COLLECTIVE EFFECTS OF RADIATIVELY INTERACTING ULTRACOLD ATOMS IN AN OPTICAL RESONATOR

PH.D. THESES

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Introduction

The appearance of high quality optical microresonators 10 years ago allowed the experimental study of light–matter interaction on a microscopic level. Cavity Quantum Electrodynamics (CQED) has not only opened up conceptually new ways in the manipulation of atomic internal and external degrees of freedom, but also created a new microscopic system consisting of cold atoms trapped inside the resonator and strongly interacting with a single cavity-field mode. This system realizes a controllable coupled dissipative quantum dynamics on a generic level, that may give rise to applications in quantum optics and in quantum informatics.

The strong coupling regime of CQED is attained, where the strength of the atom–field coupling (characterized by the single-photon Rabi frequency) exceeds the rate of dissipation in the system (set by spontaneous emission and cavity decay). This means that an atom and the cavity field exchange an excitation quantum many times before it is dissipated into the environment. They lose their identities and form a single new object, literally an atom–photon molecule. In case of strong coupling, the motion of a single atom can significantly modify the field which builds up inside the resonator, and *vice versa*, the field acts back on the atomic motion through light forces. In this way, a coupled dynamics appears between the atomic external degrees of freedom and the resonator field.

Strong coupling in the optical domain was first achieved by the group of J. Kimble at Caltech [Hood et al., 1998], and a year later by the Rempe group at MPQ Garching [Münstermann et al., 1999]. From a laser-cooled atomic sample, they kicked out single atoms with a laser towards the cavity volume at a velocity of a few meters per second. By measuring the transmission of the resonator, they could detect the presence of each atom between the mirrors. In an improved experiment, the single atom trajectories were reconstructed with a few micrometers spatial resolution from the time-resolved analysis of the transmitted field intensity [Pinkse et al., 2000, Hood et al., 2000]. The mechanical forces on the atoms stemming from the atom–cavity interaction were also demonstrated. The next challenge was to understand these forces and to exploit them for atom cooling and trapping.

Intensive theoretical investigations were started in the group of H. Ritsch in Innsbruck on the light forces in optical cavities. In the strong coupling regime, they found a novel cooling mechanism, the *cavity cool-*

ing [Horak et al., 1997]. The atom is so strongly coupled to a given mode of the cavity that they share all dissipation channels of the system, thus the atomic kinetic energy can be dissipated through the photon loss channel of the cavity mode. In collaboration with H. Ritsch, our group also investigated the cooling process. In the limit where the pumping laser is detuned far below the atomic resonance, the far-off-resonance trapping scheme can be generalized for the cavity field, which can simultaneously trap and cool arbitrary polarizable objects [Vukics and Domokos, 2005]. The experimental demonstration of cavity cooling was reported in Refs. [McKeever et al., 2003, Maunz et al., 2004]. Later, deterministic transport of atoms into the resonator mode volume is realized, and three dimensional trapping of a single atom was achieved with trapping times up to 17 seconds [Nussmann et al., 2005].

Over recent years, the study of many-body phenomena in optical resonators has attracted considerable attention. For several atoms, light–matter interaction in the strong coupling regime is, by nature, a many-body problem. As all atoms are coupled to the same cavity mode, one of them feels the change in the field that is caused by the others. Thus, the field of the resonator mediates an atom–atom interaction, which does not decay with the interatomic distance and the atom density. However, even in a dilute atomic gas, it gives rise to interesting collective effects. At first, three groups set up experiments for studying collective behaviour in optical resonators. Vuletić at the MIT succeeded in trapping a few thousand atoms inside a confocal multimode cavity [Black et al., 2003]. In Tübingen and in Hamburg, Zimmermann [Kruse et al., 2003] and Hemmerich [Nagorny et al., 2003] obtained nearly 10^6 atoms inside a ring cavity, which has two counter-propagating degenerate modes.

More recently, a peculiar atom–cavity system has been realized at the ETH in Zurich [Brennecke et al., 2007]. The Esslinger group succeeded in trapping a Bose-Einstein condensate (BEC) of 10^5 Rb atoms inside an ultra-high finesse optical microcavity. A similar system was built with ultracold Rb atoms by Stamper-Kurn at Berkeley [Gupta et al., 2007]. Working in the far-detuned limit, they attained strong dispersive coupling between the atoms and the field. Even a single atom of the condensate realizes strong coupling to the field, hence the collective coupling of so many atoms is enormous. On the one hand, this means that the cavity is highly sensitive to the dynamics of the BEC, so for instance it can be used as a stroboscope to monitor matter-wave motion [Brennecke et al., 2008]. On the other hand, the field creates both coherent and incoherent excitations in the BEC. The cavity-mediated long-range atom–atom interaction has significant effects as it is in the same order of magnitude or even larger than the collisional interaction between the atoms. This exotic quantum many-body system shows a number of generic collective effects, which are still being explored both by theory and by experiment.

Objectives

The main goal of my Ph.D. work was the theoretical understanding of many-body effects occurring in a cloud of cold atoms which is dispersively coupled to the field of a high-finesse optical cavity. A specific problem was the description of the spatial self-organization of ultracold bosons. The phenomenon was discovered for classical atoms by P. Domokos in Innsbruck [Domokos and Ritsch, 2002]. The initially homogeneous atom cloud illuminated from the side had evolved – by spontaneous symmetry breaking – into one of the two possible regular patterns which scatter maximum field into the cavity *via* Bragg scattering. Within a year, the phenomenon was demonstrated experimentally by V. Vuletić at the MIT [Black et al., 2003]. The study of self-organization continued in our Budapest group, where J. Asbóth set up a simple mean-field model for a thermal atomic gas, and found that the phenomenon is a continuous phase transition in the thermodynamic limit [Asbóth et al., 2005]. My first task was to generalize his model to a ring resonator with two counter-propagating modes (Thesis I). This work was motivated by the Hamburg and Tübingen experiments, and it established a connection between self-organization and collective atomic recoil lasing [Javaloyes et al., 2004]. My next goal was to describe self-organization for a zero-temperature Bose-Einstein condensate by a mean-field model for the coupled matter and radiation fields (Thesis II). The actuality of my study is shown by the recent experiment of the Esslinger group, which has supported my findings [Baumann et al., 2010].

