ANALYSIS OF QUASI-ISOMETRIC POLYGONAL BUCKLING SHAPES OF SPHERICAL SHELLS

A brief summary of the dissertation submitted to the Budapest University of Technology and Economics in partial fulfilment of the requirements for the degree of Doctor of Philosophy

Dániel Vető

Supervisor:
István Sajtos

Budapest University of Technology and Economics
Department of Mechanics, Materials and Structures

2016

Introduction

Large deflection analysis of thin elastic shells plays an essential role in mechanics and other fields of science, e.g. research on colloid particles (*Tsapis et al, 2005*), nano-sized spherical structures (*Falco et al, 2011*), or living spherical cells (*Hutchinson et al, 2016*). Generally, it is not easy to determine the connection between the load and the deflection of shells. Even now there exists a noticeable difference between theoretical and experimental results, which cannot be explained only by the imperfections, material nonlinearities and other effects that were not taken into account. Most differences can originate from the incompleteness of theoretical models.

Figure 1.: Buckling shapes of a ping-pong ball: circular shape at smaller deflection, polygonal shape at larger deflection (photo by the author)

Researchers used to consider axisymmetric buckling shapes for shells (*Evkin, 2005*, *Kollár-Dulácska, 1984*, *Pogorelov, 1988*, *Thang, 1989*, *Wolmir, 1962*, *Zhu et al, 2002*), although buckling shapes are usually non-axisymmetric (polygonal), as it can be seen in some recent works (*Antman, 2005*, *Audoly-Pomeau, 2010*, *Galpin et al, 2008*, *Grolleau et al, 2008*, *Knoche-Kierfeld, 2014*, *Pauchard-Rica, 1998*, *Vaziri-Mahadevan, 2008*, *Vaziri, 2009*), or in everyday life, e.g. buckling of ping-pong balls,

Figure 1. It is stated by the researchers that the problem of buckling of spherical shells is not solved yet; therefore, the topic needs further investigation (*Audoly-Pomeau, 2010*, *Vaziri, 2009*). There are several results available in references for non-axisymmetric (polygonal) buckling of point-loaded spherical shells. Although the same situation is being examined, the statements are sometimes different. The problem of spherical shell buckling is still current, with many substantial questions answered differently or not yet answered.

The goal of my research is to determine the buckling shape of point-loaded spherical shells. Additionally, the load-deflection function is also to be determined. To achieve these results, an analytical model was developed, which shows us the possible buckling shapes for spherical shells, and also the load-deflection diagrams connected to them. In the research, only regular polygons were taken into account among the possible non-axisymmetric shapes. These shapes have discrete symmetry of revolution. An approximate model – based on engineering intuition – was also developed, which considers only the point where the axisymmetric buckling shape transforms into a shape that has discrete symmetry of revolution. This shows which type of polygon is chosen by the shell at this point. Experimental and numerical (FE) results verify my models.

Analysis of axisymmetric buckling shapes

Many relevant and interesting statements are available about inextensional (isometric) deformations of surfaces in references (*Audoly, 2000*, *Croll, 1975*, *Hegedűs, 1998*, *Ivanova-Pastrone, 2002*, *Pogorelov, 1988*). The buckling shape of spherical shells can be approximated by isometric transformations of the original surface. Surfaces in reality have nonzero thickness. In the case of thin surface structures, typically locally inextensional (locally isometric) or quasi-inextensional (quasi-isometric) deformations occur. Based on simple considerations, it can be stated that there is no possibility for continuous inextensional deformations between buckling shapes with axisymmetry and discrete symmetry of revolution (*Vető-Sajtos, 2016b*).

Axisymmetric buckling shapes are considered in many publications on the buckling of spherical shells. The buckled part of the surface can be assumed to be an inverted spherical surface (*Pogorelov, 1988*, *Knoche, 2014*). Based on this assumption, the load-deflection function can be determined. In the case of concentrated force, the results of Pogorelov are valid (*Pogorelov, 1988*), while in the case of parallelly distributed load, new results are obtained (*Vető-Sajtos, 2009*). According to my solution, the load-deflection diagram has a minimum point, which belongs to the lower critical load of the shell. This critical load value shows good agreement with the critical load values found in references (*Kollár-Dulácska, 1984*, *Dulácska, 1987*). The results are contained in Principal results 1 and 2.

