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Tool path generation for pocket machining with uniform tool load

PhD dissertation booklet

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The reviews of the doctoral dissertation and the minutes of the defence are available at the Dean's Office of the Budapest University of Technology and Economics, Faculty of Mechanical Engineering.

1. Description of the research topic, main objectives

During my PhD. research, I focused on developing and optimizing the pocket milling technology. The most challenging issue in pocket milling is that the material to be removed is difficult to reach and therefore, optimal cutting conditions are difficult to ensure. Since this characteristic of pocket machining is manifested primarily in roughing, my work focused on further developing roughing pocket milling strategies, within this I concentrated on the planning of optimal tool paths. Since rouging, on average, takes about 50% of the machining time, but can sometimes require up to 5-10 times the time required for finishing, it is worth placing much emphasis on path planning [1].

Although there are various pocket machining methods, I have focused only on the most commonly used 2.5D, Z-level strategies. When further developing the path planning process, the indirect goal was to reduce the machining costs. In metal cutting, there are different methods for implementing this, but in my research, I dealt only with the possibilities of further development, related to the level of operation element. In other words, I considered that the machining type and environment, the machine tool and the tools are given. Under these conditions, manufacturing costs can be minimized by reducing machining time and/or increasing tool life. Accordingly, the explicit research objective was to determine the appropriate tool path and the appropriate cutting parameters.

Based on the literature and industrial practice, the primary goals when planning pocket milling tool paths are to ensure uniform tool load and continuous path curvature (see Section 2.1). Ensuring a uniform tool load is vital for machining time, otherwise the choice of cutting parameters would be limited by peak loads. Moreover, the impulsively changing load would place a more heavy strain on the tool, leading to a reduction in tool life. Ensuring a continuous path curvature is necessary due to the limitations of the accelerations of servo drives. The reason for this, is that in the vicinity of sharp changes in direction, the actual feed rate can deviate significantly from the programmed value, which increases the machining time. The full assurance of uniform tool load and continuous path curvature can be achieved by using hybrid tool paths, where spiral and trochoidal path segments are combined. Although CAM systems can create tool paths that meet the criteria listed above, this does not mean that they provide an optimal solution for 2.5D pocket roughing in terms of machining time. This can be explained by the fact that the path generation methods follow strict rules that, while providing a significant improvement in both machining time and tool life compared to the traditional contour parallel strategy, no real optimization is performed during the path planning process. As a result, this issue is still an active area of research today, because any development that reduces machining time can result in significant cost savings.

The main objective of my research was to develop a tool path generation strategy that provides a uniform tool load that can be used to rough 2.5D pockets bounded by an arbitrarily shaped contour and provides minimal machining time at a given load. I implemented spiral-like and trochoidal strategies by considering the development of the cutter engagement and the path curvature. During the development of the path generation algorithm, I considered the pocket contour, the tool diameter, the cutting speed, the axial depth of cut, the desired cutter engagement and the allowable feed rate for the given engagement as input parameters. For determining the optimal tool path, the optimization criterion was to achieve minimum machining time, and the optimization constraints were to control the tool load and the path curvature. However, with the help of the developed algorithm, it would also be possible to optimize the tool diameter, the cutting speed and axial depth of cut, because by running it several times it is possible to perform a secondary optimization. The methods presented do not consider the possibility of multi-tool machining or multi-immersion single-tool roughing. Furthermore, I did not cover the treatment of islands and semi-open pockets. However, the applied path generation principles remain valid in these cases as well, i.e. there is no practical obstacle to extend the developed algorithm to these geometries.

Summarizing the above, I formulated the following research goals and tasks: (1) a mathematical description of tool paths providing a constant cutter engagement, (2) the implementation of a minimum radius of curvature during path generation, (3) the connection of the methods developed for the previous two points to generate pocket milling tool paths, (4) the investigations of further development opportunities to reduce machining time.

2. State of the art, research methods

In the following chapter, the literature related to the topic and the methods used in simulation and cutting experiments will be presented.

