PhD dissertation
-Thesis Book-

**Localization Error Based
Near Time-optimal Path Planning Process,
for Mobile Robots**

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I. Introduction

Almost all living beings possess some navigation skills which then depending on the level of intelligence can be instinctive or conscious. Some navigation schemes used in mobile robot techniques in many cases have been developed on the basis of observing the navigation schemes of several animals (see BUG algorithms [1], Ant system algorithms [4]). Designers are trying to create more and more perfect algorithms for automatically guided devices, only the question remains, what is perfect?

If we take into account the human sense organs, we will see that sensors used in robotics in large measure exceed the borders of human sensing, and nevertheless a human being is orientating better and faster than a highly developed mobile robot equipped with super high-tech sensors. The answer is hidden in the difference between human thinking and *algorithm-based* planning. The autonomously operating mobile robots function with a *decision mechanism* which can select the right algorithm only from a given set of algorithms.

I believe that the level of a robot's intelligence is highly determined by the set of algorithms it works with as well as the development state of these algorithms.

Within robotization, the branch of science dealing with mobile robotics has experienced progressive development over the last decade. We started with vehicles running on fixed path and have arrived to autonomous guided vehicles (AGV) running in an unknown environment. These "equipments" – in order to be able to provide their best operation –, are supported by the highest technology of our days. For travelling, the robot has to be equipped with some locomotion (legs, wheels, belt, etc.), and for its driving we need highly precise motors and driving systems. Now, after having obtained all of these things, the robot is able to move, however it still misses "intelligence".

The intelligence of the robot is created by a control system with several feedbacks, which system can be organized hierarchically, as follows:

The main components of a mobile robot system can be:

physical components:
- the locomotion and the body of the robot
- external and internal sensors, motors, power supplies
- the robot's vision system (if we separate it from the sensory system)
- processors, memories and communication units

logical components:
- the control module of the locomotion
- the module for evaluating sensor data
- behaviour selection module
- path planning and path optimization module

From the list above, each part could be a separate, independent field of research. Moreover, in some parts several good solutions are hiding, and only during further research it appears which method was/will be the most powerful for the different applications.

To find the most efficient method and to spare some money and time, we have to prepare the **model** of the given system. In the field of mobile robot research, the exact and accurate models play a very important role in the developing of new path planning algorithms and navigation schemes. Thanks to these models, we are able to refine the instruction sequence of navigation schemes, we can produce various analyses and on their basis we can determine localization check points, we can propose different marker arrangements, or suggest different localization or sensory systems.

The subject of the dissertation:

The evolutional direction of mobile agents travelling in free space of the environment is tending from the agents controlled by people, to the fully automatic AGVs. If we want to create some categories of this development, we can classify it into three categories: *1.)* Agents controlled by humans; *2.)* Half-autonomous agents (human + machine control); *3.)* Fully autonomous agents. As for the theme of this dissertation, it belongs to this 3rd category.

The main subject of my research is to develop such a path planning process (algorithm) which starts at the positioning accuracy of the marker-based navigating systems, and in a priori known environment with the user's given *maximal localization error* determines the **navigation area**[1] (*NA*) of the environment, and then based on directed graphs builds up the so-called **graph-map**[2]. After that, the algorithm according to specified optimization conditions searches for the optimized trajectory between the given start and goal positions on the above-mentioned graph-map, then *constructs* the final trajectory on the chosen *graph-like path*, in form of a *B-spline* curve.

The proper operation of the algorithm and the correctness of my theses I am trying to justify through a simulation system, built up and run on the mathematical world model of the environment. The modelling is created by a geometric representation of the environment (see later, Work Space - *WS*), with the necessary decomposition. As a consequence, some forms and expressions appear in discrete forms in the dissertation, and the inaccuracy of the model also has to be taken into account, depending on its decomposition.

