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BOOKLET OF PHD THESES

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# Dynamic analysis and tracking control of underactuated multibody systems

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## Preliminaries and Outline of the Thesis

This Ph.D. thesis deals with the modelling and control issues of underactuated dynamical systems. Dynamical systems with less independent control input than degrees of freedom are called underactuated.

The underactuated systems are typically multibody systems with complicated kinematic structure. The so-called differential algebraic equations are useful in the mathematical description of these kind of systems. In this case the coordinates are dependent, which makes difficult the stability investigation of these systems. In the thesis a novel approach is presented which makes possible the direct eigenvalue analysis of differential algebraic equations, and the stability of given equilibrium can be determined.

In trajectory tracking control of underactuated systems, the effect of the internal dynamics on the whole controlled system causes the most challenging problem. In several cases this is a destabilizing effect, which makes the task of the controller infeasible, thus the original task has to be modified necessarily. The methods found in the related literature are based on the intuitive modification of the original task. In this thesis an alternative approach is presented, which is based on stability analysis of the controlled system, called as the method of blended servo-constraints. In the stability analysis of the internal dynamics the above mentioned analysis technique of the differential algebraic equations is a useful technique.

In addition, for the control of underactuated systems a novel technique is presented, which is based on a periodic variation of the different control objectives. This method is called as the method of periodically varied servo-constraint. This method could effectively enhance the dynamical properties of underactuated systems, which have potentially unstable internal dynamics. This periodic controller can be used in other problems effectively as well, for example in case of the handling of driving torque saturation in trajectory tracking problems.

In the final chapter as practical multibody problem, the human running is investigated. In that chapter the energy efficiency of the running is in focus. It is shown that the techniques of multibody systems are useful in the field of biomechanics as well. The investigation shows how the energy consumption of human running is related with different parameters of the motion.

## New results 1.

When holonomic multibody systems are modeled with dependent coordinates their dynamics is described by a system of index-3 differential algebraic equations (DAE). The direct eigenvalue analysis of these systems results in spurious, non-physical eigenvalues due to the applied geometric constraints. Therefore, for the stability analysis the modeled system is often re-parametrized in terms of minimum set of generalized coordinates. This requires an additional step and the non-unique selection of the new generalized coordinates.

## Thesis 1.

The method proposed in this thesis makes the eigenvalue analysis possible without the re-parametrization of the dynamic equations as follows:

- Index reduction is carried out to obtain an index-1 DAE.
- The acceleration level constraint equations are modified by a sensitivity function.
- The first order form system is linearized around the investigated equilibrium.
- The eigenvalues are calculated.
- Applying a sensitivity analysis as a final step the non-physical eigenvalues are discarded.

In case of a sampled data system prior to the eigenvalue investigation, the solution for one sampling period is generated and the discrete state transition mapping is constructed. Then the spurious eigenvalues can be discarded with the sensitivity analysis. It is a practical, convenient selection, if the sensitivity parameters are equal to or smaller than the Baumgarte-parameters which fit to the investigated system.

Related journal publication:

L. Bencsik, L.L. Kovács, and A. Zelei, “Stabilization of internal dynamics of underactuated systems by periodic servo-constraints,” *International Journal of Structural Stability and Dynamics*, 2017, 14 pages, paper id: 1740004.

Other related publication:

[2]

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## New results 2.

In underactuated systems unstable internal dynamics may exist, which makes the classical computed torque trajectory tracking impossible. It is possible to overcome this problem with the following technique which called as the method of blended servo-constraints. The application of the method was demonstrated by simulation using the sliding pendulum benchmark example, and the method was successfully applied for the trajectory control of the same system in an experiment. Furthermore, the method was proved to be useful in the trajectory tracking control of the underactuated service robot Acroboter.

## Thesis 2.

The method of blended servo-constraints uses two different sets of servo-constraints: one of these,  $\hat{\gamma}$ , describes the desired trajectory, while the other one,  $\gamma_s$ , modifies the original task such that their linear combination  $\gamma = \kappa \hat{\gamma} + (1 - \kappa)\gamma_s$  becomes feasible, where  $\kappa$  is an adjustable parameter that can be used to find the balance between accuracy and stability. The stabilizing servo-constraint set  $\gamma_s$  must contain the desired values of the coordinates that corresponds to the original internal dynamics.

In case of the sliding pendulum the fastest decay of the oscillations induced by the internal dynamics can be achieved if  $\kappa = 0.6$ , while in case the Acroboter robot the fastest decay corresponds to the value  $\kappa = 0.8$ .

Related journal publication:

L.L. Kovács and L. Bencsik, “Stability case study of the acroboter underactuated service robot,” *Theoretical and Applied Mechanics Letters*, vol. 2, no. 4, 2012, (Article 043004).

Other related publications:

[4], [5], [6], [7], [8], [9].

