

# OPTIMIZATION IN GEOTECHNICAL ENGINEERING: KEY TOPICS AND PRACTICAL TOOLS

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**Abstract.** Optimization plays a central role in the further development of geotechnical engineering practices, enabling cost-effective, sustainable and technically sound solutions. Teaching optimization effectively requires a structured approach tailored to the different learning needs of undergraduate, master's and PhD students. This paper presents a comprehensive framework for teaching optimization in geotechnical engineering that includes key topics, pedagogical strategies, and practical tools. By covering the fundamental concepts, computational techniques, and advanced applications of optimization, this framework aims to prepare students for both academic and professional excellence. The example shows the application of an optimization model to determine the critical safety factor of an infinite soil slope resting on impermeable rock.

## 1 INTRODUCTION

Geotechnical engineering deals with the design and management of earth structures such as foundations, embankments, slopes and tunnels under complex physical and economic boundary constraints. Optimization methods offer systematic approaches to achieve optimal solutions in terms of cost, safety and environmental sustainability. In addition, optimization methods are used to solve complex soil mechanical problems, e.g. to determine the critical failure surface, the required embedment depth of the retaining wall or to determine the probability of failure in reliability-based design. In order to provide students with the skills required for modern geotechnical practice, a well-structured educational framework is essential. This paper outlines the key topics that should be included in such a curriculum and presents strategies for the effective delivery of this content at different levels of education [1]. The example shows the application of an optimization model to determine the critical safety factor of an infinite soil slope resting on impermeable rock.

## 2 KEY TOPICS IN OPTIMIZATION FOR GEOTECHNICAL ENGINEERING

An essential part of teaching optimization in geotechnical engineering is the introduction to the fundamental principles underlying optimization methods. Students should understand the basic concepts of objective functions, decision variables and constraints [1]. These concepts form the basis for more advanced topics, including types of optimizations such as linear, nonlinear, integer, stochastic optimization, multi-objective optimization and multi-parametric

optimization [2, 3, 4]. Understanding the basic principles is essential, while applying the most appropriate optimization method to a geotechnical problem is one of the most important steps in solving the problem. It is also important to emphasize the role of optimization in geotechnical engineering, especially in achieving cost efficiency, safety improvements and environmental sustainability.

Applications of optimization in geotechnical engineering cover a wide range of areas. In shallow foundation design [5,6], optimization techniques can determine the most efficient pile [7], road pavements [8], retaining walls [9], slope stability [10], piled embankment [11], soil nailing [12], soil investigations [13] and many others. For slope stability, these methods can identify critical slip surfaces and optimize reinforcement placement [14]. Ground improvement strategies can benefit from optimization by minimizing material usage while maintaining effectiveness. Tunnel and excavation designs can achieve a balance between stability and efficiency through optimization methods. Optimal soil investigation also benefits from optimization by prioritizing critical areas while managing costs.

Mathematical and computer-aided geotechnical optimization techniques include linear programming and simplex methods, which are suitable for problems with linear relationships. Nonlinear optimization techniques are necessary to solve complex geotechnical problems with nonlinear boundary conditions [15,16]. Advanced methods such as genetic algorithms, particle swarm optimization and ant colony optimization provide powerful tools for solving multivariable and nonlinear problems [17]. The integration of finite element modelling with optimization techniques enables detailed analysis of geotechnical structures. Artificial intelligence and machine learning are emerging as transformative tools for geotechnical optimization, enabling predictive modelling and automated decision making.

Tools and software are an essential part of the teaching and application of optimization in geotechnical engineering. Commercial platforms such as PLAXIS, GeoStudio, Geo5 are commonly used for modelling and analysis. Programming tools such as MATLAB and GAMS are essential for implementing your own optimization algorithms. Visualization tools help to interpret optimization results and make them accessible to students and professionals alike. The Solver in Excel is an optimization tool that is used to find optimal solutions to decision problems by adjusting the input variables while taking constraints into account. It can minimize or maximize an objective function or reach a certain target value. Solver is commonly used for tasks such as linear programming, non-linear optimization and regression analysis.