The thesis book is divided into three main chapters and its sub-chapters. In the first chapter we can find an introduction and a general overview of the theme, while in the second we can read about the methods and resources of research, with the third part discussing new scientific results. The third chapter is divided further into sub-chapters, where the chapter first discusses the thesis, while the appropriate sub-section gives a detailed explanation. In accordance with this, the theses in this

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[1] *Definitions see in chapter III.2.*

[2] *Definitions see in chapter III.3.1.*

dissertation are *consecutive*, *see figure below*. Each sub-thesis ends with an own list of selected publication connected with the topic, and at the end of the thesis book a list of bibliography can be found.
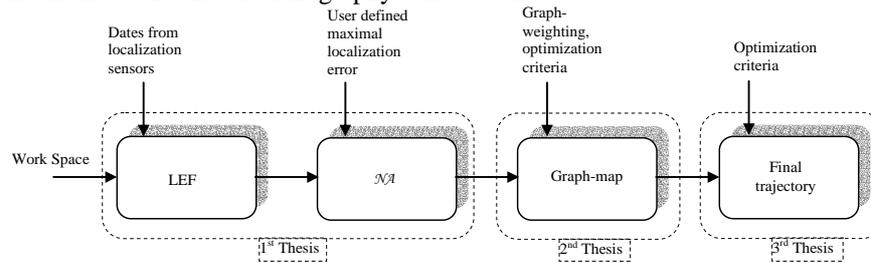

I.1. Motivation and Aims

In order of time, I began my researches with the study of the mobile robot's vision systems, and then I continued with the marker-based path planning methods in known, resp. unknown environments, its modelling, and finally based on the obtained experiences I endeavoured to produce a complex path planning process based on determined maximal localization uncertainty.

I.1.1. Motivation

Until now, the region where the mobile agent[3] was capable to travel has been established as the result of subtraction between the areas of the "workspace" minus "configured obstacles[4]". This region is known as "*reduced or **aligned free space***" (*AFS*) [2]. Where can be the problem? Let us suppose the following path planning methods: *BUG* algorithm and *TBA* algorithm [1], *SFP* algorithm [3], path planning algorithms based on *visibility* graphs, etc. The common feature of these algorithms is that each of them plans its path very close to the obstacle. In these cases, if we do not consider the localization accuracy, it can happen that the mobile agent will meet obstacles. It is true that later, in some cases, some percentual safety zones were added to the configured obstacles, but this was only an estimated value and not an exact one, which at critical scenes like narrow passages could be substantial.

This deficiency will be eliminated by the *E-Zone*, my own development. Thanks to its help, with the *given marker configuration* and *measuring system* I can definitely and exactly tell where and in what degree I have to increase the configured obstacles, moreover it discovers inside the *AFS* the so-called "black holes", where the mobile robot is unable to execute localization with expected accuracy. Further advantage is that the safety zone developed by me (E-Zone) is not uniform around the obstacle, but depending on localization accuracy it can be narrow/wide, which can be a crucial point during the path planning process (narrow passages).
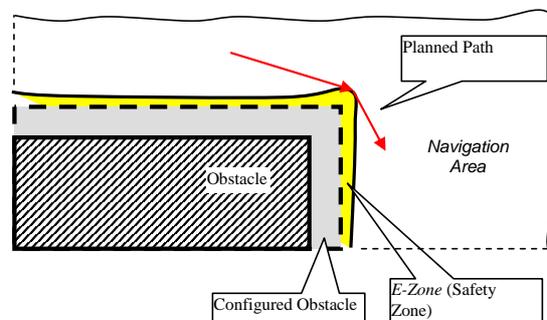
In connection with the path planning process I have the following two competitive motivations.

- The *first* is concentrating on eliminating the lack of previous processes, namely the case of weighted graphs where the developers didn't consider the dynamical properties of the path. They generally weighted only the edges of the graphs and ignored the vertices [4]. By omitting this, they left the turning radius, and the dynamics and complexity degree of the selected routes out of consideration.

- The *second* motivation is trying to exploit the advantages of Spline-like path planning [5] in such a way that besides the near time-optimal, extreme accelerations free trajectories based on the optimization criteria, also the safest or the fastest path is found.

I.1.2. Aims connected with mobile robot localization

It is well known that the difference between the planned and real position is called position error, which can derive from inaccuracies appearing during the execution of localization, to which the the systematic errors derived from locomotion are added, or other non-systematic errors, such as slipping, etc.

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[3] autonomously travelling independent mobile robot
[4] obstacle increased by "r" (where *r* is the radius encircled around the mobile robot)

Because of the limited size of the dissertation, I have no chance to analyze all the measuring systems performing positioning, this is way in modelling I have selected the one which includes some practical measuring, and that is the *marker based positioning* with *optical measuring* head. My aim is to create such a localisation error model, alongside the existing ones [6], [7], which is for the given measuring head gives the veritable localization error and its correction. As for the errors derived from the dynamical properties of the mobile robot (slipping), I will not deal with them in this model.