Modelling the buckling edge as a compressed planar ring

Axisymmetric buckling edge of spherical shells usually transforms into a polygonal edge with discrete symmetry of revolution if the loads are increased. There is a plausible analogy – based on engineering intuition – between the circular buckling edge of the spherical shell and a planar elastic bedded ring subjected to inward-pointing distributed load. Thus the buckling of a planar elastic ring subjected to inward-pointing distributed load can be analysed. Loading of the shell causes compressive force in the circumferential direction of the buckling edge, Figure 2. The ring is assumed to be bedded in radial direction by the neighbouring shell regions. The ring also has bending stiffness due to the nonzero width of the buckling edge of the shell.

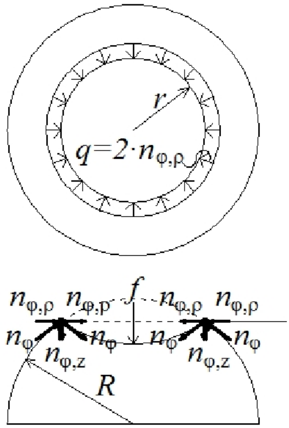

Figure 2.: Resultant forces acting to the circular buckling edge of a spherical shell in the case of concentrated force, applying membrane theory (except for the small neighbourhood of the loading point)

This model determines the possible polygonal buckling shapes that can evolve from the circle during buckling of a spherical shell with a certain radius-thickness ratio. The model considers only the transition from the circle to a regular polygon, not the possible transition from a polygon to another polygon. The numbers of sides of buckling polygons predicted by the model are in agreement with experiments (*Vető-Sajtos, 2014*, *Vető-Sajtos, 2016a*). The results are contained in Principal result 3.

Analysis of buckling shapes with discrete symmetry of revolution

An analytical model was developed (*Vető-Sajtos, 2014*, *Vető-Sajtos, 2016a*, *Vető-Sajtos, 2016b*) in order to examine the buckling of spherical shells, assuming quasi-isometric transformed shapes as buckling shapes, based on references (*Audoly-Pomeau, 2010*, *Ben Amar-Pomeau, 1997*, *Lobkovsky, 1996*, *Pauchard-Rica, 1998*, *Pogorelov, 1957*, *Pogorelov, 1988*, *Zhu et al, 2002*). The qualitative model, which was implemented in MATLAB, is able to handle not only axisymmetric buckling shapes, but buckling shapes with discrete symmetry of revolution, as well. Continuous transitions between these different shapes can be taken into consideration in the model, by means of peakedness and roundedness parameters b_1 and b_2, Figure 3. (*Vető-Sajtos, 2014*, *Vető-Sajtos, 2016a*, *Vető-Sajtos, 2016b*). This is unique among the available models. Sudden transitions are considered between circular and polygonal buckling shapes by other researchers.

The load-deflection functions corresponding to buckling shapes with axisymmetry and discrete symmetry of revolution differ from each other. The difference among load-deflection diagrams of these different functions increase as the deflection of the shells increase. This demonstrates that consideration of buckling shapes with discrete symmetry of revolution is necessary. The proposed model is able to give a more precise solution for the load-deflection function of spherical shells than the models of other researchers (*Vető-Sajtos, 2014*, *Vető-Sajtos, 2016a*, *Vető-Sajtos, 2016b*).

An additional result of the proposed model is that spherical shells with different radius and thickness show different polygonal buckling shapes. This simple model reveals that spherical shells with smaller radius-thickness ratios show smaller numbers of sides of buckling polygons, while spherical shells with larger radius-thickness ratios show larger numbers of sides of buckling polygons, Figure 4. As a summary of this part of the research, it can be clearly seen that the numbers of the sides of the buckling polygons are determined by geometric parameters (*Vető-Sajtos, 2016a*). The results are contained in Principal results 4 (excluded result 4.3) and 5.

