fuzzy Time Dependent Traveling Salesman Problem (3FTD TSP) model [10]. Second, I introduced an even more general model that allowed road uncertainties representation to become more flexible. This model was called Intuitionistic Fuzzy Time Dependent Travelling Salesman Problem (IFTDT SP) [11]. This model employs type-2 intuitionistic fuzzy sets (characterized by three-dimensional membership functions) [11]. It involves the so-called hesitation part between the membership functions and the non-membership functions representing the same uncertain components in the previous model (costs, areas of traffic jams, rush hour times) allowing a higher level of vagueness representation [12]. Third, I presented the Interval-Valued Intuitionistic Fuzzy Time Dependent Traveling Salesman Problem (IVIFTD TSP) model [13] in pursuit of considering the average collective modified costs of multiple non-deterministic factors (traffic regions and rush hours) affecting a single edge simultaneously. The IVIFTD TSP [13] uses type-2 interval-valued intuitionistic fuzzy sets. The improvement I achieved in this model was employing the aggregation concept of all the modified costs depending on their importance (weight) rather than using the max-min composition of the fuzzy factors

[11] as in the IFTD TSP, which decreased the information loss and caterer for multiple factors simultaneously.
Finally, I compared the three models efficiency and extended them on the largest instances in the literature [14,15,16,17].
In fact, my models were well received among the fuzzy community which made me eager to present more novel models and formulation in pursuit of adequately calculating the jam regions and the rush hours effect on any tour. Thus, in this model (IVFSTD TSP) [18], I moved one step forwards, by proposing another practical extension that facilitates the interval-valued fuzzy soft sets to calculate the costs between nodes and realistically quantify the traffic jam regions, and the rush hours periods effects (modified costs) on any tour.

7.3 Methodology

Preliminary definitions and operations were introduced in chapter 3 (sections 3.2.7, 3.2.8, and 3.2.9). I will build on that in this chapter.
To apply this methodology on a simple tour for the TD TSP under rush hours and jam regions. Let us assume we have 12 nodes (Fig 4.1.).

Suppose J is the set of jam factors (universe). C is the set of the modified costs (parameters), and E for set of the edges in a tour. Construct an interval-valued fuzzy soft set (F, C) over J. $\tilde{\mathrm{F}}(J)$ is a set of all interval valued fuzzy sets of J, the mapping given by:

$$F : C \to \tilde{\mathrm{F}}(J) \tag{7.1}$$

A relation matrix R_1 is constructed from the interval-valued fuzzy soft set (F, C) and called jamfactors-costs matrix.
Similarly, its complement $(F, C)^c$ gives another relation matrix, R_2, called non-jamfactors-costs matrix.

Again, we construct another interval-valued fuzzy soft set (F_1, J) over E, where $\tilde{\mathrm{F}}(E)$ is a set of all interval valued fuzzy sets of E, the mapping given by:

$$F_1 : J \to \tilde{\mathrm{F}}(E) \tag{7.2}$$

This interval-valued fuzzy soft set gives another relation matrix Q called edges-jamfactors matrix.
Then applying the composition $(\mu_{R_2 \circ Q})$ on the relation matrices R_2 with T_2 gives the edges-non cost matrix, given by:

$$\mu_{R \circ Q}(x, z) = \bigvee_y \left[\mu_Q(x, y) \wedge \mu_R(y, z)\right].$$

where $\vee$ = max and $\wedge$ = min .

The non-membership function is given by $v_{R\circ Q}(x,z) = \wedge_y\left[v_Q(x,y) \wedge v_R(y,z)\right]$.

Then we obtain two new relation matrices:

$T_1 = Q \circ R_1$, called jamfactors-edges matrix

and

$T_2 = Q \circ R_2$, non-jamfactors–edges matrix

we calculate the edges score $\boldsymbol{S_{T_1}}$and $\boldsymbol{S_{T_2}}$for and against the costs respectively:

$$\boldsymbol{S_{T_1}} = \left[\boldsymbol{max_{i,j}}\left\{\boldsymbol{ST1^{U}}\left(\boldsymbol{e_i, j_j}\right), \boldsymbol{ST2^{L}}\ \boldsymbol{R_1}\left(\boldsymbol{j_j, c_j}\right)\right\}, \forall(\boldsymbol{i\ =\ 1, 2, \ldots . n}), (\boldsymbol{j\ =\ 1, 2, \ldots\ldots, n})\ \right] \quad (7.3)$$

$$\boldsymbol{S_{T_2}} = \left[\boldsymbol{max_{i,j}}\left\{\boldsymbol{ST1^{L}}\left(\boldsymbol{e_i, j_j}\right), \boldsymbol{ST2^{U}}\ \boldsymbol{R_1}\left(\boldsymbol{j_j, c_j}\right)\right\}, \forall(\boldsymbol{i\ =\ 1, 2, \ldots . n}), (\boldsymbol{j\ =\ 1, 2, \ldots\ldots, n})\ \right] \quad (7.4)$$

Then we find:

$$maxCost_{ei} = \max_j ST1\left(e_i, j_j\right) - ST2\left(e_i, j_j\right)$$

Which indicates the modified cost that is affecting the edge the most.

To put this algorithm in sequential steps:

1. Input the interval valued fuzzy soft sets $(\boldsymbol{F}, \boldsymbol{C})$ and $(\boldsymbol{F}, \boldsymbol{C})^{\boldsymbol{c}}$ over the sets $\boldsymbol{J}$ of jam factors, where $\boldsymbol{C}$ is the set of costs. Also write the soft jam factors knowledge $\boldsymbol{R_1}$ and $\boldsymbol{R_2}$ representing the relation matrices of the IVFSs $(\boldsymbol{F}, \boldsymbol{C})$ and $(\boldsymbol{F}, \boldsymbol{C})^{\boldsymbol{c}}$ respectively.
2. Input the IVFSs $(\boldsymbol{F_1}, \boldsymbol{J})$ over the set $\boldsymbol{E}$ of edges and write its relation matrix $\boldsymbol{Q}$.
3. Compute the relation matrices $\boldsymbol{T_1} = (\boldsymbol{Q} \circ \boldsymbol{R_1})$ = and $\boldsymbol{T_2} = (\boldsymbol{Q} \circ \boldsymbol{R_2})$
4. Compute the costs scores $\boldsymbol{S_{T_1}}$, $\boldsymbol{S_{T_2}}$
5. Find max score for each edge, $\boldsymbol{maxCost_{ei}} = \boldsymbol{\max_j}[\boldsymbol{ST1}\left(\boldsymbol{e_i, j_j}\right) - \boldsymbol{ST2}\left(\boldsymbol{e_i, j_j}\right)]$. Then we conclude that the edge $\boldsymbol{e_i}$ is affected by cost $\boldsymbol{maxCost_{ei}}$
6. If $\boldsymbol{maxCost_{ei}}$ is affected by more than one modified cost value, then we can take the weighted average and all the total value to the overall cost of the tour as additional cost, or repeat the process by reassessing the jam factors affecting each edge.

7.3.1 Applying The IVFSTD TSP Model on Simple TD TSP Case.

Suppose there is three edges (e_1, e_2 and e_3) under study, each one of them is affected by different jam factors, whether jam regions in the city centers or rush hours during the day. For the set of jam factors we will assume we have 4 types of possible factors represented in this set. $J = \{j_1, j_2, j_3, j_4\}$ as universal set.

Set $C = \{c_1, c_2\}$, where c_1 is the modified costs (parameters) associated with the first jam factor, and c_2 is the associated cost for the second jam factor.

Suppose that $F(c_1) = \{\langle j_1, [0.7,1]\rangle, \langle j_2, [0.1,0.4]\rangle, \langle j_3, [0.5,0.6]\rangle, \langle j_4, [0.2,0.4]\rangle\}$. $F(c_2) = \{\langle j_1, [0.6,9]\rangle, \langle j_2, [0.4,0.6]\rangle, \langle j_3, [0.3,0.6]\rangle, \langle j_4, [0.8,0.1]\rangle\}$, and so on for the rest of the edges as seen below. The interval-valued fuzzy soft set (F, C) is a parametrized family referred to as $\{F(c_1), F(c_2)\}$ of all interval-valued fuzzy soft sets over the set J and are determined from experts for the location under study. Hence, the fuzzy soft set (F, C) gives an approximate description of the interval-valued soft traffic costs and their jam factors. This interval-valued fuzzy soft set (F, C) and its complement $(F, C)^{\mathrm{c}}$ are represented by two relation matrices R_1 and R_2 in our previous discussion, called non-jamfactors-costs matrix and non-jamfactors-costs matrix respectively.

$$R_1 = \begin{array}{c} \\ j_1 \\ j_2 \\ j_3 \\ j_4 \end{array} \begin{array}{c} \begin{array}{cc} c_1 & \quad c_2 \end{array} \\ \begin{bmatrix} [0.7,1] & [0.6,0.9] \\ [0.1,0.4] & [0.4,0.6] \\ [0.5,0.6] & [0.3,0.6] \\ [0.2,0.4] & [0.8,0.1] \end{bmatrix} \end{array}$$

$$R_2 = \begin{array}{c} \\ j_1 \\ j_2 \\ j_3 \\ j_4 \end{array} \begin{array}{c} \begin{array}{cc} c_1 & \quad c_2 \end{array} \\ \begin{bmatrix} [0.0, 0.3] & [0.1, 0.4] \\ [0.6, 0.9] & [0.4, 0.6] \\ [0.4, 0.5] & [[0.4, 0.7] \\ [0.6, 0.8] & [[0.0, 0.2] \end{bmatrix} \end{array}$$

Then

$$R_{1\mathrm{L}} = \begin{array}{c} \begin{array}{cc} c_1 & \quad c_2 \end{array} \\ \begin{bmatrix} [0.7,0.6] \\ [0.1,0.4] \\ [0.5,0.3] \\ [0.2,0.8] \end{bmatrix} \end{array} \quad R_{1\mathrm{U}} = \begin{array}{c} \begin{array}{cc} c_1 & \quad c_2 \end{array} \\ \begin{bmatrix} [1,0.9] \\ [0.4,0.6] \\ [0.6,0.6] \\ [0.4,0.1] \end{bmatrix} \end{array}$$

$$R_{2\mathrm{L}} = \begin{array}{c} \begin{array}{cc} c_1 & \quad c_2 \end{array} \\ \begin{bmatrix} [0.0,0.1] \\ [0.6,0.4] \\ [0.4,0.4] \\ [0.6,0.0] \end{bmatrix} \end{array} \quad R_{2\mathrm{U}} = \begin{array}{c} \begin{array}{cc} c_1 & \quad c_2 \end{array} \\ \begin{bmatrix} [0.3,0.4] \\ [0.9,0.6] \\ [0.5,0.7] \\ [0.8,0.2] \end{bmatrix} \end{array}$$

Now we take as a universal set where e_1, e_2 and e_3 represents the first three edges respectively and $J = \{j_1, j_2, j_3, j_4\}$ as the set of jam factors. Suppose that

$$F_1(j_1) = \{\langle e_1, [0.6,0.9]\rangle, \langle e_2, [0.3,0.5]\rangle, \langle e_3, [0.6,0.8]\rangle\},$$
$$F_2(j_2) = \{\langle e_1, [0.3,0.5]\rangle, \langle e_2, [0.3,0.7]\rangle, \langle e_3, [0.2,0.6]\rangle\},$$
$$F_3(j_3) = \{\langle e_1, [0.8,0.1]\rangle, \langle e_2, [0.2,0.4]\rangle, \langle e_3, [0.5,0.7]\rangle\},$$
$$F_4(j_4) = \{\langle e_1, [0.6,0.9]\rangle, \langle e_2, [0.3,0.5]\rangle, \langle e_3, [0.2,0.5]\rangle\}.$$

Then, the interval-valued fuzzy soft set (F_1, J) is another parameterized family of all interval-valued fuzzy soft sets gives a collection of approximate description of the edges-costs in any given tour. This interval-valued fuzzy soft sets (F_1, J) represents a relation matrix Q called edges–costs matrix given by Q (the columns here represent j_1, j_2, j_3 and j_4 respectively). Then applying composition $(\mu_{R_1 \circ Q})$ on the relation matrices R_1 with Q we get two matrices T_1 gives the edges-cost matric. Then applying the composition $(\mu_{R_2 \circ Q})$ on the relation matrices R_2 with T_2 gives the edges-non cost matrix.

Also, in Table 1, shows how the added modified cost for each edge that is being affected by a jam factor (rush hours and jam regions) and what are the costs added to each edge. After applying the Max operation, we can define which is the jam factor with the highest cost on each edge.

$$Q = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 \\ e_8 \\ e_9 \\ e_{10} \\ e_{11} \\ e_{12} \end{array} \begin{array}{cccc} j_1 & j_2 & j_3 & j_4 \\ \hline [0.6,0.9] & [0.3,0.5] & [0.8,1] & [0.6,.9] \\ [0.3,0.5] & [0.3,0.7] & [0.2,0.4] & [0.3,0.5] \\ [0.6,0.8] & [0.2,0.6] & [0.5,0.7] & [0.2,0.5] \\ [0.6,0.9] & [0.3,0.5] & [0.8,1] & [0.6,.9] \\ [0.3,0.5] & [0.3,0.7] & [0.2,0.4] & [0.3,0.5] \\ [0.6,0.8] & [0.2,0.6] & [0.5,0.7] & [0.2,0.5] \\ [0.6,0.9] & [0.3,0.5] & [0.8,1] & [0.6,.9] \\ [0.3,0.5] & [0.3,0.7] & [0.2,0.4] & [0.3,0.5] \\ [0.6,0.8] & [0.2,0.6] & [0.5,0.7] & [0.2,0.5] \\ [0.6,0.9] & [0.3,0.5] & [0.8,1] & [0.6,.9] \\ [0.3,0.5] & [0.3,0.7] & [0.2,0.4] & [0.3,0.5] \\ [0.6,0.8] & [0.2,0.6] & [0.5,0.7] & [0.2,0.5] \end{array}$$

The fuzzy relation Q has upper and lower bounds, such that $Q = [Q_L, Q_U]$. Then

$$Q_L = \begin{pmatrix} 0.6 & 0.3 & 0.8 & 0.6 \\ 0.3 & 0.3 & 0.2 & 0.3 \\ 0.6 & 0.2 & 0.5 & 0.2 \\ 0.6 & 0.3 & 0.8 & 0.6 \\ 0.3 & 0.3 & 0.2 & 0.3 \\ 0.6 & 0.2 & 0.5 & 0.2 \\ 0.6 & 0.3 & 0.8 & 0.6 \\ 0.3 & 0.3 & 0.2 & 0.3 \\ 0.6 & 0.2 & 0.5 & 0.2 \\ 0.6 & 0.3 & 0.8 & 0.6 \\ 0.3 & 0.3 & 0.2 & 0.3 \\ 0.6 & 0.2 & 0.5 & 0.2 \end{pmatrix}$$

$$Q_U = \begin{pmatrix} 0.9 & 0.3 & 0.1 & 0.9 \\ 0.5 & 0.7 & 0.4 & 0.5 \\ 0.8 & 0.6 & 0.7 & 0.5 \\ 0.9 & 0.3 & 0.1 & 0.9 \\ 0.5 & 0.7 & 0.4 & 0.5 \\ 0.8 & 0.6 & 0.7 & 0.5 \\ 0.9 & 0.3 & 0.1 & 0.9 \\ 0.5 & 0.7 & 0.4 & 0.5 \\ 0.8 & 0.6 & 0.7 & 0.5 \end{pmatrix}$$