Under the localisation accuracy I mean the *accuracy of measuring* between the differences of the real and estimated position of the robot. This measuring is very important in the path planning process, because all other calculations are based on these data.

My further aim with the developed model is in case of a given marker configuration to determine the expected localisation uncertainty *in each point* of the AFS, and from these uncertainties (given by *x,y* coordinates of the WS) to create the so-called *"Localisation Error Field – LEF"* covering the whole AFS. The LEF is nothing else but a localisation error map over the AFS, and by this map I have an opportunity to tell:

- the expected localisation error anywhere over the AFS

- in the measuring: from which marker pairs I can expect the minimal localisation error.

I.1.3. Aims connected with path planning

The present dissertation is talking about a new, until now not used path planning method. My aim, referring to path planning, is to create a path planning method whith starts out from the data of LEF and the user-given predetermined maximal localisation error (MLEF=ε_{max}), on the basis of which a navigation area (*NA*) of the WS is created, with a final route generated on it, which can be the *safest* or the *fastest* route between the *start* and the *target* positions, based on optimization criteria. The essential questions of the path planning method are that with given MLEF and given marker-configuration:

- Does any path exist between the *start* and the *target* positions?
- If it does not exist, what is the reason for it?
- If there exists more than one route, which of them has the minimal complexity, which is the fastest and safest, and which is the near time-optimal path?

II. Methods and Resources of Research

At selecting the methods of research I tried to select such method by which the "level of intelligence" of the given agent can be increased, based on the created *model*. If I want to increase the level of intelligence of anything, it is very important to known the present one. Thus, in the early phases of my research I had been occupied by the survey of the referred and hopefully complete literature. After this followed the comparison of the studied systems and methods, based on different criteria and also highlighting its weaknesses or strengths. After analyzing these features and considering the projects running in our laboratory (*LABRADOR, SENATOR* [T1-5]) in those times could come the planning of the *simulation system*, the way I imagined it.

II.1. Applied methods

During my research I elaborated the mathematical model of device used for distance measuring, which is a *rotating measuring head* equipped with a laser-light source and a CCD video sensor (CCD Camera+laser beam, this system in literature is known as "Light striping" method). This system with the active triangulation determines the distance between the given target point and the centre of the measuring head. In my case the target point is a special marker located at a known place of the environment. The special marker ensures the most precise reflection of the emitted laser beam.

For lack of full and detailed knowledge of the system, it is impossible to create the mathematical model of the measuring head and the proper *error-function* belonging to it. First, I have had to recognise what kind of errors I have to take into account, and then I have to work out the suitable error-function, and finally check the proper operation of the model. The most typical parameters of the CCD video sensor are: *focal length*, *resolution of the CCD chip*, pixel fault of the CCD chip, optical distortion of the camera. Typical parameters for laser-light source: *angle resolution* of the rotating laser-light source, errors originating from laser beam dispersion. Another important parameters, linked with the rotating measuring head is the *distance* between the CCD video sensor and the laser-light source. The analysis of the mentioned parameters can be seen in the appropriate chapters of the dissertation.

II.2. Applied resources

As mobile robotics is a relatively new, though rapidly developing field of research, the adequate literature required for the research was obtainable only from special conferences, proceedings, journals or overseas literature. Without any order of importance I can mention the American *MIT*, *Stanford*, *CalTeh-JPL*, *Cornell* universities, as well as European universities such as *IPR* from Karlsruhe, *USATU* from Russia, and *TU* of Poznan, with which we have a joint project named *Copernicus*. Additionally, I followed the mobile robot researches at the technical universities of Porto (*Machado*), Wien (*Elmenreich, Kopacek*), Zagreb (*Ribaric*), Bratislava (*Frankovic*), and Ljubljana (*Zlajpah*) with attention.

With regards to the image processing and robotization I have obtained very much from the courses held on *BUTE*, mostly in the fields of robot control[5], computer graphics[6], and machine vision[7]. In the *MoMic* laboratory at the BUTE I had the

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[5] prof. dr. Béla Lantos, prof. dr. János Somló

possibilities to verify and check the correctness of my theoretical results and experimentation by measurements.

III. New Results and Theses

In this section the theses will be shown and in the sub-sections their detailed description is given.

