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Booklet of thesis statements

STABILIZATION OF TWO-WHEELED VEHICLES:
TRAILERS AND ELECTRIC SCOOTERS

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INTRODUCTION

Motivation

Vehicle handling and stability are critical factors in road transport. Unfortunately, numerous road accidents occur every day, many of which are related to the lateral stability of vehicles. Drivers often do not choose the speed in accordance with the traffic regulations, the dynamic characteristics of the vehicle, and the environmental conditions. In other cases, unwanted vibrations may appear in real traffic that were not suspected during the design and testing of the vehicle. The mechanical modeling of classical vehicle combinations (truck, truck-semitrailer, etc.) should remain a relevant topic in vehicle dynamics to reduce the risk of serious accidents related to these vehicles. Beyond this, with the spread of micromobility vehicles (electric scooters, electric unicycles, segways, skateboards, etc.), there is a need for a new research direction of vehicle dynamics to understand how these vehicles move and how they should be controlled. This dissertation aims to achieve results in both topics, namely, the dynamics of two-wheeled trailers and e-scooters are in the focus.

The appropriate mechanical and mathematical modeling and the analysis with proper mathematical tools of these vehicle systems can lead to challenging engineering tasks from numerous perspectives. The vehicle systems in question are often complex multi-degree-of-freedom dynamical systems with coupled vibration modes. In addition, a spatial mechanical model is often required to describe the nonlinear dynamics appropriately. The complicated nature of the contact between the wheels of the vehicle and the ground has to be handled, too, which is a fairly complex task. Hence, most of the previous studies are limited to numerical analysis, e.g., using multibody dynamics software and commercial computer codes. In this thesis, low-degree-of-freedom models of two-wheeled vehicles are constructed and investigated, which can be managed analytically but still capture the spatial dynamics and the relevant vibrations of the vehicles. Namely, the thesis aims to fill the research gap between the simple in-plane and the complex multibody models.

Unwanted vibrations can emerge after the design stage of the vehicle, although extensive field tests are always included. In general, the design of vehicles is

mixed second-degree terms related to the tire forces appearing in the governing equations (*Thesis statement 1*).

An unsafe towing speed range with practical relevance is situated at low center of gravity positions. The commonly used simple trailer models, in which the pitch motion and/or the roll motion are neglected, are not conservative regarding safety and are of limited use. The full spatial model is more realistic, which is confirmed by laboratory experiments with a small-scale trailer towed on a conveyor belt. The full spatial model can also explore the critical domains of rocking motion associated with the loss of contact of the tires. Therefore, the full spatial model is recommended to explore all the possible, safety-relevant nonlinear vibrations of towed, two-wheeled trailers (*Thesis statement 2*).

A linear state feedback controller is also designed to eliminate the unsafe towing speed range using differential braking of the trailer. The dead zone and the controller's feedback delay are considered. Appropriate values of the control gains are determined for a certain towing speed. For the realistic trailer setup with a low position of the center of gravity, the controller can eliminate the unsafe zone even with a moderate feedback delay and dead zone of the controller. However, the same controller setup performs poorly for a high center of gravity position (*Thesis statement 3*).

In the second part of the thesis, the stability of riderless e-scooters is investigated. The spatial mechanical model that we use is based on the Whipple bicycle model, see Fig. 2. The multibody system comprises four rigid bodies with several geometric and kinematic constraints. Hence, the nonlinear governing equations are derived using Kane's method. The balancing of the e-scooter at zero longitudinal speed is achieved with hierarchical linear continuous-time state feedback controllers with multiple delays. In the case of control with steering, the e-scooter is balanced by applying steering torque to the handlebar. In the case of control with driving, the front wheel is turned to the $\pi/2$ steering angle position, and driving torque is applied to the front wheel.

One of the geometric constraints is manifested in the form of a complicated nonlinear equation for the lean, the steering, and the pitch angles. Approximate formulas are derived for the pitch motion to eliminate this geometric constraint. It is shown that our approximations provide the exact linearized equations of

