*Gépészet 2008*
*Budapest, 29-30.May 2008.*

*G-2008-G-01*

# SYSTEM THEORY MODELL OF HEAT PUMPS

**Dr. László Garbai**
Professor, Department of Building Service Engineering and Process Engineering, Budapest University of Technology and Economics
H-1111. Budapest, Műegyetem rkp. 3. Tel: (1) 463 2405, e-mail: tanszek@epgep.bme.hu

**Szabolcs Méhes**
Ph.D. student, Department of Building Service Engineering and Process Engineering, Budapest University of Technology and Economics
H-1111. Budapest, Műegyetem rkp. 3. Tel: (1) 463 2405, e-mail: sz.mehes@gmail.com

***Abstract:*** *Heat pumps can be used for different purposes in building services such as the heating and cooling of buildings or even for ventilation or to produce domestic hot water. This paper proposes to set up a system theory model to facilitate the understanding of heat pumps and define the relations between the various components. These relations describe the connections between certain components and determine the parameters of the components using different decisions and input data. Only system theory modelling is capable of providing an exact definition of the various working points, and enables the making of decisions, required to ensure optimal management.*

***Keywords:*** *System theory model, Input variables, Output variables, Decision variable.,*

## 1. INTRODUCTION

Heat pumps are modern energy tools that help pump heat from the environment (by investing external energy) from a lower temperature to a higher temperature usable in terms of energy. Heat pumps in fact work on the principle of refrigerators, operating by the inverse Carnot circular process. Compressor heat pumps use electricity while absorption heat pumps are heat energy operated. Heat pumps are complex systems. Their energy-related and economic evaluation and the assessment of the economic efficiency of their application can only be carried out based on system theory and through the means of system theory. We must also rely on the methodology of operations research and decision theory. Our paper will give an overview of the basics of system theory and economic decision theory as well as their use in the description and optimization of the operation of heat pumps. The aim of the optimization is to minimize the function describing the operational costs of heat pumps and/or the costs of their energy consumption. In the present case, we can only produce the models due to the limited length of the paper. The effective optimization calculations will constitute a further phase of our work.

Authors specialized on system theory modelling and dynamic optimization of energy systems by means of the application of discrete dynamic programming are e.g. Sieniutycz [7], and Cheung et al. [5]. Authors, specialized on up to date mathematical- economic investigation of the application of heat pump systems and geothermic energy are Katsunori et al. [8], Krope et al. [9]. We have not met with the extension of mathematical system theory on heat pump systems in the specialized literature.

## 2. THE PRINCIPLE OF SYSTEM THEORY

A number of authors have been engaged in the formulation of mathematical and methodological principles of the system theory in the past decades. In this particular paper we rely first of all on Nemhauser's and Bellmann's scientific achievements as they provide the clearest summary of the mathematical system theory so far. The system is understood as connected components that form a uniform whole, act as a whole and are capable of independently performing a certain task. It has well-regulated and/or self-regulating stable working points. The system can be open or closed. Left alone, closed systems always achieve a state of equilibrium while open systems only reach a dynamic state of equilibrium provided certain conditions are fulfilled [3].

The system is made up of components connected and operated by mass and energy as well as signal and information flows. Components are characterized by their condition, function of state, input and output variables, decision variables, results of decision-making as well as the transformation relations and function relations between the above.

Component: the smallest part of the system performing an independent task. Only its inputs, outputs and the functions between these are taken into consideration.

X – set of the component's inputs,
Y – set of the component's outputs,
T – set of the component's output functions,

Function between outputs and inputs:

$$t_i\,(x_j) = y_k,$$
$$t_i \in T,\ x_j \in \mathrm{X} \text{ és } y_k \in \mathrm{Y}.$$

Structure: a momentary state of a given system and the definition which components belong to the investigated system and what kind of relationships exists between them.

Process: the series of changes of state occurring in the system is called a process.

System operation: the components of the system form different relationships during operation. The relationships among the components of a given system at a given moment are expressed by the momentary state of the system.

Components that do not belong to the system yet influence its operation are called system environment. The system operates to transform input signals into output signals. The correlation that describes how the given system transforms an incoming sign into an outgoing sign is called the mapping function of the system. The function that describes how the stage of the system changes as a result of a given input signal is called the temporary function of the system.

In the mathematical description of the system we performed an output-input analysis on the system components.

## 3. OVERVIEW OF THE TYPES OF THE SYSTEM

According to Nemhauser [2] the systems may be serial or loop. Serial systems include simple serial, diverging and converging systems while loop systems are either feed forward or backward. The following section gives a brief overview of these systems.

### 3.1. Serial system

Fig. 1 shows a serial system.

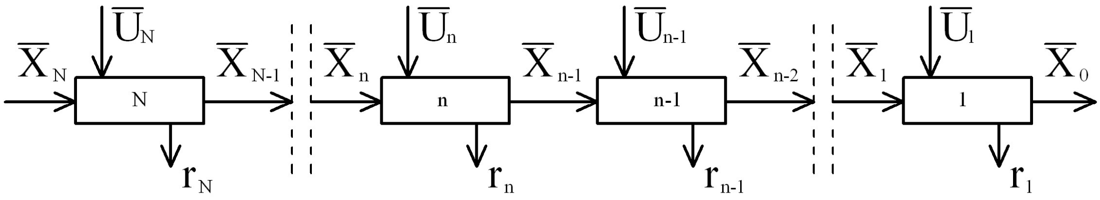


**Fig. 1.** Serial system [2]

As mentioned above the relations between input and output among the components are described by transformation relations. The example of the N and N+ 1 stage shows the mathematical relation of output and input in Fig. 2.