III.1. First Thesis

Thesis 1: *I have developed a **method** for ensuring the obstacle-free area in the WS, based on **user defined** special requirements, which method consists of a new **model** producing n a localisation error field and an **algorithm** making use of this model.*

Thesis 1.1: *I have developed a new **model** for determining the expected value of the localisation uncertainty for the current position of the mobile robot. The initial set of the procedure are apart from specifications of the sensory system (features and deficiencies of the sensors used for localisation) the description of the WS with the marker's displacement; and the procedure results in the **Localisation Error Field** (**LEF**) of the environment.*

Thesis 1.2: *I have developed a new **algorithm** for determining the continuous navigation area obtaining obstacle-free trajectories, while keeping to special user-defined requirements, which algorithm is starting from the LEF and user-defined requirements (maximal localisation uncertainty referring to the different localisation points/regions of the WS ($\varepsilon_{max(i)}$)) and creates the **continuous navigation area** ($\mathcal{NA}$) of the WS.*
The $\mathcal{NA}$'s ensure the possibilitiy of obstacle-free path planning, while maintaining the special conditions (keeping the maximal localisation uncertainty) created by the user.

Definitions and expressions occurring in this thesis:

<u>*Definition*</u>: *The Localisation Error Field (LEF) is a scalar function with two variables [$\varepsilon = f(x,y)$], which with the developed error function gives on the "z" coordinate the localisation uncertainty – belonging to the current point of the planar WS given by (x,y) coordinates –, measured from the selected marker pairs.*

<u>*Definition*</u>: *The navigation area (NA) is a set of points of AFS given by (x,y) coordinates, for which is valid that the measured localisation error (ε) is less or equal than the user-defined maximal one (ε_{max}). $\{\varepsilon \leq \varepsilon_{\max}\}$; $\{NA \subseteq AFS \subseteq FS \subseteq WS\}^{8}$.*

<u>*Definition*</u>: *Let be signed by $\mathcal{NA}$ the **continuous navigation area**. Then $\mathcal{NA}$ is accessible for **R** mobile robot if: for ($\forall x, y \in \mathcal{NA}$) is valid that any "x" can be connected with any other "y" (where $x \neq y$) in such a way that each point of the connecting line or curve is an element of $\mathcal{NA}$, moreover, if we are working with a non point-represented robot, the following condition has to be valid: $d(\overline{\mathcal{NA}}^{(i)}, \overline{\mathcal{NA}}^{(i+1)}) > 2r$, where "r" is a radius of the circle around the mobile robot.*

III.1.1.   Detailed explanation of thesis-1

In the localisation process of an AGV, what plays a very important role is the distance measurement from the predefined points (*markers*) of the WS. This measurement has to be the most accurate, because on the basis of this is determined the deviation from the planned path and henceforth the correction, moreover this gives the basis for all calculations connected with navigation. In case of known localisation sensory system and marker's displacement it is possible to create the *Localization Error Field* of the WS. This is the subject of the first part of the 1st thesis. The second part describes an algorithm which determines the area where the *user-defined maximal localisation error*, like the demand, is valid.

III.1.1.a. Detailed explanation of thesis 1.1

In this sub-thesis I am investigating – through a mathematical model of a digitalized workspace equipped by an artificial marker's system –, the conditions of accuracy connected with distance measurements. The device providing distance measurement in mobile robotics is generally a rotational measuring head with a given absolute/relative error.

In the devised simulation system, the systematic errors of the previously mentioned rotational measuring head will be analyzed. The analysis, with a little modification[9], can be applied for the different types of measuring systems. The error-function is proportional to the area of localisation uncertainty derived from positional inaccuracy.

$$H^{(i,i+k)} = \left(\int_{x_1}^{x_3} g_{i(j)}(x)dx + \int_{x_3}^{x_4} g_{i+k(j)}(x)dx \right) - \left(\int_{x_1}^{x_2} g_{i(j+1)}(x)dx + \int_{x_2}^{x_4} g_{i+k(j+1)}(x)dx \right);$$

where: *i, k={1,...,max. nr. of markers}, j={1,...,4},*

Additional topics of the 1st sub-thesis:
 1.1   Justification of the correct operation of the stated error-function.
 1.2   Establishing the margin-conditions of the measuring, namely between what limits I can use the developed error-

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[6] prof. dr. László Szirmay-Kalos
[7] dr. László Vajta
[8] FS- Free Space, AFS – Aligned Free Space, WS – Work Space
[9] different sensory system ⇒ different error-function (generalization)

function.