2.1. Literature review

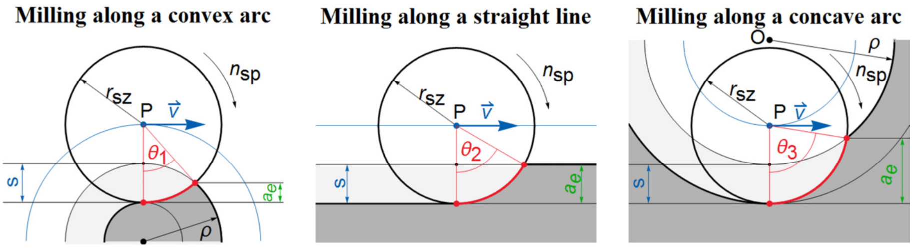
To describe the issue of pocket milling, M. Held [2] formulated the following definition: the pocket geometry and the technology goals and constraints are given; based on these, (1) the appropriate tool diameter, (2) the appropriate technological parameters, and (3) the appropriate tool path have to be determined. In my doctoral research, from these subtasks of pocket milling problem, I focused on determining the appropriate tool path. The following aspects must be taken into account when planning the tool path: (1) geometrical criterion: besides meeting the requirements for dimensional and shape accuracy and surface roughness, the entire allowance must be removed, (2) material removal criterion: appropriate cutting parameters must be applied to ensure efficient material removal (adequate chip formation and tool life), (3) mechanical criterion: the cutting force and energy must not exceed the limits due to the mechanical capabilities of the tool, the fixture and the machine tool. [3]

By using innovative cutting technologies such as high-feed milling (HFM) [4], high-performance milling (HPM) [5] or high-speed milling (HSM) [6], machining time and manufacturing costs can be significantly reduced. As the allowance material is difficult to reach in pocket milling, high-speed milling technology can be highlighted among the techniques mentioned above [7]. However, HSM technology requires the provision of appropriate conditions. The machine tool, the tool, the tool holder, the material of the workpiece and the tool path must all be suitable even under conditions where the cutting speed can reach up to 5-10 times the conventional values [8]. Tool paths of high-speed machining must meet two criteria: (1) the path must be smooth enough, and (2) it must provide a uniform tool load during machining [9] [10].

When the tool reaches a corner, it must slow down, change the direction of movement, and accelerate again when leaving the corner [11]. In the case of an inappropriate tool path with a cutting speed and a feed rate accustomed to HSM technology, the actual machining time can reach up to 3-8 times longer compared to the value calculated from the programmed feed rate [12]. In addition to the increase in machining time, the removal of material along sharp corners should also be avoided due to technical aspects of chipping [13] [14]. For this reason, when using HSM technology, the toolpath must be at least G^1 continuous, but in the case of C^2 continuity, even the acceleration functions will also be continuous [15]. Accordingly, for high-speed machining, it is advisable to use cubic polynomial spline curves that provide second-order mathematical continuity, significantly reducing the path tracking time [16]. It is worth to provide a smooth path even if it leads to an increase in tool path length, because in many cases, the path tracking time is reduced nevertheless [17].

By controlling the cutting force, the tool can work close to the maximum allowable load during machining, resulting in maximum productivity [18]. A further advantage of only small fluctuations in the cutting force is that there is less risk of tool breakage and vibration, providing longer tool life [19]. Therefore, the stability of the cutting process is essential for efficient machining [20] [21]. It is also worth noting that the experiences of research describing the milling process can only be used if the radial immersion of the tool is known and is nearly constant [22]. However, in pocket milling, where milling along curves and corners occurs, keeping the tool load constant is a major challenge in tool path planning.

The most effective parameter for characterizing the relationship between the tool and the workpiece is the cutter engagement angle (θ) [23] [24], which can be defined as the central angle corresponding to the arc length of the tool in contact with the workpiece.



Milling along a convex arc **Milling along a straight line** **Milling along a concave arc**

Figure 2.1 The evolution of engagement angles at different types of path sections

In the case of cutting along a straight line, the stepover (s) is equal to the radial immersion ($a_e = s$), i.e. the stepover directly determines the engagement angle. However, when milling along arcs and corners, the radial immersion of the tool and the radial immersion differ (see Figure 2.1) [25]. As the radial immersion changes, the cross-section of the chip also changes [26], which directly affects the cutting force [27]. Consequently, in the case of a constant stepover, the engagement angle decreases along convex arcs ($\theta_1 < \theta_2$), and increases along concave arcs ($\theta_3 > \theta_2$), as well as the cutting force [28]. Even multiple growths can occur along with concave corners, although a small increase in engagement angle can also be dangerous when cutting difficult-to-machine materials [29].

There are two solutions for eliminating tool load fluctuations: feed control and radial immersion control [30]. However, controlling the feed rate can not solve some of the problems. On the one hand, the machine tool must achieve continuous decelerations/accelerations [11], and the occasional large engagement angle can still be a source of vibrations and thermal shock [31]. An increase in the cutting temperature can be observed even if the engagement angle increases only slightly, which decreases the tool life [32]. Therefore, maximum efficiency can only be achieved by modifying the shape of the tool path [33] [34]. At the same time, feed control can effectively complement modern path generation strategies, where the radial immersion of the tool fluctuates only slightly [9]. For controlling the radial immersion, the tool path must be modified. In return, this approach also allows maintaining uniform cutting conditions along the path. This can be ensured by keeping the engagement angle constant [35]. The advantage of this approach is that a constant feed rate can be used; therefore the path generation remains independent of the workpiece material and the tool geometry, so there is no need to reckon with the inaccuracy of the models describing the tool load [36].

