ITERATIVE DESIGN OF STRUCTURED UNCERTAINTY MODELS AND ROBUST CONTROLLERS FOR LINEAR SYSTEMS

Theses of Ph.D. dissertation

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1 Motivation

In safety-critical applications guaranteed stability and performance against all disturbances and changes in the system’s dynamics are expected. In the robust control theory of linear time-invariant (LTI) systems ([16, 17, 6]) the notion of uncertain system has been introduced. An uncertain system is defined by a set of unknown systems with bounded $\mathcal{H}_\infty$ norm. For the closed-loop system consisting of the uncertain system and a controller, performance inputs (disturbances, reference signals) and performance outputs are defined. The system is robust stable (RS) if it is stable for every element of the set. Robust performance is the largest (worst-case) gain of the uncertain system.

The uncertain system can be described by the feedback interconnection of a nominal model and an unknown system. The unknown system can be unstructured consisting of a full LTI system matrix, or a structured block-diagonal matrix where each block can be LTI or linear time-varying (LTV) system or constant parameter matrix. The more details of the real plant can be taken into account the smaller can be the resulting model set and so the conservatism of the controller.

Usually, the structured uncertainty model is constructed based on physical considerations. The sizes of disturbances and uncertainty blocks are characterized by frequency-dependent weighting functions. Their choice strongly influences the robustness of the designed system, since too small weights may lead to instability or too large ones lead to poor performance. Weighting functions can be invalidated based on measurement data ([11, 13, 3]), but these validation methods does not provide how-to guide for weighting function selection.

The design of weighting functions involves much experimenting and heuristic solutions based on deep engineering insight. There does not exist a method that automatically designs structured uncertainty models based on measurement data and that improves robust performance by taking the performance specifications into account.

For this problem, a solution is proposed in the theses.
2 Basic notions and applied methods

The thesis and the underlying work is built upon the following methods and tools from systems and control theory. Throughout this document the systems have multiple inputs and multiple outputs, the signals are vector valued, but dimensions are not marked for clarity of notation.

The largest singular value of a matrix $A$ is denoted by $\bar{\sigma}(A)$. The block-diagonal matrix containing the matrices $A_1, A_2, \ldots, A_n$ is denoted by $\text{diag}\{A_1, A_2, \ldots, A_n\}$. $\mathbb{R}$ and $\mathbb{C}$ denote the set of real and complex numbers, respectively. $\mathcal{R}$ is the field of real rational transfer functions. Set $\mathcal{RL}_\infty$ contains transfer functions $H \in \mathcal{R}$, where $\|H\|_\infty := \sup_{\omega \in \mathbb{R} \cup [\infty]} \bar{\sigma}(H(j\omega)) < \infty$ is satisfied. A subspace of this is $\mathcal{RH}_\infty$, the set of transfer function analytic on the open right half plane. $\mathcal{L}_2$ denotes the space of finite energy signals $f : j\mathbb{R} \mapsto \mathbb{C}$, where the norm is defined by $\|f\|_2 := \int_{-\infty}^{\infty} f(j\omega)^* f(j\omega) d\omega$.

2.1 Structured singular value

In robust control theory the uncertain controlled system shown in Figure 1 is considered. Unknown or neglected dynamics is denoted by $\Delta$ and belongs to set $S_{\Delta_u} := \{\Delta \in \mathcal{RH}_\infty \mid \Delta = \text{diag}\{\Delta_1, \ldots, \Delta_r\}\}$, which consists of structured block-diagonal, stable and real-rational
transfer matrices. System $G$ contains the nominal model of the system, its interconnection structure, uncertainty weighting functions describing the size and shape of the uncertainty and weighting functions specifying performance. $K$ denotes an LTI controller, $z_p$ is the performance output of the closed loop, $w_p$ is the performance input (disturbances, reference signals, etc.). Let the system containing the generalized plant and the controller be denoted by $M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$, which can be written as a lower linear fractional transformation $M = F_L(G, K) := G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21}$, where $G$ is partitioned accordingly. The uncertain controlled system is denoted by $F_U(M, \Delta) := M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12}$.