1.3 Concerning the shape and size of the localisation uncertainty, studying the possible substitutions, for the sake of a simpler calculation of the size the localization uncertainty (substitution with a parallelogram).

$$H_{\tau}^{(i,i+k)} = \frac{\mu_i \mu_{i+k}}{\sin \beta} = \frac{(\pm \rho_i).(\pm \rho_{i+k})}{\sin \beta};$$

1.4 I have developed a simulation system which on the basis of the created error-function constructs the Localisation Error Field (LEF) of the workspace with any kind of configuration.

In the possession of this knowledge has the result of the 1st sub-thesis been completed, which is the digitalized LEF of a geometric model of an optional WS. The initial parameters are the point represented mobile robot, the distance errors of the rotational measuring head, the WS with given marker's arrangement, obstacle dimensions and localisation. The results are the LEF with maximal/minimal localisation's error belonging to the given x, y position of WS, the marker pairs from which we can expect the minimal/maximal localisation error, and the correction value derived from the localisation error. The construction of the LEF, with the appropriate correction of the error-function for the given sensory system, can be applicable for any sensory system.

Selected publications linked to the sub-thesis 1.1

[T11-1] I. Nagy, L. Vajta, Cs. Gürtler: *"Vision based navigation for multi-agent mobile robot system in the transport domain"* / Proc. INES 97, IEEE Int. Conf., Budapest, 1997.

[T11-2] I.Nagy, L. Vajta: *„Trajectory Planning Based on Position Error Analysis and Fault Area Modeling"* /Proc. ICAR2001, IEEE Int. Conf. Hungary, Budapest, 2001.

[T11-3] I. Nagy: *„Marker-Based Mobile Robot Positioning in Respect of Displacement of the Markers* ", /Proc. INES'02, IEEE, Int. Conf. on Int. Eng. Systems, pp 63-68, (Opatija), Croatia, 2002./

[T11-4] I. Nagy, A.L. Bencsik: *"Fault Analysis of Mobile Robot Positioning in Respect of Amount and Displacing of the Markers",* /Proc. RAAD '02, International Workshop on Robotics in Alpe-Adria-Danube Region, (Balatonfüred) Hungary 2002.

[T11-5] Nagy I.: *„Mobilis robotok pozíciómérési hibaelemzése a markerszám és a markerelhelyezés figyelembevételével (Senator és LabRador projektek)* " /Hungelektro 2002, Nemzetközi Elektronikai Tehnológiai Szakkiállítás és Konferencia, 2002. ápr. 23-25, Programfüzet, CD-ROM/

[T11-6] I. Nagy, könyvfejezet: *„Intelligent Systems at the Service of Mankind",* Volume 1, ISBN 3-935798-25-3, pp.:99-109, cikk: *„Marker-Based Mobile Robot Positioning in Respect of Displacement of the Markers",* Publisher: Ubooks, Germany, 2004.

III.1.1.b. Detailed explanation of thesis 1.2

In this sub-thesis a newly defined navigational area of the WS will be introduced providing free travel for the mobile robot, based on user-determined demand-system concerned in maximal localization error.

The 1st input function of the sub-thesis is the user-defined maximal localisation error ($\varepsilon_{max(i)}$), while the 2nd one is the previously determined LEF ($LEF=\varepsilon$). Since the input functions belong to the same WS, by joining them I can get their mutual situations. From this situation it is possible to determine the *continuous navigation area* ($\mathcal{NA}$) of the aligned free space of the WS, where is valid the user-defined demand-system, respectively, the *Error Zones* (*E-Zone*) unsuitable for navigation.

$$\forall x, y \in AFS \text{ where } i \neq j : \begin{cases} \varepsilon \leq \varepsilon_{\max(i)}; & \text{Area suitable for path planning : NA} \\ \varepsilon > \varepsilon_{\max(j)}; & \text{Area non-suitable for path planning : E-Zone} \end{cases}$$

Selected publications linked to the sub-thesis 1.2

[T12-1] Nagy, L. Vajta: *„Trajectory Planning for Multi-agent System Based on Traffic Density and Position Error Analysis"* /Proc. INES 2000., pp: 261-265, IEEE Int. Conf. (Portoroz) Slovenia, 2000.

[T12-2] I. Nagy, A.L. Bencsik: *„Model-Based Path Planning Algorithm in Respect of the APF and AEF of the Environment* ", /Proc. INES'03, IEEE Int. Conf. On Int. Eng. Systems, pp: 551-556, vol. 2., (Assiut-Luxor) Egypt 2003.

