MŰEGYETEM 1782

Budapest University of Technology and Economics
Department of Control Engineering and Information Technology

László Lemmer

Modeling and Control of a Class of Multi-Variable Mechatronic Systems with Kinematic Constraints

---

Kinematikai korlátozásokkal rendelkező többváltozós mechatronikai rendszerek egy osztályának modellezése és irányítása

---

Ph.D. Thesis

Supervisor:
dr. Bálint Kiss

Budapest, September 25, 2009

1 Introduction, Objectives

A persistent and accelerating improvement of mechatronic systems can generally be observed in the automotive industry, where the electronic actuators and digital technology is used for the further enhancement of road safety and driving enjoyment. One of the mechatronic devices supporting the driver of the modern car is the electronic steering system where servomechanisms were introduced to reduce the steering effort required from the driver.

By today, power steering systems have spread in automotive industry because numerous factors (such as greater vehicle mass and wider tires) increase the required steering effort. The traditional power steering systems are hydraulic systems equipped with an engine-driven pump and are called *hydraulic power assisted steering systems* (HPAS). Newer solutions provide the hydraulic pressure with the help of a pump that is driven by an electric motor. These systems are called *electro-hydraulic power assisted steering systems* (EHPAS). The major advantage of EHPAS systems over the conventional hydraulic systems is that the direct influence on the hydraulic pressure enables a more sophisticated assist strategy. Obviously, the direct use of an electrical motor coupled to the rack or to the steering column allows an even more flexible assistance. Another distinct advantage of these so-called *electric power assisted steering systems* (EPAS) lies in their fuel efficiency. HPAS and EHPAS systems both have to maintain a pressure in the hydraulics constantly but EPAS systems consume energy only during power assistance. On the other hand, EPAS systems increase the energy demand from the electrical network of the car and therefore the size of vehicles where they can be used is limited. The possibilities provided by EPAS systems enable the implementation of many unprecedented features such as variable assistance depending on the driving conditions (such as steering angle, steering angle velocity, speed and acceleration of the vehicle and each wheel and yaw rate) or the choice between different boost curves with the aim of optimizing the assist force depending on driving situation. With appropriate control algorithms, several novel functionalities can be achieved on EPAS systems e.g. the compensation of the effects of side-wind forces on the vehicle or park assist. All these features offer more and more comfort, security, and convenience for the driver but EPAS systems are also lighter and easier to package and install than the hydraulic alternatives. One example for the feedback control of an EPAS steering system is presented in [3] where the assist torque is determined by a two-degree-of-freedom control law. Nonetheless, logistical benefits can be still further increased allowing actuation independently of the driver. Steer-by-wire steering systems are the most common representatives of this freest type of steering assistance. In steer-by-wire steering systems, both of the steering wheel and the rack can move independently, both of them introduce one degree of freedom. Nevertheless, the critical role of the steering system in road vehicles is to make us appreciate that solutions are preferred where the mechanical link between the steering wheel and the rack is maintained to ensure system safety. Consequently, a solution is required where the degrees of freedom of the complete steering system is increased, however, the mechanical link between the components is preserved. Such steering systems are already available on the market where a planetary gear with two input shafts and one output shaft was added. One input shaft is linked to the steering column of a HPAS system, while the other engages with an additional electric motor. These steering systems are planed to make it possible to change the steering angle in addition to the angular position determined by the steering wheel with an extra steering angle generated by the electric motor, if necessary. In other words, the complete system is able to modify the steering kinematics, increasing or reducing the effective steering angle on the wheels, and to provide the correct steering ratio for the respective driving situation.

One of the most important producer of steering systems, namely ThyssenKrupp Presta Steering also took the decision to develop a similar system. From different possible mechanical setups, they chose one where the additional degree of freedom was introduced with the use of a harmonic drive built into the so-called *super imposed actuator* (SIA) unit. Also the steering system is called SIA steering system after the SIA unit. In contrast to the variable ratio steering systems based on HPAS systems, the base of the SIA steering system is an EPAS system and so not only on the additional electric motor but also on the rack servo motor it is possible to apply forces determined by a sophisticated control algorithm. For this reason a practical goal of my research was the achievement of the decoupling control for this steering system in order to decouple the steering wheel from the rack (with the steered road wheels) and thus the full realization of a virtual steer-by-wire steering system.