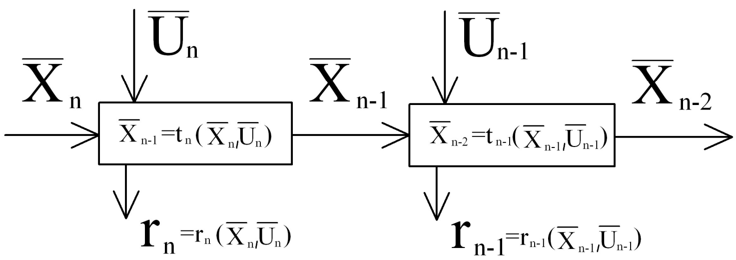


**Fig. 2.** Transformation relations at the decision stage of a serial system [2]

In the case of a serial system the relations between the stages provide a description of the relation between the output as well as the input and the decisions. In general, the mathematical relationship between the output, the input and decision is as follows:

$$X_{n-1} = t_n(X_n, U_n),\ n = 1, \ldots, N. \tag{1}$$

Results at stages are a function of input and decisions:

$$r_n = r_n(X_n, U_n),\ n = 1, \ldots, N, \tag{2}$$

where: $X_n$ – set of input functions,
$X_{n-1}$ – set of output functions,
$U_n$ – set of decision functions,
$r_n$ – results at stage,
$R_n$ – the entire result from the first stage to stage n, which constitutes the objective function of system operation as well.

$$R_n = r_n(X_n, U_n) \circ r_{n-1}(X_{n-1}, U_{n-1}) \circ \ldots \circ r_1(X_0, X_1, U_1), \tag{3}$$

For the general interpretation of operation „$\circ$" we found two useful ways, arithmetic addition and multiplication, if fulfilled: $r_n(X_n, U_n) \geq 0$ for all $X_n$ and $U_n$. Results at stages can be written as a function of the input stage of stage N and the decisions of the certain stations:

$$R_n = R_n(X_n, X_0, U_n, \ldots, U_1). \tag{4}$$

Using this we can determine the optimal operation of the system:

$$O_n(X_n) = \min_{U_n, \ldots, U_1} \{r_n(X_n, U_n, \ldots, U_1). \tag{5}$$

The optimization of the system is carried out using Bellmann's optimization principle for dynamic programming. The basic recursive scheme of dynamic programming is presented below.

The optimized objective functions of the partial systems are defined.

For the first stage:

$$O_1(X_1) = \min_{U_1} r_1(X_1, X_0, U_1), \tag{6}$$

For an interim stage:

$$\begin{aligned} O_n(X_n) &= \min_{U_n} \left[ r_n(X_n, U_n) \circ O_{n-1}(X_{n-1}) \right] \\ O_n(X_n) &= \min_{U_n} \left[ r_n(X_n, U_n) \circ O_{n-1}(t_n(X_n, U_n)) \right] \end{aligned} \quad n = 2, \dots, N, \tag{7}$$

For the last stage:

$$\begin{aligned} O_N(X_N) &= \min_{U_N} \left[ r_N(X_N, U_N) \circ O_{N-1}(X_{N-1}) \right] \\ O_N(X_N) &= \min_{U_N} \left[ r_N(X_N, U_N) \circ O_{N-1}(g_N(X_N, U_N)) \right] . \end{aligned} \tag{8}$$

provided $X_{N+1} = g_N(X_N, U_N)$. The optimal solution for the system of stage N can be determined stage by stage. The optimal operation of the system and the optimum of objective function (3) can be determined as follows: starting from stage 1 and going backwards backward recursive optimization is performed. The pre-determined output $X_0$ is recorded, input $X_1$ is considered a parameter and decision $U_1$ is defined for all possible $X_1$ values that provides the predetermined value of $X_0$ and the optimum of result $r_1$. This function is called function $O_1$. After this we go over to the stage 2. Input $X_1$ is eliminated by transformation relation $t_1$, objective function $O_2$ is set down for partial system including stages 1 and 2, which is the composition of $O_1$ and result $r_2$ of stage 2. This way $O_2$ is the function of decision $U_2$ and input $X_2$. $O_2$ can be optimized according to decision variable $U_2$ in the function of input $X_2$ as parameter. Recursively and step by step we reach the first stage of the system, stage N. The general interim optimization step is presented by equation (7).

### 3.2. Basic non-serial systems

Non-serial systems are usually used in industrial technological processes and are automatically controlled. As mentioned above there are four kinds of basic non-serial systems.

#### 3.2.1. Diverging Branch System

Diverging branch systems have two outputs or more by station, each of which is the input for a serial system (Fig. 3).

