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THESIS BOOKLET

Quantum information protocols for spin qubits: from double quantum dots to large-scale implementations

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Background

Quantum computers operating with quantum bits (or qubits) have the potential to solve problems that are intractable for classical computers, but the achievement of this great potential is hindered by errors in the physical realization. A reasonable solution is offered by topological protection which is the intriguing feature of nontrivial condensed matter. A qubit encoded in the degenerate ground state of such a system is shown to be protected against local disorder and noise [1, 2]. The simplest example is the Kitaev chain [3] which describes a one-dimensional topological superconductor. Quantum information could be stored in Majorana zero modes localized at the end of the chain robustly and manipulation is also possible with spatial exchange due to the non-Abelian statistics of these mid-gap excitations [4]. Unfortunately, the fabrication of such topological systems is an extremely challenging task.

Quantum error correction offers an alternative solution by the redundant encoding of a small number of logical qubits in a greater number of physical qubits. One of the most prominent quantum error correction codes is the surface code [5], which is based on a two-dimensional qubit grid and requires only nearest-neighbor connectivity. The surface code encodes a single logical qubit and it shows threshold behavior, which means that if the strength of physical errors is below a threshold, then the code performs increasingly better as the code size (the number of physical qubits used for the logical encoding) is increased. The error threshold of the surface code is around 1% for realistic (circuit-level) noise, which is within reach for state-of-the-art solid-state quantum hardware, e.g. superconducting qubits [6].

In recent years, semiconductor spin qubits have emerged as an other promising platform for future quantum computers. A single electron or a single hole confined in a semiconductor quantum dot has a small characteristic size (10-100 nm) [7] compared to superconducting qubits (100 microns) [8]. Moreover, spin qubits also have high-fidelity universal quantum control (single-qubit and two-qubit gate fidelities above 99.5%) [9], the capability of operation above one kelvin [10], and their fabrication exploits today's highly advanced technologies of the semiconductor industry [11]. First error correction experiments with semiconductor spin qubits realized the phase-flip repetition code [12, 13] with a single round of measurement and without feedforward; however, full-fledged quantum error correction (e.g. surface code) has yet to be realized.

Several architectural proposals have been made for scaling up semiconductor spin qubit platforms. One difficulty of controlling, e.g., a few thousand qubits on a single chip is a large number of control lines: this number scales linearly with the qubit count. A crossbar control architecture was proposed to overcome this issue, where the number of control lines scales as the square root of the qubit count [14], making it a good candidate for realizing large-scale quantum computers. In this architecture, the qubits are placed on a 2D grid of quantum dots, and they can also move from one site to the neighboring one. This could effectively increase the qubit connectivity, which is a huge advantage, however it imposes an extra challenge on the physical implementation of a quantum code, since one has to find appropriate routing as well between the encountered qubit configurations.

Besides experimental progress, another important line of research is the simulation of quantum error correction codes to forecast their future performance. The effect of Pauli errors during quantum error correction is well understood due to the efficient simulability [15] and the existence of mappings to classical disordered spin models [16]. However, the exploration of more realistic error models is an ever-present challenge. Recently, efficient large-scale numerical simulations of coherent errors became feasible, due to a newly developed technique [17]. This technique was originally used to investigate the effect of uniform coherent errors in the surface code, then follow-up works extended this to arbitrary planar-graph surface codes [18] and the presence of phenomenological readout errors [19]. These studies have numerically established the threshold behavior of the surface code in the presence of coherent errors, assuming uniform unwanted rotations on the data qubits.

Objectives and summary of research

My PhD research aimed to promote the development of a well-functioning quantum computer, focusing mainly on semiconductor spin qubit-based platforms. I carried out three research projects accordingly, all of which established theoretical background for the implementation of quantum information protocols.

Recent proposals for realizing the Kitaev chain are based on quantum dot-superconductor hybrid structures and appear to be attainable by experiments [20, 21]. As a first step towards this implementation, I investigated a simple system, a conventional s-wave Josephson junction with a double quantum dot as a weak link. Measurement data by my experimental collaborators showed that the sign of the critical current follows an even-odd pattern and the magnitude is significantly decreased in the triplet spin configuration of the double quantum dot. We set out to investigate these two phenomena by means of numerical simulation and perturbation theory using a zero-bandwidth model for the superconducting leads.

As a second major objective of my PhD work, in the context of simulating quantum error correction, motivated by the $1/f$ noise of condensed matter systems, I introduced a novel coherent noise model of phase damping where random angle coherent qubit rotations arise from Larmor frequency fluctuations. This model is relevant not only for semiconductor spin qubits but for any solid-state qubit platform, where the leading information loss mechanism is dephasing. Qubit dephasing is usually modelled by phase flip errors, therefore it is natural to compare the new model with the well-known model of independent single-qubit Pauli phase-flip errors. Assessing the performance of quantum error correction codes in the presence of this random coherent noise is also crucial.