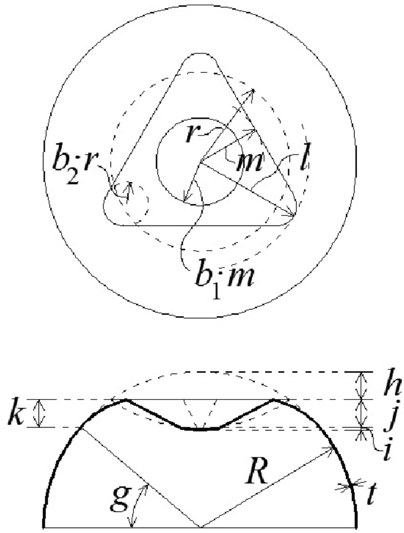

Figure 3.: Top view and section of buckling shape with discrete symmetry of revolution, as considered in the model (the variables belonging to the geometry of the buckled shell can be examined)

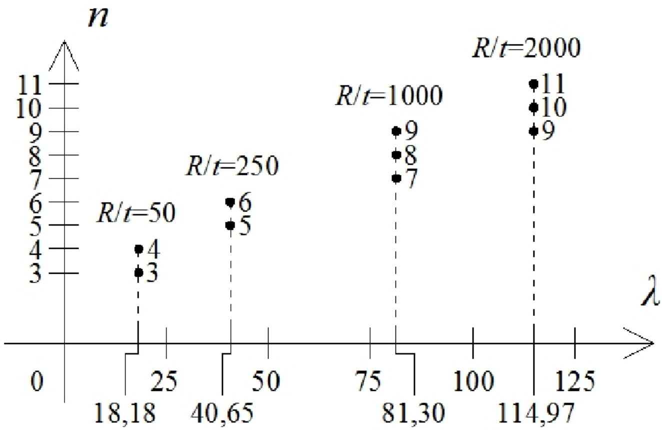

Figure 4.: Numbers of sides of buckling polygons, depending on the relative thickness parameter λ

Experiments and FE analyses

Generally, experimental verification of analytical results is very important in research. Ping-pong balls and so-called Lénárt spheres were tested in my experiments (*Vető-Sajtos, 2014*, *Vető-Sajtos, 2016a*), Figure 5. Perfect agreement was found between the results of the performed experiments and the analytical results in the case of ping-pong balls. On the other hand, in the case of Lénárt spheres, the agreement between experimental and analytical results was found to be imperfect. The reason is still unknown, but there are many effects (material inhomogeneities, effects of the support, loading speed, etc.) that can have influence on buckling polygons, which are not taken into account in the analytical models. In addition, the probable reason can be the non-regularity of buckling polygons, or the effect of the change of the width of the buckling edge, which were not considered in the models, but appeared in experiments.

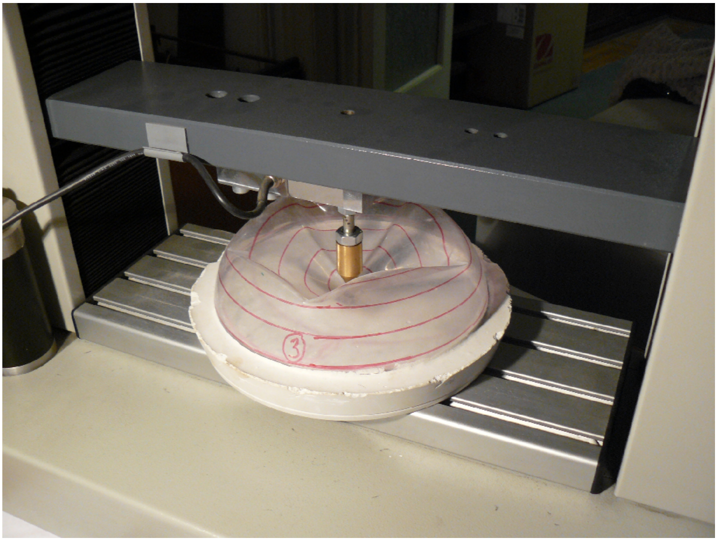

Figure 5.: Photo of the testing process of a Lénárt sphere (test specimen No. 3)

Buckling of point-loaded spherical shells was also examined by means of finite element analyses. The ANSYS results of shells having geometric and material properties of ping-pong balls and the results of the analytical model show very good agreement (*Vető-Sajtos, 2016a*). The results are contained in Principal result 4.3.

