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Stability and bifurcation analysis of mechanical systems subject to digital position control

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Introduction

Position and trajectory control of robots are very important tasks in many industrial applications. The positioning must be fast, precise, reliable and stable in order to guarantee high standards of production. This topic has been a subject of intensive studies for decades, but it is still an interesting one, in virtue of the increasing demands from any field using robots and of the increasing number of their applications. Robotic systems are usually equipped with digital controllers, while their dynamic analysis is often treated using continuous-time (analog) approaches and models; indeed, the sampling frequency can be so high, that from an engineering point of view it can be considered continuous. However, these approaches do not provide always conservative estimates for stability.

In the last decade several authors studied the stability of digital control systems, finding surprising results about the stability domains, which are strongly influenced by the sampling frequency. Industrial robots often present instabilities that cause the arm to oscillate around the desired position. The stability is then reached through several tests aimed at "calibrating" the control system, which generally imply a reduction of the control gains. This approach, although works in many cases, neither allows to predict the stability properties of the system, nor to have optimal values of the control gains with respect to fast speed of convergence and precision of positioning.

The finite stiffness of connecting joints is another elements, often underestimated, that can compromise the stability of robotic systems. Although their stiffness may be very high, in some cases it should be taken into account in the mechanical model. In the case of harmonic drives, the teflon teeth introduce a small angular displacement between the motor and the driven link. Similarly, belts or long shafts introduce a non negligible elasticity. Usually, the elasticity introduced by these elements, can be considered concentrated at the joint with an elastic or a visco-elastic term.

Although the problem of the control of robotic arms with elastic joints is well known and widely investigated, very few authors take into account the effect of digitization of the control force, due to the presence of digital processors. Besides the delay introduced by the digital controller, which can be often neglected in modern processors due to the high value of the sampling frequency, this can still generate resonances with the natural frequency of the system.

Besides stability, another aspect that will be investigated in this research is the bifurcation behavior of the system at the loss of stability. Although most of the industrial systems of any kind are designed to work in stable conditions, it is important to study the effects of instabilities in order to know the damages caused by instabilities and estimate their danger, especially in cases of close human cooperation. In particular, subcritical bifurcations can compromise the robustness of the stability of the system, generating unexpected motions, in conditions where stability has been previously experienced. In some cases such events can have catastrophic consequences.

The rich dynamics experienced in cases of instabilities is another aspect that motivated us to investigate the bifurcation behavior of digitally controlled system. Multi-frequency quasiperiodic and chaotic attractors can occur, which cannot be explained by a linear analysis and can be hardly understood from simple numerical investigations of nonlinear models.

Aims of the work

The work can be divided in four main parts: a stability analysis and a bifurcation analysis of digital position control, a section dedicated to the act-and-wait controller, a special periodic controller of recent development, and a section dedicated to experimental investigations.

The aim of the first part consists in analyzing the effects of digitalization and associated delay on single link robotic arm with a rigid or an elastic joint (Fig. 1), with respect to stability and speed of convergence. In the investigation, the differences between the collocated and the non-collocated cases are highlighted. The aim of the bifurcation analysis consists in investigating the nature of the Neimark-Sacker (NS) bifurcation, occurring at the loss of stability of digitally controlled systems. In particular, the objective of the work is to define in which circumstances the bifurcation is supercritical and in which it is subcritical. The analysis has been carried out for both single and double NS bifurcations, first analytically and then numerically.

The chapter dedicated to the act-and-wait controller aims at investigating the nonlinear behavior of this controller. Although its advantages in a linear range are already well known, there are no previous studies of analysis of the controller in nonlinear models. The chapter dedicated to the experimental work aims at verifying some of the results obtained

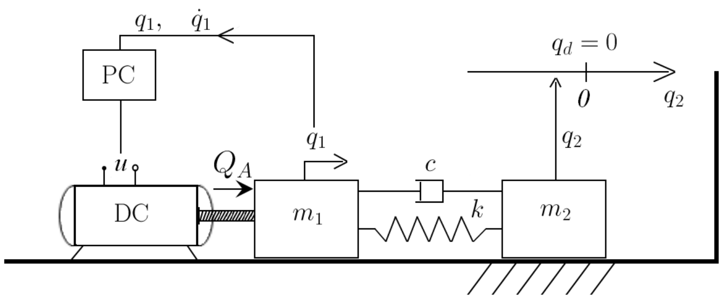

Figure 1: 2 DoF model of a single link robotic arm with an elastic joint, subject to PD control.

in other parts of the dissertation, through an experimental investigation of the so-called Aeropendulum.

Theses

Thesis 1

Considering a single link robotic arm with an elastic joint (Fig. 1), subject to PD position control, it has been proven that the digitization and the delay introduced by the digital processor of the controller have negative effects with respect to stability in the case of a collocated system. On the contrary, in the case of a non-collocated system (in the figure, the processor would read in input q_2 and not q_1), the digitization does not cause any significant loss of stability. Experiments on a digitally controlled aeropendulum confirm the validity of the applied mechanical model.