Using the appropriate tool path is crucial for production efficiency [37]. Even when machining free-form surfaces during roughing, the 2.5D Z-level strategy is generally used [38]. In that approach, the machining is executed parallel to the X-Y plane, and the only movements along the Z-axis are for positioning to the next machining plane. Thus, the path planning task is simplified to a planar problem. During pocket machining, the material within a given contour has to be removed. In addition to some very rarely used methods, there are two classical 2.5D pocket milling method: directional-parallel (also known as zigzag) and contour-parallel [2]. However, these solutions can not ensure a smooth tool path and a uniform tool load. Besides the basic strategies, there are more complex solutions that have emerged as achievements of computer-aided technologies [39]. The most crucial advance in modern path generation solutions is that the evolution of the engagement angle and the path curvature are considered during path generation. Commonly used solutions for machining pockets are the spiral-like strategy [40], which is also excellent for high-speed machining, since the path does not contain corners and sharp changes of direction [41], and the trochoidal strategy, which allows continuous curvature and well-controlled tool load [42]. Because the two strategies work effectively under opposite conditions, combining the two strategies is a standard practice. Paths created by combining two or more path generation strategies are called hybrid tool paths.

Without modern path generation strategies, high-speed machining would have been impracticable for pocket milling tasks. However, the actual implementation of the machining time optimization has not yet been solved, so this area is the subject of further research.

2.2. Simulation and cutting experiments

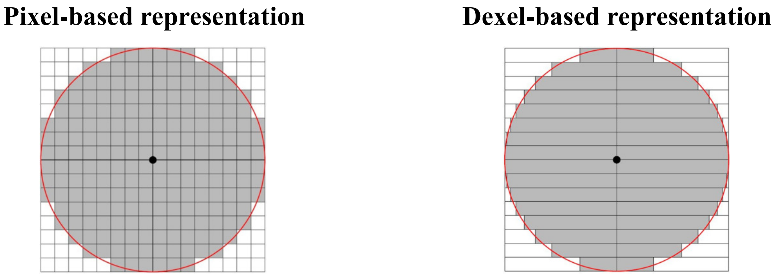
The most challenging part of pocket milling is that with traditional strategies, the material removal process is continuously varying along the path. Based on the literature, the following are the most effective indicators for characterizing the varying material removal conditions: (1) engagement angle, (2) material removal rate, (3) maximum and average chip thickness, and (4) the cutting force. To investigate these parameters, I used both simulation and experimental methods when evaluating machining strategies.

Several simulation software is commercially available for the analysis of CNC machining. Nowadays, even if not in large numbers, there are already high-level software packages that can also perform the technological analysis of milling operations. It would have been reasonable to use one of these softwares during my PhD. research. However, none of them was available at the Department of Manufacturing Science and Technology. Since I had to describe mathematically the cutting conditions along the path during the development of tool path generation algorithms. I avoided buying an expensive software, instead, I created a self-developed method in Wolfram Mathematica.

During the development of the simulation method, the primary objective was to examine the instantaneous tool load even in general-shaped tool paths. Since the analytical description of material removal is only possible in simple cases, for complex tool paths I have developed an algorithm based on a discrete model. The new algorithm can be applied to 2.5D milling operations and can determine the instantaneous value of the material removal rate, engagement angle, and maximum and average chip thickness.

For performing the simulation, the first step is to discretize the workpiece and the tool in the machining plane, i.e. in the X-Y plane, which is perpendicular to the tool axis. For this purpose, it would be reasonable to use a pixel-based representation, especially if the evolution of the allowance material is required to be shown, graphically. Applying this method is a standard practice. However, during the development of the algorithm, I realized that introducing a dexel-based representation could significantly reduce the computational requirement if we only want to use the algorithm to analyse the cutting characteristics. This is because the geometrical models of the workpiece and the tool can be represented with 1D vectors instead of 2D matrices. After the models are created, the tool can traverse along the tool path by taking steps with a given increment. During this procedure, in the X-Y plane the common cross-section of the workpiece and the tool can be determined at each step. If the axial depth of cut and the feed rate are known, the material removal rate and the other cutting indices to be examined can be calculated from the area of the sections determined step by step.