<u>Recursive equations:</u>

1. diverging branches from 11 to M1

$$\begin{aligned} O_{11}(X_{11}) &= \min_{U_{11}} \left\{ r_{11}(X_{11}, U_{11}) \right\} \\ O_{M1}(X_{M1}) &= \min_{U_{M1}} \left\{ \begin{matrix} r_{M1}(X_{M1}, U_{M1}) \circ \\ O_{M-1,1}\left[ t_{M1}(X_{M1}, U_{M1}) \right] \end{matrix} \right\} , \end{aligned} \tag{9}$$

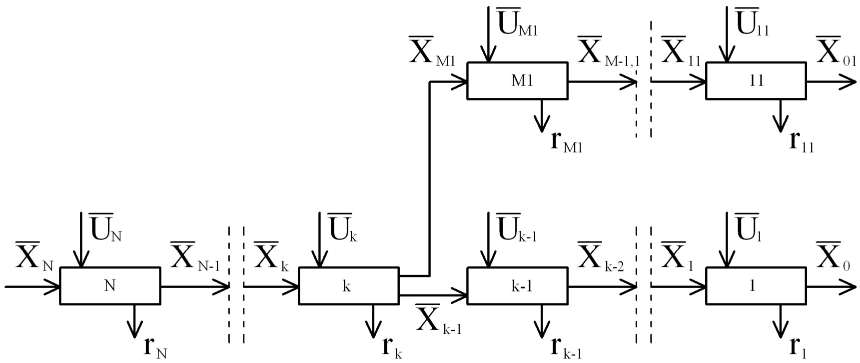


**Fig. 3.** Diverging branch system [2]

2. diverging branches from 1 to k-1

$$O_1(X_1) = \min_{U_1} \{r_1(X_1, U_1)\}$$

$$O_{k-1}(X_{k-1}) = \min_{U_{k-1}} \left\{ \begin{matrix} r_{k-1}(X_{k-1}, U_{k-1}) \circ \\ O_{k-2}\left[t_{k-1}(X_{k-1}, U_{k-1})\right] \end{matrix} \right\}, \qquad (10)$$

3. for diverging stage k

$$O_k(X_k) = \min_{U_k} \left\{ r_k(X_k, U_k) \circ O_{k-1}\left[t_k(X_k, U_k)\right] \circ O_{M1}\left[t_k^{/}(X_k, U_k)\right] \right\}, \qquad (11)$$

4. for stage N

$$\mathrm{O_N(X_N) = \min_{U_N} \{r_N(X_N, U_N) \circ O_{N-1}[t_N(X_N, U_N)]\}}. \qquad (12)$$

### 3.2.2. Converging Branch System

In converging branch systems two serial systems or more form the input of a serial system (Fig. 4).

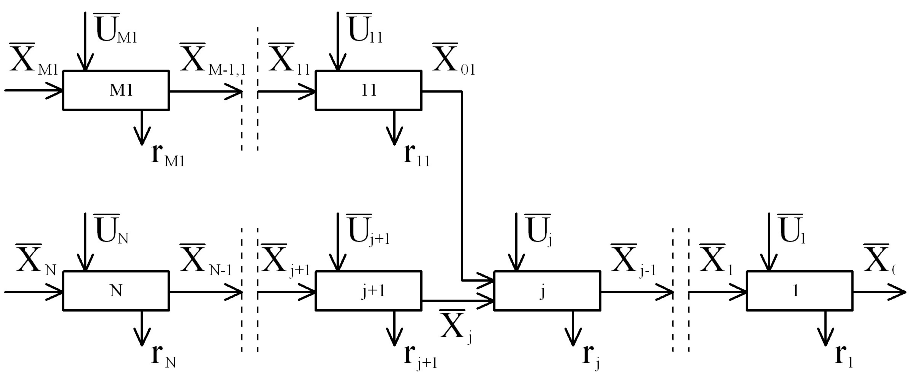


**Fig. 4.** Converging branch system [2]

In converging branch systems the recursive equations for the various system theory components are set down similarly to diverging branch systems.

For stage j:

$$O_j(X_j, X_{01}) = \min_{U_j} \left\{ r_j(X_j, X_{01}, U_j) \circ O_{j+1}\left[t_j(X_j, X_{01}, U_j)\right] \right\}. \qquad (13)$$

### 3.2.3. Feed Forward Loop System

In feed forward loop systems a braches diverges from a stage of the serial system then joins the original serial system again at a later branch (Fig. 5).