Robust stability and performance of the system in Figure 1 can be analyzed by structured singular value $\mu$. In each frequency, function $\mu_\Delta(M_{11}) : \mathcal{RH}_\infty \mapsto \mathbb{R}$ maps $M_{11}(j\omega)$ into a real number. The inverse of this number is the norm of the smallest destabilizing structured uncertainty.\(^1\) If uncertainty $\Delta \in \mathcal{S}_\Delta$ satisfies $\|\Delta\|_\infty \leq \beta^{-1}$, then the system in Figure 1 is robust stable if and only if $\mu_\Delta(M_{11}) < \beta$. Robust performance can be analyzed with the help of a generalized stability test: for all uncertainty $\Delta \in \mathcal{S}_\Delta$ satisfying $\|\Delta\|_\infty \leq \beta^{-1}$ the robust performance is bounded as $\|F_U(M, \Delta)\|_{\infty} \leq \beta$ if and only if $\mu_{\Delta_a}(M(j\omega)) < \beta$ for all $\omega$, where $\Delta_a = \text{diag}\{\Delta, \Delta_p\}$, and $\Delta_p \in \mathcal{RH}_\infty : z_p \mapsto w_p$ is a fictive uncertainty block.

The computation of $\mu$ is NP-hard, in general, for this reason an upper-bound is calculated:

$$\mu_{\Delta_a}(M(j\omega)) \leq \inf_{D_L, D_R} \bar{\sigma}(D_L(j\omega)M(j\omega)D_R(j\omega)^{-1}),$$

where block-diagonal matrices $D_L, D_R$ satisfy $D_L^{-1}\Delta_a D_R = \Delta_a$. In controller synthesis problems $\bar{\sigma}(D_LMD_R^{-1})$ is minimized in scaling matrices $D_L, D_R$ and controller $K$. This problem is usually solved by the so-called D-K iteration [17].

The level of robust performance ($\beta$) and the size of the uncertainty ($\beta^{-1}$) can be specified separately with the help of the function skew $\mu$.

\(^1\)As a generalization of Small Gain Theorem it can be stated that a sufficient condition of RS is that $\mu_\Delta(M_{11})\bar{\sigma}(\Delta) < 1$, $\forall \omega$. 
which has been applied in analysis problems ([4, 5, 8]). For all uncertainty \( \Delta \in S_{\Delta u} \) with \( \| \Delta \|_\infty \leq 1 \) robust performance is satisfied with \( \| \mathcal{F}_U(M, \Delta) \|_\infty \leq \gamma \) if and only if \( \mu_{\Delta u}(M(j\omega)\text{diag}\{\gamma I_{w_u}, I_{w_p}\}) < \gamma \) for all \( \omega \).

### 2.2 Checking consistency of structured uncertainty models in frequency-domain

\[
\begin{pmatrix}
U_{11} & U_{12} & U_{13} \\
U_{21} & U_{22} & U_{23}
\end{pmatrix}
\]

\( U = \)

Figure 2: Uncertain system

Assume that the unknown system \( T : u \mapsto y \) is modelled by the system in Figure 2. The unknown dynamics and the disturbances are normalized by weighting functions \( W_\Delta = \text{diag}\{W_{\Delta,1} I, \ldots, W_{\Delta,\tau} I\} \) and \( W_d = \text{diag}\{W_{d,1}, \ldots, W_{d,n_d}\} \), respectively. The input and output of the model are denoted by \( u \) and \( \hat{y} \). Assume that measurement data \( (u(j\omega_k), y(j\omega_k)) \) are given in discrete frequency points \( \omega_k \). The uncertain model is consistent with the data, if there exists a \( \Delta \in S_{\Delta u} \), \( \| \Delta \|_\infty \leq 1 \) and a \( d \in L_2 \), \( \| d \|_2 \leq 1 \) that \( \hat{y}(j\omega_k) = y(j\omega_k) \) is satisfied [13, 9]. A sufficient condition of this is the existence of a parameter \( \theta_k \in \mathbb{C} \) parameterizing the subspace of variables \( w_{u0} \) and \( d_0 \) that solve \( \hat{y}(j\omega_k) = y(j\omega_k) \) such that