Same concept, we combine the relation matrices $R1_L$, $R1_U$ and $R2_L$, $R2_U$ with Q_L and Q_U as follows :

$$T1_L = Q_L \circ R1_L \text{ and } T1_U = Q_U \circ R1_U$$

$$T1_L = \begin{pmatrix} 0.6 & 0.6 \\ 0.3 & 0.3 \\ 0.6 & 0.6 \\ 0.6 & 0.6 \\ 0.3 & 0.3 \\ 0.6 & 0.6 \\ 0.6 & 0.6 \\ 0.3 & 0.3 \\ 0.6 & 0.6 \\ 0.6 & 0.6 \\ 0.3 & 0.3 \\ 0.6 & 0.6 \end{pmatrix} \quad T1_U = \begin{pmatrix} 0.9 & 0.9 \\ 0.5 & 0.6 \\ 0.8 & 0.8 \\ 0.9 & 0.9 \\ 0.5 & 0.6 \\ 0.8 & 0.8 \\ 0.9 & 0.9 \\ 0.5 & 0.6 \\ 0.8 & 0.8 \\ 0.9 & 0.9 \\ 0.5 & 0.6 \\ 0.8 & 0.8 \end{pmatrix}$$

$$T2_L = Q_L \circ R2_L \text{ and } T2_U = Q_U \circ R2_U$$

$$T2_L = \begin{pmatrix} 0.6 & 0.4 \\ 0.3 & 0.3 \\ 0.4 & 0.4 \\ 0.6 & 0.4 \\ 0.3 & 0.3 \\ 0.4 & 0.4 \\ 0.6 & 0.4 \\ 0.3 & 0.3 \\ 0.4 & 0.4 \\ 0.6 & 0.4 \\ 0.3 & 0.3 \\ 0.4 & 0.4 \end{pmatrix} \quad T2_U = \begin{pmatrix} 0.8 & 0.7 \\ 0.7 & 0.6 \\ 0.6 & 0.7 \\ 0.8 & 0.7 \\ 0.7 & 0.6 \\ 0.6 & 0.7 \\ 0.8 & 0.7 \\ 0.7 & 0.6 \\ 0.6 & 0.7 \\ 0.8 & 0.7 \\ 0.7 & 0.6 \\ 0.6 & 0.7 \end{pmatrix}$$

To calculate the negated values for each edge and non-belonging (negation or the complement) to a specific modified cost, we apply:

$$T3_L = Q_L \circ (1 - R1_L) \text{ and } T3_U = Q_U \circ (1 - R3_U)$$

$$T3_L = \begin{pmatrix} 0.6 & 0.7 \\ 0.3 & 0.3 \\ 0.5 & 0.5 \\ 0.6 & 0.7 \\ 0.3 & 0.3 \\ 0.5 & 0.5 \\ 0.6 & 0.7 \\ 0.3 & 0.3 \\ 0.5 & 0.5 \\ 0.6 & 0.7 \\ 0.3 & 0.3 \\ 0.5 & 0.5 \end{pmatrix} \quad T3_U = \begin{pmatrix} 0.6 & 0.4 \\ 0.6 & 0.4 \\ 0.6 & 0.4 \\ 0.6 & 0.4 \\ 0.6 & 0.4 \\ 0.6 & 0.4 \\ 0.6 & 0.4 \\ 0.6 & 0.4 \\ 0.6 & 0.4 \\ 0.6 & 0.4 \\ 0.6 & 0.4 \\ 0.6 & 0.4 \end{pmatrix}$$

$$T4_L = Q_L \circ (1 - R2_L) \text{ and } T4_U = Q_U \circ (1 - R2_U)$$

$$T4_L = \begin{pmatrix} 0.6 & 0.6 \\ 0.3 & 0.3 \\ 0.6 & 0.6 \\ 0.6 & 0.6 \\ 0.3 & 0.3 \\ 0.6 & 0.6 \\ 0.6 & 0.6 \\ 0.3 & 0.3 \\ 0.6 & 0.6 \\ 0.6 & 0.6 \\ 0.3 & 0.3 \\ 0.6 & 0.6 \end{pmatrix} \quad T4_U = \begin{pmatrix} 0.6 & 0.7 \\ 0.5 & 05 \\ 0.7 & 0.6 \\ 0.6 & 0.7 \\ 0.5 & 0.5 \\ 0.7 & 0.6 \\ 0.6 & 0.7 \\ 0.5 & 0.5 \\ 0.7 & 0.6 \\ 0.6 & 0.7 \\ 0.5 & 0.5 \\ 0.7 & 0.6 \end{pmatrix}$$

To calculate $S_{T1_L} = max_{i,j}\ ([T_{1_L}(e_i, c_j), T_{4_L}(e_i, c_j)])$, $S_{T1_U} = max_{i,j}\ ([T_{1_U}(e_i, c_j), T_{4_U}(e_i, c_j)])$, in our example, ($i = 1,2, \dots 12$) the edges count and ($j = 1,2$) for the costs.

$$S_{T1_L} = \begin{pmatrix} 0.6 & 0.6 \\ 0.3 & 0.3 \\ 0.6 & 0.6 \\ 0.6 & 0.6 \\ 0.3 & 0.3 \\ 0.6 & 0.6 \\ 0.6 & 0.6 \\ 0.3 & 0.3 \\ 0.6 & 0.6 \\ 0.6 & 0.6 \\ 0.3 & 0.3 \\ 0.6 & 0.6 \end{pmatrix} \qquad S_{T1_U} = \begin{pmatrix} 0.9 & 0.9 \\ 0.5 & 0.6 \\ 0.8 & 0.8 \\ 0.9 & 0.9 \\ 0.5 & 0.6 \\ 0.8 & 0.8 \\ 0.9 & 0.9 \\ 0.5 & 0.6 \\ 0.8 & 0.8 \\ 0.9 & 0.9 \\ 0.5 & 0.6 \\ 0.8 & 0.8 \end{pmatrix}$$

Similarly $S_{T2_L} = max_{i,j}\ ([T_{2_L}(e_i, c_j), T_{3_L}(e_i, c_j)])$, $S_{T2_U} = max_{i,j}\ ([T_{2_U}(e_i, c_j), T_{3_U}(e_i, c_j)])$,

($i = 1,2, \dots 12$) the edges count in this exmaple and ($j = 1,2$) for the costs.

$$S_{T2_L} = \begin{pmatrix} 0.6 & 0.7 \\ 0.3 & 0.3 \\ 0.5 & 0.5 \\ 0.6 & 0.7 \\ 0.3 & 0.3 \\ 0.5 & 0.5 \\ 0.6 & 0.7 \\ 0.3 & 0.3 \\ 0.5 & 0.5 \\ 0.6 & 0.7 \\ 0.3 & 0.3 \\ 0.5 & 0.5 \end{pmatrix} \qquad S_{T2_U} = \begin{pmatrix} 0.8 & 0.7 \\ 0.7 & 0.6 \\ 0.6 & 0.7 \\ 0.8 & 0.7 \\ 0.7 & 0.6 \\ 0.6 & 0.7 \\ 0.8 & 0.7 \\ 0.7 & 0.6 \\ 0.6 & 0.7 \\ 0.8 & 0.7 \\ 0.7 & 0.6 \\ 0.6 & 0.7 \end{pmatrix}$$

Finally, we apply:

$$S_{T1} = max_{i,j}\left([S_{T1_U}(e_i, c_j), S_{T2_L}(e_i, c_j)]\right)$$

and

$$S_{T2}=max_{i,j}\left([S_{T1_L}(e_i, c_j), S_{T2_U}(e_i, c_j)]\right)$$

$(i = 1,2, \dots 12)$the edges count in this exmaple and $(j = 1,2)$ for the costs under study.

$$S_{T1} = \begin{pmatrix} 0.9 & 0.9 \\ 0.5 & 0.6 \\ 0.8 & 0.8 \\ 0.9 & 0.9 \\ 0.5 & 0.6 \\ 0.8 & 0.8 \\ 0.9 & 0.9 \\ 0.5 & 0.6 \\ 0.8 & 0.8 \\ 0.9 & 0.9 \\ 0.5 & 0.6 \\ 0.8 & 0.8 \end{pmatrix} \qquad S_{T2} = \begin{pmatrix} 0.8 & 0.7 \\ 0.7 & 0.6 \\ 0.6 & 0.7 \\ 0.8 & 0.7 \\ 0.7 & 0.6 \\ 0.6 & 0.7 \\ 0.8 & 0.7 \\ 0.7 & 0.6 \\ 0.6 & 0.7 \\ 0.8 & 0.7 \\ 0.7 & 0.6 \\ 0.6 & 0.7 \end{pmatrix}$$

To calculate the modified cost for each edge we apply $[S_{T1} - S_{T2}]$ which gives :

$$maxCost_{ei} = (S_{T1} - S_{T2}) = \begin{pmatrix} 0.1 & 0.2 \\ -0.2 & 0.0 \\ 0.2 & 0.1 \\ 0.1 & 0.2 \\ -0.2 & 0 \\ 0.2 & 0.1 \\ 0.1 & 0.2 \\ -0.2 & 0 \\ 0.2 & 0.1 \\ 0.1 & 0.2 \\ -0.2 & 0 \\ 0.2 & 0.1 \end{pmatrix}$$

Table 7.1 jam factors costs on a simple tour's edges

$Score_{T_1}$-$Score_{T_2}$	c_1	c_2
e_1	0.1	0.2
e_2	-0.2	0
e_3	0.2	0.1

e_4	0.1	0.2
e_5	-0.2	0
e_6	0.2	0.1
e_7	01	0.2
e_8	-0.2	0
e_9	0.2	0.1
e_{10}	0.1	0.2
e_{11}	-0.2	0
e_{12}	0.2	0.1

For example, it is clear (from Table.1) that edge 3 (e_3) for example is mostly affected by cost1 (c_1) and edges 1 and 2 (e_1, e_2) are being affected by Costs c_2 mostly. We have applied the notion of interval-valued fuzzy soft sets on the TD TSP to be able to calculate the jam factors and rush hours' additional costs that affect each edge in a tour and how the additional delays are being interpreted in terms of trip costs respectively.

7.4 Summary

The results of this chapter are summarized in the booklet section (4.6).

I have applied the notion of interval-valued fuzzy soft sets for deciding about the overlapping jam factors for an edge to calculate the overall added cost tangibly. The case study has been taken to exhibit the simplicity of the technique and applicability. To validate this model, I am eager to test and validate this approach on the largest instances in the literature and compare the results with our previously extended fuzzy approaches for the TD TSP with the jam factors and the rush hours and with the state of the art solutions.

- *I introduced a complete novel Interval-Valued Fuzzy Soft Time Dependent Traveling Salesman Problem (IVFSTD TSP), by applying interval-valued fuzzy soft sets on the costs between nodes to realistically quantify the traffic jam regions, and the rush hours periods effects on any tour, which offered a closer-to-reality calculation for the travelled routes under vague and unpredictable road conditions.*

Chapter 8 The Extended TOPSIS Method for the Dial-a-ride Problem with Time Window Using Interval-Valued Intuitionistic Fuzzy Sets

8.1 Introduction

The Dial-a-ride problem with time window specifies transportation requests between pairs of origins and destinations within a specific time window [95]. Transportation is provided by a fleet of vehicles based at a common depot. The aim is to design a set of least cost vehicle routes that is capable of accommodating all requests. There are some constraints related to the vehicle capacity, route duration and the maximum tolerated (accepted) delay time by users. TOPSIS (The technique for ordered preference by similarity to the ideal solution) is one of the well-known methods for multiple attribute decision making (MADM) [96,97]. In this part, I will extend the TOPSIS method [98] into an interval-valued intuitionistic fuzzy environment to solve multiple attribute group decision making (MAGDM) problem in which all the preferences information provided by the decision-makers (in our case the passengers) are presented in the form of interval-valued intuitionistic fuzzy decision matrices. I established an optimization model to determine the weights of attributes and constructed the weighted collective interval-valued intuitionistic fuzzy decision matrix, then I determined both the interval-valued intuitionistic positive-ideal solution and the interval-valued intuitionistic negative-ideal solution. Finally, based on different distance definitions, I calculated the relative closeness of each alternative to the interval-valued intuitionistic positive-ideal solution and ranked the alternatives according to their relative closeness to the interval-valued intuitionistic positive-ideal solution and, thus, recommended the collective most desirable alternative(s).

8.2 Related Work

The Dial-a-ride routing and scheduling problem with time windows (DARPTW) is widely applied in logistics and transportations. It consists of designing vehicle routes and schedules for n users who specify pickup and delivery requests between origins and destinations. Transportation is supplied by a fleet of vehicles based at a common depot. DARPTW aim is to plan a set of minimum cost vehicle routes capable of accommodating as many users as possible, under a set of constraints within a pre-specified time window. In order to fulfil the planned requests, a set of routes must be scheduled such that each request is transported from its origin to its destination by exactly *one* vehicle [95]. It is worth mentioning here that my focus is on one-to-many shared vehicle rather than many-to-many (only one vehicle that serves multiple passengers). Since the DARPTW is a special case of the Vehicle Routing Problem with Time Windows (VRPTW), it is an NP-hard problem [96]. In fact, DARPTW is a special case of the transportation problem where each customer can have a distinct origin and destination where “advance-request” is applied, so that all the requests are received before the time of vehicle dispatching. Furthermore, “service quality constraints” must satisfy two main conditions in each ride planning. *First*, customers’ riding times should not exceed a pre-specified maximum allowed time. *Second*, the times of pick-up and delivery should not exceed the specific acceptable deviation times of these customers (which must also be pre-defined), which

is the time windows constraint. Indeed, DARPTW applications range from local area courier services, door to door delivery, cross boarder transportations and public transit which explains the growing interest in the DARPTW over the recent years.

In the literature, this problem was solved through discrete optimization algorithms [97]; more precisely, an efficient neighbourhood search procedure was used for the probabilistic DARP (dial-a-ride problem). In this approach, it was possible to update the probabilities faster if the search was conducted in a certain order. Some researchers used a weighted bipartite matching model to pair drivers and riders dynamically. Others used the Non-Dominated Sorting Genetic Algorithm, the model formulation depends on solving multiple objective DARP while trying to minimize the total tour distances, the number of empty seats, and the wage differences between the drivers. Moreover, the Tabu search heuristic with capacity and route length restrictions was applied on DARP [98]. This approach uses a sequence of adjacent solutions obtained by repeatedly removing a vertex from its current route and reinserting it into another route. Furthermore, Tripathy, T. [99] introduced the Ant Colony Optimization algorithm which was applied to minimize the fleet size required to solve a DARP. Also, one of the most popular approaches is Qu and Bard's [100], where they used a Branch and Price and Cut algorithm (B&P&C). Generally, A B&P&C algorithm can solve small to medium-size instances [101] along with a mixed-integer programming formulation. In fact, the B&P (Branch-and-Price) and the B&P&C algorithms are two of the most popular exact algorithms to solve DARPs. The essence of the B&P and the B&P&C [102,103,104,105] algorithms are the reformulation of the overall problem into a master problem and sub-problems, which enhances the computational performance of these exact algorithms. Although, multi-objective DARPs (i.e., time window, vehicle capacity, ride time, route duration, and selective visits) have been considered in the literature as shown in [102,103,104,105,106], but all those models used meta-heuristics algorithms, therefore, it was necessary to integrate a decomposition technique into multi-objective exact algorithms, in the pursuit of providing a benchmark for the results obtained by the multi-objective problem in hand.

This was the main motivation of my current research. Another reason was the fact that there had been no publications found, which dealt with the uncertainties of passenger demands while fulfilling the tour's requirements including minimizing the total mileage and maintaining service fairness (least delay time and least cost). Therefore, the need emerged for my novel model.

Indeed, uncertainty (fuzzy) is a vital feature of this problem formulation. It comes from the fact that passengers provide either imprecise information, that is, there may be some hesitation concerning their preferences, because decisions are made under time pressure, or with lack of information at a specific time, or simply because the passengers have limited attention and information processing capacities. Handling this undeniably neglected aspect of the DARP will lead to a more realistic solution that simulates real-life situations much closer than the previous approaches.