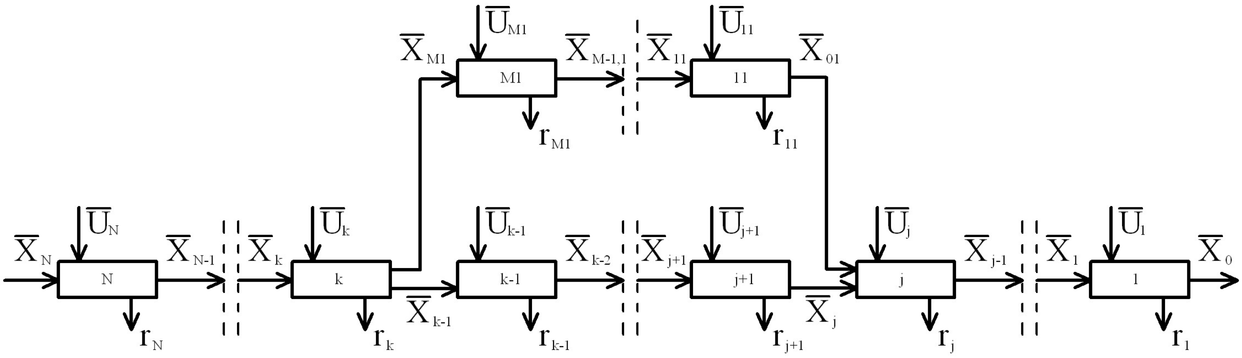


**Fig. 5.** Feed forward loop system [2]

In feed forward loop systems the recursive equations for the various system theory components are set down similarly to diverging and converging branch systems.

### 3.2.4. Feed Back Loop System

In feedback loop systems a braches diverges from a stage of the serial system then joins the original serial system again at a later branch (Fig. 6).

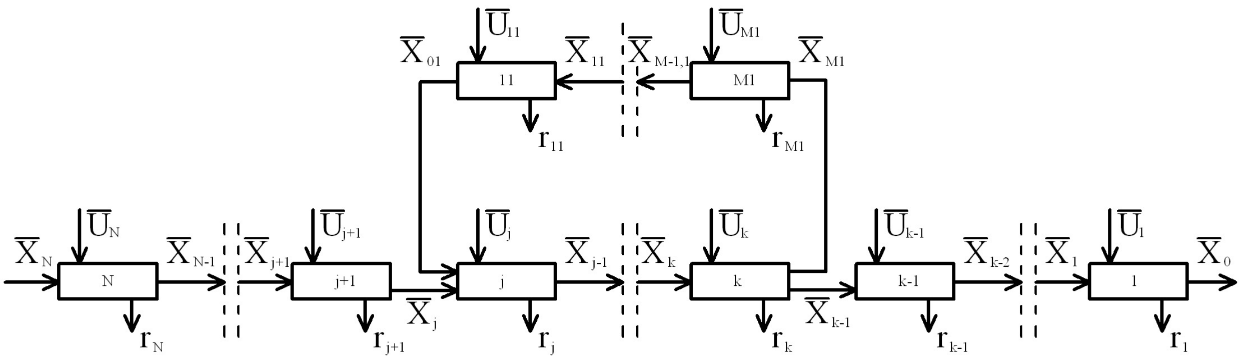


**Fig. 6.** Feed Back Loop System [2]

In feedback loop systems the recursive equations for the various system theory components are set down similarly to diverging and converging branch systems.

More complex non-serial systems can be set up if we put together basic non-serial system in different ways. A recursive scheme can be developed for each non-serial system.

### 3.2.5. Analysis of the system

The analysis of a concrete energetic-engineering system means the following:

- To break down the system into components.
- To construct the graph of the system components.
- To define the type of the system.
- To define the input and output variables as well as the decision variables.
- To define the decision result variables.
- Transformational relationships between inputs, outputs, decisions and result variables.

- To establish the objective function of the system.
- To define the extreme value of the objective function by means of dynamic programming.

**4. SYSTEM THEORY SCHEME OF THE HEAT PUMPS**

The heat pump is made up of the heat source characterized by low temperature level, the heat pump transforming the low temperature heat energy into a high temperature one, and the customer consuming heat. (Fig. 7)


**Fig. 7.** System theory scheme of the heat pump system

**4.1. Types of heat sources of heat pumps and their components in the system theory model**

Thereinafter we present the heat pump heating/cooling systems and their interpretation from a system theory point of view. In this particular paper the heat pump heating/cooling systems are broken down into three main parts. The primary side of the system i.e. its heat source will be presented firstly. After that, the machine itself and at last the customers' system will be presented. The theoretical grounds connected to all this have already been presented.

The input-, output- and decision variables are only indicated in the models without specifying the energy parameters involved.

**4.1.1. Using the heat of the Earth through earth collectors**

In this case the Earth is regarded as an infinite heat source. The maximum volume of heat obtained from the Earth depends of the volume of obtained and returned to the Earth.

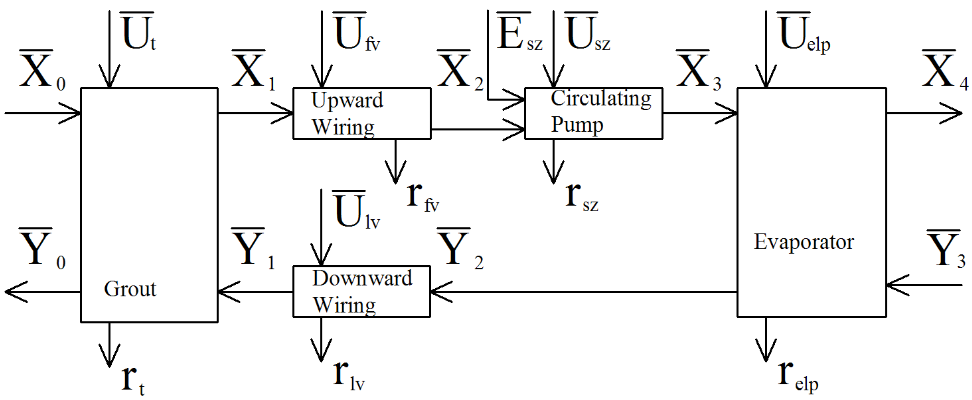


**Fig. 8.** Installing an earth collector

**4.1.2. Obtaining the heat of subsoil water through wells**

In this case the soil is regarded as an infinite source of heat (water). The required volume of water is obtained by using lifting and drain wells.

