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Theses of the PhD Dissertation

TOPOLOGY OPTIMIZATION PROBLEMS USING OPTIMALITY CRITERIA METHODS

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1. Introduction

It is rather accepted that the decision making is one of the most important fact in human's life and it always influences the life in all field of activities, it doesn't matter is it economical, political or technological. As a matter of fact human's nature always seeks for better, easier and economical solution in a safe manner, which in a good condition leads to better life's style. It is rather obvious that the demands usually contain some problems that need researches and investigations which usually give "optimal" solution(s). This phenomenon is also valid in engineering field where engineers try to find better solution in everyday life's problems and use advanced technique to solve them in secure, enhanced and economical way.

In Engineering science, unfortunately analytical solutions by using fundamental theories can only be applied to a limited class of structures of simple geometry, and direct numerical solutions require computing power which was not available until comparatively recently. For instance, in Civil Engineering a class of structure that came in for particular attention, because of its particular importance, was the skeletal structure like beam column connections that are connected each other at ends or two, three dimensional frame structures. Using basic principles it is possible to develop adequate relations between moments and the lateral forces at the end of the particular beam member and the corresponding relations and lateral displacements. A skeletal structure can be analyzed directly by combining the force displacement relationships for the individual members subjected to satisfying equilibrium and compatibility conditions at the joints. But in case of complicated structures it was impossible to solve the resulting set of linear algebraic simultaneous equations other than simple problems. The developments which were taking place in the realm of digital, computing presented the structural and stress analyst with ever increasingly powerful aid in the solution of these kinds of problems. The availability of computing power meant that a general systematic and repetitive approach was advantageous and that the solution of any resulting large set of simultaneous equations was no longer a big problem. Matrix algebra provided a basis for the efficient organization and manipulation of large quantities of data. Thus, without the need to develop any fundamentally new structural principles, the stage was set for the introduction of what are known as the matrix methods of structural analysis.

At the end of first half of this century, in aircraft industry, engineers developed clever methods in structural analysis. Matrix algebra had important role and with development of computer power by time the situation was ripe for generalization and extension of analysis methods. The rapid changes and fast developments in aircraft's structural forms were forcing for developments in analysis methods as well. It was these circumstances that the finite element method gets its modern form in the mid 1950s. As with the matrix method of structural analysis the basis of the finite element method resides in subdividing a complex structure (the theoretical model of actual structure) in to a finite number of discrete parts or regions or elements.

The above goals can be archived by the use of the mathematical programming tools. Optimization has many effects in industrial field such car, aeronautics, aerospace, building, textile, packaging industry (shape and/or material of pack), etc...

This topic is also very popular in Structural Civil Engineering field where the Engineers try to optimize the structure in different boundary condition, for different purpose.

2. Problem statement, purpose and scope

In the field of the structural optimization there are two main areas: analysis and design. Analysis principally means that the load carrying capacity has to be found with given boundary conditions (size, resistance, supports, etc…). In design, the loads and supports are given and the engineer should find the best geometry and dimension for the structure.

Engineers do usually strive for a global optimization of weight, rigidity, resistance and cost. Traditionally, engineers proceed by trial and error, and optimization is really a matter of intuition and know-how. This is of course an old-fashion, costly, and improper way of optimizing. The modern trend is to use more and more numerical software which simultaneously analyze and optimize many possible designs, making optimal design an automatic process. The most commonly used optimization technique called *Layout Optimization* which deals with optimization of *size, shape* and *topology* of the structures. Each of them has very wide range of research field and developments.

In last decade through my M.Sc. and Ph.D. studies I was working mainly in field of topology optimization and cooperating with Prof. G. Rozvany, Prof. S. Kaliszky, Prof. A. Vásárhelyi and my supervisor Dr. J. Lógó in Budapest University of Technology and Economy, Faculty of Civil Engineering, Department of Structural Mechanics. The title of my M.Sc. thesis was topology optimization with SIMP method (supervised by G. Rozvany and J. Lógó). The SIMP means *Solid Isotropic Material with Penalty*. The described method has been used extensively by Rozvany, Zhou, and Birker in 1992, but it was originally suggested by Bendsoe in 1989 as an extension of a technique employed by Rossow and Taylor in 1973. The extension of SIMP like method investigation was developed in my Ph.D. work and later it was extended to non-design area, internal and external support strengthening and finally stochastic topology optimization as one can see later among my works.