In order to handle the real-life problems mentioned above, I incorporated the extended TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) method with interval-valued intuitionistic fuzzy environment in pursuit of the best fuzzy alternatives of each criterion depicted in a simple mathematical form. In the extended TOPSIS [108] method with interval-valued intuitionistic fuzzy environment [109], all the preferences provided by passengers are presented in the form of interval-valued intuitionistic fuzzy decision matrices [14].

Afterwards, I established an optimization model to determine the weights of attributes and constructed the weighted aggregated interval-valued intuitionistic fuzzy decision matrix, and then determined the interval-valued intuitionistic [110] positive-ideal solution (PIS) maximizing the benefit criteria and the interval-valued intuitionistic [111] negative-ideal solution (NIS) minimizing the cost criteria. The PIS is composed

of all best criteria values attainable, while the NIS is constituted of all worst criteria values. Incorporating this technique, allowed me to determine the gap (distance) between the ideal alternative and each alternative found, ranking these alternatives, and ultimately choose the best alternative among all [14]. In addition, my model facilitates the importance of attributes weights in the comparison procedure, so that it ranks the alternatives according to their relative closeness to the PIS, and eventually, selects the collective most desirable alternative(s) for the whole group of passengers. Indeed, this approach does not only reach the most beneficial alternative (ride) for all group members, but also avoids losing or distorting the original decision information in the process of aggregation.
From the above reasoning, it is obvious that DARPTW is a typical Multiple attribute decision making (MADM) problem that involves minimizing the costs and maximizing the benefits. The proposed novel approach infuses extended TOPSIS [112,113] with the MADM concept to formulate an efficient DARPTW solution that overcomes all the shortages that can be found in the literature, and views DARPTW from a completely different perspective. The main features of the new model are:

1. The prediction for the best route is done by aggregating the information pair of the degrees of belonging and of not belonging of the attributes obtained from the interval-valued intuitionistic fuzzy sets.
2. Using the distance between interval sets helps reduce the information loss and include all attributes according to their respective importance (weights).
3. This approach is easily applicable in practice and can be implemented in the form of a computer program module.

8.3 Methodology

Generally, any Multiple attribute decision making (MADM) problem can be expressed in matrix format as: $O_1, O_2, \ldots . O_n$ represent the possible alternatives among users, and $u_1, u_2, \ldots . u_m$ represent the attributes whose alternative performance is measured, r_{ij} is the rating of alternative O_j with respect to attribute u_i, and w_i is the weight of attribute u_i, as below in table 8.1 [113]. In the situations where the information about attributes weights is completely known, the weight vector $w = (w_1, w_2, w_3 \ldots w_m)^T$, where $j = (1,2, \ldots . n)$; $i = (1,2, \ldots . m)$.

Table 8.1 Multiple attribute decision making

	O_1	O_2	….	O_{n-1}	O_n
u_1	r_{11}	r_{12}	…	…	r_{1n}
u_2	r_{21}	r_{22}	…	…	r_{2n}
u_3	r_{31}	r_{32}	….	….	r_{3n}
….	….	….	….	….	….
u_m	r_{m1}	r_{m2}	….	….	r_{mn}

The attributes considered in our study are the ones the passengers consider when sharing a ride; they were borrowed from the relevant literature, especially for the DARPTW problem [77,78,80,82,86,88].

8.3.1 DARPTW Attributes $u_{1..n}$

1. **Earliest departure time**
2. **Time window:** Each request and each vehicle have a time window that must be satisfied (latest arrival time)
3. **Maximum accepted delay**
4. **Shared cost:** Each request defines a maximum cost to be paid for a shared ride. The variable trip-related costs are re-allocated among the ride participants in a way that makes it beneficial for them to participate from cost reduction perspective.
5. **Non-recurring trips:** Dynamic ridesharing focuses on single, non-recurring trips. This distinguishes it from traditional carpooling or vanpooling.

Using Equations and definitions in chapter 3 (from Eq. 3.1 to Eq. 3.18), let $D[0,1]$ be the set of all closed subintervals of $[0,1]$, an IVIFS $\tilde{A}$ in X is defined as:

$$\tilde{A} = \{\langle x, \tilde{\mu}_A(x), \tilde{v}_A(x)\rangle | x \epsilon X\} \quad (8.1)$$

where $\tilde{\mu}_A(x)$: $X \rightarrow D[0,1]$ and $\tilde{v}_A(x)$: $X \rightarrow D[0,1]$, with the condition :

$$0 \leq \sup(\tilde{\mu}_A(x)) + \sup(\tilde{v}_A(x)) \leq 1, \forall x \in X \quad (8.2)$$

For each $x \in X$, $\tilde{\mu}_A(x)$ and $\tilde{v}_A(x)$ are closed intervals and their lower and upper end points are denoted by $\tilde{\mu}_{AL}(x), \tilde{\mu}_{AU}(x)$ and $\tilde{v}_{AL}(x), \tilde{v}_{AU}(x)$, respectively and can be written :

$$\tilde{A} = \{\langle x, [\tilde{\mu}_{AL}(x), \tilde{\mu}_{AU}(x)], [\tilde{v}_{AL}(x), \tilde{v}_{AU}(x)]\rangle : x \in X\} \quad (8.3)$$

where $0 \leq \tilde{\mu}_{AU}(x) + \tilde{v}_{AU}(x) \leq 1$ and $0 \leq \tilde{\mu}_{AL}(x) + \tilde{v}_{AL}(x) \leq 1$ for any $x \in X$.

For each IVIFS A in X, Park et al. [46,47,70] called $\pi_A(x)$ an intuitionistic fuzzy interval of X in A .

$$\pi_A(x) = 1 - \tilde{\mu}_A(x) - \tilde{v}_A(x) = [1 - \tilde{\mu}_{AU}(x) - \tilde{v}_{AU}(x), 1 - \tilde{\mu}_{AL}(x) - \tilde{v}_{AL}(x)] \quad (8.4)$$

Similarly, the intervals $\tilde{\mu}_A(x)$ and $\tilde{v}_A(x)$ denote the degrees of membership and non-membership of x to $\tilde{A}$, respectively. Given any $x \in X$, the couple $\langle x, \tilde{\mu}_A(x), \tilde{v}_A(x)\rangle$ is called an IVIFN [45,93,94,95].

8.3.2 IVIFNs Operations and Definitions

For convenience, Xu [43] called $\tilde{a} = \langle [a, b], [c, d] \rangle$ an interval-valued intuitionistic fuzzy (IVIFN) where $[a, b] \subset [0,1]$, $[c, d] \subset [0,1]$ and $b + d \leq 1$.

Next, I will introduce some operational laws of IVIFNs, which will be useful in the following sections [41,43,44,45,69]

Let $\tilde{a}_1 = \langle [a_1, b_1], [c_1, d_1] \rangle$, $\tilde{a}_2 = \langle [a_2, b_2], [c_2, d_2] \rangle$ *and* $\tilde{a} = \langle [a, b], [c, d] \rangle$ be three IVIFNs and $\lambda > 0$, then:

$$\tilde{a}_1 \otimes \tilde{a}_2 = \langle [a_1 a_2, b_1 b_2], [c_1 + c_2 - c_1 c_2, d_1 + d_2 - d_1 d_2] \rangle \quad (8.5)$$

$$\tilde{a}^{\lambda} = \langle [a^{\lambda}, b^{\lambda}], [1 - (1 - c)^{\lambda}, 1 - (1 - d)^{\lambda}] \rangle \quad (8.6)$$

$$\boldsymbol{\lambda \tilde{a}} = \langle [\mathbf{1} - (\mathbf{1} - \boldsymbol{a})^{\boldsymbol{\lambda}}, \mathbf{1} - (\mathbf{1} - \boldsymbol{b})^{\boldsymbol{\lambda}}], [\boldsymbol{c}^{\boldsymbol{\lambda}}, \boldsymbol{d}^{\boldsymbol{\lambda}}] \rangle \quad (8.7)$$

Xu [45,92,93,94], defined the score function s to measure an IVIFN $\tilde{a}$ as follows:

$$\boldsymbol{s(\tilde{a}) = 1/2(a - c + b - d)} \quad (8.8)$$

where $s(\tilde{a}) \in [-1,1]$. The larger the value of $s(\tilde{a})$ is, the higher is the IVIFN $\tilde{a}$. The largest IVIFN value is assumed when $s(\tilde{a}) = 1$

The accuracy function h is defined by Wei and Wang [94] in order to evaluate the accuracy degree of IVIFN $\tilde{a}$ as follows:

$$h(\boldsymbol{\tilde{a}}) = \mathbf{1/2}(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} + \boldsymbol{d}) \quad (8.9)$$

where h($\tilde{a}$) $\in$ [0,1]. Hence, the larger is value of $h(\tilde{a})$ the higher the accuracy of IVIFN $\tilde{a}$ is.
Thus, the hesitancy degree (utilizing Eq. 8.4) of the IVIFN $\tilde{a} = \langle [a, b], [c, d] \rangle$ as the midpoint of the intuitionistic fuzzy interval of $\tilde{a}$ is:

$$\boldsymbol{\pi(\tilde{a}) = 1/2((1 - a - c) + (1 - b - d))} \quad (8.10)$$

Then, the relation between the hesitation and accuracy degrees of IVIFN $\tilde{a}$ is given by:

$$\boldsymbol{\pi(\tilde{a}) = 1/2((1 - a - c) + (1 - b - d)) = 1 - h(\tilde{a})} \quad (8.11)$$

thus,

$$\boldsymbol{\pi(\tilde{a}) + h(\tilde{a}) = 1} \quad (8.12)$$

We conclude that the higher is the accuracy $h(\tilde{a})$, the lower is the hesitation $\pi(\tilde{a})$ and vice versa.
Depending on scores function Eq. (8.8) and accuracy function Eq. (8.9) we can compare two IVIFNs as follows [93,94]:

Definitions

Let $\tilde{a}_1 = \langle [a_1, b_1], [c_1, d_1] \rangle$ and $\tilde{a}_2 = \langle [a_2, b_2], [c_2, d_2] \rangle$ be two IVIFNs, then $s(\tilde{a}_1) = 1/2(a_1 - c_1 + b_1 - d_1)$ and $s(\tilde{a}_2) = 1/2(a_2 - c_2 + b_2 - d_2)$ are the scores of $\tilde{a}_1, \tilde{a}_2$, respectively and $h(\tilde{a}_1) = 1/2(a_1 + b_1 + c_1 + d_1)$ and $h(\tilde{a}_2) = 1/2(a_2 + b_2 + c_2 + d_2)$ represent the accuracy degrees for $\tilde{a}_1$ and $\tilde{a}_2$, respectively, then:

- If $(\tilde{a}_1) < s(\tilde{a}_2)$ then $\tilde{a}_1 < \tilde{a}_2$
- If $s(\tilde{a}_1) = s(\tilde{a}_2)$ then:
  - If $h(\tilde{a}_1) = h(\tilde{a}_2)$ then $\tilde{a}_1, \tilde{a}_2$ represent the same information and is denoted by $\tilde{a}_1 = \tilde{a}_2$
  - If $h(\tilde{a}_1) < h(\tilde{a}_2)$ then $\tilde{a}_1$ is smaller than $\tilde{a}_2$,denoted by $\tilde{a}_1 < \tilde{a}_2$

- Let $\tilde{a}_1 = \langle [a_1, b_1], [c_1, d_1] \rangle$ and $\tilde{a}_2 = \langle [a_2, b_2], [c_2, d_2] \rangle$ be two IVIFNs, then:

$$\boldsymbol{a_1 \leq a_2, b_1 \leq b_2, c_1 \geq c_2} \text{ and } \boldsymbol{d_1 \geq d_2 \Rightarrow \tilde{a}_1 \leq \tilde{a}_2} \quad (8.13)$$

8.3.3 The Extended TOPSIS with Interval-Valued Intuitionistic Fuzzy Sets

TOPSIS has numerous applications in the literature [114,115]. However, there are necessary steps in utilizing TOPSIS, particularly, measuring the attributes importance and each alternative performance with respect to those attributes. However, when modelling vague and uncertain real-life situations, crisp numbers are insufficient and thus inadequate (such as in the case of the problem on hand). This motivated some researchers to extend the TOPSIS approach to model fuzzy environments. For example, Jahanshaloo [109] developed an algorithmic method to extend TOPSIS for decision making problems with interval valued data. In particular, Wang and Lee [112,113] proposed two operators Up and Lo to find positive-ideal and negative-ideal solutions and used these operators to solve the fuzzy multiple-criteria group decision making problem. Chen and Tsao [108] extended the concept of TOPSIS to develop a method for solving MADM problems with interval-valued fuzzy data and compared the results using different distance measures, including Hamming distance, Euclidean distance and their normalized forms (distances between IVIFNs).

Dial-a-ride is a typical problem where the decision-makers (passengers) usually need to provide their preferences over the available alternatives (possible routes). The information provided by the decision-makers (passengers) might be vague or imprecise at a specific time, or simply, because the passengers have limited attention and information processing capacities. In such cases, it is suitable and convenient to express the decision-makers' preferences in the form of interval-valued intuitionistic fuzzy sets (IVIFSs) [111,112,113] It is obvious that the optimal alternative is the one which has the shortest distance from the positive solution and the farthest distance from the negative solution.

8.3.4 The Extended TOPSIS for MAGDM with Interval-Valued Intuitionistic Fuzzy for the Dial-a-ride Problem

In this section, I will apply the new model on a DARPTW illustrative case. Information provided by passengers is expressed in the form of interval-valued intuitionistic fuzzy decision matrices, where each of the elements is characterized by IVIFN, and the information about attribute weights is known [114].
Let $O = \{O_1, O_2, O_3 \dots O_n\}$ be the set of n alternatives, $D = \{d, d_2, d_3 \dots d_l\}$ the set of passengers l and $\lambda = \{\lambda, \lambda_2, \lambda_3 \dots \lambda_l\}^T$ the weight vector of a passenger, where $\lambda_k \geq 0$, $k = 1,2,\dots, l$, and $\sum_{k=1}^{l} \lambda_k = 1$. Let $U = \{u_1, u_2, u_3 \dots u_m\}$ be the set of m attributes. In general, the passengers need to determine the importance degrees of a set of attribute weights u_m (depending on their personal degree of importance for each attribute the weights will be defined). Thus, we suppose the passengers provide the information on the attribute in the following forms ($i \neq j$) :

- An interval form of weak ranking: $\{w_i \geq w_j\}$;
- An interval form of strict ranking: $\{w_i - w_j \geq \delta_i (> 0)\}$;
- An interval form of ranking with multiples: $\{w_i \geq \delta_i w_j\}, 0 \leq \delta_i \leq 1)$;
- An interval form: $\{\delta_i \leq w_i \leq \delta_i + \epsilon_i\}, 0 \leq \delta_i \leq \delta_i + \epsilon_i \leq 1)$;
- A ranking of differences: $\{w_i - w_j \geq w_k - w_l\}$,for $j \neq k \neq l$

I denoted the set of the known attributes weights provided by the passengers by H. Let $R^{(k)} = \left(\tilde{r}_{ij}^{(k)}\right)_{m \times n}$ be an interval-valued intuitionistic fuzzy decision matrix, provided by the passenger d_k $(k = 1,2,\dots, l)$ in the following form:

Table 8.2 IVIFN representing the performance rating of the alternative

	O_1	O_2	….	O_n
u_1	$\tilde{r}_{11}^{(k)}$	$\tilde{r}_{12}^{(k)}$	….	$\tilde{r}_{1n}^{(k)}$
u_2	$\tilde{r}_{21}^{k}$	$\tilde{r}_{22}^{k}$	…	
…..	…..	…..	…	

$\boldsymbol{u_m}$	$\tilde{r}_{m1}^{(k)}$	$\tilde{r}_{m2}^{(k)}$	…	$\tilde{r}_{mn}^{(k)}$

where $\tilde{r}_{ij}^{(k)} = \langle \left[a_{ij}^{(k)}, b_{ij}^{(k)}\right], \left[c_{ij}^{(k)}, d_{ij}^{(k)}\right] \rangle$ is the IVIFN representing the performance rating of the alternative O_j with respect to the attribute $u_i \in U$, provided by the decision-maker $d_k \in D$, such that $\left[a_{ij}^{(k)}, b_{ij}^{(k)}\right]$ indicates the degree the alternative $O_j \in O$ satisfy attribute u_i. $\left[c_{ij}^{(k)}, d_{ij}^{(k)}\right]$ indicates the degree the alternative $O_j \in O$ does not satisfy attribute u_i, where $\left[a_{ij}^{(k)}, b_{ij}^{(k)}\right] \subset [0,1]$, $\left[c_{ij}^{(k)}, d_{ij}^{(k)}\right] \subset [0,1]$, $b_{ij}^{(k)} + d_{ij}^{(k)} \leq 1, j = (1,2, \ldots . n)$; $i = (1,2, \ldots . m)$.