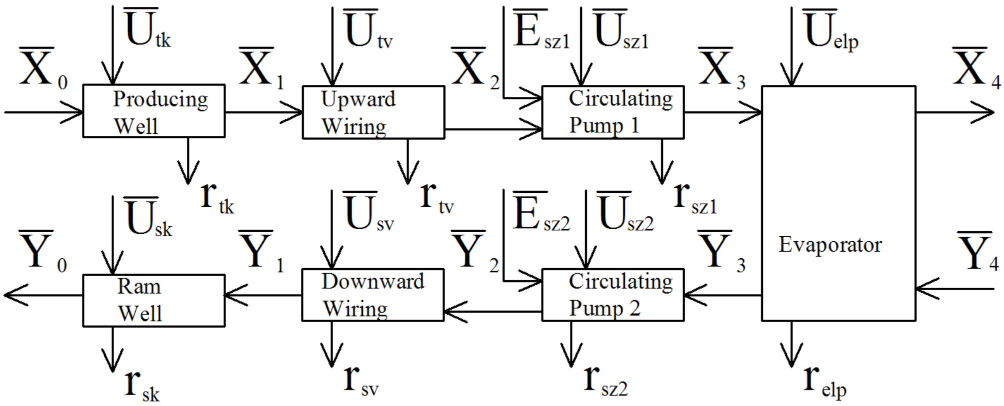


**Fig. 9.** In the case of geothermal well system

### 4.1.3. The air as a source of heat

The primary heat source of the heat pump is the air which is ventilated to the heat exchange of the heat pump and thus the required volume of heat is obtained.

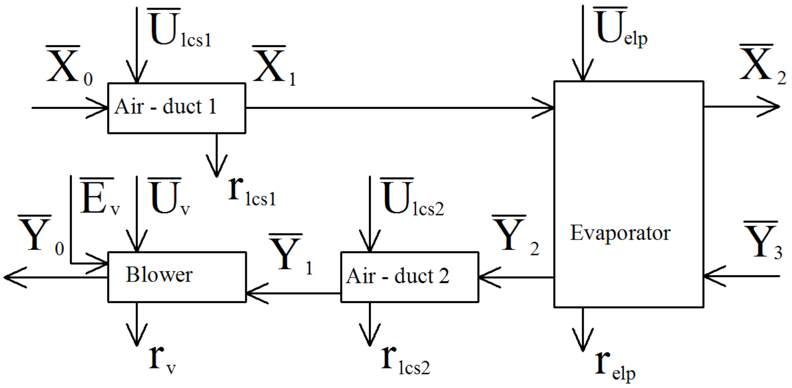


**Fig. 10.** In the case of air

### 4.1.4. The return pipe of district heating as the heat source of the heat pump

The primary heat source of the heat pump is the water flowing in the in the return pipe of district heating. The heat is obtained through a heat exchange.

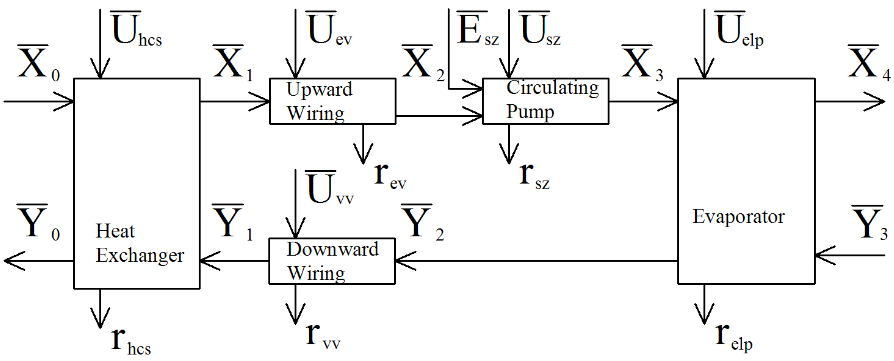


**Fig. 11.** In the case of obtaining the heat from the return pipe

### 4.2. Heat pumps of various types in the system theory model

As an equipment, the heat pump is made up of several components depending on one another. Their relationships are presented in the following system theory model.