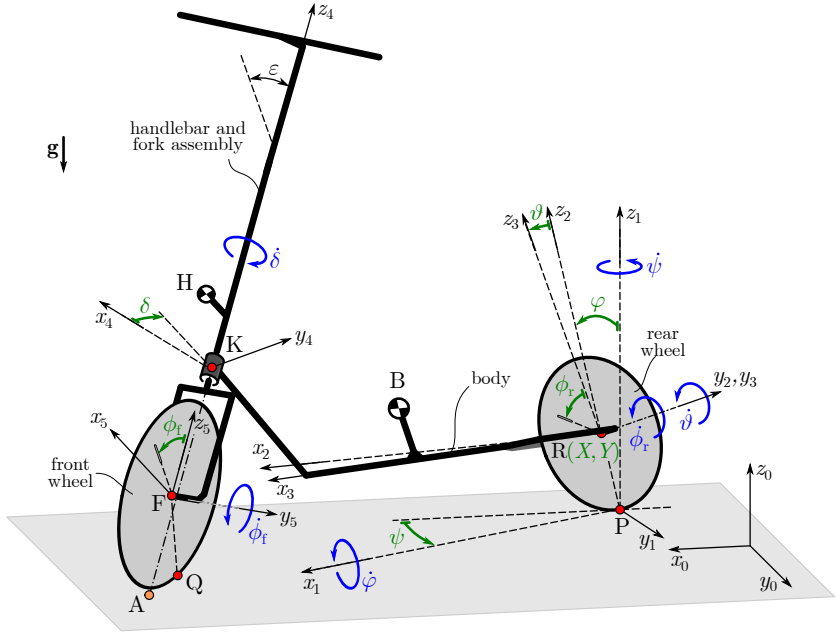


Figure 2: The spatial mechanical model of an electric scooter with the generalized coordinates, angular velocities, and reference frames.

motion (*Thesis statement 4*).

Simple linear state feedback controllers with multiple delays are implemented, and a careful analysis is accomplished with respect to the effect of the time delays and geometric parameters of the e-scooter. Thanks to the optimization process, optimal control gains of the hierarchical controllers are selected. The position of the handlebar's center of gravity is identified as a key parameter by which the performance of the controllers can be tuned (*Thesis statement 5*).

If multiple time delays are considered in the hierarchical balancing control, stabilizability plots constructed in the plane of the higher-level and the lower-level delays show that the most stable delay setup does not correspond to the delay-free cases. When the lower-level delay is increased, the higher-level delay has to be proportionally increased as well to maintain the linearly most stable system (*Thesis statement 6*).

MAIN RESULTS

Stability of towed two-wheeled trailers

Consider a full spatial four degrees-of-freedom (4 DoF) mechanical model of a towed two-wheeled trailer. The generalized coordinates of the model consist of the yaw angle ψ , the pitch angle ϑ , the roll angle φ , and the lateral displacement u of the kingpin. The kingpin is towed in the longitudinal direction with constant speed v , while it is supported by a spring and a damper in the lateral direction to imitate the lateral dynamics of the towing vehicle. The wheel suspensions of the trailer include spring-damper units; the lateral tire forces are calculated via Pacejka's creep force model considering the varying normal loads. Take into account the non-smooth effects of the wheel suspension characteristics and the detachment of the wheels from the ground.

Thesis statement 1

The linear stability of the rectilinear motion of the trailer is not affected by the pitch motion if the following criteria hold:

$$2kl^2 - mgh - \frac{mgh_0(l-a)}{l} > 0 \quad \text{and} \quad c > 0, \quad (1)$$

where k and c are the stiffness and the damping of the suspension, respectively, l is the caster length, m is the mass of the trailer, g is the gravitational acceleration, h and a characterize the vertical and horizontal positions of the center of gravity of the trailer and h_0 is the height of the kingpin. Then, the linear stability is governed by a 3 DoF subsystem with generalized coordinates ψ , φ , and u . On the contrary, based on the semi-analytical and numerical bifurcation analyses of the full spatial model, the pitch motion affects the sense of the Hopf bifurcation at the linear stability boundary. This is caused by mixed second-degree terms related to the tire forces appearing in the governing equations expanded up to third-degree terms.

Related publications: [1–6].

Thesis statement 2

For a realistic trailer setup, an unsafe towing speed range with practical relevance is situated at low positions of the center of gravity. This significantly reduces the speed range, where the rectilinear motion is globally stable. The commonly used simpler trailer models, in which the pitch motion and/or the roll motion are neglected, are not conservative with respect to safety, and they are of limited use.

Based on the semi-analytical and numerical bifurcation analyses, the full spatial trailer model is also capable of exploring the critical domains of the rocking motion, associated with the loss of contact of tires. This is confirmed by laboratory experiments with a small-scale trailer towed on a conveyor belt. The use of the full spatial model is recommended to explore all the possible, safety-relevant nonlinear vibrations of two-wheeled trailers.