To extend the TOPSIS method in the process of group decision making, first aggregation of all individual decision opinion into group opinion must be done. To do this, I used the IIFHG operator [115,116,117] to aggregate all individual interval-valued intuitionistic fuzzy decision matrices.

Let $R^{(k)} = \left(\tilde{r}_{ij}{}^{(k)}\right)_{m\times n}$ $(k = 1,2 \ldots . l)$ be the collective interval-valued intuitionistic fuzzy decision matrix $R = \left(\tilde{r}_{ij}\right)_{m\times n}$.

Where

$$\tilde{r}_{ij} = \text{IIFHG}_{\alpha,\lambda}\left(\tilde{r}_{ij}^{(1)}, \tilde{r}_{ij}^{(2)}, \ldots . \tilde{r}_{ij}^{(l)}\right) = \left(\left(\dot{\tilde{r}}_{ij}^{(\sigma(1))}\right)^{\alpha_1} \otimes \left(\dot{\tilde{r}}_{ij}^{(\sigma(2))}\right)^{\alpha_2} \otimes \ldots . \left(\dot{\tilde{r}}_{ij}^{(\sigma(l))}\right)^{\alpha_l}\right) = \langle \left[\prod_{k=1}^{n}\left(\dot{a}_{ij}^{(\sigma(k))}\right)^{\alpha_k}, \prod_{k=1}^{n}\left(\dot{b}_{ij}^{(\sigma(k))}\right)^{\alpha_k}\right], \left[1 - \prod_{k=1}^{n}\left(1 - \dot{c}_{ij}^{(\sigma(k))}\right)^{\alpha_k}, 1 - \prod_{k=1}^{n}\left(1 - \dot{d}_{ij}^{(\sigma(k))}\right)^{\alpha_k}\right] \rangle \quad (8.14)$$

where $\alpha = (\alpha_1, \alpha_2, \ldots \alpha_l)^T$ is the weight vector of IIFHG operator and $\alpha_k > 0$ $(k = 1,2 \ldots . l)$ and $\sum_{k=1}^{l} \alpha_k = 1$,

and $\dot{\tilde{r}}_{ij}^{(\sigma(k))} = \langle \left[\dot{a}_{ij}^{(\sigma(k))}, \dot{b}_{ij}^{(\sigma(k))}\right], \left[\dot{c}_{ij}^{(\sigma(k))}, \dot{d}_{ij}^{(\sigma(k))}\right] \rangle$ is the k^{th} largest of the weighted IVIFNs $\dot{\tilde{r}}_{ij}^{(k)}, j = (1,2, \ldots . n)$; $i = (1,2, \ldots . m)$. In the situations where the information about attribute weights is completely known, that is, the weight vector $w = (w_1, w_2, w_3 \ldots w_m)^T$ of the attribute $u_i, (i = (1,2, \ldots . m)$ can be completely determined in advance, then we can construct the weighted collective interval-valued intuitionistic fuzzy decision matrix $R^* = \left(\tilde{r}_{ij}^*\right)_{m\times n}$.

8.3.5 The Definition of the interval-valued intuitionistic PIS and NIS

Let J_1 be a collection of benefit attributes (i.e., the larger u_i is, the greater is the preference) and let J_2 be a collection of cost attributes (i.e., the smaller u_i is, the greater is the preference). The interval-valued intuitionistic PIS, denoted by O^* and the interval-valued intuitionistic NIS, denoted by O^- are defined as follows:

$$\boldsymbol{O^*} = \left\{\langle \boldsymbol{u_i}, \left(\max_j \tilde{\boldsymbol{r}}_{ij}{}^* \,|\boldsymbol{i} \in \boldsymbol{J_1}\right), \left(\min_j \tilde{\boldsymbol{r}}_{ij}{}^* \,|\boldsymbol{i} \in \boldsymbol{J_2}\right)\rangle \,|\boldsymbol{i} = \boldsymbol{1, 2, \ldots m}\right\}^{\boldsymbol{T}} = \{\tilde{\boldsymbol{r}}_{\boldsymbol{1}}{}^+, \tilde{\boldsymbol{r}}_{\boldsymbol{2}}{}^+ \ldots., \tilde{\boldsymbol{r}}_{\boldsymbol{m}}{}^+\}^{\boldsymbol{T}} \quad (8.15)$$

$$\boldsymbol{O^-} = \left\{\langle \boldsymbol{u_i}, \left(\min_j \tilde{\boldsymbol{r}}_{ij}{}^* \,|\boldsymbol{i} \in \boldsymbol{J_1}\right), \left(\max_j \tilde{\boldsymbol{r}}_{ij}{}^* \,|\boldsymbol{i} \in \boldsymbol{J_2}\right)\rangle \mid \boldsymbol{i} = \boldsymbol{1, 2, \ldots m}\right\}^{\boldsymbol{T}} = \{\tilde{\boldsymbol{r}}_{\boldsymbol{1}}{}^-, \tilde{\boldsymbol{r}}_{\boldsymbol{2}}{}^- \ldots., \tilde{\boldsymbol{r}}_{\boldsymbol{m}}{}^-\}^{\boldsymbol{T}} \quad (8.16)$$

Where

$\tilde{r}_i{}^+ = \langle [a_i^+, b_i^+], [c_i^+, d_i^+]\rangle$ and $\tilde{r}_i{}^- = \langle [a_i^-, b_i^-], [c_i^-, d_i^-]\rangle$, $i = 1,2, \ldots m.$

Park *et al.* [46,47] first proposed new distance measures between interval-valued fuzzy sets and compare these measures with the distance measures proposed by Burillo and Bustince [117], then extended those methods into three new methods for measuring distances between IVIFSs. The separation between alternatives can be measured by the Hamming distance, or by the Euclidean distance. We adopted definitions proposed by Park *et al*. and separation measures based on the Hamming distance with Burillo and Bustince's and Grzegorzewski extension [117,118]. In the example below, I have applied the concept using Burillo and Bustince's extension, d_1 (Eq. 8.17 and Eq. 8.18), but the reader can apply different extensions which will offer the same ranking for the alternatives at the end. Here S_{j^*} represents interval-valued intuitionistic PIS and S_{j^-} represents interval-valued intuitionistic NIS of each alternative. See the definitions below for Separation measures based on the Hamming distance:

1. The extension of the Burillo and Bustince's method, d_1 [117,118]:

$$\boldsymbol{S_{j^*}{}^{d_1} = 1/4 \sum_{i=1}^{m} \left[\left|a_{ij}^* - a_i^+\right| + \left|b_{ij}^* - b_i^+\right| + \left|c_{ij}^* - c_i^+\right| + \left|d_{ij}^* - d_i^+\right|\right]} \quad (8.17)$$

$$\boldsymbol{S_{j^-}{}^{d_1} = 1/4 \sum_{i=1}^{m} \left[\left|a_{ij}^* - a_i^-\right| + \left|b_{ij}^* - b_i^-\right| + \left|c_{ij}^* - c_i^-\right| + \left|d_{ij}^* - d_i^-\right|\right]} \quad (8.18)$$

2. The extension of modified Burillo and Bustince's method d_2 [117,118].

$$S_{j^*}{}^{d_2} = 1/4 \sum_{i=1}^{m} \Big[\left|a_{ij}^* - a_i^+\right| + \left|b_{ij}^* - b_i^+\right| + \left|c_{ij}^* - c_i^+\right| + \left|d_{ij}^* - d_i^+\right| + \Big| \left|a_{ij}^* - b_{ij}^*\right| - \left|a_i^+ - b_i^+\right| \Big| + \Big| \left|c_{ij}^* - d_{ij}^*\right| - \left|c_i^+ - d_i^+\right| \Big| \Big] \quad (8.19)$$

$$S_{j^-}{}^{d_2} = 1/4 \sum_{i=1}^{m} \Big[\left|a_{ij}^* - a_i^-\right| + \left|b_{ij}^* - b_i^-\right| + \left|c_{ij}^* - c_i^-\right| + \left|d_{ij}^* - d_i^-\right| + \Big| \left|a_{ij}^* - b_{ij}^*\right| - \left|a_i^- - b_i^-\right| \Big| + \Big| \left|c_{ij}^* - d_{ij}^*\right| - \left|c_i^- - d_i^-\right| \Big| \Big] \quad (8.20)$$

3. The extension of Grzegorzewski's method, d_H [117,118] :

$$S_{j^*}{}^{d_H} = 1/2 \sum_{i=1}^{m} \left[\max\left(\left|a_{ij}^* - a_i^+\right|, \left|b_{ij}^* - b_i^+\right|\right) + \max\left(\left|c_{ij}^* - c_i^+\right|, \left|d_{ij}^* - d_i^+\right|\right) \right], \quad (8.21)$$

$$S_{j^-}{}^{d_H} = 1/2 \sum_{i=1}^{m} \left[\max\left(\left|a_{ij}^* - a_i^-\right|, \left|b_{ij}^* - b_i^-\right|\right) + \max\left(\left|c_{ij}^* - c_i^-\right|, \left|d_{ij}^* - d_i^-\right|\right) \right] \quad (8.22)$$

8.3.6 Definition of interval-valued intuitionistic alternative closeness

The relative closeness of an alternative O_j with respect to interval-valued intuitionistic PIS O^* is defined as follows:

$$C_{j^*} = \frac{S_{j^-}}{S_{j^*} + S_{j^-}}, (j = 1, 2, \dots n) \quad (8.23)$$

The bigger the closeness coefficient C_{j^*} is, the better the alternative O_j will be, as the alternative O_j is closer to the interval-valued intuitionistic PIS O^*. Therefore, the alternatives O_j $(j = 1,2, \dots n)$ can be ranked according to their respective closeness coefficients, so that the best alternative can be selected.

8.3.7 Attribute Weights

Let $R = \left(\tilde{r}_{ij}\right)_{m\times n}$ be the collective interval-valued intuitionistic fuzzy decision matrix. Then, we call $S = \left(s_{ij}\right)_{m\times n}$ the score matrix of $R = \left(\tilde{r}_{ij}\right)_{m\times n}$

where

$$\boldsymbol{s_{ij} = s(\tilde{r}_{ij}) = 1/2(a_{ij} - c_{ij} + b_{ij} - d_{ij})}\ ,\ (\boldsymbol{i = 1, 2, 3, .. m}),\ (\boldsymbol{j = 1, 2, 3, .. n}) \qquad (8.24)$$

and s($\tilde{r}_{ij}$) is called the score of $\tilde{r}_{ij}$.
Based on the score matrix, we present the overall score values of each alternative O_j $(j = 1,2, \ldots . n)$ as:

$$\boldsymbol{s_j(w) = \sum_{i=1}^{m} w_i s_{ij}}\ ,\ (\boldsymbol{j = 1, 2, \ldots n}) \qquad (8.25)$$

The greater the value of $s_j(w)$ is, the better is the alternative O_j (i.e., the more profitable it is). When we only consider the alternative O_j, then a reasonable vector of the attribute weights $w = (w_1, w_2, w_3 \ldots w_m)^T$ should be determined. Thus, we establish the following optimization model (**F**) to maximize $s_j(w)$:

$$(\mathbf{F}) \rightarrow \textbf{Maximize } \boldsymbol{s_j(w) = \sum_{i=1}^{m} w_i s_{ij}} \qquad (8.26)$$

subject to : $w = (w_1, w_2, w_3 \ldots w_m)^T \in H$, $w_i \geq 0$, $i = 1 \ldots m$, $\sum_{i=1}^{m} w_i = 1$.

By solving model (**F**) we obtain the optimal solution $w^{(j)} = \left(w_1^{(j)}, w_2^{(j)}, \ldots\ldots, w_m^{(j)}\right)^T$ corresponding to the alternative O_j . However, in the process of determining the weight vector $w = (w_1, w_2, w_3, \ldots\ldots, w_m)^T$ we need to consider all the alternatives O_j $(j = 1,2, \ldots\ldots, n)$ as a whole. Thus, we construct weight matrix $W = \left(w_i^{(j)}\right)_{m \times n}$ representing the optimal solution $w^{(j)} = \left(w_1^{(j)}, w_2^{(j)}, \ldots\ldots, w_m^{(j)}\right)^T$, $(j = 1,2, \ldots\ldots n)$ as:

$$W = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & .. & w_1^{(n)} \\ w_2^{(1)} & w_2^{(2)} & .. & w_2^{(n)} \\ . & .. & .. & .. \\ w_m^{(1)} & w_m^{(2)} & .. & w_m^{(n)} \end{pmatrix}$$

and we calculate the normalized eigenvector $\omega = (\omega_1, \omega_2, \ldots\ldots, \omega_n)^T$ by the matrix $(S^T W)^T S^T W$, and then we construct a combined weight vector as follows:

$$\boldsymbol{w = W\omega = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & .. & w_1^{(n)} \\ w_2^{(1)} & w_2^{(2)} & .. & w_2^{(n)} \\ . & .. & .. & .. \\ w_m^{(1)} & w_m^{(2)} & .. & w_m^{(n)} \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ .. \\ \omega_n \end{pmatrix} = \omega_1 w^{(1)} + \omega_2 w^{(2)} + \ldots . + \omega_n w^{(n)}} \qquad (8.27)$$

and thus, we derive the weight vector $w = (w_1, w_2, w_3 \dots w_m)^T$ of the attributes $u_k (k = 1,2, \dots . m)$.

