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Real-Time Motion Planning for Nonlinear Nonholonomic Mechatronic Systems Using Time-Scaling

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Nemlineáris anholonom mechatronikai rendszerek valósidejű mozgástervezése időskálázás használatával

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PhD Thesis

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1 Introduction, objectives

Several technical products and processes in the area of electrical and mechanical engineering are showing an increasing integration of mechanics with digital electronics and information processing. This integration is realized between the components (hardware) and the information-driven functions (software), resulting in integrated systems often called mechatronic systems. Their development involves finding an optimal balance between the basic mechanical structure, sensor and actuator implementation, automatic information processing and overall control [Ise08].

Given an overall goal and based on the sensed information an appropriate action has to be determined which will serve as reference. The calculation of this reference is usually referred to as trajectory or motion planning [Lat91, LaV06]. Once the desired behavior is defined, a control algorithm is used to ensure that behavior. The sensing, motion planning and tracking control methods for mobile robots are parts of the navigation task.

Motion planning is an important subtask in the navigation of autonomous robotic systems. Several robots are designed to move and operate without the help of a human operator hence they are required to be able to plan their motions themselves. This is solved by the motion planning.

To solve the trajectory planning problem, several information have to be given or estimated. One needs to know the properties of the system (e.g. the motion equation of the robot), its environment or, alternatively, the robot has to be able to map it during its operation. The task is now to plan an appropriate motion between two given configurations such that some additional constraints (e.g. collision avoidance, maximal allowable control signals) are satisfied as well. The motion is defined by a path geometry and by a velocity profile which describes the time distribution along the reference path.

The ideal planning method satisfies several properties:

- It always finds the solution if it exists.
- If there exists several solutions, it chooses the optimal one according to some cost function (e.g. the shortest path or the path with the lowest energy cost).
- It determines if there is no solution.
- It calculates a feasible path, i.e. all the given constraints are satisfied.
- It can calculate the reference path in real-time.

Some of the properties above are conflicting with each other in particular in the case of nonlinear and/or nonholonomic systems [Dul98, Blo03]. (Roughly speaking, a mechanical system is said to be nonholonomic if the vector space of the possible motion directions at a given configuration is restricted such that the restriction cannot be converted into an algebraic relation between configuration variables. Different kinematic models of a car are typical examples of nonholonomic systems since the direction of the car displacement is determined by its orientation.) It is usually time-consuming to calculate an optimal solution such that all constraints are satisfied. If an algorithm with less computational demand is selected, it is probably not able to find always a solution which fulfills all the requirements or the method is not able to find any solution at all in a complicated problem.

Due to the several practical applications and the difficulties mentioned above, the motion planning problem nowadays remains a popular research area. Its results are used in practice. For example one of these popular applications is the intelligent vehicle control.

The main goal of my work is to give effective, real-time motion planning methods for some classes of problems, which can be applied in specific applications as well.

The motivation behind my research work was given partly by a practical problem, namely we wanted to control a commercial passenger car in real-time. Due to the properties of the vehicles, I focused on nonlinear and nonholonomic mechatronic systems. I address two problems in the thesis: the development of motion planning algorithms for nonholonomic robots which can be used in real-time and the elaboration of adequate control strategies to perform special tasks.

1.1 Motion Planning Task

The exact formulation of the motion planning task is the following [LaV06]:

- A world (or workspace) $\mathcal{W}$ is given (e.g. $\mathcal{W} \subseteq \mathbb{R}^2$ or $\mathcal{W} \subseteq \mathbb{R}^3$).
- Obstacles are defined in the world. The subset of the workspace, which is occupied by the obstacles is denoted by $\mathcal{O} \subset \mathcal{W}$.
- A rigid robot or a collection of m links move in the world $\mathcal{W}$. $\mathcal{A}$, respectively $\mathcal{A}_1, \mathcal{A}_2 \ldots \mathcal{A}_m$ denote the sets in $\mathcal{W}$ occupied by the robot, respectively by the links. ($\mathcal{A}$ and $\mathcal{A}_i$ are finite, simply connected and closed in $\mathcal{W}$.)
- The configuration of the robot is described by a set of variables. The admissible values of these variables, denoted by $\mathcal{C}$ span the configuration space of the robot. The mapping $\mathcal{A} : \mathcal{C} \to \mathcal{W}$ gives the points of the robot in the workspace for a given configuration in $\mathcal{C}$. (For a planar robot one may choose $\mathcal{C} \subseteq SE(2)$. $SE(2)$ denotes the special Euclidean group in the plane.) Let $\mathcal{C}_{obs}$ (obstacle region) denote the image of $\mathcal{O}$ under the inverse mapping of $\mathcal{A}$. From this, the free space $\mathcal{C}_{free}$ is defined as $\mathcal{C}_{free} = \mathcal{C} \setminus \mathcal{C}_{obs}$.
- The motion of the robot is described by the kinematic equation $\dot{c} = f(c, u)$ where $c \in \mathcal{C}$ is the configuration of the robot and u is the applied control signal.

- The distance of two configurations is given by a distance metric: $\mathcal{D} : \mathcal{C} \times \mathcal{C} \to [0, \infty)$.
- The motion starts at an initial configuration: $c_I \in \mathcal{C}_{free}$.
- The goal configuration, respectively goal region, is also given: $c_G \in \mathcal{C}_{free}$, respectively $\mathcal{C}_G \subset \mathcal{C}_{free}$.
- A continuous path $\Gamma : \mathbb{R} \to \mathcal{C}$ is said to be a trajectory if

$$\Gamma = \{\Gamma(t) | \forall t_i \in [0, T_f], \Gamma(t_i) \in \mathcal{C}_{free} \text{ \& } \Gamma(0) = c_I \text{ \& } \Gamma(T_f) = c_G \text{ (or } \Gamma(T_f) \in \mathcal{C}_G) \text{ \& } \exists u, \frac{d\Gamma(t)}{dt} = f(\Gamma(t), u)\},$$

  where T_f denotes the traveling time during the motion and u is the applied control signal.
- Γ may not exist if the initial and final configurations cannot be connected.
- Some other constraints could be defined as well such as
  - Bounds on control inputs (e.g. $u_{min} \leq u \leq u_{max}$)
  - Geometrical constraints (e.g. $h_i(c) \neq 0$)
  - Nonholonomic constraints (e.g. $h_j(c, \dot{c}) = 0$)
  - Kinodynamic constraints (e.g. $h_k(c, \dot{c}, \ddot{c}) \leq 0$)
- A trajectory $\Gamma(t)$ is said to be feasible if it satisfies these constraints.

In [CR87] it is shown that some typical motion planning problems (e.g. shortest path problem according to the euclidean metric in the 3 dimensions between two points amid polyhedral obstacles or the 2 dimensional dynamic motion planning problem with bounded velocity) are NP-hard. Therefore it is worth studying the planning problem only for specific systems instead of giving a general solution or to use probabilistic methods to reduce the computational demand.

1.2 Research Methodology

The motivation of my research was given by a practical application. The goal was to plan a path and to design a tracking controller for a passenger car. This special system led me to study the nonlinear nonholonomic mechatronic systems.

A part of the research was done with the Advanced Vehicles and Vehicle Control Knowledge Center, where I was able to cooperate with some industrial partners (mainly with the ThyssenKrupp Presta Hungary Kft).

The research had several phases. After the literature overview, novel methods were developed. The effectiveness of these algorithms were shown by theoretical results, and both in simulations and real application.

The research was started by the overview of the literature. A short summary of it is presented in my dissertation. Several methods were studied, their advantages and imperfections were collected focusing first of all on the requirement of the real-time calculation. My goal was to extend the existing algorithms or to work out novel path planning methods, which can be used to calculate a feasible reference motion for nonholonomic systems in general and for car-like mobile robots in particular in real-time. To design novel methods I exploited the advantages of the existing solutions.

The properties of the designed algorithms were analyzed by the methods of control theory and mathematics. For example the computational demand, the modified system attributes (e.g. controllability, stability) were studied.

All the designed algorithms were implemented. Their effectiveness were also analyzed in simulations. The algorithms were implemented in the MATLAB/Simulink environment. To try the applications in a real-time environment the rapid prototyping tools of dSPACE (AutoBox and ControlDesk) were used according to the wish of our industrial partner.