The other focus of my work was on the quantum fluctuations of a BEC inside an optical resonator around the mean-field solution. Photon field of the cavity couples to the excitations of the condensate, hence polariton modes appear in the normal-mode excitations of the system. I calculated the spectrum of these collective excitations in the transverse pump geometry, which provided additional insight into the phase transition (Thesis

III). The polariton modes inherit the properties of the cavity mode, therefore they are lossy and they are driven by the quantum fluctuations accompanying the photon loss. Thus, even in the steady-state, these excitation modes have a non-zero population, which corresponds to a finite number of atoms that are out of the macroscopically occupied condensate state. I revealed this new type of condensate depletion, which also describes the inaccuracy of the mean-field approximation (Thesis IV).

The investigations in my thesis were highly motivated by the Esslinger experiments in Zurich. My goal was to provide a deeper understanding of the phenomena produced by their BEC–cavity system. In a recent experiment they induced coherent matter wave oscillations in a single excitation mode of the BEC by simply tuning the cavity frequency [Brennecke et al., 2008]. This excitation mode realizes the opto-mechanical coupling with the field, which gives rise to optical bistability. My motivation was to set up a quantum theory for this classically critical open quantum system, therefore I completed their model by taking into account the quantum back-action noise of the cavity in terms of a quantum master equation (Thesis V).

Finally, the mean-field model, which I previously used to study self-organization, is recognized to allow for answering a long-lasting question that whether dipole-dipole interaction can prevent a thermal gas from reaching quantum degeneracy in a far-off-resonance optical dipole trap [Barrett et al., 2001]. This long-range atom–atom interaction is mediated by the free space radiation modes, and for strong laser field, it is expected to give rise an instability of a dense atom cloud (Thesis VI).

New scientific achievements

I Using a mean-field approach, I studied the spatial self-organization of a cold gas of linearly polarized particles transversely pumped by an off-resonant laser and interacting with the counterpropagating radiation modes of a high-Q optical ring resonator. In the thermodynamic limit, the canonical distribution together with the optical dipole potential exerted on the atoms by the cavity field provides a self-consistent equation for the density of the gas, that I solved by an iterative numerical method. I found three stable thermodynamic phases: i) a uniform distribution, ii) a self-organized Bragg-lattice and iii) a lattice with defects. I mapped these phases on a phase diagram as a function of the pump strength and the particle density [1].

II In terms of the Gross-Pitaevskii mean-field theory, I described the self-organization of a Bose-Einstein condensate laser-driven from the transverse direction and dispersively interacting with the field of a high-finesse linear optical cavity. Above a critical pump intensity, the homogeneous condensate evolves into a stable pattern, whose periodicity is given by the cavity wavelength. The analytic expression for the transition point reveals that below threshold the homogeneous phase is stabilized mainly by the atom–atom collisions besides the small kinetic energy of the gas [3].

III Considering small fluctuations around the mean-field solution, I calculated the collective excitation spectrum of the BEC–cavity system in the transverse pumping geometry. The critical point of self-organization is clearly manifested in the spectrum by the degeneracy of one excitation mode with the ground state of the system. Below threshold, I obtained the spectrum analytically. Above threshold, the spectrum qualitatively depends on the density of the atoms, because in the strong coupling regime, where classically defect sites appear in the optical potential, new type of BEC excitations can take place [3].

IV Quantum fluctuations of the resonator field drive the excitation modes of the compound BEC–cavity system, which results in a finite number of atoms outside the macroscopically occupied condensate state. For cavity pumping, I determined this excess noise depletion in the steady state by solving the Heisenberg-Langevin equations for the quasinormal mode excitations. In the weak coupling limit, I obtained both analytical and numerical results. I found that the steady-state depletion depends mostly on the cavity detuning and photon loss rate setting the noise level in the system. The depletion is independent of the total number of atoms, and it does not significantly depend on the strength of the atom-field interaction and on the photon number [4].

V I derived a quantum master equation for a single mode excitation of a Bose-Einstein condensate by an optical cavity mode. The presented model originates from the opto-mechanical coupling between the atom and radiation field modes, however the cavity field is eliminated from the system. The resulting equation for the single BEC excitation mode accounts for the dissipative part of the dynamics including friction and diffusion effects. By the numerical simulation of the master equation, I found that the measurement-induced back-action noise impedes the observation of quantum tunneling in the classically bistable regime. Nevertheless, coherent matter wave oscillations are possible, where diffusion leads to a non-exponential dephasing effect, whose magnitude decreases with the increase of the oscillation amplitude [5].

VI In a mean-field approach, I calculated the effect of the dipole-dipole interaction on the far-off-resonance optical dipole trapping of cold atoms. I found that the mean-field interaction energy is negative for a pancake-shaped cloud, therefore it deepens the center of the trap potential. Above a critical peak density, the thermal motion cannot stabilize the gas against self-contraction and an instability occurs, which manifests itself in the collapse of the cloud. The boundary of the stable equilibrium region expressed in terms of the ratio of the trap depth to the temperature shows a power law dependence on the atom number with exponent -0.4. I determined the maximum achievable peak density in a dipole trap as a function of the system parameters and compared it to the limit of quantum degeneracy [2].

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