Finally, I made efforts to bring quantum error correction with spin qubits one step closer to physical realization. To run any quantum code (or algorithm) on a digital quantum computer, the corresponding circuits have to be compiled to the given gate set of that processor. As the surface code is compatible with the layout of the crossbar architecture, I aimed to compile the surface code to this architecture. Namely, to decompose the circuits needed for error detection in terms of the native gates of the spin qubit architecture, engineer a routing and scheduling protocol for the operations, and also a pulse sequence, which realizes surface-code quantum error correction in a crossbar spin qubit array.

Novel scientific findings

My PhD research is detailed in the thesis, and the corresponding novel findings are summarized in the following thesis statements:

1. I investigated a conventional s-wave Josephson junction with a double quantum dot as a weak link. Based on numerical simulations and analytical results in the perturbative regime I predicted an even-odd pattern in the (equilibrium) critical current. The cause of the switching between the even and odd patterns is identified as the change of the ground-state fermion parity. The sign of the supercurrent is uniquely determined by the ground state, up to leading order in perturbation theory. In the (1,1) charge sector of the serially coupled double quantum dot, the magnetically induced singlet-triplet ground-state transition has a significant effect on the supercurrent: the Josephson current carried by the triplet ground state at high magnetic field is much suppressed compared to the current carried by the singlet ground state at low magnetic field. I proved, in the framework of perturbation theory, the existence of strong triplet blockade in two different limiting cases: the large-superconducting-gap limit ($\Delta \gg U$), and the strong-Coulomb-repulsion limit ($U \gg \Delta$). Also in the perturbative framework, I have computed the number of processes contributing to the Josephson current for triplet and singlet states. This process counting indicates partial triplet blockade in the intermediate regime ($\Delta \approx U$). The even-odd effect and triplet blockade were confirmed by experiment. I took part in the interpretation of the experiment and the estimation of model parameters fitting the measurement data. I also proposed ways to utilize triplet blockade, for the readout of spin qubits and coupling to superconducting qubits.

   Related publications: I. and II.

2. In the context of multi-cycle quantum error correction, I introduced the error model of quasistatic phase damping, which is a simplified model describing Larmor-frequency fluctuations due to $1/f$ noise. These Larmor-frequency fluctuations amount to unwanted coherent rotations whose axis is uniform, but the rotation angle is random across the qubit register. I proved that quasistatic phase damping error is equivalent to independent single-qubit Pauli phase-flip errors in the case of a single cycle of error detection or error correction. However, I found that for multiple cycles, quasistatic phase damping is distinct from independent phase-flip errors. I numerically investigated the performance of the surface code as quantum memory in the presence of quasistatic random coherent errors as well as (phenomenological) readout errors. I performed large-scale numerical experiments and established the surface code error correction threshold using a minimum-weight perfect matching decoder, as $p_{\mathrm{th}} = 2.85\%$. This is close to the threshold with independent phase-flip errors and readout errors. At the threshold, the logical error rate for quasistatic phase damping combined with readout errors is around 7%, but for the combination of phase-flip and readout errors it is slightly higher, around 8.5%. I also determined the break-even boundary line for the distance-3 surface code on the parameter plane of the physical error rate and the readout error rate.

   Related publication: III.

3. I decomposed the surface code stabilizer measurement circuits in terms of single-qubit and two-qubit gates that are native to the crossbar spin-qubit architecture. I described a routing and scheduling protocol for the implementation of such a stabilizer measurement cycle. I identified a pulse sequence that realizes this protocol in the crossbar spin qubit array. As an important ingredient, I developed and implemented a verification algorithm that confirms that the gate-voltage changes in the abstract pulse sequence induce the desired real-space routing of the electrons in the quantum dot array. I transformed the abstract pulse sequence to a physical pulse sequence and systematically and quantitatively evaluated the logical error rates, taking into account idle qubit errors as well. I showed a concrete, realistic parameter set that enables below-threshold surface code operation.

   Related publication: IV.

Publications related to the thesis statements

I. D. Pataki, and A. Pályi, Even-odd effect and triplet blockade in a double quantum dot Josephson junction. Proceedings of the PhD workshop of Physics Doctoral School at the Faculty of Natural Sciences, Budapest University of Technology and Economics (2020).

II. D. Bouman, R. J. J. van Gulik, G. Steffensen, D. Pataki, P. Boross, P. Krogstrup, J. Nygård, J. Paaske, A. Pályi, and A. Geresdi, Triplet-blockaded Josephson supercurrent in double quantum dots. Physical Review B **102**(22), 220505(R) (2020).

III. D. Pataki, Á. Márton, J. K. Asbóth, and A. Pályi, Coherent errors in stabilizer codes caused by quasistatic phase damping. Physical Review A **110**(1), 012417 (2024).

IV. D. Pataki, and A. Pályi, Compiling the surface code to crossbar spin qubit architectures. Physical Review B **111**(11), 115307 (2025).

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