Some novel results were also analyzed in a real application. A commercial Ford Focus type passenger car equipped with an Electronic Power Assist Steering (EPAS) system was the test car. The implemented methods were run on an AutoBox which was connected to the CAN (Controller Area Network) bus of the vehicle.

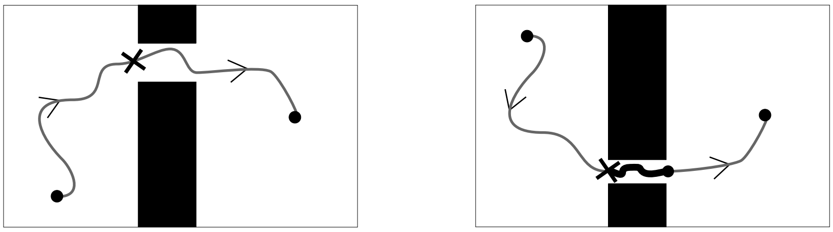
2 Summary of the New Scientific Results

2.1 Probabilistic Motion Planning Methods for Nonholonomic Mechatronic Systems

The literature suggests to use probabilistic methods if real-time path planning is required for nonholonomic systems. These algorithms build a graph in the configuration space of the robot, the nodes of this graph are selected randomly. In such cases the motion planning problem can be solved by searching for a path between two nodes in the graph.

In this thesis group the Rapidly Exploring Random Tree (RRT) [LaV98] is considered. This algorithm is a randomized path planning method specifically designed for robots with nonholonomic constraints. First, I studied how this method can be applied for such systems, where the configuration space can be decomposed into submanifolds (strata) such that different motion equations hold true for each stratum (e.g. walking robots). Such systems are called stratified systems [Goo97].

In special situations (e.g. moving through a narrow passage) the convergence of the RRT



(a) Key configuration (b) Key section

Figure 1: A narrow passage with key configuration or key section

can be poor. I worked out an extended version of the RRT, where key configurations or key sections are used for path planning (see Figure 1).

Another version of the RRT method was also developed to modify the time distribution along the reference path in case of moving obstacles. If the motion of the obstacles are known, the reference velocity profile can be changed using time-scaling, such that the robot is able to avoid collisions (see Figure 2).

Thesis Group 1 *Based on the probabilistic RRT algorithm, I developed new effective motion planning methods for moving obstacles and/or for stratified systems.*

Related publications: [1, 8–10, 13]

Thesis 1.1 *Based on the RRT algorithm, I worked out a novel probabilistic motion planning method for nonholonomic mechatronic systems by introducing key configurations and key sections in the configuration space. I showed that the convergence of the RRT algorithm is not corrupted by using the key configurations and key sections, moreover, there are complex environments where my method finds a path although it is not possible for the RRT algorithm with the same parameters. The key configurations and key sections increase the effectiveness of the algorithm in complex subspaces where the known obstacles are located close to each other. I gave a detailed evaluation and comparison about the computational demand of the RRT and my algorithm.*

Related publications: [8, 9]

Thesis 1.2 *I showed for the stratified nonholonomic systems that the motion planning method given in Thesis 1.1 with appropriate modifications can be used for the motion planning problem specified in the bottom stratum. I defined elementary displacement primitives and corresponding control inputs such that they connect two points of the bottom stratum (passing through the points of some higher dimensional strata). I showed that the properties presented in Thesis 1.1 (convergence, computational demand) hold true in the case of the stratified systems as well.*

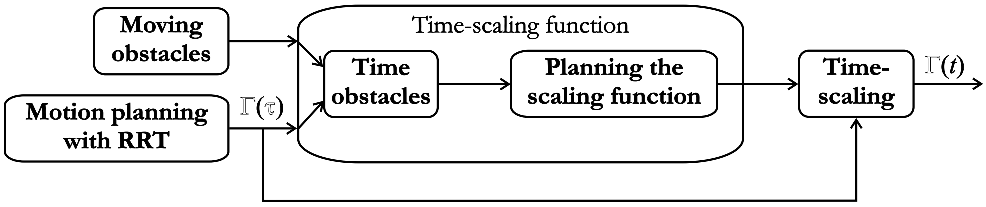


Figure 2: RRT based motion planning with moving obstacles using time-scaling

Related publications: [1, 10]