1.1 Research Methodology

The steering system can be modeled as a multi-body system where the motion of the system is constrained by kinematic motion constraints. The use of the harmonic drive in differential gearing configuration is a novel application. The mathematical modeling of the dynamics of this system implied the definition of a more general system class. I analyzed this more general case with regard to modeling aspects and modified system attributes such as controllability and observability.

With the results of this more general study, I gave the equations of motion of the harmonic-drive system in two setups. In the first setup, the harmonic drive is modeled as a holonomic scleronomic constraint that limit the motion three bodies (each with one degree of freedom). In the second setup, the harmonic drive is augmented with a spring that results in a generalized spring. Finally, I derived the nonlinear dynamical model of the new steering system of ThyssenKrupp Presta Steering that includes universal joints, as well.

I implemented the noninteracting control algorithms for all systems according to the steps in the modeling process. The accomplishments in the closed-loop performances were proven with simulation studies.

During my research, I worked in cooperation with the Advanced Vehicles and Vehicle Control Knowledge Center and ThyssenKrupp Presta Steering. The later parter offered me the opportunity to use their hardware devices such as a test bench, their *electronic control units* (ECU) with motor control software for the control of the electric motors mounted to the test bench and the rapid prototyping (hardware-software) tools of dSPACE (AutoBox and ControlDesk), which they apply for development and real-time product tests. The measurements for the identification of the parameters as well as for the verification of the closed-loop behaviour in the harmonic-drive system, were performed with the use of these pieces of equipment.

2 Summary of the New Results

2.1 Modeling of Constant Linear Constrained Mechanical Systems

Many processes can be described by linear second-order multi-variable differential equations of the form $H\ddot{q} + D\dot{q} + Sq = Fu$. The most common examples of processes to be modeled with such differential equations are mechanical and electrical processes. The analogy between them is given in Table 1 for the coefficient matrices on the left-hand side and the coefficient F is called input mapping matrix. The motion of such systems is often constrained by holonomic scleronomic linear constraints of the form $Cq = 0$. The equations of motion of a mechanical setup with

Notation	Mechanical interpretation	Electrical interpretation
H	inertia	inductivity
D	damping	resistance
S	stiffness	elastance
q	displacement	quantity of charge
u	force/torque	voltage

Table 1: Analogy between mechnical and electrical systems

a harmonic drive, which plays a central role in the SIA steering system, have the above form and the mechanical setup belongs to this system class.

Thesis Group 1. *I have defined a class of linear second-order multi-variable systems with constant coefficient matrices where the evolution of the configuration variables is restricted by linear holonomic scleronomic constraints. I have given the transformation for the elimination of constraint forces and redundant coordinates from the equations of motion. I have given a procedure for the determination of the transformation matrix. I have given the projection for the calculation of the constraint forces, too.*

Related publications: [6, 8, 10]

Thesis 1.1. *I have shown that the redundant variables and the constraint torques in the equations of motion of the systems in the defined system class can be reduced with a linear transformation applied to the input mapping matrix and congruent transformations applied to the other matrices. The matrix of the transformations is built of the column vectors that span the right null space of the coefficient matrix C in the kinematic constraints.*

Related publications: [6, 8, 10]

Thesis 1.2. *I have given bounds for the eigenvalues of the transformed matrices with the help of a set of basis vectors in the right null space of the coefficient matrix of the kinematic constraints. I have given a procedure, as well, that builds the matrix of the transformation to the reduced form, which transforms the inertia matrix to the identity matrix.*

Related publications: [6]

Thesis 1.3. *I have shown that the closed form of the constraint forces in this system class is given by a linear transformation that is a projection. The matrix of the* *projection can be expressed with the inertia matrix and the matrix of the kinematic constraints.*

Related publication: [6, 8, 10]