Sensitivity analysis and decision making are important topics in the optimization curriculum. Students should learn how to evaluate the sensitivity of parameters and quantify uncertainties. Multi-objective optimization techniques and Pareto efficiency concepts help in balancing conflicting objectives, e.g. cost and safety. Risk-based optimization approaches provide a framework for dealing with uncertainties and prioritizing safety in geotechnical designs.

### **3 PEDAGOGICAL STRATEGIES BY EDUCATIONAL LEVEL**

At the undergraduate level, the primary objective is to develop a basic understanding of optimization principles and to introduce basic applications. Students should be exposed to visual aids and simplified examples that illustrate optimization concepts, such as optimizing the design of a retaining wall or slope. Interactive tools such as Excel can be used to demonstrate linear programming in a user-friendly way. Real world geotechnical scenarios should be

presented to link theoretical knowledge with practical applications. Assessments at this level may include problem-solving exercises and mini-projects that encourage collaboration and critical thinking.

For Masters students, the focus shifts to building computational skills and tackling complex real-world problems. Case studies and project-based learning are effective ways to accomplish this. Students should be exposed to tools such as MATLAB, GAMS, and commercial software to solve nonlinear optimization problems and explore advanced methods such as genetic algorithms and multi-objective optimization. Detailed design projects and technical reports may serve as assessments, focusing on the integration of technical, economic and environmental considerations.

At the doctoral level, the goal is to improve the state of the art through research-driven optimization. Students should work on advanced topics such as metaheuristics, sustainability-oriented optimization and the development of novel algorithms for specific geotechnical challenges. Collaboration with industry partners can enable engagement with current problems and practical applications. Assessments should focus on original research contributions, including publications in academic journals and at conferences, as well as the development of innovative tools or frameworks.

#### 4 OPTIMIZATION MODEL FOR SLOPE STABILITY

The example considers an infinite soil slope resting on impermeable rock, with groundwater present in the slope and seepage occurring parallel to the underlying rock, as illustrated in Figure 1. To determine the slope's safety factor, all forces acting on the slope must be in equilibrium. This is achieved by calculating the appropriate safety factor (SF) and the force ( $Q_\phi$ ). To ensure force equilibrium, an optimization approach is applied. The optimization model is defined by Equations 1 to 9, with the objective function represented by Equation 8. The goal is to achieve vertical equilibrium of forces, requiring the objective function to equal zero. An additional constraint, ensuring horizontal force equilibrium, is provided by Equation 9, which is also set to zero. The optimization model is developed in Excel. The input data include a unit weight of soil ( $\gamma_k$ ) of  $20 \text{ kN/m}^3$ , a clay soil thickness (H) of 5 m, and a water table located 1 m below the ground surface ( $h_w = 4 \text{ m}$ ). The hill slopes at  $v = 1 \text{ m}$  (vertical) to  $n = 4 \text{ m}$  (horizontal), with a soil friction angle ( $\phi'_k$ ) of  $25^\circ$  and cohesion ( $c'_k$ ) of  $2 \text{ kPa}$ . Groundwater has a weight density ( $\gamma_w$ ) of  $10 \text{ kN/m}^3$  and flows parallel to the ground surface. Figure 2 presents the input data, the optimal solution, and the Solver settings used for the optimization. The critical safety factor for the given input data is  $\text{SF} = 1.1825$ . Additionally, finite element modelling (FEM) was applied to determine the factor of safety for the same example, resulting in a safety factor of  $\text{SF} = 1.164$ . The critical failure line obtained from the FEM analysis is shown in Figure 3.

$$\phi'_d = \text{atan} \left( \frac{\tan \phi'_k}{\text{SF}} \right) \quad (1)$$

$$c'_d = c'_k / \text{SF} \quad (2)$$

$$s = \sqrt{1^2 + n^2} \quad (3)$$

$$i = 1/s \quad (4)$$

$$G' = (H - h_w) * \gamma_k + h_w * (\gamma_k - h_w) \quad (5)$$

$$F = i * \gamma_k * h_w \quad (6)$$

$$T_c = c'_d * (1/\cos \beta) \quad (7)$$

$$\sum V = 0: G' + F * \sin \beta - T_c * \sin \beta - Q_\varphi * \cos(\varphi - \beta) = 0 \quad (8)$$

$$\sum H = 0: F * \cos \beta - T_c * \cos \beta - Q_\varphi * \sin(\varphi - \beta) = 0 \quad (9)$$