Thesis 1.3 *Based on the RRT algorithm, I worked out a motion planner for dynamic environments which contain known moving obstacles as well. For single-tree algorithms, I defined a distance like quasimetric which takes the traveling time and the maximal allowable input signal also into account. I showed that this quasimetric fulfills the triangle inequality. For multi-tree RRT methods, I worked out a time-scaling based motion planning algorithm where the bounds of the control input signals influence the form of the time-scaling function.*

Related publication: [13]

2.2 Planning of Continuous-Curvature Paths for Special (Car-Like) Nonholonomic Robots

A general path planning method was discussed above which is able to solve the path planning problem for any nonholonomic system. Now the task is to get a more effective planning algorithm but only for a special class of systems, namely for car-like robots.

The goal is to construct a motion planning algorithm which can calculate a trajectory in real-time and takes the typical constraints of car-like robots (e.g. minimal turning radius, continuous-curvature) into account.

My work is mainly based on the continuous-curvature steer method [FS04] which calculates a path using clothoid arcs, hence the robot does not have to stop to change the orientation of the front wheels. The restriction in this method is that the initial and final configurations are not arbitrary. The curvature has to be zero there (see Figure 3(a)).

In this thesis group I study how to generate continuous-curvature paths for car-like wheeled mobile robots joining arbitrary initial and final configurations (also with nonzero initial and final curvatures). The task is to plan a feasible trajectory which satisfies the equations of the kinematic model and the bounds on the variables (upper bounds on the curvature and curvature derivative). An example is depicted in Figure 3(b).

A method that allows changing in real-time the time distribution along the path while leaving unchanged its geometry was also studied. Such a time-scaling method is used to reduce computational demand.

Thesis Group 2 *I gave a real-time solution for the continuous-curvature path planning problem of car-like mobile robots between arbitrary initial and final configurations.*

Related publications: [12, 14, 15, 18, 19]

Thesis 2.1 *I worked out a novel, low level motion planning method for wheeled mobile robots between two arbitrary configurations (containing arbitrary curvatures) where the curvature varies continuously along the calculated reference path. The method takes the upper bounds on the curvature and curvature derivative into consideration. I defined some path planning primitives such as straight lines, circular arcs and general continuous-curvature turns. I gave the equations describing the geometry of the primitives, using Fresnel integrals for continuous-curvature turns. I gave the set of the Simple Continuous-Curvature paths (SCC-paths) which can be constructed from these primitives. I showed that using the SCC-paths, the low level planning algorithm can be solved for high level motion planning methods (e.g. the Probobilistic RoadMap).*

Related publications: [14, 15, 18, 19]

Thesis 2.2 *I worked out a real-time method for the calculation of continuous-curvature paths with the method presented in Thesis 2.1 such that the approximation of the Fresnel integrals is based on a precalculated lookup table. I showed that the whole path can be determined using elementary and trigonometrical mathematical operations based on the values stored in one single lookup table. I gave an upper bound on the precision error of the real-time calculations depending on the resolution of the lookup table containing the Fresnel integral values. I showed that the constant reference velocity, which was selected to reduce computational demand, can be modified using time-scaling.*

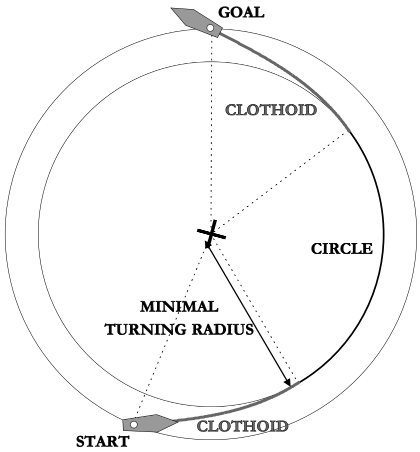
Related publications: [12, 15]