2.2 Controllability and Observability of Constant Linear Mechanical Systems

Underactuated mechanical systems are systems that have less number of actuators than degrees of freedom. The variable gear ratio steering system of ThyssenKrupp Presta Steering, namely the SIA steering system has 3 degrees of freedom but only 2 electric motors are mounted to actuate the whole system. Accordingly, the SIA steering system is an underactuated mechanical system. The question naturally arises, under which conditions the whole system is controllable. Another economic question, which arises in the industrial production concerns the minimal number of actuators (sensors) such that the system is still controllable (observable) and the objectives of the control can be fulfilled. This thesis group addresses controllability and observability properties of the system class defined in Thesis Group 1 with a special approach.

Thesis Group 2. *I have given lower bounds for the number of actuators (sensors) in constant linear mechanical systems such that the system is controllable (observable). The analyses are based on the Hautus rank tests [Hau69].*

Related publications: [6]

Thesis 2.1. *I have shown that the minimum for the number of linearly independent inputs (outputs) in a controllable (observable) state-space model is the maximal number of the linearly independent eigenvectors of the system matrix belonging to one of its eigenvalues. I have given a procedure that terminates with an appropriate input (output) matrix.*

Related publications: [6]

Thesis 2.2. *I have shown that the number of the independent torque inputs (position outputs) in a controllable (observable) constant linear mechanical system is not less than* $\max_{\lambda}\{n - \operatorname{rank}(\lambda^2 H + \lambda D + S)\}$ *where* n *stand for the degrees of freedom of the system and the maximum is taken over all eigenvalues of the system matrix of the state-space model. Obviously, at least 1 torque input (position output) is required always for the controllability (observability).*

Related publications: [6]

Thesis 2.3. *I have shown that constant linear mechanical systems with pure damping (i.e. zero stiffness matrix) are controllable (observable) if and only if the number of the independent torque inputs (position outputs) equals the number of the degrees of freedom.*

Related publications: [6]

Thesis 2.4. *I have shown that the number of the independent torque inputs (position outputs) in a controllable (observable) constant linear mechanical system is at least the highest multiplicity in the eigenvalues of the stiffness matrix.*

Related publications: [6]

2.3 Control of the SIA Steering System

The SIA steering system of ThyssenKrupp Presta AG is a multi-body system with special kinematic constraints. The aim of the preceding thesis groups was to create a framework for the modeling and analysis of mechanical setups such as the SIA steering system that makes the development of a control algorithm possible. This thesis group applies the previous results to the SIA steering system.

Thesis Group 3. *I have given the equations of motion of the SIA unit. I have identified the parameters of this harmonic-drive system and achieved the objectives of the steering system application on this system. After the augmentation of the harmonic-drive system with a spring, I accomplished the decoupling control of the resulting underactuated system. I have given the nonlinear model of the SIA steering system including universal joints and I have developed the noninteracting control for it.*

Related publications: [1, 2, 4, 5, 7–10]

Thesis 3.1. *With the application of Thesis 1.1, I have given the dynamical model of a multi-body system where three bodies, each with one rotational degree of freedom, are connected by a harmonic drive. With the use of Thesis 1.3, I have expressed the constraint torques in the system. I have identified the frictional parameters assuming linear (viscous) and nonlinear (Coulomb) friction terms based on measurements. With the use of the identified parameters, I implemented a feedback law that controls the angular position of one axis and achieves the actuation of another axis with* *an additive torque term (force feedback for the driver). I have verified the control law with simulation analysis as well as with measurements on the test bench of ThyssenKrupp Presta Steering.*

Related publications: [2, 4, 7, 9]

Thesis 3.2. *I have shown that the harmonic drive with a spring on one of its axes can be modeled as a generalized spring that actuates all of the connected three bodies. I have achieved the decoupling control [Gil69, GP69] of this underactuated mechanical system with two outputs. I have verified the closed-loop performance with simulation study.*

Related publications: [1, 5]

Thesis 3.3. *I have given the nonlinear model of the complete SIA steering system including universal joints. I have shown that this model is differentially flat [FLMR99, FLMR95, Lan03] but the flat output does not coincide with the output, which corresponds to the control objectives. After linearization, I developed the non-interacting control law for this underactuated system and I verified the robustness of the control algorithm with a simulation study with respect to parameter changes that increase the modeling error arising from the nonlinearities.*

Related publications: [8, 10]

3 Conclusion

The dissertation summarizes the results of the research work towards the accomplishment of the decoupling control for the SIA steering system of ThyssenKrupp Presta Steering in order to obtain a virtual steer-by-wire steering system with the maintenance of the mechanical link that can provide system safety if the control system collapses because of a system failure.