\[
\begin{align}
|W_{\Delta,i}(j\omega_k)| & \geq \frac{|w_{u0,i}(j\omega_k, \theta_k)|}{|z_{u,i}(j\omega_k, \theta_k)|}, & i = 1, \ldots, \tau \\
|W_{d,i}(j\omega_k)| & \geq |d_{0,i}(j\omega_k, \theta_k)|, & i = 1, \ldots, n_d
\end{align}
\]

is satisfied.
2.3 Analysis and synthesis using integral quadratic constraints

The integral quadratic constraints are general forms for characterizing uncertainty, stability and performance of (possibly nonlinear) systems. An IQC is an inequality of the form $\Sigma(x) \geq 0$, where $\Sigma(x)$ is an integral quadratic function that can be defined in the frequency-domain as

$$
\Sigma(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(j\omega)^* \Pi(j\omega) x(j\omega) d\omega
$$

via a measurable Hermitian bounded mapping $\Pi$ called multiplier: $\Pi(j\omega) \in \mathbb{C}^{n_x \times n_x}$, $\|\Pi(j\omega)\| \leq c$, where $c$ is a constant, and $x \in \mathcal{L}_2$.

For example the following abstract performance theorem is considered.

**Theorem:** (An abstract performance characterization) Given an uncertainty set $S_\Delta$, an integral quadratic function $\Sigma(x)$ with multiplier $\Pi(j\omega)$ and a performance specification

$$
\Sigma_p \left( \begin{bmatrix} w_p \\ z_p \end{bmatrix} \right) \leq -\epsilon \|w_p\|_2^2 \text{ for all } w_p \in \mathcal{L}_2,
$$

where $\Sigma_p$ is an arbitrary mapping $\Sigma_p : \mathcal{L}_2 \mapsto \mathbb{R}$ satisfying $\Sigma_p \left( \begin{bmatrix} 0 \\ z_p \end{bmatrix} \right) \geq 0$. Suppose that all $\Delta \in S_\Delta$ in Figure 1 satisfy the IQC

$$
\Sigma \left( \begin{bmatrix} w_u \\ z_u \end{bmatrix} \right) \geq 0
$$

Suppose there exists an $\epsilon > 0$ such that

$$
\Sigma \left( \begin{bmatrix} w_u \\ z_u \end{bmatrix} \right) + \Sigma_p \left( \begin{bmatrix} w_p \\ z_p \end{bmatrix} \right) \leq -\epsilon \left( \|w_u\|_2^2 + \|w_p\|_2^2 \right)
$$

for all $w_u, w_p \in \mathcal{L}_2$. Then $I - M_u \Delta$ has a causal inverse whose $\mathcal{L}_2$-gain is bounded uniformly for $\Delta \in S_\Delta$ (uniform robust stability) and uncertain system $\mathcal{F}_U(M, \Delta)$ satisfies the performance criterion (3).

When uncertainty set $S_\Delta$ is given, robust performance can be tested by 1.) trying to find as many IQCs (characterized by $\Pi$) as possible
which satisfy (4) for all $\Delta \in S_\Delta$, then 2) finding one among these IQCs that also satisfies (5).

When $\Delta$ is an on-line measurable LTV parameter matrix and model $G$ is an LTI system, then the feedback interconnection of $\Delta$ and $G$ is said to be an LPV system with linear fractional dependence on the scheduling parameters (LFT-LPV). If the controller also has an LFT-LPV form, the synthesis problem lead to the solution of linear matrix inequalities (LMIs) [12, 1, 2, 15].

3 New scientific results

**THESIS 1**
Structured uncertainty modelling and robust control design for LTI systems

The goal is to elaborate an algorithm that provides the weighting functions ($W_\Delta$ and $W_d$) in Figure 2 so that the uncertainty model is consistent with measurement data and the controller designed based on this model achieves as good performance as possible on the real system.