Related publications: [1–7].

Thesis statement 3

For a realistic trailer setup with a low position of the center of gravity, a linear continuous-time state feedback controller is capable of eliminating the unsafe zone by means of differential braking of the wheels. This statement holds even in the case when a moderate time delay ($\tau < 0.1$ s) and a yaw rate dead zone ($\Omega_{dz} = 0.1$ rad/s) are present. However, the same controller setup does not perform well for a high position of center of gravity. Time delays $\tau > 0.1$ s lead to unsafe (bistable) domains for such payload positions, which are originally safe without the controller.

Related publications: [1–6].

Stability of riderless electric scooters

Consider a spatial mechanical model of an electric scooter comprising four rigid bodies: the front and the rear wheels, the body of the e-scooter, and the handlebar. The generalized coordinates of the model consist of the coordinates X and Y of the center point of the rear wheel, the yaw angle ψ and the lean angle φ of the body, the steering angle δ of the handlebar, the rotational angles ϕ_f and ϕ_r of the front and the rear wheels around their axes of rotation. Consider that the wheels of the e-scooter are rolling on the horizontal rigid ground. The pitch angle ϑ is governed by a geometric constraint that gives a relationship within the lean, the steering, and the pitch angles.

The balancing of the e-scooter at zero longitudinal speed is achieved with hierarchical linear continuous-time state feedback controllers with multiple delays, by applying steering torque to the handlebar (control with steering) or by turning the front wheel to the $\pi/2$ steering angle position and applying driving torque on the front wheel (control with driving).

Thesis statement 4

Approximations exist that eliminate the complex geometric constraint within the lean φ , the steering δ , and the pitch ϑ angles, and simplify the required algebraic manipulations during the derivation of the exact linearized equations of motion.

Consider the case when the e-scooter is controlled with steering, and the rotation of its front wheel is locked. For small vibrations around the vertical position, the appropriate approximation of the body's pitch angle is

$$\vartheta \approx \frac{e}{4p} \delta^2 \sin 2\varepsilon - \frac{e}{p} \varphi \delta \cos \varepsilon, \quad (2)$$

where p is the wheelbase, e is the trail, and ε is the rake angle of the e-scooter.

In the case when the e-scooter is controlled with driving, while its handlebar is fixed in the $\delta = \pi/2$ steering angle position, and the fork

offset is zero, the suitable approximation of the body's pitch angle is

$$\vartheta \approx \vartheta_0 + \vartheta_2 \varphi^2, \quad (3)$$

where

$$\vartheta_0 = \arccos \left(\frac{R^2 \cos \varepsilon + (R \sin \varepsilon + p) \sqrt{2Rp \sin \varepsilon + p^2}}{R^2 + 2Rp \sin \varepsilon + p^2} \right), \quad (4)$$

$$\vartheta_2 = -\frac{1}{2} \frac{R + R \sin(\varepsilon - \vartheta_0) \tan(\varepsilon - \vartheta_0) - p \sin \vartheta_0}{R \sin(\varepsilon - \vartheta_0) + p \cos \vartheta_0}, \quad (5)$$

and R refers to the radii of the front and rear wheels.

Related publications: [8–13].

Thesis statement 5

The performance of the utilized controller is improved by changing the position of the handlebar's center of gravity in two different aspects:

- (a) The stable domain of the higher-level control gains is increased, and the controller is more robust against parameter mismatch if
- the handlebar's center of gravity is shifted to the front in the control with steering case,
 - the handlebar's center of gravity is lifted up in the control with driving case.
- (b) From a robustness against perturbation point of view, the handlebar's center of gravity has an optimal position, for which the rightmost characteristic exponent has the smallest real part. This optimal position is in a practically applicable range in the case of commercialized e-scooters.

Related publications: [8–13].

Thesis statement 6

Consider the specific parameters of the commercialized e-scooter and the optimal lower-level control gains investigated in this thesis. If multiple time delays are considered in the hierarchical balancing control, stabilizability plots constructed in the plane of the higher-level τ and the lower-level σ delays show that the most stable delay setup does not correspond to the delay-free case. When the lower-level delay is increased, the higher-level delay has to be proportionally increased as well in order to maintain the robustness of the system against perturbations. This statement is true for the control with steering approach when the delays are below 50 ms, i.e., $\max\{\sigma, \tau\} < 50$ ms. For the control with driving approach, the statement holds for smaller delays only, i.e., $\max\{\sigma, \tau\} < 10$ ms.

Related publications: [8–13].

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