Since the simulation is mainly focused on the analysis of the material removal rate and the engagement angle, the tool can be considered to be a simple cylinder, since the exact geometry of the tool does not affect the examined parameters. This cylinder cross-section in the X-Y plane will be equivalent to a circular disk. If this circular disk is modelled with a matrix of binary elements, then the 1-valued elements of the matrix are always next to each other. This gave the idea to describe a row's contents in the form of an interval only by describing the position of the first and last non-zero matrix elements as in the dexel-based representation. This solution is more advantageous than the pixel-based approach in two ways. On the one hand, by reducing the resolution step size, the amount of data, and the number of logical operations to be performed will increase not quadratically but linearly. On the other hand, when specifying the interval's boundaries, it is not required to follow the rounding by pixel size, but floating-point numbers can also be used. This also allows to make equal the modelled area of the circular disk to the exact cross-section of the circle.



Pixel-based representation **Dexel-based representation**

Figure 2.2 Pixel- and dexel-based representation of the tool

During my PhD. research, the results of the simulation and cutting experiments were consistent in all cases, but I felt it essential to have direct results on the development of the tool load, so I performed control measurements in each case to validate the developed algorithms. The cutting experiments were performed on a KONDIA 640B type 3-axis machining centre at the Department of Manufacturing Science and Technology, where the force acting on the workpiece was measured with a Kistler 9257B type, three-component piezoelectric dynamometer. When comparing different strategies, the machining time and maximum cutting force were the basis for comparison.

3. Summary of the research and description of the theses

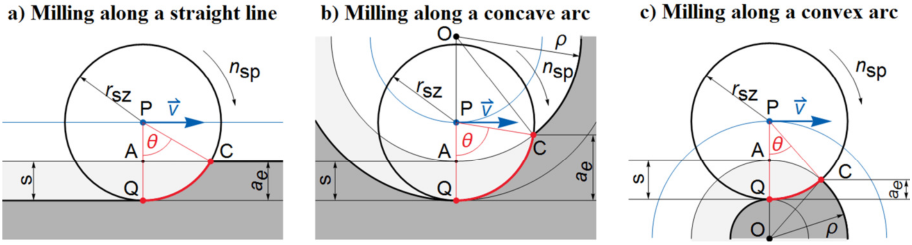
The research objective was to develop a tool path generation algorithm for roughing of 2.5D pockets bounded by an arbitrarily shaped contour, ensuring a uniform tool load. The development of the algorithm required several subtasks, which required the development of new methods and formulas.

As a first step, I developed a simulation method that can be effectively applied to the technological analysis of 2.5D pocket milling tool paths (see Section 2.2). Afterwards, I have expanded the range of mathematical formulas known from the literature that can be used to specify the relationship between engagement angle and stepover. These new formulas can be used to create tool paths that provide a constant engagement angle for both constant-curvature (see Section 3.1) and arbitrarily shaped contour curves (see Section 3.2). In the latter case, the shape of the tool path can be described by a differential equation. To solve this equation, I developed an effective numerical method (see Section 3.3). This algorithm can also be used to generate trochoidal tool paths. The efficiency of the new tool path pattern efficiency exceeds the currently known solutions (see Section 3.4).

For arbitrarily shaped pockets, I developed a method that can be used to link spiral-like and trochoidal strategies after performing a medial axis transformation (MAT). The tool path created with the self-developed algorithm was examined through simulations and machining experiments, during which it was compared with modern path generation strategies available in CAM systems (see Section 4).

3.1. Constant engagement tool path generation for contour-parallel machining of contours with constant arc curvature

For analysing the generation of tool paths that provide a constant engagement angle, it is first expedient to examine the case of contour segments with constant curvature. The following figure illustrates the geometrical conditions during milling along a straight line and circular arcs, where s is the side step, r_{sz} is the tool radius, ρ is the curvature radius of the current contour and θ is the engagement angle!



a) Milling along a straight line b) Milling along a concave arc c) Milling along a convex arc

Figure 3.1 Evolution of the engagement angle for contour segments with constant curvature

For the cases shown in Figure 3.1, formulas can be found in the literature to give the engagement angle based on the stepover. On the other hand, I have not found any expression for arcs that would specify the stepover based on the engagement angle, even though this would be necessary during the tool path planning. In the case of milling along a **straight line**, this can be described by a simple relation ($\theta(s) = \arccos(1 - s/r_{sz})$), but in the case of milling along circular arcs, the inverse function cannot be defined from the formula describing the engagement angle. Therefore, the cosine theorem written for OPC triangle should be used, on which the formula for the engagement angle was also based [35]:

$$|\overline{OC}|^2 = |\overline{PO}|^2 + |\overline{PC}|^2 - 2\,|\overline{PO}|\,|\overline{PC}|\cos(\measuredangle \mathrm{OPC}) \qquad (1)$$