8.3.8 Applying the TOPSIS Method extended to Interval-Valued Intuitionistic Fuzzy Sets for Multi-Attribute Group Decision-Making for a simple case of the Dial-a-ride Problem

In this section, I will apply multiple attribute group decision making technique to determine which route should be taken to satisfy all criteria according to the passengers' preferences (different attributes and their weights). The driver must decide which route is the most feasible one among the available alternatives O_j $(j = 1,2,3,4)$, which might be adopted according to the following five attributes:

(1) Earliest departure time (u_1)
(2) delay (u_2)
(3) Non-recurring trips (u_3)
(4) minimum cost (u_4)
(5) Time window (u_5)

Let $J = \{u_1, u_2, u_3, u_4, u_5\}$ be the set of five attributes, and assume that u_1, u_2, u_3 and u_5 are benefit attributes and u_4 is the cost attribute. That is, $J_1 = \{u_1, u_2, u_3, u_5\}$ and $J_2 = \{u_4\}$. There are four passengers who are involved in evaluating the situation: d_1 , d_2 , d_3 and d_4 (four was chosen, since it is the average number of passengers in a medium capacity car). The passengers' weight vector is $\lambda = (0.3, 0.2, 0.3, 0.2)^T$. The passengers d_k $(k = 1,2,3,4)$ represent, respectively, the characteristics of the alternatives O_j $(j = 1,2,3,4)$ by the IVIFNs $r_{ij}^{(k)}$, $(i = 1,2,3,4,5$ and $j = 1,2,3,4)$ with respect to the attributes u_i $(i = 1,2,3,4,5)$, listed in Tables 8.3 to 8.6 (i.e., interval-valued intuitionistic fuzzy decision matrices $R^{(k)} = \left(r_{ij}^{(k)}\right)_{5\times4}$, $(k = 1,2,3,4)$).
Let us assume that the information on the attribute weights given by the passengers is shown as follows:

- d_1: $w_1 \leq 0.3$, $0.2 \leq w_3 \leq 0.5$;
- d_2: $0.1 \leq w_2 \leq 0.2$, $w_5 \leq 0.4$;
- d_3: $w_3 - w_2 \geq w_5 - w_4$, $w_4 \geq w_1$;
- d_4: $w_3 - w_1 \leq 0.1$, $0.1 \leq w_4 \leq 0.3$;

Then the set H of the known information on the attribute weights provided by the passengers is:

$H = \{w_1 \leq 0.3, 0.2 \leq w_3 \leq 0.5, 0.1 \leq w_2 \leq 0.2, w_5 \leq 0.4, w_3 - w_2 \geq w_5 - w_4,$
$w_4 \geq w_1, w_3 - w_1 \leq 0.1, 0.1 \leq w_4 \leq 0.3\}$.

Based on the analysis and definitions in the above sections, in the following section I will apply the presented model on multiple attribute interval-valued intuitionistic fuzzy group decision making in the pursuit of choosing the accumulative aggregated PIS alternative for all the passengers:

- **Step 1**. Utilize the IIFHG operator (let α =$(0.155,0.345,0.345,0.155)^T$ be its weight vector derived by the normal distribution [100] based method Eq. (8.14)) which aggregates the individual interval-valued intuitionistic fuzzy decision matrices $R^{(k)} = \left(r_{ij}^{(k)}\right)_{5\times4}$ $(k = 1,2,3,4)$ into the collective interval-valued intuitionistic fuzzy decision matrix $R = \left(r_{ij}\right)_{5\times4}$ (see Table 8.7).

- **Step 2.** Calculate the score matrix $S = \left(s_{ij}\right)_{5\times4}$ of the collective interval-valued intuitionistic fuzzy decision matrix *R* (Table 8.8):

- **Step 3**. Use the model **(F)** to obtain the optimal weight vectors $w^{(j)} = \left(w_1^{(j)}, w_2^{(j)}, w_3^{(j)}, w_4^{(j)}, w_5^{(j)}\right)^T, (j = 1,2,3,4)$ corresponding to the alternatives $O_j = (1,2,3,4)$

$w^{(1)}$=$(0.2667, 0.1, 0.3667, 0.2667, 0)^T$
$w^{(2)} = (0.16, 0.1, 0.26, 0.16, 0.32)^T$
$w^{(3)}$ = $(0.1, 0.2, 0.2, 0.25, 0.25)^T$
$w^{(4)}$ = $(0.3, 0.1, 0.2, 0.3, 0.1)^T$

Table 8.3 Interval-valued intuitionistic fuzzy decision matrix $\boldsymbol{R^{(1)}}$

	$\boldsymbol{O_1}$	$\boldsymbol{O_2}$	$\boldsymbol{O_3}$	$\boldsymbol{O_4}$
u_1	[[0.5, 0.6], [0.2, 0.3]	[0.3, 0.4], [0.4, 0.6]]	[0.4, 0.5], [0.3, 0.5]	[0.3, 0.5], [0.4, 0.5]
u_2	[0.3, 0.5], [0.4, 0.5]	[0.1, 0.3], [0.2, 0.4]	[0.7, 0.8] , [0.1, 0.2][	[0.1, 0.2], [0.7, 0.8]
u_3	[0.6, 0.7], [0.2, 0.3]	[0.3, 0.4], [0.4, 0.5]	[0.5, 0.8], [0.1, 0.2]	[0.1, 0.2], [0.5, 0.8]
u_4	[0.5, 0.7], [0.1, 0.2]	[0.2, 0.4], [0.5, 0.6]	[0.4, 0.6]] [0.2, 0.3]	[0.1, 0.2], [0.5, 0.8]
u_5	[0.1, 0.4], [0.3, 0.5]	[0.7, 0.8], [0.1, 0.2]	[0.5, 0.6], [0.2, 0.3]	[0.2, 0.3], [0.5, 0.6]

Table 8.4 Interval-valued intuitionistic fuzzy decision matrix $\boldsymbol{R^{(2)}}$

	$\boldsymbol{O_1}$	$\boldsymbol{O_2}$	$\boldsymbol{O_3}$	$\boldsymbol{O_4}$
u_1	[0.4, 0.5], [0.2, 0.4]	[0.3, 0.5], [0.4, 0.5]	[0.4, 0.6], [0.3, 0.4]	[0.3, 0.4], [0.4, 0.6]
u_2	[0.3, 0.4], [0.4, 0.6]	[0.1, 0.3], [0.3, 0.7]	[0.6, 0.8], [0.1, 0.2]	[0.1, 0.2], [0.6, 0.8]
u_3	[0.6, 0.7], [0.1, 0.2]	[0.3, 0.4], [0.4, 0.5]	[0.7, 0.8], [0.1, 0.2]	[0.1, 0.2], [0.7, 0.8]
u_4	[0.5, 0.6], [0.1, 0.3]	[0.2, 0.3], [0.6, 0.7]	[0.4, 0.6], [0.3, 0.4]	[0.3, 0.4], [0.4, 0.6]
u_5	[0.1, 0.3], [0.3, 0.5]	[0.6, 0.8], [0.1, 0.2]	[0.5, 0.6], [0.2, 0.4]	[0.2, 0.4], [0.5, 0.6]

Table 8.5 Interval-valued intuitionistic fuzzy decision matrix $\boldsymbol{R^{(3)}}$

	$\boldsymbol{O_1}$	$\boldsymbol{O_2}$	$\boldsymbol{O_3}$	$\boldsymbol{O_4}$
u_1	[0.4, 0.7], [0.1, 0.2]	[0.4, 0.5], [0.2, 0.4]	[0.2, 0.4], [0.3, 0.4]	[0.3, 0.4], [0.2, 0.4]
u_2	[0.3, 0.5], [0.3, 0.4]	[0.2, 0.4], [0.4, 0.5]	[0.6, 0.8], [0.1, 0.2]	[0.1, 0.2], [0.6, 0.8]
u_3	[0.6, 0.7], [0.1, 0.2]	[0.4, 0.5], [0.3, 0.4]	[0.5, 0.7], [0.1, 0.3]	[0.1, 0.3], [0.5, 0.7]
u_4	[0.5, 0.6], [0.1, 0.3]	[0.1, 0.2], [0.7, 0.8]	[0.5, 0.7], [0.2, 0.3]	[0.2, 0.3], [0.5, 0.7]
u_5	[0.3, 0.5], [0.4, 0.5]	[0.6, 0.7], [0.2, 0.3]	[0.6, 0.8], [0.1, 0.2]	[0.1, 0.2], [0.6, 0.8]

Table 8.6 Interval-valued intuitionistic fuzzy decision matrix $\boldsymbol{R^{(4)}}$

	$\boldsymbol{O_1}$	$\boldsymbol{O_2}$	$\boldsymbol{O_3}$	$\boldsymbol{O_4}$
u_1	[0.6, 0.7], [0.2, 0.3]	[0.4, 0.5], [0.4, 0.5]	[0.4, 0.5], [0.3, 0.4]	[0.3, 0.4], [0.4, 0.5]
u_2	[0.3, 0.4], [0.3, 0.4]	[0.1, 0.2], [0.2, 0.3]	[0.6, 0.7], [0.1, 0.3]	[0.1, 0.3], [0.6, 0.7]],
u_3	[0.7, 0.8], [0.1, 0.2]	[0.3, 0.4], [0.5, 0.6]	[0.5, 0.8], [0.1, 0.2]	[0.1, 0.2], [0.5, 0.8]
u_4	[0.5, 0.6], [0.1, 0.3]	[0.2, 0.3], [0.4, 0.6]	[0.4, 0.5], [0.2, 0.3]	[0.2, 0.3], [0.4, 0.5]
u_5	[0.1, 0.2], [0.5, 0.7]	[0.6, 0.7], [0.1, 0.2]	[0.5, 0.6], [0.3, 0.4]	[0.3, 0.4], [0.5, 0.6]

Table 8.7 Collective interval-valued intuitionistic fuzzy decision matrix $\boldsymbol{R}$

	$\boldsymbol{O_1}$	$\boldsymbol{O_2}$	$\boldsymbol{O_3}$	$\boldsymbol{O_4}$
u_1	$\begin{bmatrix}0.4385,\\0.6199\end{bmatrix}, \begin{bmatrix}0.1549,\\0.2848\end{bmatrix}$	$\begin{bmatrix}0.3502,\\0.4797\end{bmatrix}, \begin{bmatrix}0.3114,\\0.4681\end{bmatrix}$	$\begin{bmatrix}0.3516,\\0.4906\end{bmatrix}, \begin{bmatrix}0.2940,\\0.4214\end{bmatrix}$	$\begin{bmatrix}0.3000,\\0.4170\end{bmatrix}, \begin{bmatrix}0.3114,\\0.4887\end{bmatrix}$
u_2	$\begin{bmatrix}0.3000,\\0.4573\end{bmatrix}, \begin{bmatrix}0.3404,\\0.4710\end{bmatrix}$	$\begin{bmatrix}0.1138,\\0.3010\end{bmatrix}, \begin{bmatrix}0.2511,\\0.4773\end{bmatrix}$	$\begin{bmatrix}0.6395,\\0.7711\end{bmatrix}, \begin{bmatrix}0.0980,\\0.2263\end{bmatrix}$	$\begin{bmatrix}0.1000,\\0.2103\end{bmatrix}, \begin{bmatrix}0.6012,\\0.7678\end{bmatrix}$
u_3	$\begin{bmatrix}0.6116,\\0.7117\end{bmatrix}, \begin{bmatrix}0.1089,\\0.2083\end{bmatrix}$	$\begin{bmatrix}0.3379,\\0.4387\end{bmatrix}, \begin{bmatrix}0.3872,\\0.4887\end{bmatrix}$	$\begin{bmatrix}0.5213,\\0.7804\end{bmatrix}, \begin{bmatrix}0.0980,\\0.2083\end{bmatrix}$	$\begin{bmatrix}0.1000,\\0.2366\end{bmatrix}, \begin{bmatrix}0.5577,\\0.7569\end{bmatrix}$
u_4	$\begin{bmatrix}0.5000,\\0.6395\end{bmatrix}, \begin{bmatrix}0.0980,\\0.2567\end{bmatrix}$	$\begin{bmatrix}0.1758,\\0.3134\end{bmatrix}, \begin{bmatrix}0.5305,\\0.6496\end{bmatrix}$	$\begin{bmatrix}0.4387,\\0.6252\end{bmatrix}, \begin{bmatrix}0.2263,\\0.3262\end{bmatrix}$	$\begin{bmatrix}0.2103,\\0.3109\end{bmatrix}, \begin{bmatrix}0.4050,\\0.5613\end{bmatrix}$
u_5	$\begin{bmatrix}0.1323,\\0.3623\end{bmatrix}, \begin{bmatrix}0.3747,\\0.5482\end{bmatrix}$	$\begin{bmatrix}0.6395,\\0.7521\end{bmatrix}, \begin{bmatrix}0.1089,\\0.2083\end{bmatrix}$	$\begin{bmatrix}0.5452,\\0.6502\end{bmatrix}, \begin{bmatrix}0.1770,\\0.3005\end{bmatrix}$	$\begin{bmatrix}0.1849,\\0.3121\end{bmatrix}, \begin{bmatrix}0.5031,\\0.6118\end{bmatrix}$

Table 8.8 Collective score matrix *S*.

	$\boldsymbol{O_1}$	$\boldsymbol{O_2}$	$\boldsymbol{O_3}$	$\boldsymbol{O_4}$
u_1	0.3093	0.0252	0.0634	-0.0415
u_2	- 0.0270	-0.1568	0.5431	- 0.5294
u_3	0.5030	-0.0496	0.4977	- 0.4890
u_4	0.3924	-0.3454	0.2557	- 0.2225
u_5	- 0.2141	0.5372	0.3589	- 0.3089

Table 8.9 Weighted collective interval-valued intuitionistic fuzzy decision matrix $\boldsymbol{R^{(*)}}$

	$\boldsymbol{O_1}$	$\boldsymbol{O_2}$	$\boldsymbol{O_3}$	$\boldsymbol{O_4}$
u_1	$\begin{bmatrix}0.1125,\\0.1814\end{bmatrix}, \begin{bmatrix}0.6798,\\0.7711\end{bmatrix}$	$\begin{bmatrix}0.0853,\\0.1264\end{bmatrix}, \begin{bmatrix}0.7855,\\0.8547\end{bmatrix}$	$\begin{bmatrix}0.0857,\\0.1303\end{bmatrix}, \begin{bmatrix}0.7762,\\0.8363\end{bmatrix}$	$\begin{bmatrix}0.0711,\\0.1056\end{bmatrix}, \begin{bmatrix}0.7855,\\0.8623\end{bmatrix}$
u_2	$\begin{bmatrix}0.0437,\\0.0737\end{bmatrix}, \begin{bmatrix}0.8738,\\0.9100\end{bmatrix}$	$\begin{bmatrix}0.0150,\\0.0438\end{bmatrix}, \begin{bmatrix}0.8411,\\0.9116\end{bmatrix}$	$\begin{bmatrix}0.1199,\\0.1685\end{bmatrix}, \begin{bmatrix}0.7476,\\0.8303\end{bmatrix}$	$\begin{bmatrix}0.0131,\\0.0291\end{bmatrix}, \begin{bmatrix}0.9383,\\0.9675\end{bmatrix}$
u_3	$\begin{bmatrix}0.2183,\\0.2766\end{bmatrix}, \begin{bmatrix}0.5614,\\0.6646\end{bmatrix}$	$\begin{bmatrix}0.1018,\\0.1396\end{bmatrix}, \begin{bmatrix}0.7811,\\0.8299\end{bmatrix}$	$\begin{bmatrix}0.1746,\\0.3261\end{bmatrix}, \begin{bmatrix}0.5461,\\0.6646\end{bmatrix}$	$\begin{bmatrix}0.0271,\\0.0679\end{bmatrix}, \begin{bmatrix}0.8589,\\0.9300\end{bmatrix}$

u_4	$\begin{bmatrix}0.1560,\\0.2210\end{bmatrix},\begin{bmatrix}0.5664,\\0.7169\end{bmatrix}$	$\begin{bmatrix}0.0462,\\0.0879\end{bmatrix},\begin{bmatrix}0.8563,\\0.8998\end{bmatrix}$	$\begin{bmatrix}0.1318,\\0.2135\end{bmatrix},\begin{bmatrix}0.6952,\\0.7602\end{bmatrix}$	$\begin{bmatrix}0.0561,\\0.0871\end{bmatrix},\begin{bmatrix}0.8016,\\0.8682\end{bmatrix}$
u_5	$\begin{bmatrix}0.0228,\\0.0706\end{bmatrix},\begin{bmatrix}0.8524,\\0.9068\end{bmatrix}$	$\begin{bmatrix}0.1530,\\0.2030\end{bmatrix},\begin{bmatrix}0.6972,\\0.7747\end{bmatrix}$	$\begin{bmatrix}0.1203,\\0.1571\end{bmatrix},\begin{bmatrix}0.7545,\\0.8223\end{bmatrix}$	$\begin{bmatrix}0.0327,\\0.0591\end{bmatrix},\begin{bmatrix}0.8942,\\0.9232\end{bmatrix}$