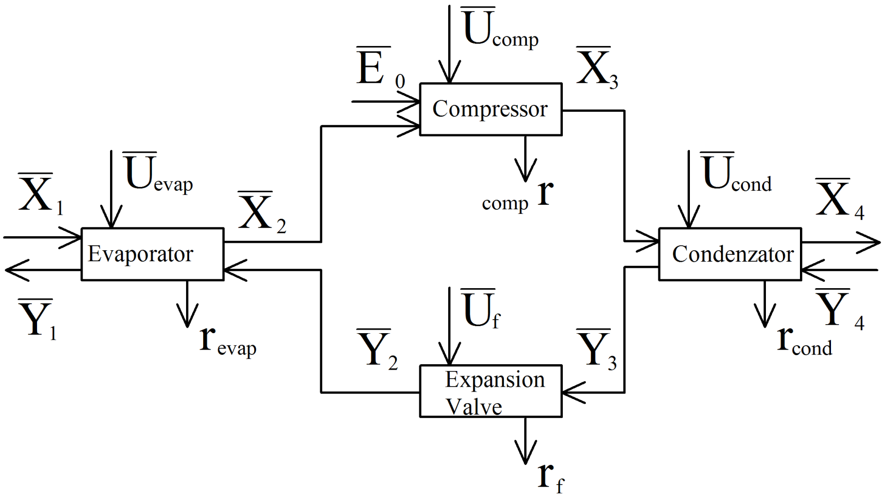


**Fig. 12.** System theory model of an electricity-powered heat pump

The above model only applies to heat pumps powered by electricity. The electricity input is described by $\bar{e}_0$ in the figure.

Fig. 14 shows the system theory model of the gas engine-powered heat pump where gas $\bar{g}_0$ is seen as input, powering the heat pump and performing mechanical work. This work drives the mechanical compressor of the heat pump.

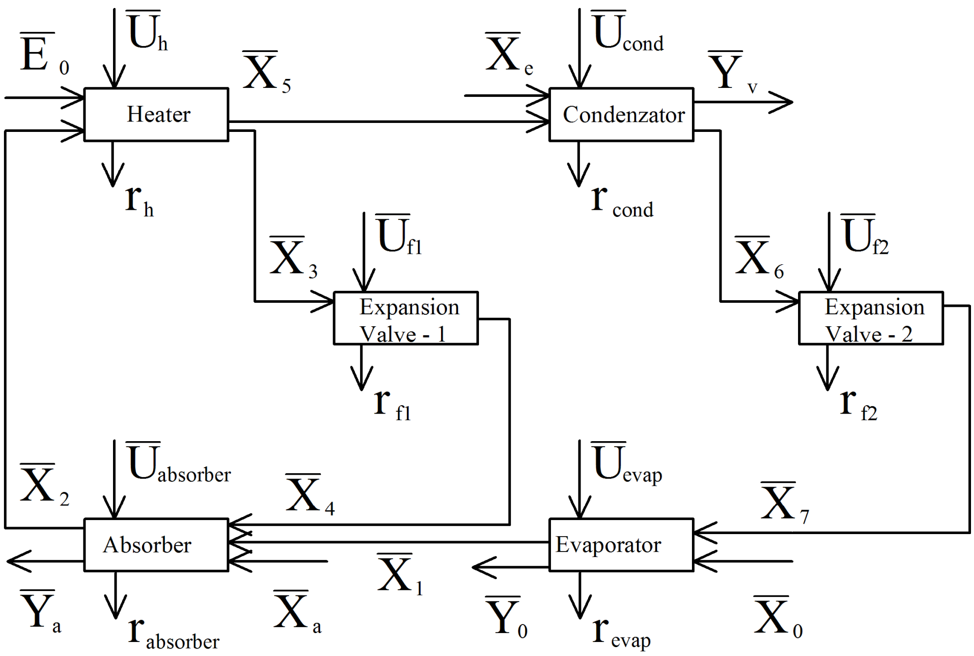


**Fig. 13.** System theory model of the absorption heat pump

The next type of heat pump is the absorption heat pump, which is highly similar to absorption refrigerators. Absorption heat pumps are preferred to traditional electricity-powered heat pumps, as they require less electricity. In the absorption heat pump's circular process, electricity is only needed for the circulating pump, used to transport rich solution. Figure 13 presents the system theory model of the absorption heat pump.