2.3 Planning of Time Distribution to a Pre-designed Path Using Time-Scaling

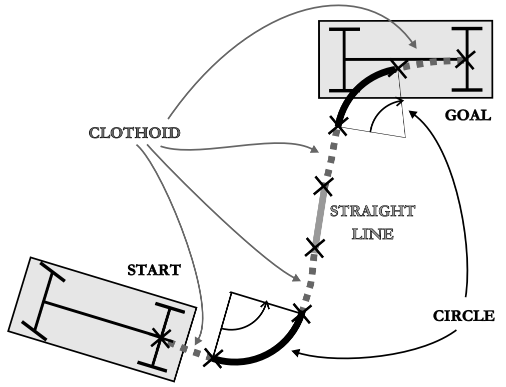
Some motion planning methods separate the planning of the path geometry and the designing of a velocity profile (i.e. the time distribution) for the path. The goal of this separation is usually to reduce computational burden.

If no time distribution was calculated for the reference path or the designed velocity profile has to be modified during the motion, one should use a velocity planning method which is able to calculate the modifications without changing the geometry of the reference path. This task can be solved by the time-scaling [Hol84, DN90].

In this thesis group a novel on-line time-scaling concept is presented. My time-scaling function is selected based on the closed loop behavior of the system.



(a) CC-steer. The start and the goal curvatures are zeros.



(b) Path for parallel parking with nonzero initial curvature

Figure 3: Continuous-curvature path

First, the tracking error is used to modify the time-scaling function, which influences the time distribution along the reference path. This method can be used both for robot manipulators and car-like robots.

The other method uses the time-scaling to transform a system. (This system is the single input kinematic car.) The transformed system can have some useful properties which can be used in the tracking controller design. This time-scaling function is also determined based on the closed loop behavior of the system (see Figure 4).

Thesis Group 3 *I developed new methods for the modification of the time distribution along the reference path. The time-scaling function is determined based on the closed loop behavior of the system and/or depending on some external signals. I demonstrated that using the time-scaling, the system can be transformed such that it has useful properties (e.g. differentially flat whence feedback linearizability) which can be exploited during the controller design.*

Related publications: [2–6, 11, 14, 16, 18]

Thesis 3.1 *I worked out a novel adaptive method which modifies on-line the velocity profile of a reference path with given geometry, based on the tracking error. I gave different methods for the selection of the time-scaling function (piecewise constant, constant-linear, or hyperbolic time-scaling functions). I demonstrated that the developed method can be used in closed loop control as well and it may accelerate or decelerate the velocity of the reference depending on the tracking error. I demonstrated on the kinematic model of a car-like robot that the time-scaling of the reference does not change the controllability of the system obtained by the linearization of the tracking error dynamics.*

Related publications: [2, 11, 16]

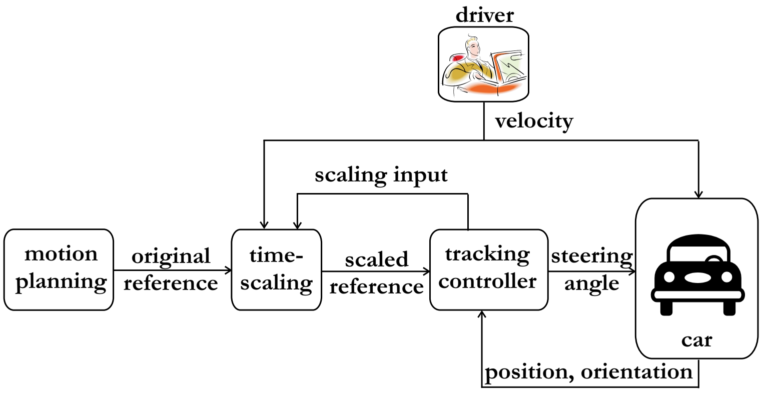


Figure 4: Control loop including time-scaling

Thesis 3.2 *I defined a new class of transformations based on time-scaling such that the relationship between the old and the transformed times is determined based on the states of the system and additionally on a new time-scaling input. The scaling input can be calculated by the tracking controller in closed loop based on the tracking error and it can influence the modification of the reference velocity profile. I showed that using a transformation which belongs to this class, the single input kinematic model of a car can be linearized using a state feedback, i.e. it becomes differentially flat.*