I was entitled to cover the following tasks through my Ph.D. work:

- To create new algorithms based on the Kuhn-Tucker condition in order to solve the optimal design (topology) problems with thousands of design variables (make the numerical procedures capable for industrial application as well),
- Computer program which solves constraint optimization problems with thousands of design variables (an ordinary optimization programs could solve about 1000 design variables),
- Create a new algorithms in order to find new type of topologies,
- Investigate the effects of the additional internal and external support for optimal topologies,
- Investigate the effects non-design domain for optimal topology (for technical reason the design area should be divided into two parts, design and non-design one),
- Probabilistic effects (probabilistic boundary conditions, probabilistic loading) in topology optimization.

2.1. Main Assumptions

The following assumptions are considered in my entire dissertation work:

- Small displacement theory is used,
- Material is linear elastic,

- The load is one-parametric, static,
- Stability problems are not considered.

2.2. Solution strategy

The topology optimization that I have been involved in was started by the M. Michell in the beginning of 19^{th} century. Nowadays the topology optimization is one of the most “popular” topics in the field of optimal design. A great number of papers indicate the importance of the topic. Generally, the reviews of these papers trace back only to the last 15 years and mostly the reports which were published earlier remain “hidden” and “undiscovered”.

The past century has produced impressive improvements in power and efficiency of mathematical programming techniques, as applied to general structural design problem, but these methods pay for their generality with rapid increase in the number of computational requirements such as the increase of the number of design variables and number of constraints. This computational burden tends to restrict their usefulness to problems from ten to a few hundred of design variables. Attempts to apply numerical search procedures to resize problems have failed due to the fact of the large number of design variables involved or due to the huge computational expenses.

The special approaches which have solved such problems successfully are known from the literature as optimality criteria methods. Optimality criteria methods are based on radically different thinking from those applied in the development of the mathematical programming methods (MP). Most MP methods concentrate on obtaining information from conditions around the current design point in design space in order to find the answer to two questions: in what direction and how far to go to best reduce the value of the objective function directly. This is repeated until no more reduction is produced in the iterations within some selected tolerance. On the other hand, optimality criteria methods, exact or heuristic, derive or state conditions which characterize the optimum design, then find or change the design to satisfy those conditions while indirectly optimizing the structure. The optimality criteria method approach results in finding the close neighbourhood of the optimum usually very quickly depending on certain conditions.

The procedure can be divided into four steps: Step 1. derives the optimality criteria equations – they can be intuitive criteria (fully stress design (FSD), simultaneous failure mode design (SFMD), uniform strain energy density design (USEDD), constant internal force distribution design (CIFDD), etc..), or mathematically defined optimality criteria equations (classical optimality criteria method (COC), dual optimality criteria method (DOC), general optimality criteria method (GOC), etc…). Step 2. is the iteration procedure for the design variables. Step 3. is the iteration procedure for the Lagrange multipliers. Step 4. is the computer program implementation.

Basically my research work was divided into two main parts, topology optimization with deterministic problems and topology optimization of stochastic problems.

In the first case, all the structural design data were given in deterministic form while in the second case the design data contained some deterministic data and some probabilistic data or constraint.

In engineering practice it is know that the support condition and the load position plays a very important and sensitive role on final shape of the structure and eventually weight of the structure.

This was one of the reasons that engineers start to think and investigate the optimum topology of the structures. As the topology optimization became rapidly expanding field in structural mechanics field, several methods and conditions was introduced to investigate the different conditions on the structures. I also investigate some tasks during my M.Sc. study and they were extended in my Ph.D. study that can be found in chapter two.

As a matter of interest stochastic topics catch my attention and I did some investigation on topology optimization taking into consideration stochastic effect. The detail of investigation and achieved result can be found in chapters three and four.