The kinematic properties of the components in this mechanical setup implied the introduction of a system class that led to more general theoretical results in the field of mathematical modeling and analysis of the controllability and observability properties. These results were successfully applied to the SIA steering system and to other similar mechanical setups.

Publication of New Results

[1] László Lemmer. The decoupling of a harmonic-drive-spring system for position and torque control on two different axes. In *Proceedings, 15th Mediterranean Conference on Control and Automation*, Athens, Greece, June 2007.

[2] László Lemmer. Hullámhajtómű-rendszerek modellezése és szabályozása. In *Proceedings, Tavaszi Szél 2007 Conference*, Budapest, Hungary, May 2007.

[3] László Lemmer, István Jánosi, and Bálint Kiss. Two-degree-of-freedom controller design for electrical steering systems. In *Proceedings, microCAD 2004 International Scientific Conference*, Miskolc, Hungary, March 2004.

[4] László Lemmer and Bálint Kiss. Modeling, identification, and control of harmonic drives for mobile vehicles. In *Proceedings, IEEE International Conference on Mechatronics*, Budapest, Hungary, July 2006.

[5] László Lemmer and Bálint Kiss. Decoupling of harmonic drive systems in differential mode configuration. In *Proceedings, 8th International Carpathian Control Conference*, Štrbské Pleso, Slovak Republic, May 2007.

[6] László Lemmer and Bálint Kiss. Underactuated mechanical systems with constant linear constraints. *Systems Science*, 2008. submitted for publication.

[7] László Lemmer and Bálint Kiss. *Intelligent Engineering Systems and Computational Cybernetics*, chapter Control of Differential Mode Harmonic Drive Systems, pages 223–233. Springer-Verlag, 2009.

[8] László Lemmer and Bálint Kiss. Noninteracting control of a steering system. *Periodica Polytechnica*, 2009. accepted for publication.

[9] László Lemmer, Bálint Kiss, and István Jánosi. Modelling, identification, and control of harmonic drives for automotive applications. In *Proceedings, 10th International Conference on Intelligent Engineering Systems*, London, UK, June 2006.

[10] László Lemmer and Bálint Kiss. Modeling and control of a steering system including a universal joint and a harmonic drive. *European Journal of Mechanical and Environmental Engineering*, 1:7–13, 2009.

References

[FLMR95] M. Fliess, J. Lévine, Ph. Martin, and P. Rouchon. Flatness and defect of nonlinear systems: Introductory theory and examples. *International Journal of Control*, 61(6):1327–1361, 1995.

[FLMR99] M. Fliess, J. Lévine, Ph. Martin, and P. Rouchon. A Lie-Bäcklund approach to equivalence and flatness of nonlinear systems. *IEEE Transactions on Automatic Control*, 44(5):922–937, 1999.

[Gil69] Elmer G. Gilbert. The decoupling of multivariable systems by state feedback. *SIAM Journal on Control*, 7(1):50–63, 1969.

[GP69] Elmer G. Gilbert and John R. Pivnichny. A computer program for the synthesis of decoupled multivariable feedback systems. *IEEE Transactions on Automatic Control*, 14(6):652– 659, December 1969.

[Hau69] M. L. J. Hautus. Controllability and observability conditions of linear autonomous systems. In *Kon. Ned. Akad. Wetensch.*, volume 72 of *A*, pages 443–448, 1969.

[Lan03] Béla Lantos. *Irányítási rendszerek elmélete és tervezése II.* Akadémiai Kiadó, Budapest, Hungary, 2003.