Principal results

Principal result 1

(relevant publication: (*Vető-Sajtos, 2009*))

For spherical shells subjected to parallelly distributed load (which is distributed uniformly along the horizontal projection of the surface), considering the buckling shape as an axisymmetric quasi-isometric transformed shape of the shell, I determined analytically the load-deflection function in the post-buckling state and the lower critical load. The value of the lower critical load is in good agreement with analytical results obtained by different methods available in literature.

Principal result 2
(relevant publication: (*Vető-Sajtos, 2016b*))

I proved that two spherical shell caps (which can realise inextensional deformations), derived from the same sphere by intersection with a plane, are not able to perform inextensional deformations if their edges are joined in a way that the caps are located at the same side of the plane of the edges, except for the case of two half-spheres. Consequently, the axisymmetric buckling shape of spherical shells cannot be transformed inextensionally into a buckling shape with discrete symmetry of revolution, if the buckled part is smaller than the half of the sphere.

Principal result 3
(relevant publications: (*Vető-Sajtos, 2014, Vető-Sajtos, 2016a*))

I developed a qualitative model based on the analogy between the circular buckling edge of the point-loaded spherical shell and a planar elastic bedded ring subjected to inward-pointing distributed load. The analogy is verified by the physical behaviour of spherical shells. I showed that in the case of a certain spherical shell the possible number of sides of the buckling polygons can be determined by the model, which considers only the transition from the buckling shape with axisymmetry to the buckling shape with discrete symmetry of revolution. The results are supported by experiments in literature.

Principal result 4

(relevant publications: (*Vető-Sajtos, 2014*, *Vető-Sajtos, 2016a, Vető-Sajtos, 2016b*))

Using quasi-isometric transformed shapes as buckling shapes, I developed an analytical energy function, which can be handled numerically to examine the post-buckling behaviour of spherical shells, considering shapes with axisymmetry and discrete symmetry of revolution as well.

4.1 The proposed model can be used to analyse the transition between buckling shapes with axisymmetry and discrete symmetry of revolution. The transition is achieved by the possibility of continuous change of roundedness and peakedness parameters corresponding to the geometry of the buckled surface.

4.2 It was shown by the proposed model that the load-deflection functions corresponding to buckling shapes with discrete symmetry of revolution bifurcate from the equilibrium path of the axisymmetric buckling shape. The buckling shapes with discrete symmetry of revolution correspond to lower energy levels compared to the axisymmetric buckling shape.

4.3 According to the proposed model, the behaviour of spherical shells shows good agreement with the results of the performed experiments and finite element analyses, concerning the number of sides of the buckling polygons. The load-deflection diagrams of the performed experiments are in correspondence with the analytical results of the proposed model if the value of deflection does not exceed one-third of the radius of the sphere.

Principal result 5

(relevant publication: (*Vető-Sajtos, 2016a*))

I showed that in the case of buckling of spherical shells the possible number of sides of the buckling polygons increases monotonically with the radius-thickness ratio, according to numerical analyses.

Publications connected to the principal results

Vető, D., Sajtos, I. (2009) Application of geometric method to determine the buckling load of spherical shells, *Pollack Periodica*, **4/2**, 123-134

Vető, D., Sajtos, I. (2014) Geometriai módszer alkalmazása gömbhéjak horpadásának vizsgálatához (Application of geometric method to examine the buckling of spherical shells, in Hungarian), *Építés-Építészettudomány*, **42/3-4**, 241-259

Vető, D., Sajtos, I. (2016a) Theoretical, numerical and experimental analysis of polygonal buckling shapes of spherical shells, *Journal of the IASS* – submitted for publication

Vető, D., Sajtos, I. (2016b) Gömbhéjak poligonális horpadási alakjának vizsgálata (Examination of polygonal buckling shapes of spherical shells, in Hungarian), *Műszaki Szemle*, **68** – accepted for publication