[T12-3] I. Nagy: *„Path Planning Algorithm, Based on User Defined Maximal Localization Error*",/ Periodica Polytechnica, BUTE, Hungary, 2005.

III.2. Second Thesis

Thesis 2: *I have developed a new procedure suitable for generating oriented weighted-graphs, which starts at the continuous navigation area ($\mathcal{NA}$) of the WS, producing an oriented weighted graph, where the weighting of the **edges** are **determined through a polynomial**, its members being determined on the basis of user demands, while the weighting of the **nodes** is determined by the **decomposition of the nodes**. The level of complexity of the certain routes is given by adding the final weighting of segments and nodes participating in the selected route.*

Definitions:

Segment's end point: (*Graph node*) *This is the starting or ending point of the segment, or; at the meeting point of two segments, the end point of the 1st segment and the starting point of the 2nd segment, which points are in coincidence with each other.*

Cross point: (*Graph node*) *Meeting point of more than two segments.*

Edge: (*Graph edge*) *A line, connecting two segment's end points, or cross points.*

III.2.1. Detailed explanation of thesis-2

The weighted graphs are generally used in mobile robotics where the weighting is established concerning the length of the path-segments, which resulted in omitting the dynamical properties of the trajectory [4].

In this thesis I will show how we can consider the dynamical properties of the practicable routes, by the disintegration of the graph-nodes of the graph built up on the AFS of the WS. As the oriented weighted graph is constructed on the $\mathcal{NA}$ of the workspace, it contains the really travelable routes in form of network of oriented weighted road segments (path) and nodes. This is where the expression "*graph-map*" originates from.

The formal mathematical description of the graph-map constructed by me is:

$$H = \left\langle \{M_1, M_2, G\}, C \right\rangle ;$$

what is a hybrid map (*H*), composed of two metric (geometric) maps (M_1, M_2) and a topological map (*G*). Where M_1, is a geometrical map of the WS, M_2 is a geometrical map of the $\mathcal{NA}$, and *C* represents the connection between the different maps. The *G* topology I have received by the linearization of the *Voronoi* diagrams of $\mathcal{NA}$.

Weighting of the edges – In contrast with weighted graphs used until now, where only the edges have been weighted based on the length of the segments (path) and in this way some other important parameters have been left out of consideration, I introduced a *polynomial* for the edge's weighting, where the numbers of the members are not fixed and can be changed based on the optimization requirements of the user. The first step at the weighting is to determine the *optimization conditions*, and then assume the factor of significance ($w_{(n)}$) to each member. Let us take an example for better comprehension. Let us weight the $S_{(i)}$ segment (edge) of the graph-map, where the optimization criteria are: the length ($S_{(i)}L$), the accuracy respect of localisations ($S_{(i)}ARe$), and the traffic density ($S_{(i)}TrD$). In this case for the weighting of $S_{(i)}$ can be written:

$$Ws_{(i)} = w_1 S_{(i)}L + w_2 S_{(i)}ARe + \ldots + \ldots + w_{(n)} S_{(i)}TrD$$

Weighting of the nodes – Based on previous definition the graph nodes are the segment's end points and the cross points of the graph. The novelty of my graph weighting is that the dynamical properties of the path are taken into consideration. With this procedure we can consider the complexity of the planned path that is the relative incidence and sharpness of the turns. The procedure is the following: ***decomposing the graph-nodes*** based on Figure TF-1, and calculating the weights of the participating *sub-segments* signed by "*n*". In this way, each cross-point will have ***multiple weighting***, from which the final weighting will be created during the path planning (depending on the sharpness of the turn). The weighting of the sub-segments is derived from angle (α) enclosed between two consecutive segments (edges), based on the following. Let the angle be α, be valid between $[0^0, 180^0]$, and besides this it has to be valid that sharp-angled turns have to have higher weighting than blunt-angled ones, and straight path have to be zero weight. Consequently, the weighting of the "*n*" signed sub-segment of the Jth node, in knowledge of the angle α, is:

$$Wn_{(J)} = w_n |180 - \alpha^0_{(i,j)}|$$

where the indexes *i,j* indicate the numbers of entering and passing segments of the node. After the weighting of the segments and nodes of the graph-map, and choosing the *start* and *goal* coordinates, it is possible to calculate the *cost function* of each route between the mentioned coordinates. From the calculated cost-functions, with selecting the minimal one, we can get the route with minimal complexity (for more explanation, see chapter III.3.).
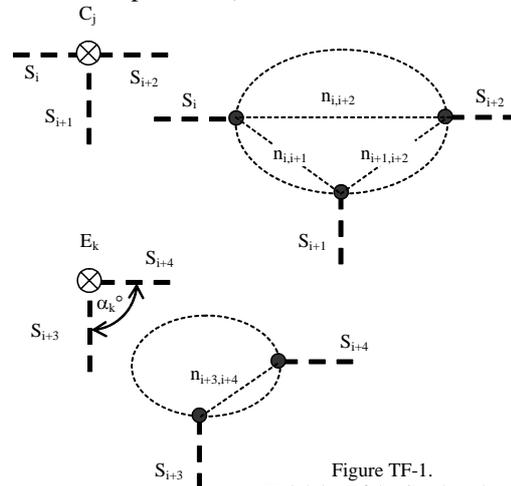
Figure TF-1.
Weighting of the Graph nodes

Selected publications linked to the thesis 2

[T2-1] I. Nagy, L. Vajta, Cs. Gürtler: *"Trajectory Planning for Mobile Robots Based on Dynamical Models"* / Proc. INES 97., IEEE Int. Conf., Budapest, 1997.

[T2-2] I. Nagy L. Vajta: „*Local Trajectory Optimization Based on Dynamical Properties of Mobile Platform*", /Proc. INES'01, IEEE, Int. Conf. on Intelligent Eng. Systems, pp 285-290, (Helsinki) Finland 2001.

[T2-3] I. Nagy, L. Vajta: „*Path planning algorithms for mobile robots equipped with navigation sensors*", Proc. of the INES 99, IEEE Int. Conf. on Intelligent Engineering Systems, ISBN 80-88964-25-3, pp: 43-50, Slovakia (Stará Lesná) 1999.

III.3. Third Thesis

Thesis 3: *I have developed a new B-spline based path planning algorithm which along the selected route (based on optimization criteria) constructs the final trajectory in such a way that considering the velocity of task-executing and the safety of the path it gives the near-optimal solution, while the condition for the algorithm is that each point of the final trajectory (B-*

spline) has to be inside of $\mathcal{NA}$, and the final trajectory has to ensure the unbroken run of the mobile robot, with eliminating the extreme linear or angular accelerations, in contrary to so far used brokenfull procedures.

III.3.1. Detailed explanation of thesis 3

The B-spline-based path planning was the subject of more publications [5], but nowadays it somehow seems to have been pushed into the background. In spite of this fact it can be particularly important for the elimination of the extreme linear and angular accelerations of the mobile robot and from the aspect of improving the dynamical properties of the mobile robot's motion. Computer graphics and the fast growth of computer animation opened a new area in this type of motion planning.

Calculation of the cost function

Let $S,C \in \mathcal{NA}$, $(S \neq C)$ where S is a starting and C is a goal coordinate. I search for the graph-nodes visible from S and C ($SG_{(g)}$, $CG_{(h)}$) and connect them appropriately with segments ($SSG_{(g)}$) ($CCG_{(h)}$), and then perform the weighting of the given segments:

$$w_{(g)}SSG_{(g)} = w_{(g)}SSG_{(g)}L + w_{(g)}SSG_{(g)}A\,\mathrm{Re};$$
$$w_{(h)}CCG_{(h)} = w_{(h)}CCG_{(h)}L + w_{(h)}CCG_{(h)}A\,\mathrm{Re};$$

where g is the number of graph-nodes visible from S, and h is the same from C. Between the $SG_{(g)}$ and $CG_{(h)}$ graph-nodes a *network of routes* is formed, which is nothing else but an oriented graph between the nodes $SG_{(g)} \rightarrow CG_{(h)}$. It is possible to calculate the cost functions of all the possible routes between the mentioned graph-node pairs, and store the results in the *matrix of network routes*, with dimensions $g \times h \times k$, where k is the number of possible routes between two selected $SG_{(g)}$, $CG_{(h)}$ graph-node pairs. The route, with minimal cost function between the $SG_{(g)}$, $CG_{(h)}$ graph-node pairs, gives us the pair of two significant nodes SG_{min} resp. CG_{min}. Between them, inside of the route's network, is the route with minimal complexity. From now on, the calculation of the relevant cost function between the S and C coordinates, with given g, h and k, is composed from the following components:

$$KF = w_{(g)}SSG_{(g)} + w_{(h)}CCG_{(h)} + \underbrace{\left(\sum_i Ws_{(i)} + \sum_J Wn_{(J)} \right)\Bigg|_k}_{\substack{\text{Weighting of the selected route} \\ \text{inside of the route's network}}}$$

In accordance with optimization conditions the selection of final trajectory will be performed on this graph-system.