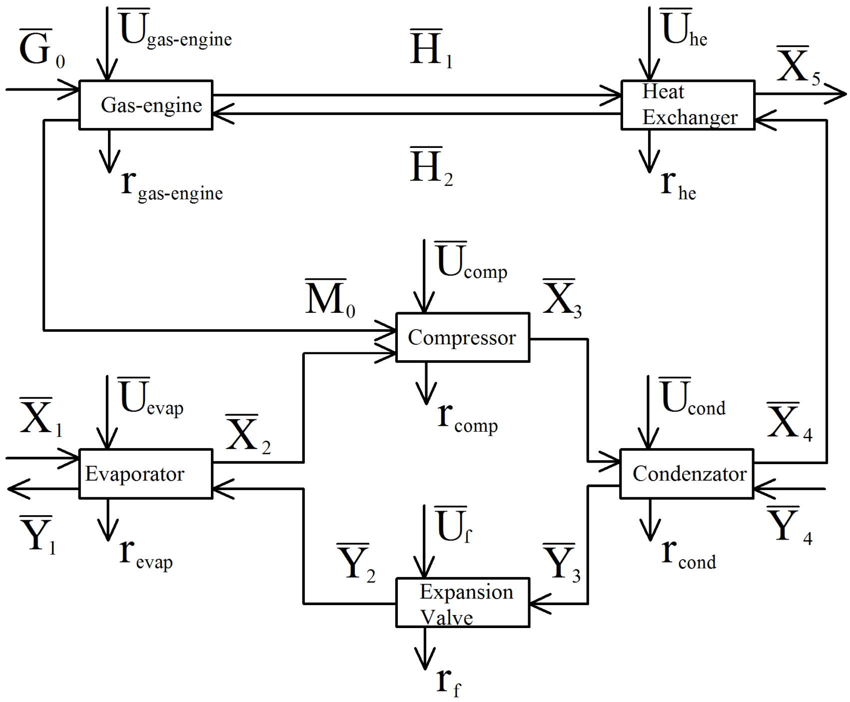


**Fig. 14.** System theory model of a gas engine fired heat pump

### 4.3. Components of the customers' (heating/cooling) system in the system theory model

Just as the heat pump and the primary circuits the heating system can also be described in terms of system theory.

The heating system connected to the heat pump is made up of a circulating pump, buffer tank and the radiator.

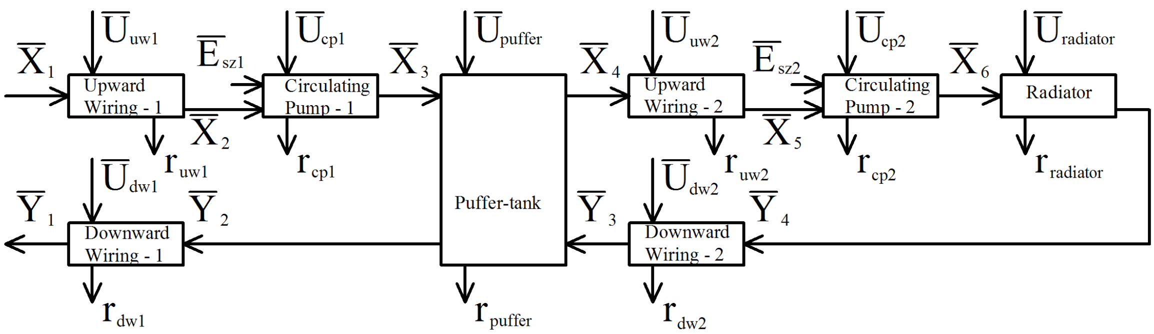


**Fig. 15.** System theory model of customer's heating/cooling system

## 5. OPTIMIZATION OF A HEAT PUMP SYSTEM

The evaluation and optimization of heating/cooling systems with heat pump are carried out using the above presented system theory model. When building the model the end stage and the final output such as the heat demand of the room or the building need to determine. It is determined how the different variables are correlated i.e. the transformation equations are established between the input and the output. Decision variables need to be defined too, concerning the investment, implementation, management and regulation of the various system components. Cost functions need to be specified for the investment and management of the different components. After this we use backward analysis, the optimization recursive function equation is established for the various components, usually starting from the heat demand. If the heating system with heat pump is managed by time decisions the process must be broken down to a definite number infinite time steps. The method of discrete dynamic programming is used to optimize the investigated time regulatory process.