Here we construct the weight matrix

$$W = \begin{pmatrix} 0.2667 & 0.16 & 0.1 & 0.3 \\ 0.1 & 0.1 & 0.2 & 0.1 \\ 0.3667 & 0.26 & 0.2 & 0.2 \\ 0.2667 & 0.16 & 0.25 & 0.3 \\ 0 & 0.32 & 0.25 & 0.1 \end{pmatrix}$$

Then

$$(S^T W)^T S^T W = \begin{pmatrix} 0.3455 & 0.2621 & 0.2835 & 0.2848 \\ 0.2621 & 0.2634 & 0.2683 & 0.2266 \\ 0.2835 & 0.2683 & 0.2808 & 0.2423 \\ 0.2848 & 0.2266 & 0.2423 & 0.2365 \end{pmatrix}$$

- **Step 4**. Calculate the normalized eigenvectors ω of the matrix $(S^T W)^T S^T W$

$$\omega = (0.2764, 0.2390, 0.2519, 0.2326)^T$$

- **Step 5.** Use Eq. (8.27) to derive the weight vector *w***:**

$$w = W\omega = \begin{pmatrix} 0.2667 & 0.16 & 0.1 & 0.3 \\ 0.1 & 0.1 & 0.2 & 0.1 \\ 0.3667 & 0.26 & 0.2 & 0.2 \\ 0.2667 & 0.16 & 0.25 & 0.3 \\ 0 & 0.32 & 0.25 & 0.1 \end{pmatrix} \begin{pmatrix} 0.2764 \\ 0.2390 \\ 0.2519 \\ 0.2326 \end{pmatrix} =$$

$$(0.2069, 0.1252, 0.2604, 0.2447, 0.1627)^{\mathrm{T}}$$

Table 8.10 Separation measures for the example. Here, the separation measures are based on the Hamming distance

	$S_{j^*}^{d_1}$	$S_{j^-}^{d_1}$	$S_{j^*}^{d_2}$	$S_{j^-}^{d_2}$	$S_{j^*}^{d_H}$	$S_{j^-}^{d_H}$
$\boldsymbol{O_1}$	0.4385	0.3877	0.5201	0.4300	0.4982	0.4251
$\boldsymbol{O_2}$	0.3339	0.4655	0.4002	0.5395	0.4002	0.5395
$\boldsymbol{O_3}$	0.2337	0.5657	0.2687	0.6713	0.2687	0.6713
$\boldsymbol{O_4}$	0.6447	0.1551	0.7445	0.1846	0.7441	0.1846

Table 8.11 The relative closeness of each alternative in the example

	Value	Rank	Value	Rank	Value	Rank
	$C_j^{d_1}$		$C_j^{d_2}$		$C_j^{d_H}$	
O_1	0.4692	3	0.4526	3	0.4604	3
O_2	0.5823	2	0.5741	2	0.5741	2
O_3	0.7076	1	0.7141	1	0.7141	1
O_4	0.1939	4	0.1987	4	0.7141	4

- **Step 6.** Calculate the collective weighted interval-valued intuitionistic fuzzy decision matrix $R^* = \left(\tilde{r}_{ij}^*\right)_{m\times n}$ (see Table 8.9).

- **Step 7**. Utilize Eq. (8.15) and Eq. (8.16) to determine the interval-valued intuitionistic PIS O^* and NIS O^-:

$$\boldsymbol{O}^* = \begin{Bmatrix} \langle [0.1125, 0.1814], [0.6798, 0.7711] \rangle, \\ \langle [0.1199, 0.1685], [0.7476, 0.8303] \rangle, \\ \langle [0.1746, 0.3261], [0.5461, 0.6646] \rangle, \\ \langle [0.0462, 0.0879], [0.8563, 0.8998] \rangle, \\ \langle [0.1530, 0.2030], [0.6972, 0.7747] \rangle \end{Bmatrix}^T$$

and

$$\boldsymbol{O}^- = \begin{Bmatrix} \langle [0{:}\,0711;\ 0{:}\,1056], [0{:}\,7855;\ 0{:}\,8623] \rangle, \\ \langle [0{:}\,0131;\ 0{:}\,0291], [0{:}\,9383;\ 0{:}\,9675] \rangle, \\ \langle [0{:}\,0271;\ 0{:}\,0679], [0{:}\,8589;\ 0{:}\,9300] \rangle, \\ \langle [0{:}\,1560;\ 0{:}\,2210], [0{:}\,5664;\ 0{:}\,7169] \rangle, \\ \langle [0{:}\,0327;\ 0{:}\,0591], [0{:}\,8942;\ 0{:}\,9232] \rangle \end{Bmatrix}^T$$

- **Step 8. C**alculate the separation measures S_{J^*} and S_{J^-} (Eq. 8.17 to Eq. 8.22) for each alternative O_j $(j = 1,2,3,4)$ from the interval-valued intuitionistic PIS O^* and the interval-valued intuitionistic NIS O^-, respectively, based on the distance measure (Table 8.10).

- **Step 9**. Utilize Eq. 8.23 to calculate the relative closeness C_{j^*} of each alternative O_j $(j = 1,2,3,4)$ to the interval-valued intuitionistic PIS O^* with the chosen separation measures, thus, $C_j^{d_1}, C_j^{d_2}$ *and* $C_j^{d_H}$ in Table 8.11.

Finally, rank the preference order of alternatives O_j $(j = 1,2,3,4)$ in Table 8.11, according to the relative closeness to the interval-valued intuitionistic PIS O^*. Obviously, the most desirable alternative is O_3.

8.4 Results and Discussion

As illustrated through the example above, using multiple attribute group decision making problem to determine the PIS alternative for all passengers, satisfying all criteria while taking in consideration the users own preference. Indeed, this approach does not only determine the most beneficial alternative for all group members (the "best" ride), but also avoid losing or distorting the original decision information in the process of aggregation.

8.5 Summary

The results of this chapter are summarized in the booklet section (4.7).

This novel representation of the extended dial-a-ride problem as multiple attribute decision making under the assumption of an interval-valued intuitionistic fuzzy environment using the extended Technique for Ordered Preference by Similarity to the Ideal Solution method, does not only determine the most beneficial alternative for all group members (the "best" ride), but also avoids losing or distorting the original decision information because it employs the weighted aggregation operation. In fact, this model handles passengers' demands with uncertainties while fulfilling the tour's requirements, which is a vital but completely overlooked aspect in the literature. It facilitates all individual's interval-valued intuitionistic fuzzy decision matrices into a collective interval-valued intuitionistic fuzzy decision matrix. Then, employing the interval-valued intuitionistic fuzzy hybrid geometric mean operator, guarantees less information loss. Indeed, aggregating the information pair of the degrees of belonging and of not belonging of the attributes obtained from the intuitionistic fuzzy sets by the users, makes it easy to rank the alternatives using the scoring matrix of the collective interval-valued intuitionistic fuzzy decision. Afterwards, choosing the closest alternative to the positive-ideal solution (and farthest from the negative-ideal solution) as the collectively most feasible and most desirable alternative for all passengers.

This model is leading to an effective and practical way to deal with dial-a-ride problem, it is (somewhat) computationally not as expensive as the meta-heuristics and heuristics approaches presented in the literature. However, I must test it on larger instances in the future for comparisons.

- *I introduced a novel mathematical formulation for the extended Dial-a-ride problem as multiple attributes decision making under the assumption of an interval-valued intuitionistic fuzzy environment. I incorporated the extended Technique for Ordered Preference by Similarity to the Ideal Solution (TOPSIS) method to determine the most beneficial alternative for all group members (the "best" ride).To avoid information distortion; I used the collective interval-valued intuitionistic fuzzy hybrid geometric mean (IIFHG) operator to build the collective decision matrix and to calculate each alternative's closeness from the interval-valued intuitionistic positive-ideal solution (PIS) using Park's extension for Hamming distance. Afterwards, I ranked the alternatives according to their closeness coefficients using scoring matrix to ultimately select the unanimous most desirable alternative(s) efficiently.*

Chapter 9 Future Work

9.1 Extension of the IFTD TSP on Large Universal Instances

To assess the IFTD TSP (Intuitionistic Fuzzy Time Dependent Traveling Salesman Problem) model applicability on real life scenarios, I will run my C++ intuitionistic model after integrating it with DBMEA and GA for optimization and run it on the largest instances in the history (Eil51, Bier127, Eil76, Eil101 s250_1 and s250_2)

9.2 Extension of the IVIFTD TSP on Large Universal Instances

To assess the IVIFTD TSP (Interval-Valued Intuitionistic Fuzzy Time Dependent Traveling Salesman Problem) model applicability on real life scenarios, I continue to develop and enhance my C++ modules for the interval-valued intuitionistic fuzzy set model after integrating it with DBMEA and GA for optimization and run it on the largest instances in the history (Eil51, Bier127, Eil76, Eil101 s250_1 and s250_2)

9.3 Extension of the IVFSTD TSP on Large Universal Instances

To assess the IVFSTD TSP (Interval-Valued Fuzzy Soft Time Dependent Traveling Salesman Problem) model applicability on real life scenarios, I will run my C++ module for interval-valued fuzzy set model after integrating it with the DBMEA and the GA for optimization and run it on the largest available instances in the history (Eil51, Bier127, Eil76, Eil101 s250_1 and s250_2)

9.4 Thorough Comparative Study for the Extended IVIFTD TSP, IVIFTD TSP and IVFSTD TSP on Large Universal Instances

Lastly, I will run all my models and integrate with the DBMEA and the GA for optimization and run it on the largest available instances in the history (Eil51, Bier127, Eil76, Eil101 s250_1 and s250_2) and compare the results of the runs between all my fuzzy-based novel models.

Chapter 10 Thesis Summary

In this thesis work, I introduced and compared various novel fuzzy-based extensions for the Time Dependent Travel Salesman Problem under vague conditions (such as the rush hours periods and the jam regions); namely, the 3FTD TSP (Triple Fuzzy Time Dependent Travelling Salesman Problem), the IFTD TSP (intuitionistic Fuzzy Time Dependent Travelling Salesman Problem), the IVIFTD TSP (interval-valued intuitionistic Fuzzy Time Dependent Travelling Salesman Problem), and the IVFSTD TSP (interval-valued Fuzzy Soft Time Dependent Travelling Salesman Problem). Those models offer alternative extensions of the abstract TD TSP formulation with traffic regions and the time dependent rush hours. The 3FTD TSP represents the jam regions and rush hours costs by type-1 fuzzy sets. The extension of the 3FTD TSP which is applied, optimized and compared thoroughly on the largest instances presented in the literature (Eil51, Bier127, Eil76, Eil101 s250_1 and s250_2), shows the applicability and efficiency of the model.

The IFTD TSP offers higher degrees of association and lower degrees of non-association of the jam factors and the rush hours as well as lower degrees of hesitation to any edge costs by employing the intuitionistic fuzzy sets.

The IVIFTD TSP employs the interval-valued intuitionistic fuzzy sets. This model was able to decrease the information loss by employing the IIFWAA (interval-valued intuitionistic fuzzy weighted arithmetic mean aggregation) operator to aggregate on interval-valued intuitionistic fuzzy modified costs influenced by the jam regions and the rush hours. This model, takes in consideration multiple jam factors simultaneously, rank them according to their closeness from the PIS, then it selects the ultimate route(s) based on the scoring matrix for the normalized Park's distance between the IVIFS(s) for the extended TD TSP.

Lastly, the IVFS TD TSP model, that incorporates the interval-valued fuzzy soft sets to calculate multiple jam factors and rush hours effects on the modified tour costs.

Through my work, each model was presented on a simple example first to validate it mathematically, then it was applied and extended on the largest available instances in the literature. The results of the runs indicate that my models effectively simulate real-life scenarios and successfully quantify the traffic jam regions and the rush hours periods added costs with minimum information loss.

After applying each fuzzy-based model we introduced, each one slightly differs in the optimal solution it suggests (including the total costs). Thus, each one of those proposed approaches uniquely contributes to a more adequate (and unique) calculation of the jam regions and rush hours under vague and uncertain circumstances, yet it is hard to choose solely the best model as each one offers undisputable advantages.

The presented results are applicable in both academic and industrial fields. The results were tested and validated on the largest available universal benchmarks (Eil51, Bier127, Eil76, Eil101 s250_1 and s250_2) and compared with the state-of-the-art solutions such as Schneider's approach. The run results have shown applicability and universality where the models were able to simulate real-life TD TSP cases with rush hours and jam factors affecting actual tours and offering more accurate solutions that converge in a reasonable time even with large jam factors values (rush hours and jam regions). The (quasi)-optimal path was found for the suggested tours under study using the DBMEA and the GA.

Furthermore, the main purpose of applying the interval-valued intuitionistic fuzzy sets with the extended TOPSIS method on the Dial-a-ride problem with time window (which is a prime example of the TD TSP) was to simulate the most complicated model I have proposed (IVIFTD TSP) on another variation of the TD TSP; Dial-a-Ride with time window under uncertainty. Which is a common transportation problem. The results only proved the claim of practicality of my proposed models on general cases and numerous application areas. Hence, my proposed extended models proven to be beneficial, applicable, efficient and scalable on real-life scenarios. All my publications were well received and acknowledged within the fuzzy community.

References

[1] Applegate, D.L., Bixby, R.E., Chvátal, V., Cook, W.J.: The Traveling Salesman Problem: A Computational Study, Princeton University Press, Princeton, pp. 1-81.

[2] Christos H. Papadimitriou,The Euclidean travelling salesman problem is NP-complete,Theoretical Computer Science,Volume 4, Issue 3,1977,Pages 237-244.

[3] Polynomial time approximation schemes for Euclidean traveling salesman and other geometric problems, Sanjeev Arora , Journal of the ACM, Volume 45, Issue 5, Sept. 1998 pp 753–782.

[4] Schneider, J.: The Time-Dependent Traveling Salesman Problem, PhysicaA. 314, pp.151-155, 2002.

[5] Alberto V. Donati, Roberto Montemanni, Norman Casagrande, Andrea E. Rizzoli, Luca M. Gambardella,Time dependent vehicle routing problem with a multi ant colony system,European Journal of Operational Research,Volume 185, Issue 3,2008,1174-1191.

[6] Zhang, T., Chaovalitwongse, W.A. & Zhang, Y. Integrated Ant Colony and Tabu Search approach for time dependent vehicle routing problems with simultaneous pickup and delivery. J Comb Optim 28, 288–309 (2014). https://doi.org/10.1007/s10878-014-9741-1.

[7] Fox, K.R., Gavish, B., Graves, S.C., 1980. An n-constraintformulation of the (time-dependent) traveling salesman problem. Operations Research 28 (4), 1018–1021.

[8] Malandraki, C., Dial, R.B., 1996. A restricted dynamic programming heuristic algorithm for the time dependent traveling salesman problem. European Journal of Operational Research 90, 45–55.

[9] Zadeh, L.A: Fuzzy sets, Information and Control, 8 (3), pp. 338–353, 1965.

[10] Kóczy, L.T., Földesi, P., Tüű-Szabó, B., Ruba Almahasneh: Modeling of Fuzzy Rule-base Algorithm for the Time Dependent Traveling Salesman Problem, Proceedings of the IEEE International Conference on Fuzzy Systems (FUZZ-IEEE),2019.

[11] Ruba Almahasneh , Tüű-Szabó, B., Földesi, Kóczy, L.T., Intuitionistic Fuzzy Model of Jam Regions and Rush Hours for te Time Dependent Traveling salesman Problem. Proceedings of the IFSA World Congress and NAFIPS Annual Conference,2019.

[12] Almahasneh, R., Tuu-Szabo, B., Foldesi, P., Koczy, L.T. (2020). Quasi-Optimization of the Time Dependent Traveling Salesman Problem by Intuitionistic Fuzzy Model and Memetic Algorithm. In: Llanes Santiago, O., Cruz Corona, C., Silva Neto, A., Verdegay, J. (eds) Computational Intelligence in Emerging Technologies for Engineering Applications. Studies in Computational Intelligence, vol 872. Springer, Cham.