For **concave arcs**, the following substitutions can be made in the previous formula: $|\overline{OC}| = \rho$, $|\overline{PO}| = \rho + s - r_{sz}$, $|\overline{PC}| = r_{sz}$, $\measuredangle \mathrm{OPC} = \pi - \theta$, while for **convex arcs** the following substitutions are required: $|\overline{OC}| = \rho$, $|\overline{PO}| = \rho - s + r_{sz}$, $|\overline{PC}| = r_{sz}$, $\measuredangle \mathrm{OPC} = \theta$. Considering the boundary condition $s \in [0, 2r_{sz}]$, by performing the appropriate simplifications a uniform relation can be derived to describe the stepover based on the engagement angle.

Thesis I.:

For milling of planar contour segments with constant curvature, the stepover which provides the engagement angle θ can be described with the following formula depending on the contour's radius of curvature:

$$s(\theta, \rho) = r_{sz}(1 - cos\theta) + k\rho - k\sqrt{\rho^2 + {r_{sz}}^2(cos^2\theta - 1)}$$

where s is the stepover, r_{sz} is the tool radius, ρ the contour's radius of curvature, and k is -1 in the case of concave arcs, 1 in the case of convex arcs.

Own publications related to the thesis: [JÁ1-JÁ8] [JÁ10] [JÁ12]

The above formula can only be applied to contour segments with constant curvature. In the following part, it will be shown how to specify the tool path that provides a constant engagement angle for an arbitrarily shaped contour.

3.2. Constant engagement tool path generation for arbitrarily shaped contours

Assume that the radius r_{sz} of the tool, the engagement angle θ to be used and a planar curve $\vec{c}(t)$ denoting the current contour of the workpiece are given. Based on these, the planar curve $\vec{p}(t)$ must be determined, along which the resulting engagement angle will be equal to θ. To solve this planar problem, the tool can be considered as a circular disk with radius r_{sz}.

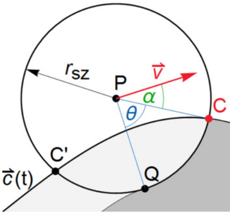


Figure 3.2 Determining the appropriate feed direction to provide the desired engagement angle

From the figure above, it can be deduced that the appropriate feed direction can be given by rotating the vector $\overrightarrow{CP}$ by angle $\beta = \pi/2 + \theta$. Since the function describing the feed vector is the same as the first derivative of the tool path curve ($\vec{v}(t) = \vec{p}'(t)$), a differential equation can be formulated from the contour curve ($\vec{c}(t)$) and the engagement angle to give the appropriate tool path ($\vec{p}(t)$).

Based on the above considerations, the corresponding feed direction can be determined at any point P located within a distance r_{sz} relative to the contour of the workpiece. An example of the vector field representing the appropriate feed directions is shown in Figure 3.3.

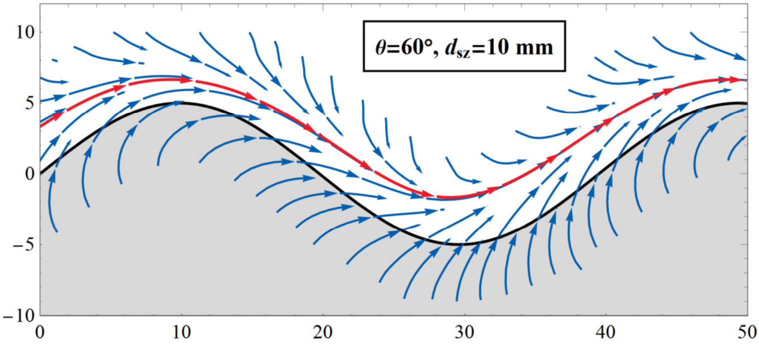


Figure 3.3 The vector field representing the appropriate feed directions

After selecting a starting point, the task can be described as an initial value problem:

Thesis II.:

For milling of arbitrarily shaped planar contours, the tool path which provides a constant engagement angle θ can be determined by solving the following initial value problem:

$$\vec{p}(t) = \vec{c}(t) + R\left(\frac{\pi}{2} + \theta\right) \cdot \vec{p}\,'(t) \cdot \frac{r_{sz}}{|\vec{p}\,'(t)|}$$

$$\vec{p}(0) = P_0$$

where $\vec{p}(\mathrm{t})$ is the tool path's vector function, $\vec{c}(\mathrm{t})$ is the contour curve's vector function, r_{sz} is the tool radius, P_0 is the starting point of the toolpath corresponding to the starting point of the contour $\vec{c}(0)$, and $R\left(\frac{\pi}{2} + \theta\right)$ is the rotational matrix:

$$R\left(\frac{\pi}{2} + \theta\right) = \begin{bmatrix} \cos\left(\frac{\pi}{2} + \theta\right) & -\sin\left(\frac{\pi}{2} + \theta\right) \\ \sin\left(\frac{\pi}{2} + \theta\right) & \cos\left(\frac{\pi}{2} + \theta\right) \end{bmatrix}$$

Own publications related to the thesis: [JÁ6] [JÁ9] [JÁ11] [JÁ12]

Although this problem can be solved analytically only in the case of elementary contours, after selecting a starting point, the tool path providing a constant engagement angle can be determined using numerical methods.

3.3. Numerical tool path generation algorithm to ensure constant engagement angle

To solve the problem geometrically, further analysis of Figure 3.2 is needed. This figure illustrates the conditions belonging to the parameter t_i. Based on this, the condition belonging to the parameter t_{i+1} must be determined. In that case, the point P_i belonging to the parameter t_i and the tangential unit vector $\vec{v}_i$ belonging to this point are known. These supplemental conditions are illustrated in Figure 3.4.

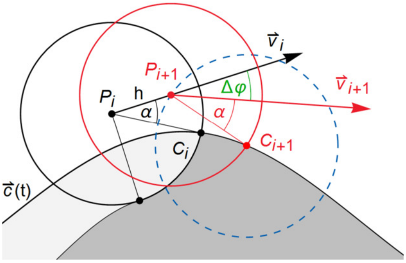


Figure 3.4 Construction of one tool path segment

The numerical tool path generation algorithm can be summarized in the following scientific statement:

Thesis III.:

For milling of arbitrarily shaped planar contours, the tool path, which provides a constant engagement angle θ can be determined by the following numerical algorithm.

Assume, that the parametric vector equation of the current boundary of the workpiece to be machined $\vec{c}(t)$ is given, where $t \epsilon [t_{min}, t_{max}]$. If the tool radius ($r_{sz}$), the starting position of the tool (P_0) from which the tool touches the starting point of the contour ($C_0 = \vec{c}(t_{min})$) and the numerical step size Δt) are known, the tool path's consecutive points can be determined by performing the following steps:

Step 0: Initialization: $t_i = t_{min}$, $i = 0$

Step 1: Rotate vector $\overrightarrow{P_i C_i}$ around P_i by angle $\alpha = \pi/2 - \theta$, and thereby obtain tangent vector $\vec{v}_i$

Step 2: in function $\vec{c}(t)$, substitute parameter $t_i + \Delta t$, and thereby obtain point C_{i+1}

Step 3: determine the intersection point of the circle with a centre point C_{i+1} and a radius r_{sz} and of a half-line from point P_i along vector $\vec{v}_i$ (if there are more points, then take the point which is closer to P_i), and thereby obtain point P_{i+1}

Step 4: increase parameter ($t_i = t_i + \Delta t$) and step index ($i = i + 1$)

Step 5: if the end of the contour is not reached ($t_i \le t_{max}$), go back to Step 1

The computational efficiency of the algorithm can be further increased, if

- adaptive step size is used instead of constant step size
- a cubic spline instead of a first degree polyline is used for connecting the points
- for considering the direction of the next step the Midpoint method is used instead of the explicit Euler method.

Own publications related to the thesis: [JÁ9] [JÁ11-JÁ13]

The algorithm can generate a tool path for arbitrarily shaped contours through simple geometric operations that provide a constant engagement angle. As opposed to the available geometrical methods, the algorithm can be used in the case of diverse machining conditions, not exclusively in the case of constant machining conditions: the algorithm also functions in the case of transition sections by creating suitable tool paths. Furthermore, the method has both the general applicability characteristic of pixel-based methods and the simplicity and fast operation of geometric methods (especially if further development solutions are also implemented). The performed experiments also proved that in the case of tool paths with constant engagement angle, the tool load also remains constant, thereby providing maximum productivity.

3.4. Generating trochoidal tool path patterns that provide a constant engagement angle

The algorithm described in the previous thesis was developed to determine the tool path providing a constant engagement angle, based on the boundary of the actual state of the workpiece. Until the tool reaches the final contour of the workpiece, the only task is to reduce the allowance material. Nonetheless, the algorithm can also be applied to tasks where multiple turns are needed. In this case, the workpiece boundary must be recalculated after each pass, and the limits given by the final geometry must be considered when determining the path. Using these extensions, it is also possible to design trochoidal tool paths in which the engagement angle remains at the nominal value for as long as possible, as opposed to conventional trochoidal tool path patterns where the tool load is continuously variable.