Since the weighting functions of the uncertainty are not fixed, we have to define the notion of performance. According to the robust control theory, the performance is defined as the $H_\infty$-norm of the system $w_p \mapsto z_p$. In the present problem, performance channel $w_p \mapsto z_p$ can be altered through weighting function $W_d$ of the disturbances: in this way the performance criterion is not a predefined and fixed criterion.

We assume that true system $T$

$$y = T(\bar{d}, u), \quad T \in S_T, \quad \bar{d} \in S_{\bar{d}} \subset L_2$$

is stable and its input-output data $(u, y)$ can be described in every finite experiment by model $U$ in Figure 2. Physical disturbances $\bar{d}$ and true system $T$ belong to bounded sets $S_{\bar{d}}$ and $S_T$, respectively. By placing the system into the closed-loop configuration and augmenting it to generalized plant $G_T$, we can define performance output $\tilde{z}_p$ and, in this way, the performance of the controlled system can be defined as
follows

\[ \bar{\gamma}(K) := \sup_{T \in S_T, \bar{d} \in S_d, \bar{r} \in \mathcal{B}L_2} \| \bar{z}_p \|_2, \quad \bar{z}_p = \mathcal{F}_L(G_T, K)(\bar{d}, \bar{r}), \]  

(7)
i.e. the performance is the worst-case value of \( \| \bar{z}_p \|_2 \) that can be obtained for all possible physical disturbances and changes in the system’s dynamics. The RP criterion can be formulated also based on the model:

\[ \bar{\gamma}_s(K, W) := \sup_{w_p \in \mathcal{B}L_2, \Delta \in \mathcal{B}S_{\Delta u}} \| \bar{z}_p \|_2 = \| \mathcal{F}_L(\mathcal{F}_U(G_0 W, \Delta), K) \|_{\infty}, \]  

where \( W = \text{diag}\{W_\Delta, W_d, I\} \), \( G = G_0 W \) and \( \mathcal{F}_L(\mathcal{F}_U(G_0 W, \Delta), K) \) is the transfer function of the uncertain closed-loop. The following lemma establishes the connection between the two criteria

**Lemma: (an upper bound)** If \( W \) is consistent with the data of all experiments with controller \( K \), then \( \bar{\gamma}_s(K, W) \geq \bar{\gamma}(K) \).

The lemma suggests the use of closed-loop data to find a consistent model with weighting function \( W \) that minimizes robust performance criterion \( \bar{\gamma}_s(K, W) \). The consistency condition of the lemma cannot be guaranteed without a priori information on the system. This is true, anyway, in every validation problems: a future experiment can always invalidate the model. The tuning of the weighting functions is an optimization problem subject to consistency constraints, which can be connected to the control synthesis algorithm, thus, we arrive at the proposed D-K-W iteration method as an extension of the D-K iteration. This extension, however, requires the modification of both the search for scalings (\( D \)) and the control design (\( K \)) steps when the control design is performed in accordance with the actual modelling condition \( \| \Delta \|_{\infty} \leq 1 \). In this case the optimization criterion must be scaled by \( \text{diag}\{\gamma I, I\} \), which results in a skew \( \mu \) synthesis problem.

When the D-K-W iteration method is placed in an outer iteration loop where closed-loop experiments are performed with the designed controller, then we arrive at the algorithm proposed in the first thesis:
Algorithm 1.

A. Initialization

B. D-K-W iteration
   1) Scaled \( \mathcal{H}_\infty \) controller synthesis (\( K \))
   2) Optimization of uncertainty model (\( W \))
   3) Skew \( \mu \) analysis by finding scalings (\( D \))

C. Closed-loop experiments

D. Update of the uncertainty model

When the skew \( \mu \) criterion is applied, the following theorem can be stated from which the important properties of the proposed algorithm follow.

**Theorem:** Given experimental data set denoted by \( E^N \). Based on these data a controller \( K^N \) and a consistent model characterized by weighting functions \( W^N \) are designed by using the D-K-W iteration algorithm. The guaranteed performance level skew \( \mu \) is \( \bar{\gamma}_s(K^N) \). A new experiment is performed with controller \( K^N \) on the true system. The gathered data set is \( E_{N+1} \), and the realized performance \( \tilde{\gamma}_s^{N+1} := \| \tilde{z}_{p,N+1} \|_2 \). Then, the following implications hold.