[13] Ruba Almahasneh , Tüű-Szabó, B., Földesi, Kóczy, L.T., Extension of the Time Dependent Travelling Salesman Problem with Interval Valued Intuitionistic Fuzzy Model Applying Memetic Optimization Algorithm. ISMSI '20: Proceedings of the 2020 4th International Conference on Intelligent Systems, Metaheuristics & Swarm Intelligence, Pages 111–118,2020.

[14] Almahasneh, Ruba; Kóczy, László T. , Extended TOPSIS Method for the Dial-a-ride Problem with Time Window Using Interval-Valued Intuitionistic Fuzzy Sets, International Journal of Fuzzy Systems, under review.

[15] Almahasneh, Ruba ; Tuu-Szabo, Boldizsár ; Foldesi, Peter ; Koczy, Laszlo T., Fuzzy Set Based Models Comparative Study for the TD TSP with Rush Hours and Traffic Regions, In: Information Processing and Management of Uncertainty in Knowledge-Based Systems , Springer International Publishing (2020) pp. 699-714. Paper: Chapter 55 , 16 p., DOI Scopus .

[16] Tüű-Szabó, B., Ruba Almahasneh,Földesi, P.,Kóczy, L.T.: A software tool for solving the Traveling Salesman Problem and related non-fuzzy and fuzzy optimization problems, Proceedings of the 14th European Symposium on Computational Intelligence and Mathematics (ESCIM 2022), October 2nd-5th, 2022, under publication.

[17] Almahasneh, Ruba; Tüű-Szabó, Kóczy, László T., Optimization of the Extended Triple Fuzzy Time Dependent Travelling Salesman Problem Model with Discrete Bacterial Memetic Algorithm, Fuzzy Sets and Systems journal, under review.

[18] Ruba Almahasneh,Kóczy, L.T.: Extension of The Time Dependent Travelling Salesman Problem Model with Interval-Valued Fuzzy Soft Sets, Proceedings of the 14th European Symposium on Computational Intelligence and Mathematics (ESCIM 2022), October 2nd-5th, 2022, under publication.

[19] Fuzzy Chukwuemeka C. Nwobi-Okoye, Member, IAENG , Stanley Okiy, Francis I. Obidike., Fuzzy Based Solution to the Travelling Salesman Problem: A Case Study, Proceedings of the World Congress on Engineering and Computer Science 2017 Vol II WCECS 2017, October 25-27, 2017, San Francisco, USA.

[20] Taş, D.; Gendreau, M.; Jabali, O.; Laporte, G. The Traveling Salesman Problem with Time-Dependent Service Times. Eur. J. Oper. Res. 2016, 248, 372–383.

[21] Hurkała, J. Time-Dependent Traveling Salesman Problem with Multiple Time Windows. In Position Papers of the 2015 Federated Conference on Computer Science and Information Systems; Ganzha, M., Maciaszek, L., Paprzycki, M., Eds.; ACSIS: Marlton, NJ, USA, 2015; Volume 6, pp. 71–78.

[22] Vu, D.M.; Hewitt, M.; Boland, N.; Savelsbergh, M. Solving Time Dependent Traveling Salesman Problem with Time Windows. In Proceedings of the 7th International Workshop on Freight Transportation and Logistics, ODYSSEUS 2018, Cagliari, Italy, 3–8 June 2018.

[23] Kóczy, L.T.; Földesi, P.; Tüű-Szabó, B. Enhanced discrete bacterial memetic evolutionary algorithm—An efficacious metaheuristic for the traveling salesman optimization. Inf. Sci. 2018, 460–461, 389–400.

[24] Ban, H.B. An efficient two-phase metaheuristic algorithm for The Time Dependent Traveling Salesman Problem. RAIRO-Oper. Res. 2019, 53, 917–935.

[25] Vu, D.M.; Hewitt, M.; Boland, N.; Savelsbergh, M. Dynamic Discretization Discovery for Solving the Time-Dependent Traveling Salesman Problem with Time Windows. Transp. Sci. 2019, 1–18, doi:10.1287/trsc.2019.0911.

[26] Balázs, K.; Botzheim, J.; Kóczy, L.T. Comparison of Various Evolutionary and Memetic Algorithms, Integreted Uncertainty Management and Applications; Springer: Berlin/Heidelberg, Germany, 2010; Volume 68, pp. 431–442.

[27] Tüű-Szabó, B.; Földesi, P.; Kóczy, L.T. Discrete Bacterial Memetic Evolutionary Algorithm for the Time Dependent Traveling Salesman Problem. In Proceedings of the International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems (IPMU 2018), Cadíz, Spain, 11–15 June 2018; Springer: Cham, Switzerland, 2018; pp. 523–533.

[28] Roy, A.; Manna, A.; Maity, S. A novel memetic genetic algorithm for solving traveling salesman problem based on multi-parent crossover technique,Decision Making. Appl. Manag. Eng. 2019, 2, 100–111.

[29] Parvathi Rangasamy, Muhammad Akram, S. Thilagavathi, Intuitionistic fuzzy shortest hyperpath in a network,Information Processing Letters,Volume 113, Issue 17,2013,Pages 599-603.

[30] N. Anitha, C. Vijayalakshmi, Implementation of Fuzzy Intuitionistic Algorithm for Traveling Salesman Problem, EAI Endorsed Transactions on Energy Web, 2018, 5(18):154817.

[31] Holland, John H. Adaptation in natural and artificial systems: an introductory analysis with applications to biology, control, and artificial intelligence. MIT press, 1992.

[32] Nawa, N.E.; Furuhashi, T. Fuzzy System Parameters Discovery by Bacterial Evolutionary Algorithm. IEEE Trans. Fuzzy Syst. 1999, 7, 608–616.

[33] Moscato, P. On Evolution, Search, Optimization, Genetic Algorithms and Martial Arts—Towards Memetic Algorithms; Technical Report Caltech Concurrent Computation Program; California Institute of Technology: Pasadena, CA, USA, 1989, p. 826.

[34] Quoc Phan Tan, A Genetic Approach for Solving Minimum Routing Cost Spanning Tree Problem, IJMLC 2012 Vol.2(4): 410-414 ISSN: 2010-3700

[35] Beasley, D., Bull, D. R. and Martin, R. R. (1993), "An overview of genetic algorithms: Part 2, research topics", University Computing , Vol. 15, pp. 170–181.

[36] Mendel, J.M.; John, R.I.B. Type-2 Fuzzy Sets Made Simple. IEEE Trans. Fuzzy Syst. 2002, 10, 117–127.

[37] Mendel, J.M.; John, R.I.B.; Liu, F. Interval Type-2 Fuzzy Logic Systems Made Simple. IEEE Trans. Fuzzy Syst. 2006, 14, 808–821.

[38] Gau, W.L.; Buehrer, D.J. Vague Sets. IEEE Trans. Syst. Man Cybernet 1993, 23, 610–614.

[39] Atanassov, R.K. Intuitionistic Fuzzy Sets. Fuzzy Sets Syst. 1986, 20, 87–96.

[40] A.V.Patel, B.M.Mohan Fuzzy Sets and Systems,Volume 132, Issue 3, 16 December 2002, Pages 401-409

[41] K. Atanassov, G. Gargov, Interval-valued intuitionistic fuzzy sets, Fuzzy Sets Syst. 31 (1989) 343–349.

[42] Biswas, R. On Fuzzy Sets and Intuitionistic Fuzzy Sets. Notes Intuit. Fuzzy Sets 1997, 3, 3–11.

[43] Xu, Z.S. Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making. Control Decis. 2007, 22, 215–219.

[44] Xu, Z.S.; Chen, J. Approach to group decision making based interval-valued intuitionistic judgment matrices. Syst. Eng. Theory Pract. 2007, 27, 126–133.

[45] Xu, Z.S.; Chen, J. On geometric aggregation over interval-valued intuitionistic fuzzy information. In Proceedings of the Fourth International Conference on Fuzzy Systems and Knowledge Discovery, Haikou, Hainan, China, 24–27 August 2007; IEEE Computer Society Press: Washington, DC, USA, 2007; pp. 466–471.

[46] J.H. Park, K.M. Lim, J.S. Park, Y.C. Kwun, "Distances between interval-valued intuitionistic fuzzy sets," J. Phys.: Conf. Ser., 96, 2008, 012089.

[47] D.G. Park, Y.C. Kwun, J.H. Park, I.Y. Park, Correlation coefficient of interval-valued intuitionistic fuzzy sets and its application to multiple attribute group decision making problems, Math. Comput. Model. 50 (2009) 1279–1293.

[48] P. Grzegorzewski, "Distance between intuitionistic fuzzy sets and/or interval-valued fuzzy sets on the Hausdorff metric," Fuzzy Sets Syst., 148, 2004, 319–328.

[49] Mamdani, E.H.: Application of Fuzzy Algorithms for Control of Simple Dynamic Plant, IEEE Proc., Vol. 121, No. 12, pp. 1585-1588, 1974.

[50] K. Hirota, Concept of probabilistic sets, Fuzzy Sets and Systems 5 (1981) 31-46.

[51] E. Czogala, On distribution function description of probabilistic sets and its application in decision making, Fuzzy Sets and Systems 10 (1983) 21-29.

[52] Kanika Mittal, Amita Jain, Kunwar Singh Vaisla, Oscar Castillo, Janusz Kacprzyk, A comprehensive review on type 2 fuzzy logic applications: Past, present and future, En-gineering Applications of Artificial Intelligence,Volume 95,2020.

[53] Shuvasree Karmakar, Mijanur Rahaman Seikh, Oscar Castillo,Type-2 intuitionistic fuzzy matrix games based on a new distance measure: Application to biogas-plant im-plementation problem, Applied Soft Computing,Volume 106,2021.

[54] D.A. Molodtsov, Soft set theory - first results, Computers and Mathematics with Applications, 37 (1999), 19 - 31.

[55] P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, Computers and Mathematics with Applications, 45 (2003), 555 – 562.

[56] D. A. Molodtsov, Soft set theory-first results, Comput. Math. Appl., 37 (1999), 19-31.

[57] D. A. Molodtsov, The description of a dependence with the help of soft sets, J. Comput. Sys. Sc. Int., 40(6) (2001), 977-984.

[58] D. A. Molodtsov, The theory of soft sets (in Russian), URSS Publishers, Moscow, 2004.

[59] D. A. Molodtsov, V. Yu. Leonov and D. V. Kovkov, Soft sets sechnique and its application, Nechetkie Sistemi I Myakie Vychisleniya, 1(1) (2006), 8-39

[60] Z. Pawlak, Rough sets, Int. J. Comput. Inform. Sci., 11 (1982), 341-356.

[61] D. Chen, E.C.C. Tsang, D.S. Yeung and X. Wang, The parameterization reduction of soft sets and its applications, Computers and Mathematics with Applications, 49 (2005), 757 - 763.

[62] Z. Kong, L. Gao and L. Wang, Comment on "A fuzzy soft set theoretic approach to decision making problems", J. Comput. Appl. Math., 223 (2009), 540-542.

[63] Z. Xiao, Y. Li, B. Zhong and X. Yang, Research on synthetically evaluating method for business competitive capacity based on soft set, Stat. Methods. Med. Res., (2003), 52-54

[64] Z. Xiao, L. Chen, B. Zhong and S. Ye, Recognition for soft information based on the theory of soft sets, In: J. Chen ,eds., Proceedings of ICSSSM-05, 2 (2005), 1104-1106.

[65] A. R. Roy and P. K. Maji, A fuzzy soft set theoretic approach to decision making problems, J. Comput. Appl. Math., 203 (2007), 412-418

[66] M. M. Mushrif, S. Sengupta and A. K. Ray, Texture classification using a novel, soft-set theory based classification, Algorithm. Lecture Notes In Computer Science, 3851 (2006), 246-254.

[67] D. Pei and D. Miao, From soft sets to information systems, In: X. Hu, Q. Liu, A. Skowron, T. Y. Lin, R. R. Yager, B. Zhang ,eds., Proceedings of Granular Computing, IEEE, 2 (2005), 617-621.

[68] T. Som, On the theory of soft sets, soft relation and fuzzy soft relation, Proc. of the National Conference on Uncertainty: A Mathematical Approach, UAMA-06, Burdwan, (2006), 1-9.

[69] Q. M. Sun, Z. L. Zhang and J. Liu, Soft sets and soft modules, In: G. Wang, T. Li, J. W. Grzymala-Busse, D. Miao, A. Skowron, Y. Yao, eds., Rough Sets and Knowledge Technology, RSKT-08, Proceedings, Springer, (2008), 403-409.

[70] P.K. Maji, R. Biswas, A.R. Roy, Fuzzy soft sets, Journal of Fuzzy Mathematics, 9 (2001), 589 - 602.

[71] A. Mukherjee and S. B. Chakraborty, On intuitionistic fuzzy soft relations, Bull. Kerala Math. Assoc., 5(1) (2008), 35-42.

[72] Z. Xiao, K. Gong and Y. Zou, A combined forecasting approach based on fuzzy soft sets, J. Comput. Appl. Math., 228 (2009), 326-333.

[73] X. Yang, D. Yu, J. Yang and C. Wu, Generalization of soft set theory: from crisp to fuzzy case, In: Bing-Yuan Cao ,eds., Fuzzy Information and Engineering: Proceedings of ICFIE2007, Advances in Soft Computing 40, Springer, (2007), 345-355.

[74] Y. Zou, Z. Xiao, Data analysis approaches of soft sets under incomplete information, Knowl-Based Syst. 21 (2008) 941–94.

[75] N. Cagman and S. Enginoglu, Soft matrices and its decision makings,Computers and Mathematics with Applications, 59 (2010), 3308 - 3314.

[76] N. Cagman F. Citak, and S. Enginoglu, Fuzzy parameterized fuzzy soft set theory and its applications, Turkish Journal of Fuzzy Systems, 1 (2010),21 - 35.

[77] X. Yang, T. Y. Lin, J. Yang, Y. Li and D. Yu, Combination of interval-valued fuzzy set and soft set, Comput. Math. Appl., 58 (2009), 521-527.

[78] Z. Kong, L. Gao, L. Wang and S. Li, The normal parameter reduction of soft sets and its algorithm, Comput. Math. Appl., 56 (2008), 3029-3037.

[79] Y. Zou and Y. Chen, Research on soft set theory and parameters reduction based on relational algebra, In: Intelligent Information Technology Application, IITA, 1 (2008), 152 - 156

[80] P.K. Maji, A.R. Roy, R. Biswas, An application of soft sets in a decision making problem, Computers and Mathematics with Applications, 44 (2002), 1077 - 1083.

[81] P.K. Maji, R. Biswas, A.R. Roy, Fuzzy soft sets, J. Fuzzy Math. 9 (3) (2001) 589–602.

[82] X.B. Yang, D.J. Yu, J.Y. Yang, C. Wu, Generalization of soft set theory: From crisp to fuzzy case, in: B.Y. Cao (Ed.), Proceeding of the second international conference on fuzzy information and engineering, May, 23–16, Guangzhou, 2007, in: Advance on Soft Computing, vol. 40, Springer-Verlag, Berlin, Heidelberg, 2007, pp. 345–354.

[83] Liu, Y., Jiang, W. A new distance measure of interval-valued intuitionistic fuzzy sets and its application in decision making. Soft Comput 24, 6987–7003 (2020).

[84] Chen SJ., Hwang CL. (1992) Fuzzy Multiple Attribute Decision Making Methods. In: Fuzzy Multiple Attribute Decision Making. Lecture Notes in Economics and Mathematical Systems, vol 375. Springer, Berlin, Heidelberg.