These results are detailed in Chapters 2 and 8 of the dissertation.

■ Related publications: [3,10].

Thesis 2

Considering a single-DoF (degrees of freedom), purely inertial model of a single link robotic arm subject to digital PD position control, it has been shown that the system loses stability through a supercritical NS bifurcation. This occurs if a smooth saturation of the control force is taken into account. However, an asymmetry of the saturation, with the consequent introduction of second order terms, can make the bifurcation switch from supercritical to subcritical. The level of asymmetry necessary for the transition from supercritical to subcritical has been defined analytically and confirmed numerically.

Furthermore, a similar scenario has been obtained for a two DoF system in correspondence of a double NS bifurcation. Also in this case the critical values have been defined both analytically and numerically with an excellent agreement between the two cases.

Note the practical importance of subcriticality for the engineering practice: the existence of an unstable solution in the stable region of the trivial solution, consequent to a subcritical bifurcation, generally compromises the robustness of the stability, causing unexpected motions unpredictable from a linear analysis.
These results are detailed in Chapters 4 and 5 of the dissertation.
■ Related publications: [1,5,6,8,9,11].

Thesis 3

The analysis of one and two DoF systems, modeling a simple robotic arm subject to digital PD position control, showed the existence of 2-torus and 3-torus quasiperiodic motions, often experienced in real applications but hardly explained. These motions have been explained and predicted both analytically and numerically.

Furthermore, numerical investigations showed the appearance of chaotic motions as a consequence of an increasingly asymmetric saturation of the control force. In particular, it has been described as a transition to chaos through torus breakdown, highlighting the typical features usually experienced in this kind of route to chaos.
These results are detailed in Chapters 4, 5 and 6 of the dissertation.
■ Related publications: [1,5,6,8,11].

Thesis 4

We applied the so-called act-and-wait controller to a single DoF model of a single link robotic arm, which consists in a particular controller that periodically turns on (act) and off (wait), in order to reduce the number of poles of a delayed system. Our investigation confirmed its advantages regarding an enlarged stable region and a faster convergence. In addition, still unknown disadvantages of the method were shown. In particular it was verified that in correspondence to the loss of stability the system controlled with the act-and-wait controller experiences much larger oscillations than the same system controlled in a traditional way. Furthermore, numerical simulations showed the existence of lots of nontrivial solutions within the region of stability of the trivial solution. These exist also in the case of supercritical bifurcations

at the loss of stability, which makes them very hard to be identified even with a standard nonlinear approach. Nevertheless, they compromise the robustness of the stable solution in a non-negligible way.
These results are detailed in Chapter 7 of the dissertation.
■ Related publications: [7,12].

Publications

Journal papers:

[1] Habib, G., Rega, G., Stepan, G., Nonlinear bifurcation analysis of a single-DoF model of a robotic arm subject to digital position control. *J. Comput. and Nonlin. Dyn.* **8**(1) (2012) pp. 011009

[2] Habib, G., Rega, G., Stepan, G., Analytical investigation of single and double Neimark-Sacker bifurcations. *Periodica Polytechnica* **56**(1) (2012) pp.13–22.

[3] Habib, G., Rega, G., Stepan, G., Stability analysis of a two-DoF system subject to proportional-derivative digital position control. To appear in *J. Vib. and Control*

[4] Gottlieb, O., Habib, G., Non-linear model-based estimation of quadratic and cubic damping mechanisms governing the dynamics of a chaotic spherical pendulum. *J. of Vibration and Control.* **18**(4) (2011) pp. 536-547.

[5] Habib, G., Rega, G., Stepan, G., Bifurcation analysis of a two-DoF mechanical system subject to digital position control. Part I: theoretical investigation. Under review at *Nonlinear Dynamics*.

[6] Habib, G., Rega, G., Stepan, G., Bifurcation analysis of a two-DoF mechanical system subject to digital position control. Part II: effects of asymmetry and transition to chaos. Under review at *Nonlinear Dynamics*.

[7] Habib, G., Rega, G., Stepan, G., Nonlinear investigation of the act-and-wait control concept applied to a single-DoF robotic arm. In preparation.

Conference papers:

[8] Habib, G., Rega, G., Stepan, G., Stability and bifurcation analysis of robots subject to digital position control: 7th ENOC, 24-29 July 2011, Rome, Italy (2011).

[9] Habib, G., Rega, G., Stepan, G., Supercritical bifurcation of the digital position control of a single-dof robot model under symmetric nonlinear force: AIMETA 2012, 12-15 September 2011, Bologna, Italy (2011).

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[12] Habib, G., Rega, G., Stepan, G., Improving the digital position control of a single-DoF system via the act-and-wait control concept: EUROMECH Colloquium 532 – Time-Periodic Systems Current Trends in Theory and Application, 27-30 August, Frankfurt, Germany (2012).