1. \( W^N \) is consistent with \( E_{N+1} \) \( \implies \) \( \tilde{\gamma}_s^{N+1} \leq \bar{\gamma}_s(K^N) \)
2. \( \tilde{\gamma}_s^{N+1} > \bar{\gamma}_s(K^N) \) \( \implies \) \( W^N \) is not consistent with \( E_{N+1} \).

The first implication expresses that if a new experiment does not falsify the model with \( W(\Theta^N) \), then the performance of the control is, as expected, below the guaranteed level \( \bar{\gamma}_s(K^N) \). The second implication states that if the realized performance in the new experiment exceeds the guaranteed level, then the model is necessarily falsified, which leads to the continuation of the algorithm, increasing the uncertainty set and so improving robust performance of the controller.

The theorem implies, furthermore, that data from unstable or bad performing closed-loop system can be utilized for improving the model. It is assumed that instability can be detected by monitoring some variables, in which case the possibly destabilizing controller can be switched off and a known stabilizing controller or control input can be switched on. The experiments with unstable systems show necessarily weaker
performance as compared to the guaranteed performance level, for this reason the model is invalidated and the algorithm continues with increased uncertainty weighting functions.

The contribution can be summarized in the following thesis:

**Thesis 1**

An iterative algorithm has been elaborated for LTI models with the aim of shaping structured uncertainty models and designing robust controllers based on measurement data. The algorithm handles unstable experiments ensuring safe improvement of guaranteed robust performance on the true, unknown system. For additive uncertainty structures, the algorithm is formulated as a series of LMI problems. The steps of skew $\mu$ synthesis has been elaborated as part of the algorithm. It has been proved that performance degradation beyond the guaranteed level implies falsification of the actual model, which forces the continuation of the algorithm.

Own publications related to thesis 1: [RB05b, RB06b, RB06a, RGSB08, Röd09, RBar, RG10]. Details of the thesis are presented in Chapter 4 of the dissertation.

**THESIS 2**

Structured uncertainty modelling and robust control design for LPV systems

A wide class of nonlinear systems can be approximated by LPV models, which facilitates the application of LPV control theory guaranteeing stability and performance for nonlinear systems. In the last two decades, more and more LPV applications have been emerged mainly in the vehicle industry (see e.g. [7, 10, 14]). The heuristic design of the uncertainty weighting functions came, however, more difficult, as compared to the LTI case, due to the extra design work related to the LTV scheduling parameters. For this reason, the automatic optimization of weighting functions subject to consistency constraints may be more useful in case of LPV models.
LPV systems cannot be represented in the frequency-domain. It would be obvious to define consistency constraints in the time-domain, but this would imply handling variables and matrices of dimensions about the data length. As a consequence, only very short experiments could be utilized due to the computational burden. The advantage of frequency-domain consistency conditions (1)-(2) is that the test consists of a series of small complexity problems where nominal model $U_{23}$ does not play a role. Due to the latter property it is possible to apply the frequency-domain consistency conditions to LPV models, provided the uncertainty model $(U_{ij}, ij \neq 23)$ does not depend on the scheduling parameters. The frequency-domain tuning of the weighting functions has a further precondition: the existence of the RP criterion in the frequency-domain. It can be shown that RP condition (5) can be rewritten in the frequency-domain also for LFT LPV systems.

The developed method for designing weighting functions and robust LPV controllers is summarized by the following algorithm

**Algorithm 2.**

A. **Initialization**

B. **D-K-W iteration**
   1) LPV controller synthesis ($K$)
   2) Optimization of uncertainty model ($W$)
   3) Skew $\mu$ analysis by finding scalings ($D$)

C. **Closed-loop experiments**

D. **Update of the uncertainty model**

The control design step can be performed in the time-domain by Scherer’s method [12]. The tuning of the weighting functions and scalings are carried out in the frequency-domain. In order the lemmas and theorems of Chapter 3 to be valid also for Algorithm 2, the multipliers of the IQCs has to be parameterized according to the skew $\mu$ criterion.