[85] Y.J. Lai, T.Y. Liu, C.L. Hwang, TOPSIS for MODM, Eur. J. Oper. Res. 76 (3) (1994) 486–500.

[86] M.A. Abo-Sinha, A.H. Amer, Extensions of TOPSIS for multi-objective large-scale nonlinear programming problems, Appl. Math. Comput. 162 (2005) 243–256.

[87] Palash Dutta , Hrishikesh Boruah , Tazid Ali , Fuzzy Arithmetic with and without Using α – cut method : A comparative study, International Journal of Latest trends in Computing , 2011,vol 2, issue 1, 99-107.

[88] Voskoglou, M.Gr.: Comparison of the COG Defuzzification Technique and Its Variations to the GPA Index, American Journal of Computational and Applied Mathematics, 6(5), pp. 187-193, 2016.

[89] Chakraverty S., Sahoo D.M., Mahato N.R. (2019) Defuzzification. In: Concepts of Soft Computing. Springer, Singapore

[90] Berenji H.R. (1992) Fuzzy Logic Controllers. In: Yager R.R., Zadeh L.A. (eds) An Introduction to Fuzzy Logic Applications in Intelligent Systems. The Springer International Series in Engineering and Computer Science (Knowledge Representation, Learning and Expert Systems), vol 165. Springer, Boston, MA.

[91] Pedrycz, W., Gomide, F.: Fuzzy systems engineering—toward human-centric computing. Wiley, New Jersey

[92] Iancu I. A Mamdani type fuzzy logic controller. Fuzzy logic-controls, concepts, theories and applications. 2012 Mar 28;15(2):325-50.

[93] Dong Gun Park, Young Chel Kwun, Jin Han Park, Il Young Park,Correlation coefficient of interval-valued intuitionistic fuzzy sets and its application to multiple attribute group decision making problems,Mathematical and Computer Modelling, Volume 50, Issues 9–10, 2009, Pages 1279-1293.

[94] Alkhazaleh, S. and Salleh, A. (2014) Fuzzy Soft Expert Set and Its Application. Applied Mathematics, 5, 1349-1368. doi: 10.4236/am.2014.59127.

[95] Marcus Posada, Henrik Andersson & Carl H. Häll, The integrated dial-a-ride problem with timetabled fixed route service, Public Transport volume 9, pages217–241, (2017).

[96] VictorPimenta, AlainQuilliot, HélèneToussaint, DanieleVigo, Models and algorithms for reliability-oriented Dial-a-Ride with autonomous electric vehicles, European Journal of Operational Research Volume 257, Issue 2, 1 March 2017, Pages 601-613.

[97] Sophie N. Parragh, Jorge Pinho de Sousa, Bernardo Almada-Lobo, The Dial-a-Ride Problem with Split Requests and Profits, Transportation Science Vol. 49, No. 2, 2014.

[98] Michel Gendreau, Alain Hertz, Gilbert Laporte, A Tabu Search Heuristic for the Vehicle Routing Problem,1994 Management Science,Vol. 40, No. 10.

[99] Tripathy, T., Nagavarapu, S.C., Azizian, K., Ramasamy Pandi, R., Dauwels, J.: Solving dial-a-ride problems using multiple ant colony system with fleet size minimisation. In: Chao, F., Schockaert, S., Zhang, Q. (eds.) UKCI 2017. AISC, vol. 650, pp. 325–336. Springer, Cham (2018). https://doi.org/10.1007/978-3-319-66939-7 28Trans. Evol. Comput. 7(2), 117–132 (2003). https://doi.org/10.1109/TEVC.2003.

[100] Qu, Y., Bard, J.F.: A branch-and-price-and-cut algorithm for heterogeneous pickup and delivery problems with configurable vehicle capacity. Transp. Sci. 49(2), 254– 270 (2014). https://doi.org/10.1287/trsc.2014.0524.

[101] Cordeau, J.F.: A branch-and-cut algorithm for the dial-a-ride problem. Oper. Res. 54(3), 573–586 (2006). https://doi.org/10.1287/opre.1060.0283.

[102] Yuan Qu, Jonathan F.Bard, The heterogeneous pickup and delivery problem with configurable vehicle capacity, Transportation Research Part C: Emerging Technologies Volume 32, July 2013, Pages 1-20.

[103] Yuan Qu, Jonathan F. Bard, A Branch-and-Price-and-Cut Algorithm for Heterogeneous Pickup and Delivery Problems with Configurable Vehicle Capacity, Transportation ScienceVol. 49, No. 2, (2015).

[104] Guerreiro P.M.M., Cardoso P.J.S., Fernandes H.C.L. (2019) Applying NSGA-II to a Multiple Objective Dial a Ride Problem. In: Rodrigues J. et al. (eds) Computational Science – ICCS 2019. ICCS 2019. Lecture Notes in Computer Science, vol 11540. Springer, Cham. https://doi.org/10.1007/978-3-030-22750-0_5.

[105] Sophie N.Parragh, Introducing heterogeneous users and vehicles into models and algorithms for the dial-a-ride problem, Transportation Research Part C: Emerging Technologies Volume 19, Issue 5, August 2011, Pages 912-930.

[106] VictorPimenta,AlainQuilliot, HélèneToussaint,DanieleVigo, Models and algorithms for reliability-oriented Dial-a-Ride with autonomous electric vehicles, European Journal of Operational Research Volume 257, Issue 2, 1 March 2017, Pages 601-613.

[107] C.T. Chen, Extensions of the TOPSIS for group decision-making under fuzzy environment, Fuzzy Sets Syst. 114 (2000) 1–9.

[108] T.Y. Chen, C.Y. Tsao, The interval-valued fuzzy TOPSIS method and experimental analysis, Fuzzy Sets Syst. 159 (2008) 1410–1428.

[109] G.R. Jahanshaloo, F. Hosseinzadeh Lotfi, M. Izadikhah, An algorithmic method to extend TOPSIS method for decision-making problems with interval data, Appl. Math. Comput. 175 (2006) 1375–1384.

[110] Young-JouLai1Ting-YunLiu, Ching-LaiHwang, TOPSIS for MODM, European Journal of Operational Research Volume 76, Issue 3, 11 August 1994, Pages 486-500.

[111] K. Atanassov, Operators over interval-valued intuitionistic fuzzy sets, Fuzzy Sets Syst. 64 (1994) 159–174.

[112] G.W. Wei, X.R. Wang, Some geometric aggregation operators on interval-valued intuitionistic fuzzy sets and their application to group decision making, Proc. 2007 ICCIS (2007) 495–499.

[113] Z.S. Xu, J. Chen, On geometric aggregation over interval-valued intuitionistic fuzzy information, in: Proceedings of Fourth International Conference on Fuzzy Systems and Knowledge Discovery (FSKD'07) vol. 2, 2007, pp. 466–471.

[114] F. Torfi, R.Z. Farahani, S. Rezapour, Fuzzy AHP to determine the relative weights of evaluation criteria and fuzzy TOPSIS to rank the alternatives, Appl. Soft Comput. 10 (2010) 520–528.

[115] T. Yang, C.C. Hung, Multi-attribute decision making methods for plant layout design problem, Robotics Comput. Integrated Manuf. 23 (2007) 126–137.

[116] Y.J. Wang, H.S. Lee, Generalizing TOPSIS for fuzzy multiple-criteria group decision-making, Comput. Math. Appl. 53 (2007) 1762–1772.

[117] P. Burillo, H. Bustince, "Entropy on intuitionistic fuzzy sets and interval-valued fuzzy sets," Fuzzy Sets Syst., 78, 1996, 305–316.

[118] Z.S. Xu, An overview of methods for determining OWA weights, Int. J. Intell. Syst. 20 (2005) 843–865.

Chapter 11 Publications

11.1 International Journals

J5. **Almahasneh, Ruba** ; Tüű-Szabó, Boldizsár; Kóczy, László T., *Extension of the Triple Fuzzy Time Dependent Travelling Salesman Problem Model with Discrete Bacterial Memetic Optimization Algorithm*, Fuzzy Sets and Systems journal, under review

J4. **Almahasneh, Ruba** ; Kóczy, László T., *Extended TOPSIS Method for the Dial-a-ride Problem with Time Window Using Interval-Valued Intuitionistic Fuzzy Sets*, International Journal of Fuzzy Systems, under review

J3. **Almahasneh, Ruba** ; Tüű-Szabó, Boldizsár ; Kóczy, László T. ; Földesi, Péter, *Optimization of the Time-Dependent Traveling Salesman Problem Using Interval-Valued Intuitionistic Fuzzy Sets*, AXIOMS 9 : 2 p. 53 (2020), DOI DOI (WoS Scopus)

J2. **Almahasneh, Ruba** ; Tuu-Szabo, Boldizsar ; Foldesi, Peter ; Koczy, Laszlo T., *Quasi-Optimization of the Time Dependent Traveling Salesman Problem by Intuitionistic Fuzzy Model and Memetic Algorithm*, STUDIES IN COMPUTATIONAL INTELLIGENCE 872 pp. 239-253. Paper: Chapter 14 , 15 p. (2020), DOI (WoS Scopus)

J1. **Ruba, S.Almahasneh** ; László, T. Kóczy, *Intuitionistic Fuzzy Rule-base Model for the Time Dependent Traveling Salesman Problem*, INTERDISCIPLINARY DESCRIPTION OF COMPLEX SYSTEMS 18 : 3 pp. 352-359. , 8 p. (2020), (DOI WoS)

11.2 Book Chapters

BC2. Zargar, Hisham ; **Almahasneh, Ruba** ; Kóczy, László T., *Automatic Recognition of Handwritten Urdu Characters*, In: István, Á. Harmati; László, T. Kóczy; Computational Intelligence and Mathematics for Tackling Complex, Problems 3 ,Cham, Svájc : Springer, Cham (2022) 220 p. pp. 165-175. Paper: Chapter 19, 11p, DOI

BC1. **Almahasneh, Ruba** ; Tuu-Szabo, Boldizsár ; Foldesi, Peter ; Koczy, Laszlo T., *Fuzzy Set Based Models Comparative Study for the TD TSP with Rush Hours and Traffic Regions*, In: Information Processing and Management of Uncertainty in Knowledge-Based Systems , Springer International Publishing (2020) pp. 699-714. Paper: Chapter 55 , 16 p., DOI Scopus

11.3 International Conference Papers

C8. Tüű-Szabó, B., **Ruba Almahasneh**,Földesi, P.,Kóczy, L.T.: A software tool for solving the Traveling Salesman Problem and related non-fuzzy and fuzzy optimization problems, Proceedings of the 14th European Symposium on Computational Intelligence and Mathematics (ESCIM 2022), October 2nd-5th, 2022, under publication

C7. **Almahasneh, R**. ; Koczy, L.T., *Extension of The Time Dependent Travelling Salesman Problem Model with Interval-Valued Fuzzy Soft Sets*, 14th European Symposium on Computational Intelligence and Mathematics (ESCIM 2022). Under publication.

C5. **Almahasneh, R**. ; Foldesi, Peter ; Koczy, L.T., *Mixed Arabic and English Reviews Sentiment Analysis with LSTM Recurrent Neural Network*, 12th IEEE International Conference on Cognitive Infocommunications (CogInfoCom 2021),pp. 179-185 (2021)

C4. **Almahasneh, R**. ; Tuu-Szabo, B. ; Foldesi, P. ; Koczy, L.T., *Extension of the Time Dependent Travelling Salesman Problem with Interval Valued Intuitionistic Fuzzy Model Applying Memetic Optimization Algorithm*, In: ACM, 2020 4th International Conference on Intelligent Systems, Metaheuristics and Swarm Intelligence (ISMSI 2020) , New York (NY), Amerikai Egyesült Államok : Association for Computing Machinery (ACM) (2020) pp. 111-118. Paper: 3396490 , 8 p., DOI (WoS Scopus)

C3. **Almahasneh, Ruba**; Tuu Szabo, Boldizsar ; Foldesi, Peter ; Koczy, T. Laszlo, *Quasi-optimization of the Time Dependent Travelling Salesman Problem by Intuitionistic Fuzzy Model and Memetic Algorithm1*, In: Computational, Intelligence in Emerging Technologies for Engineering Applications , Springer International Publishing (2020) pp. 239-253., 15 p.

C2. **Almahasneh, Ruba** ; Tuu-Szabo, Boldizsar ; Foldesi, Peter ; Koczy, Laszlo T., *Intuitionistic Fuzzy Model of Traffic Jam Regions and Rush Hours for the Time Dependent Traveling Salesman Problem*, ADVANCES IN INTELLIGENT SYSTEMS AND COMPUTING 1000 pp. 123-134. Paper: Chapter 12 , 12 p. (2019), DOI (WoS Scopus)

C1. Kóczy, T László ; Foldesi, Peter ; Tuu, Szabo Boldizsar ; **Almahasneh, Ruba,** *Modeling of Fuzzy Rule-base Algorithm for the Time Dependent Traveling Salesman Problem*, In: IEEE (szerk.) 2019 IEEE International Conference on Fuzzy Systems (FUZZ -IEEE) , New York (NY), USA, IEEE (2019) , 6 p., DOI Scopus

11.4 Awards and Grants

1) **Fuzzy youth Award 2021** by Hungarian Fuzzy Association.

2) International Fuzzy Systems Association World Congress (IFSA),2019, ***Outstanding paper award, "***Intuitionistic Fuzzy Model of Jam Regions and Rush Hours for the Time Dependent Traveling Salesman Problem, International Fuzzy Systems Association World Congress (IFSA), pp 123-134.***",**

3) **EUSFLAT** Grants for IPMU 2020.

4) "**Higher Education Institutional Excellence Program – Digital Industrial Technologies Research at University of Győr (UDFO/47138-1/2019-ITM)",** and M. F. acknowledges the

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11.5 Collaboration Research Groups

1. SAP Hungary Ltd., BlockChain for ERP
2. AIMS5.0

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List of Abbreviations

3FTDTSP: Triple Fuzzy Time Dependent Travelling Salesman Problem.
B&P: Branch-and-Price.
B&P&C: Branch and Price and Cut algorithm.
DBMEA: Discrete Bacterial Memetic Evolutionary Algorithm.
DARPTW: The Dial-a-ride routing and scheduling problem with time windows.
DARP: Dial-a-ride problem.
FOU: footprint of uncertainty.
GA: Genetic Algorithm.
IVIFS: interval-valued intuitionistic fuzzy set.
IFS: intuitionistic fuzzy set.
IVIFNs: interval-valued intuitionistic fuzzy numbers.
IVFS: interval-valued fuzzy set.
IVIFS interval-valued intuitionistic fuzzy Sets.
IIFWA: interval-valued intuitionistic fuzzy weighted arithmetic mean aggregation.
IIFOWA: the interval-valued intuitionistic fuzzy ordered weighted mean aggregation.
IIFHA: and the interval-valued intuitionistic fuzzy hybrid aggregation.
IIFWG: the interval-valued intuitionistic fuzzy weighted geometric mean.
IIFOWG: the interval-valued intuitionistic fuzzy ordered weighted geometric mean.
IIFHG: the interval-valued intuitionistic fuzzy hybrid geometric mean operators.
IVIFTD TSP: interval-valued intuitionistic Fuzzy Time Dependent Travelling Salesman Problem.
IVFSTD TSP: interval-valued Fuzzy Soft Time Dependent Travelling Salesman Problem.
IFTD TSP: intuitionistic Fuzzy Time Dependent Travelling Salesman Problem.
TD TSP: Time Dependent Travelling Salesman Problem.
TSP: Travelling Salesman Problem.
TOPSIS: The technique for ordered preference by similarity to the ideal solution.
MADM: multiple attribute decision making.
MAGDM: multiple attribute group decision making.
MCGDM: multiple-criteria group decision making problem.
NIS: negative-ideal solution.
PIS: positive-ideal solution.