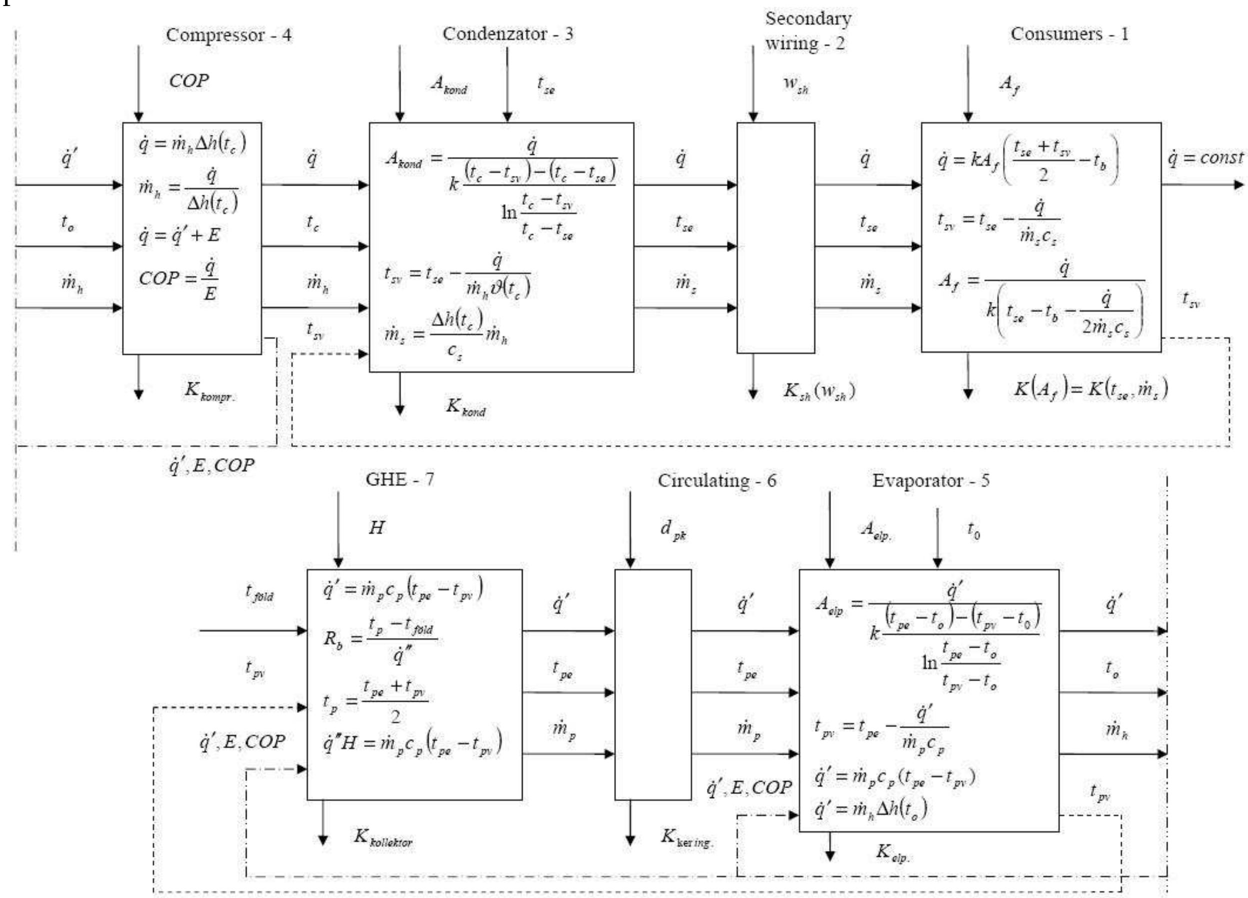


**Fig. 16.** System theory model of heating/cooling system

Figure 16 shows the decision model of heating/cooling system of electric driven geothermal heat pump. We can clearly see the input, output and decision variables. Between the inputs and outputs the transformation equations are written, together with the economic results of each stage. The further workout of the model is done numerically with the help of Excel tables, because the optimal functions "O" analytically can not be done, only numerically, in the form of matrixes. Further analysis of this model will be presented in our next papers.

## 6. CONCLUSION

The grounds of the dynamic programming [1], [2], [4], [6] and its application for heat pump heating/cooling systems have been presented in our paper. In our other papers we will study the possibilities of precise application of dynamic programming equations on the system theory model of a particular heat pump heating/cooling system. Two types of models can be established. We will fulfil the analysis of the system from the point of view of energetic in one of them, while we will minimize the operating costs of the system by means of the other.

As a result of our literary research we can claim that heat pump heating/cooling systems have not been approached from mathematical system theory aspect so far. The application of discrete dynamic programming, multistage decision systems and decomposition optimization techniques provide new investigation possibilities.

**SYMBOLS**

| | | |
|---|---|---|
| k | - | Heat transfer coefficient, |
| A | - | Heat transfer surface area, |
| t | - | Temperature, |
| $\dot{q}$ | - | Heat flux, |
| $\dot{m}$ | - | Mass flow, |
| c | - | Specific heat, |
| h | - | Enthalpy, |
| E | - | Electric power, |
| COP | - | Coefficient of Performance, |
| R | - | Thermal resistance, |
| H | - | Dept, |
| O | - | Optimal function, |
| $\overline{X}$ | - | Input variables, |
| $\overline{Y}$ | - | Output variables, |
| $\overline{U}$ | - | Decision variables, |
| r | - | Economic result. |

**REFERENCES**

*Books:*
[1] Bellman, R. – Dynamic programming, Princeton University Press, Princeton, 1957.
[2] Nemhauser, A. G. – Introduction to Dynamic Programming, Wiley, New York, 1966.
[3] Szintay, I. –Rendszerelmélet, rendszerszervezés I. (System Theory, System Organization), Nemzeti tankönyvkiadó, 1993, Budapest, Hungary.

*Journals:*
[4] Trujillo, D. M., Busby, H. R. - Optimal regularization of the inverse heat-conduction problem, Journal of Thermo physics and Heat Transfer (ISSN 0887-8722), vol. 3, Oct. 1989, p. 423-427., October 1989.
[5] Cheung, R., Powell, W. B. - An algorithm for multistage dynamic networks with random arc capacities, with an application to dynamic fleet management, Operations Research 44(6), 951–963, 1996.
[6] Archibald, T. W., Buchanan, C. S., McKinnon, K. I. M. & Thomas, L. C. - Nested Benders decomposition and dynamic programming for reservoir optimisation, J. Operating Research Soc. 50, 468–479, 1999.
[7] Sieniutycz, S., Kubiak, M. – Dynamical energy limits in traditional and work-driven operations I. Heat-mechanical systems, International Journal of Heat and Mass Transfer, Volume 45, pages 2995-3012, 2002.
[8] Katsunori, N., Takao, K., Sayaka, T. – Development of a design and performance prediction tool for the ground source heat pump system, Applied Thermal Engineering, Volume 26, pages 1578-1592, 2006.

*Conference Proceeding:*
[9] Kulcar, B., Goricanec, D., Krope, J., - Economy of exploiting heat from low-temperature geothermal sources using a heat pump, Energy and Buildings, Volume 39, 2007.