References

Antman, S. (2005) *Nonlinear problems of elasticity*, Springer, New York

Audoly, B. (2000) Élasticité et géométrie: de la rigidité des surfaces à la délamination en fil de telephone (in French), PhD thesis, École Normale Supérieure, Département de Physique, Paris, 2000

Audoly, B., Pomeau, Y. (2010) *Elasticity and geometry*, Oxford University Press, Oxford

Ben Amar, M., Pomeau, Y. (1997) Crumpled paper, *Proceedings of the Royal Society London A*, **453**, 729-755

Croll, J. G. A. (1975) Towards simple estimates of shell buckling loads, *Der Stahlbau*, **9**, 283-285

Dulácska, E. (1987) Kupolahéjak horpadása (Buckling of dome shells, in Hungarian), *Építés-Építészettudomány*, **19/3-4**, 305-309

Evkin, A. Yu. (2005) Large deflections of deep orthotropic shells under radial concentrated load: asymptotic solution, *International Journal of Solids and Structures*, **42**, 1173-1186

Falco, C., Baccile, N., Titirici, M-M. (2011) Morphological and structural differences between glucose, cellulose and lignocellulosic biomass derived hydrothermal carbons, *Green Chemistry*, **13**, 3273-3281

Galpin, B., Grolleau, V., Umiastowski, S., Rio, G., Mahéo, L. (2008) Design and application of an instrumented projectile for load measurements during impact, *International Journal of Crashworthiness*, **13/2**, 139-148

Grolleau, V., Galpin, B., Penin, A., Rio, G. (2008) Modeling the effect of forming history in impact simulations: evaluation of the effect of thickness change and strain hardening based on experiments, *International Journal of Crashworthiness*, **13/4**, 363-373

Hegedűs, I. (1998) *Héjszerkezetek* (Shell structures, in Hungarian), Műegyetemi Kiadó, Budapest

Hutchinson, C. A. et al. (2016) Design and synthesis of a minimal bacterial genome, *Science*, **357/6280**, 1414/1-11

Ivanova, J., Pastrone, F. (2002) *Geometric method for stability of non-linear elastic thin shells*, Kluwer Academic Publishers, Boston

Knoche, S. (2014) Instabilities and shape analyses of elastic shells, PhD thesis, TU Dortmund, Faculty of Physics, Dortmund

Knoche, S., Kierfeld, J. (2014) The secondary buckling transition: wrinkling of buckled spherical shells, *European Physical Journal E*, **37/7**, 1-21

Kollár, L., Dulácska, E. (1984) *Buckling of shells for engineers*, Akadémiai Kiadó, Budapest

Lobkovsky, A. E. (1996) Structure of crumpled thin elastic membranes, PhD thesis, University of Chicago, Department of Physics, Chicago

Pauchard, L., Rica, S. (1998) Contact and compression of elastic spherical shells: the physics of a 'ping-pong' ball, *Philosophical Magazine B*, **78/2**, 225-233

Pogorelov, A. V. (1957) *Die Verbiegung konvexer Flächen* (in German), Akademie-Verlag, Berlin

Pogorelov, A. V. (1988) *Bendings of Surfaces and Stability of Shells* (translation from Russian), American Mathematical Society, Providence

Thang, C. Q. (1989) A gömbhéj szimmetrikus horpadása (Symmetric buckling of spherical shells, in Hungarian), *Építés-Építészettudomány*, **21/1-2**, 95-108

Tsapis, N. et al. (2005) Onset of buckling in drying droplets of colloidal suspensions, *Physical Review Letters*, **94**, 018302/1-4

Vaziri, A., Mahadevan, L. (2008) Localized and extended deformations of elastic shells, *Proceedings of the National Academy of Sciences*, **105/23**, 7913-7918

Vaziri, A. (2009) Mechanics of highly deformed elastic shells, *Thin-Walled Structures*, **47**, 692-700

Wolmir, A. S. (1962) *Biegsame Platten und Schalen* (in German), VEB Verlag für Bauwesen, Berlin

Zhu, E., Mandal, P., Calladine, C. R. (2002) Buckling of thin cylindrical shells: An attempt to resolve a paradox, *International Journal of Mechanical Sciences*, **44**, 1583-1601