## New results 3.

A novel periodically switched controller was proposed for the trajectory tracking control of underactuated mechanical systems with potentially unstable internal dynamics. This technique is called as the method of periodically varied servo-constraints.

The applicability of the method was demonstrated by simulations in case of the sliding pendulum and in case of the Acroboter platform.

## Thesis 3.

In the method of periodically varied servo-constraints, the periodic switching between the different control objectives of trajectory following and stabilization is realized by the blended servo-constraints  $\gamma = \kappa(t)\hat{\gamma} + (1 - \kappa(t))\gamma_s$ , where  $\hat{\gamma}$  and  $\gamma_s$  represent the original control task and the stabilizing servo-constraints. In addition,  $\kappa(t) = \kappa(t+T)$  is a piecewise constant periodic switching function, take only the values of zero or one. It means that within a period, the controller aims to accurately realize the desired motion for a given time and the stabilizing servo-constraints are applied in the rest of the period.

The root mean square value of the trajectory following error is decreased by 37% in case of the sliding pendulum example and it is decreased by 50% in case of the Acroboter platform, when  $\kappa(t) = \kappa(t + T)$  periodic switching function is applied instead of the most stabilizing constant  $\kappa$  values.

Related journal publication:

L. Bencsik, L.L. Kovács, and A. Zelei, “Stabilization of internal dynamics of underactuated systems by periodic servo-constraints,” *International Journal of Structural Stability and Dynamics*, 2017, 14 pages, paper id: 1740004.

Other related publications:

[2], [10].

## New results 4.

Driving torque saturation is an undesired event in trajectory tracking control of mechanical systems, which makes the controlled system temporarily underactuated. To reduce the undesired dynamic effects due to this underactuation, a period motion controller was proposed. For the algorithmic tuning of the controller a new manipulability type performance measure was introduced. A detailed comparison with other techniques was carried out in case of a planar robot application. The proposed controller is described in the next thesis.

### Thesis 4.

The proposed control concept for driving torque saturation in trajectory tracking control is detailed in the followings. Until saturation occurs the classical computed torque controller is applied. During saturation, the number of servo-constraints is appropriately reduced, different sets of servo-constraints are formed, and combined into a new periodic servo-constraint signal  $\gamma_{\text{sat}}(t)$ . The periodic servo-constraint signal is formed as

$$\gamma_{\text{sat}} = \sum_{i=1}^N \kappa_i \hat{\gamma}_i \quad \text{with} \quad \kappa_i(t) = \kappa_i(t + T),$$

where  $\kappa_i(t)$  controls the switching between the different sets of servo-constraints, and  $N < \binom{l}{r}$  is the number of the considered different sets,  $\hat{\gamma}_i$ , which are subsets of the original control objectives,  $\hat{\gamma}$ . The algorithmic selection of a suitable periodic control signal is based on the introduced performance measure, which rates the different servo-constraint sets,  $\hat{\gamma}_i$ , based on their effects and importance.

Compared to different classical methods taken from the literature the average tracking error is improved by at least 20%. The maximum trajectory following error is decreased by 7% compared to the best performing classical method investigated.

Related journal publications:

[1] L. Bencsik and L.L. Kovács, “Reduction of the effect of actuator saturation with periodic servo-constraints,” *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 2016, published online in October 20, 2016, 10 pages.

[2] A. Zelei, L. Bencsik, and G. Stépán, “Handling actuator saturation as underactuation: Case study with acroboter service robot,” *Journal of Computational and Nonlinear Dynamics*, vol. 12, no. 3, 2017.

Other related publication: [13]

## New results 5.

The impact intensity in different running modes is characterized by the constrained motion space kinetic energy (CMSKE), which is absorbed by the ground-foot contact. Using the models recently proposed in the literature it is shown that lower ground-foot impact is generated when fore-foot strike occurs than in case of rear-foot strike. The corresponding CMSKE values are independent of the sign of the shank angles. This is in contrast with the specialist literature, where it was shown that landing with a positive shank angle, i.e., overstriding, causes larger impacts. Here, an improved planar model was proposed for the analysis of different modes of running, which is a minimally complex representative that describes the phenomenon of overstriding, and the effect of rear- and fore-foot strikes at the same time.

## Thesis 5.

The improved planar model contains the leg, the shank, the thigh and an additional pointmass representing the mass of the unmodelled body parts. With the proposed extended model, the proper effect of the shank angle can be captured because the absorbed kinetic energy (CMSKE) is larger in case of the positive shank angle than in case of the negative shank angle. The results confirm the physical observations. Furthermore, the proposed model shows that lower ground-foot impact is generated when fore-foot strike occurs than in case of rear-foot strike, which corresponds to the results obtained by the other models of the recent literature.

Related journal publication:

L. Bencsik and A. Zelei, "Effects of human running cadence and experimental validation of the bouncing ball model," *Mechanical Systems and Signal Processing*, vol. 89, pp. 78–87, 2016.

Other related publications:

[15], [16], [14], [17].

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