3. New scientific results

3.1. Topology Optimization in case of Extended Design Condition

A very efficient iterative algorithm was presented for topology design of continuum type structures with variable support „cost" and having a compliance constraint. The applied meshing provides a good technique to avoid the checkerboard pattern. By the use of the smooth penalization increment the obtained numerical solutions are in good agreement with the expectations. Conceptually this topology design is simple, since the algorithm does not require intensive computer storage. The number of the design variables (thousands) significantly exceeds the maximum number of variables what can be used in any kind of mathematical programming algorithm. The main disadvantage is that the buckling and other constraints are not taken into consideration during the optimization procedure but the obtained numerical topologies are good starting points for further optimal design. The support optimization technique is suitable to demonstrate the effects of strengthening of structures.

Theses I. [2]

I.a. I have extended the basic topology optimization model and algorithm to consider external and internal supports. The parametric study results shown that the elaborated algorithm was sensitive between 0 to 10000 cost range to model the full scale for external/internal reinforcement.

I.b. The elaborated topology model and algorithm gives the possibility to confirm the Prager-Rozvany layout theory in case of pre-assign (pre-existing) members.

I.c. The parametric studies confirm that the elaborated methods are suitable to prove the numerically obtained optimal topologies. The results are very similar or same as the analytically assumed ones.

3.2. Stochastic Compliance Constrained Topology Optimization Based On Optimality Criteria Method

The stochastic optimization problem was successfully solved. The introduced algorithm provides an iterative tool which allows using thousands of design variables what is impossible by the use of conventional optimization program. By the use of secondary meshing of ground elements the amount of the checker-board pattern was neglected on acceptable level.

The present stage of the research work shows that due to iterative formulation of thicknesses of the ground elements obtained in the stochastic problem it is advised to start the computation with the deterministic SIMP algorithm and when the calculated probability of the solution is not zero needs to turn to the stochastic algorithm. The data of the random variables in the problem can create such cases where the problem is non-convex. The “so-called grey” solution means that the probabilistic optimal topology is naturally lighter than the corresponding deterministic optimal topology, but the optimal shape of the structure practically the same. Some other advantage is that the designer can take into consideration some initial design uncertainties with the probabilistically given compliance limit.

Theses II. [8]

II.a. I have elaborated the model and algorithm to perform topology optimization calculation with probabilistically given compliance bounds.

II.b. The parametric studies confirm that the elaborated methods effective when the numerical procedure starts with deterministic algorithm and later on continue with stochastic algorithm.

3.3. Optimal Topologies in case of Correlated Loading

The probabilistically constrained topology optimization problem was presented. The compliance approximation gives an appropriate tool to describe probabilistically given loading. The introduced algorithm provides an iterative tool which allows using thousands of design variables what is impossible by the use of conventional stochastic optimization program. By the use of secondary meshing of ground elements the amount of the checker-board patter was avoided on an acceptable level.

The present stage of the research shows that to compute the stochastic topology needs the same computational time as in case of deterministic topology. The elaborated topology design method is more effective when the correlation is not significant (smaller than 10% of the mean values). The algorithm is rather stable and provides the convergence to reach the optimum. One can see that the covariance values and the minimum probability values have significant effect on the optimal topology. The symmetry of the design can be lost due to the effect of the covariance values. The applied method gives a wider possibility to the designer to take into consideration more realistic loading description than the deterministic topology design.

Theses III. [15]

III.a. I have elaborated the model and algorithm to perform topology optimization calculation with probabilistically given loads (Gaussian distribution).

III.b. The numerical studies confirm that the elaborated methods effective and a wide rage of problems can be solved.

III.c. The parametric study shows that the covariance values have significant effect for the optimal topologies.

III.d. The symmetry of the design can be lost due to the effect of the covariance values.

4. Utilization of the results

The elaborated work in general belongs to the category of fundamental research. The presented mechanical models, computational methods, numerical algorithms can be applied almost in all field of the life. The application of results is primarily used in auto-motor, airspace industries, packing industry. Also the building industry can benefit from the results. Significant material saving, better understanding of the behavior of the structural response can be utilized by the use of these methods.

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