The contribution can be summarized in the following thesis:

**Thesis 2**

*An iterative uncertainty modelling and robust control design algorithm*
is elaborated for LPV models with LFT dependence on scheduling parameters, structured dynamic uncertainty and disturbances. Based on time- and frequency-domain IQCs, guaranteed robust quadratic performance level is minimized by searching for an unfalsified uncertainty model and a robust LPV controller. For additive uncertainty structures, the algorithm is formulated as a series of LMI problems. Due to the advantageous properties of the algorithm, unstable experiments are handled and improvement of guaranteed robust performance on the true, unknown system is ensured.

Own publications related to thesis 2:
[RLB07, MKD+09, RGB09]. Details of the thesis are presented in Chapter 5 of the dissertation.

THESIS 3
Emergency steering of heavy trucks by front wheel braking

The goal of the research was to analyze the steering quality of the brake system of heavy trucks. This problem may arise when the driver becomes incapable of controlling the vehicle due to some lipothyphymy or drowsiness, and the only available tool for automatic intervention is the electronic brake system which can be made suitable for steering by software modification.

The steering capability of the brake system is limited, for this reason the controller is designed for normal driving situations (lateral acceleration $< 4.2 \frac{m}{s^2}$) and dry asphalt. The vehicle is assumed to be equipped with some navigation and path planning system providing yaw-rate reference for the steering controller.

Beside the good tracking performance, the control objective includes the minimization of control effort as well in order to spare the brake-lining. Based on experiments on a loaded MAN truck we have con-

\[2^\text{The control problem presented in Thesis 3 motivated the research problems of Thesis 1 and 2. Given a highly complex dynamic system. The goal is to design a robust controller based on an identified low order nominal model. The question is that how to distribute the nominal model error among components of dynamic uncertainty and disturbances.}\]
cluded that neither the heavy individual braking of a rear wheel nor of a front wheel (in case of fixed handwheel/steering angle) had significant effect on the yaw dynamics of the vehicle. Conversely, braking one of the front wheels with released handwheel, small (10%) brake pressure is sufficient to turn the steering system, and the vehicle can be well steered in a wide range of velocity.

In order to reduce implementation costs a low order linear controller is designed based on a simple nominal model. The model has to approximate the yaw-, steering- and wheel dynamics of the vehicle with sufficient accuracy in the specific control problem. Because of the presence of neglected dynamics, unknown physical parameters and strict safety requirements, the $H_\infty/\mu$ control synthesis method is chosen. The designed controller is tested on a high-fidelity simulation program.

The design method proposed for the above problem can be summarized in the following thesis:

**Thesis 3**

A control strategy, including the process of modelling, identification and control design, is elaborated for steering a vehicle by using the electronic brake system. The design involves the following steps:

I. Velocity scheduled linear model construction

1. Physically parameterized continuous-time state-space models are derived describing yaw dynamics, steering system and wheel dynamics

2. A structure estimation and model reduction method is developed which is based on physical considerations on model reduction, parameter identification and pole analysis of models with frozen velocity

II. Robust control design

1. By using linear matrix-inequalities, unfalsified uncertainty models are parameterized in a numerically tractable simple structure involving multiplicative dynamic perturbation and additive disturbance
2. Robust performance bound is minimized iteratively based on $\mu$ synthesis and shaping of uncertainty weighting functions

It is shown that 1.) the individual braking of the front wheels alone provide satisfactory steering performance in normal driving conditions with relatively low, applicable braking effort and 2.) the elaborated control design method leads to improved robust control performance compared to pure $\mu$ synthesis.

Own publications related to thesis 3: [Röd03, RB04b, RB04a, RB05b, RB05a, Röd07, RGSB08, Röd09, RBar, RGB10]. Details of the thesis are presented in Chapter 6 of the dissertation.

Publications related directly to the theses


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### Further publications


References