Using the developed algorithm, I experienced that after a few periods, the path shape stabilized, i.e. the difference between adjacent periods decreased to an almost negligible value. Of course, this is an advantage for the application because it is sufficient to define a single period that can be periodically repeated when machining the entire length of the slot.

Therefore, the algorithm can determine the cutting segments of the toolpath after recalculating the contour for each period. The final shape of the trajectory can be determined by an iteration. It has been experienced that this iteration is fast and usually provides monotonous convergence. Once the iteration reaches the final shape of the tool path pattern, it is only necessary to supplement it with linking movements and make it periodically repetitive with the appropriate trochoidal step. The iteration is illustrated in the figure below.

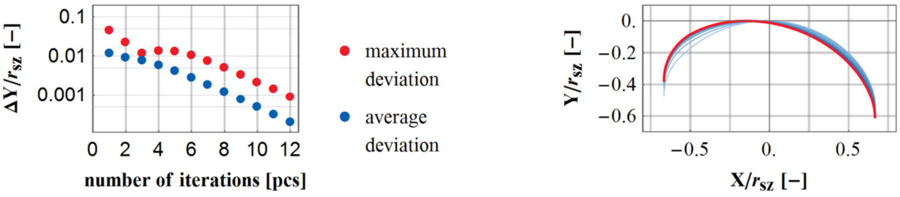


Figure 3.5 The evolution of the tool path shape during the iteration

To verify the efficiency of the developed method, cutting experiments were performed. The results showed that with the same technological parameters, the application of the developed path pattern is a significant improvement compared to both circular and cycloid-type tool paths. Thanks to the engagement angle control, the tool can work for as long as possible near the permitted tool load. Besides, with the new type of path shape, the tool path length became shorter, and the trochoidal step became longer compared to traditional strategies, thanks to which the value of the material removal rate could increase by up to 40%.

Based on the above, I made the following scientific statement:

Thesis IV.:

For trochoidal machining of constant width slots along a straight centreline, a repetitive tool path pattern can be generated that provides a constant engagement angle between the roll-in and roll-out segments. Besides providing stationary machining conditions, a major advantage of the generated path pattern is that the limit on the maximal engagement angle can be met with a significantly shorter tool path than the conventional circular and cycloid-type trochoidal strategies.

Own publications related to the thesis: [JÁ9] [JÁ11-JÁ13]

4. Utilization of the results

In the theses presented above, the algorithms for generating tool paths with a constant engagement angle were described. Evidently, it was a fundamental aspect in developing the methods that the algorithms could be applied to arbitrarily pocket geometries. In the following section, a hybrid tool path, providing a constant engagement angle, is presented using the self-developed algorithms.

A tool path generated for a star-shaped sample geometry was used for simulation and cutting experiments. To evaluate the results, modern path generation strategies of four different CAM systems were also investigated: the Waveform strategy of EdgeCAM 2017R2, the Adaptive Milling strategy of NX 12.0, the iMachining strategy of SolidCAM 2020, and the VoluMill strategy of ZW3D 2021.

During the experiments, a $\emptyset 6\ mm$ single edge carbide end milling tool was used. The cutting parameters were determined based on the tool catalogue (see Table 4.1). The starting hole diameter was 11 mm, so the volume of the material to be removed was $V = 2098\ mm^3$. Since the material removal rate can be calculated from the cutting parameters given in the table ($MRR_{nom} = 1147{,}5\ mm^3/min$), the theoretically achievable minimal machining time was $t = 110\ s$. As an experiment, I also performed a straight-line contour milling with the nominal cutting parameters, where the measured maximum cutting force was $99\ N$. On the straight sections of the connecting movements, an equally increased feed rate ($v_f = 1500\ mm/min$) was used, while the upper limit of the adjusted feed was $v_{fmax} = 1000\ mm/min$.

The radii of the roll-in and roll-out sections are uniformly $\rho = 1\,mm$ for both the self-developed strategy and the CAM systems' strategies where it was possible to specify it, so the acceleration of the machine tool was not a limiting factor.