Related publications: [3–6]

Thesis 3.3 *I demonstrated on the single input kinematic model of a car that using the transformation given in Thesis 3.2 one may design a tracking controller. I gave the equations of the controller such that the time derivatives of the vehicle velocity are required. I showed that the usage of these time derivatives can be eliminated by the linearization of the error model while local stability can be still guaranteed along the reference path.*

Related publications: [3–5, 14, 18]

3 Application

Some of the results presented in this thesis were applied and tested in a real application in the field of automatic vehicle control [19] in the Advanced Vehicles and Vehicle Control Knowledge Center in cooperation with ThyssenKrupp Presta Hungary. The goal was to build a parking assist system (PAS) for a passenger car with a human driver. The system is able to find a feasible parking place and to control the steering system to assist the parking manoeuvre. The driver generates the longitudinal velocity of the car by an appropriate management of the pedals (throttle, brake and clutch).

To ensure autonomous behavior, several tasks have to be solved. These tasks are performed by separate interconnected subsystems [17] which are depicted in Figure 5.

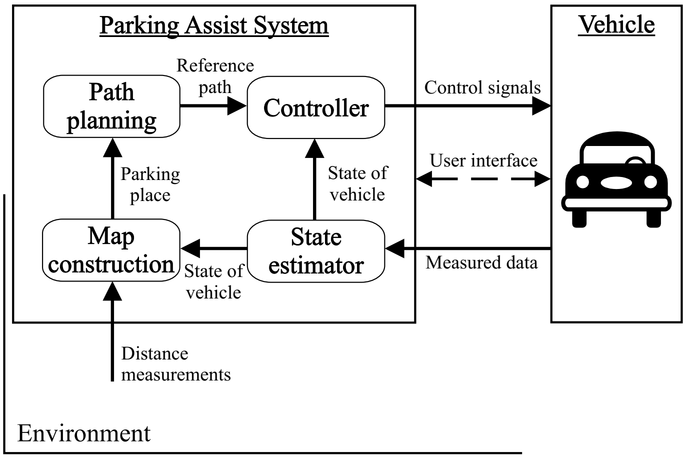


Figure 5: Components of the Parking Assist System

The ABS (Anti-lock Braking System) sensors of the vehicle can detect the displacement of the wheels of the car. Based on these data, an estimator calculates the actual position and orientation of the car in a fixed word coordinate frame. This estimated state is used by the map construction and controller modules. To draw a map, additional data is also required about the environment. Ultrasonic sensors are used to measure the distances to the surrounding obstacles. Based on these distance measurements a map can be created. One may then use simple algorithms to detect accessible parking places (if any) on the map [7].

During the motion planning, a reference path is calculated which connects the initial and the desired final configurations in one step (i.e. without changing the driving direction). In this planning phase some constraints (e.g. the nonholonomic behavior described by the model, collision avoidance, maximal values of the actuator signals) have to be taken into consideration. The continuous curvature path planning method presented in Thesis 2.1–2.2 is used to solve this problem.

Finally, the tracking control algorithm is used to track the reference path. My tracking control method with time-scaling is used here (see Thesis 3.2–3.3).

The components of the parking assist system were tested separately and together both in simulations and in the real environment. The developed methods worked sufficiently. The integration of the whole system and the final tests are in progress.

4 Conclusions

In this thesis motion planning and tracking methods for mechatronic systems were discussed. Particularly, the goal was to give real-time solutions for nonlinear nonholonomic systems (e.g. for passenger cars or car-like robots).

Time-scaling was used in the different methods for different reasons. First the probabilistic motion planning problem between moving obstacles was solved with the time-scaling method (see Thesis 1.3). Next, time-scaling was also used to separate the planning method into a path and a velocity planning part, to reduce computational complexity (see Thesis 2.1–2.2).

I developed some methods, where the time-scaling depends on the closed loop behavior of the system. The rate of time-acceleration can be influenced by the tracking error and/or by some external signals (see Thesis 3.1–3.3).

The algorithms were implemented and their effectiveness were analyzed in simulations. Some of my novel solutions were integrated into an automotive application.

Publication of New Results

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