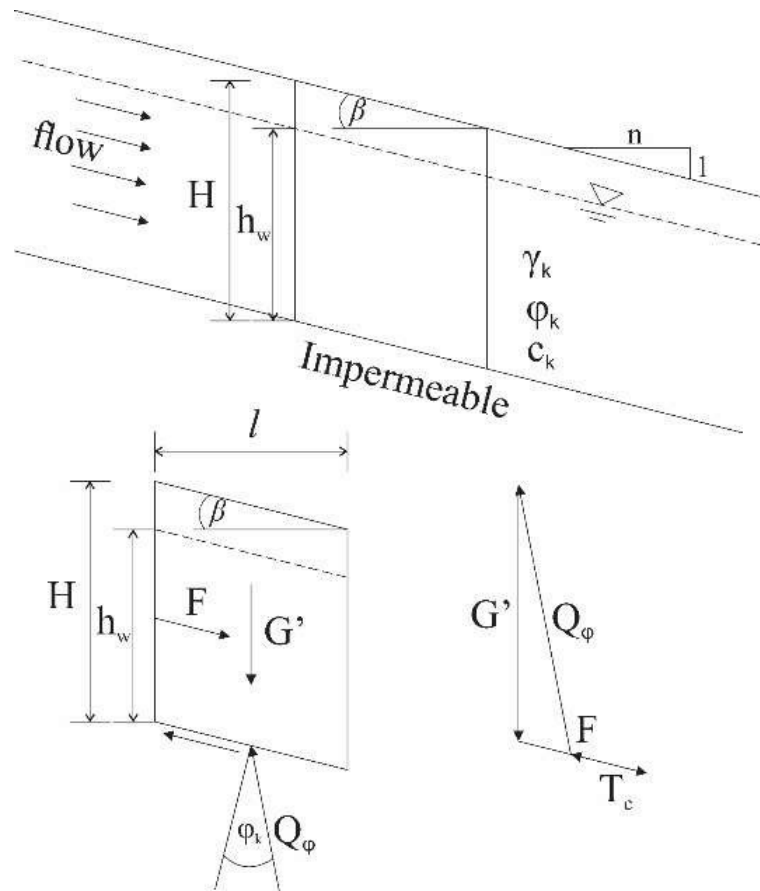


Figure 1: Stability of infinite soil slope overlying impermeable rock

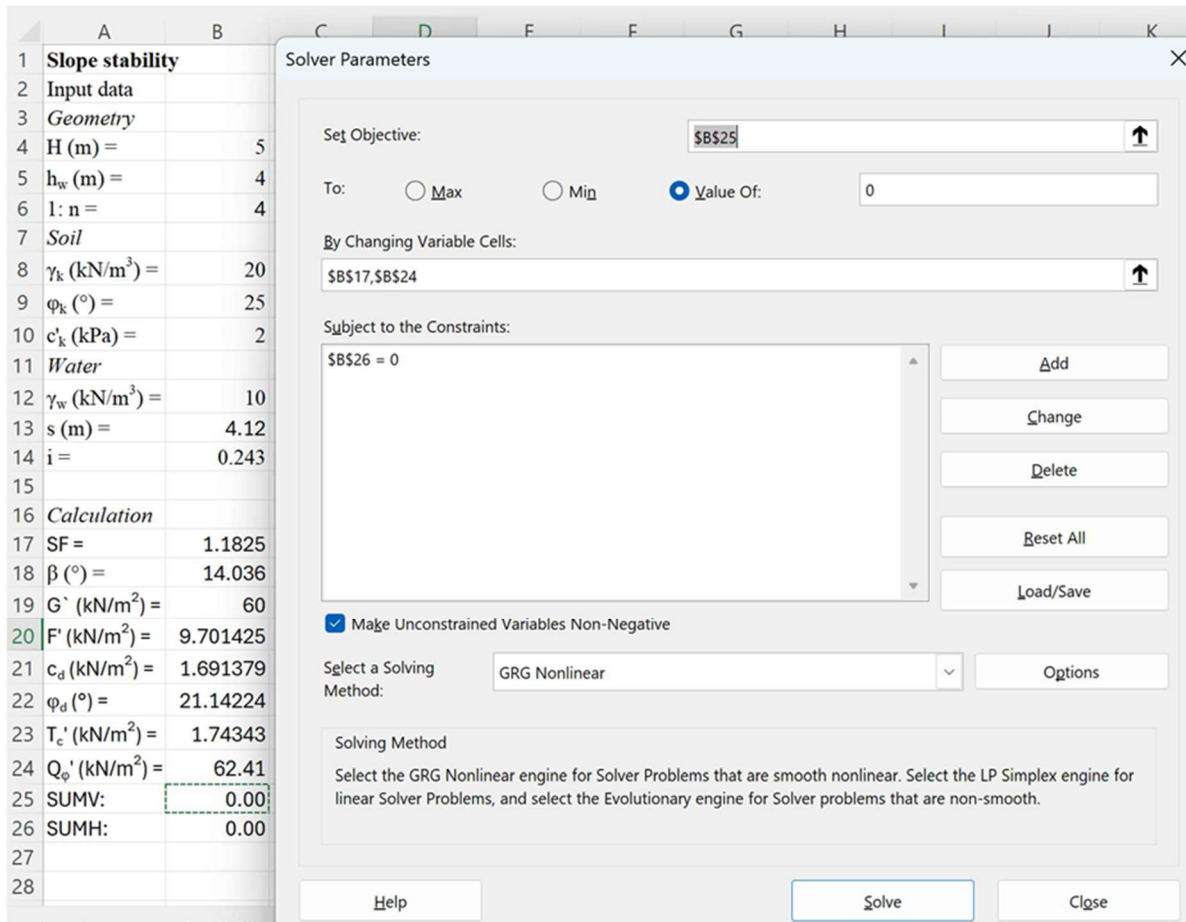


Figure 2: Optimization model for slope stability of infinite soil slope

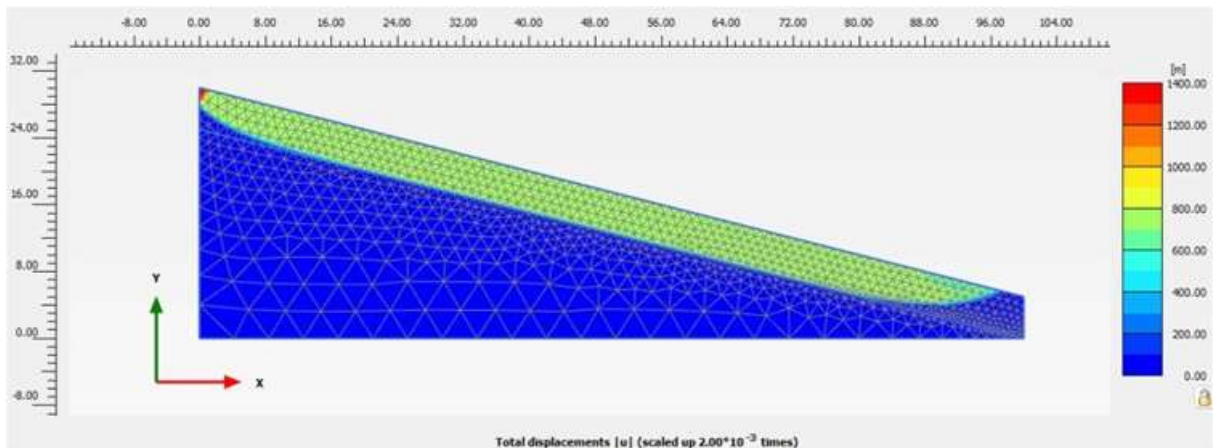


Figure 3: Slope stability problem of an infinite soil slope, modelled with FEM

## 5 CONCLUSIONS

- Optimization is a cornerstone of modern geotechnical engineering and enables efficient and sustainable construction solutions. A structured curriculum, as outlined in this paper, can provide students with the necessary knowledge and skills to effectively apply optimization techniques. By tailoring content to different levels of education and integrating theoretical, practical and research components, educators can foster a deeper understanding of optimization in geotechnical engineering and prepare students to meet industry and academic requirements.

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