Table 4.1 Cutting parameters used in the experiments

Cutting speed	$v_c = 120\,m/min$
Feed rate	$f_z = 0{,}04\,mm$ $(v_f = 255\,mm/min)$
Axial depth of cut	$a_p = 3\,mm$
Engagement angle	$\theta = 60°$

The diagrams shown in the following figure demonstrate that both the engagement angle and the material removal rate control work appropriately in the developed algorithm (the 1.5% error seen at MRR can be considered negligible). The only difference was found in the cutting force, where the maximum value was 9% higher compared to straight-line milling. This is probably because the feed control was based on keeping the material removal rate constant. The machining time was 134 s, which is only 22% higher than the theoretical minimum.

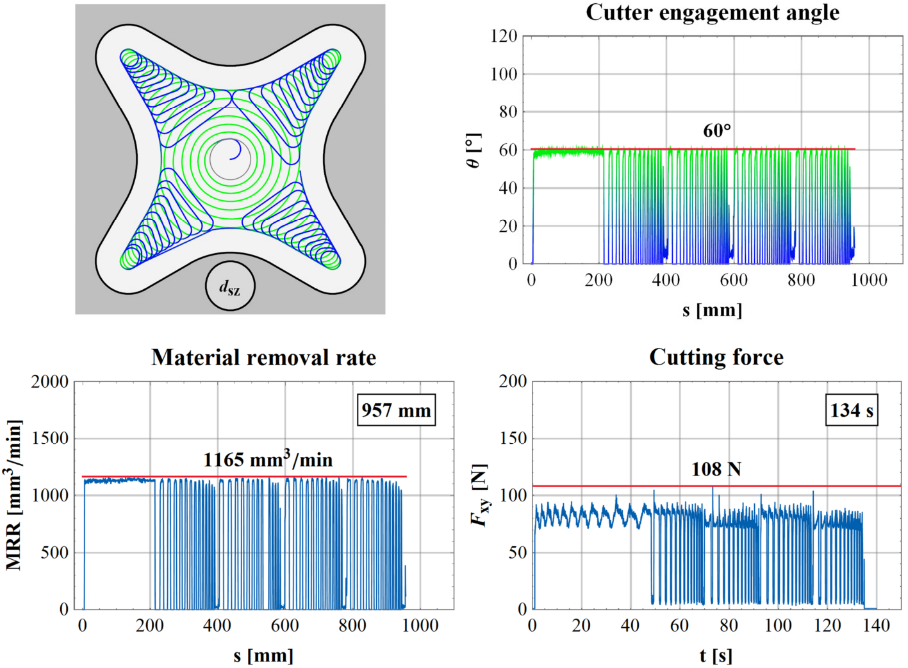


Figure 4.1 Machining the star-shaped pocket with the developed strategy

To evaluate the strategies, the maximum cutting force and the machining time were chosen as basis for comparison. In terms of cutting force, the Waveform ($96\ N$) and the Adaptive Milling strategies ($99\ N$) emerged, where both the engagement angle and the feed rate remain constant along the path, so that the force is almost the same as in milling along a straight line with the nominal parameters. The path generated with the developed algorithm has almost the same shape as these tool paths. However, I also implemented a feed rate control in the roll-in and roll-out sections, due to which the machining time became significantly shorter. Since the feed rate control was not based on a cutting force model, there was a slight increase in cutting force ($108\ N$). However, for the iMachining strategy ($119\ N$) and the VoluMill strategy ($119\ N$), which also used feed rate control, the difference were even greater. In terms of machining time, the self-developed path generation algorithm proved to be the best in the comparison ($134\ s$). The second shortest machining time was achieved with the VoluMill ($171\ s$), but it should not be forgotten that this strategy had the highest tool load. Path generation algorithms based on keeping the engagement angle constant, i.e. the Waveform ($203\ s$) and the Adaptive Milling ($196\ s$), also proved to be favourable in terms of machining time among the CAM strategies, while the iMachinig strategy ($225\ s$) lagged behind competing strategies not only in cutting force but also in terms of machining time.

Of course, this one example does not mean that such a reduction in machining time can be achieved in all cases. If the tool path contains only a few connecting movements or the difference between the nominal feed rate and the increased feed rate used on the linking sections is small, the importance of feed rate control decreases. However, this comparative analysis has shown that optimizing the feed rate at the entry and exit stages of trochoidal sections can significantly improve the machining efficiency, so the self-developed strategy can also outperform the solutions available in commercial CAM systems.

One of my main plans is to implement the developed algorithms in industrial applications as well. As a first step, a domestic company developing CNC controls has already been contacted. Besides, I see opportunities to develop the methods further for spiral-like and trochoidal-type strategies. The objective is primarily to integrate cutting force models and the consideration of the machine tool's acceleration capabilities, as these are expected to be the basis of future’s path generation algorithms.

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