Generating the safety path:

Because the network of the routes is *generated at the centre-line of $\mathcal{NA}$,* I am endeavouring to get to this route's network in *as short a route as possible*. In accordance with this the cost function is composed from these four components:

$$KF_{safe} = \underbrace{\min\!\left(w_{(g)}SSG_{(g)}\right) + \min\!\left(w_{(h)}CCG_{(h)}\right)}_{\text{The shortest path from S resp. from C to the network.}} + \underbrace{\min\!\left(\sum_i Ws_{(i)} + \sum_J Wn_{(J)} \right)\Bigg|_{SG(g)\to SG(min)}}_{\text{The shortest path between } SG_{(g)} \text{ and } SG_{(min)}} +$$

$$+ \underbrace{\min\!\left(\sum_i Ws_{(i)} + \sum_J Wn_{(J)} \right)\Bigg|_{CG(h)\to CG(min)}}_{\text{The shortest path between } CG_{(h)} \text{ and } CG_{(min)}} + \underbrace{\left(\sum_i Ws_{(i)} + \sum_J Wn_{(J)} \right)\Bigg|_{SG(min)\to CG(min)}}_{\text{weighting of the segments and nodes between } SG_{(min)} \text{ resp. } CG_{(min)}}$$

Generating the fast path:

Based on visibility, I want to get *as fast as possible* from the given S and C coordinates to the $SG_{(min)}$ resp. $CG_{(min)}$ graph-nodes.

$$KF_{fast} = \underbrace{w_{(g)}SSG_{(min)} + w_{(h)}CCG_{(min)}}_{\substack{\text{Weighting of the segments from S to } SG_{min}, \\ \text{resp. from C to } CG_{min}}} + \underbrace{\left(\sum_i Ws_{(i)} + \sum_J Wn_{(J)} \right)\Bigg|_{SG(min)\to CG(min)}}_{\text{weighting of the segments and nodes between } SG_{min} \text{ resp } CG_{min}} ;$$

Generating the curve:

The curve will be *based on optimization criteria,* and I will generate it along the selected graph-like route (given from the optimization criteria of the user) in such a way that the curve has to go through the nodes of the selected route, that is the curve of the safety path has to pass the graph-nodes of the route selected by KF_{safe}, and the curve of the fast path has to pass the nodes of the route selected by KF_{fast}. Besides these the *general conditions* for the curve are:
- has to contain S resp. C coordinates, in this way: $S,C \in b(t) \in \mathcal{NA}$;
- has to pass the graph-nodes of the selected route
- has to satisfy C^2 continuity
- has to stay inside of $\mathcal{NA}$, that is this has to be valid: $(\forall(x,y \in b(t)) \in \mathcal{NA})$

Recall that a spline b in B-spline form is given by:

$$b(t) = \sum_{j=0}^{m} b_j N_j^d(t)$$

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Suppose that the *configuration points* of the curve $[p_0,...,p_n]$ are given with their respective time values, at the graph-nodes of the selected route. I have to find the function $p_i(t)$ in such a way that it has to satisfy C^2 continuity that is in form of 3-degree polynomials.

$$p_i(t) = a_{3i}.(t-t_i)^3 + a_{2i}.(t-t_i)^2 + a_{1i}.(t-t_i) + a_0 = ? \quad ha \quad t_i \le t \le t_{i+1};$$

where „*a*" coefficients are the unknown. With introducing the simplifying $T_i = t_{i+1} - t_i$ can be written:

$$p_i = p_i(t_i) = a_0$$
$$p_{i+1} = p_i(t_{i+1}) = a_3.T_i^3 + a_2.T_i^2 + a_1.T_i + a_0 \quad ;$$
$$\text{---------------------}$$
$$p_i^{'} = p_i^{'}(t_i) = a_1$$
$$p_{i+1}^{'} = p_i^{'}(t_{i+1}) = 3a_3.T_i^2 + 2a_2.T_i + a_1$$

where, $p_i^{'}$ derivatives are the velocities. Because the C^2 continuity has to be valid:

$$p_k^{''}(t_{k+1}) = p_{k+1}^{''}(t_{k+1})$$

Expressing the second derivatives and simplifying we get:

$$T_{k+1}p_k^{'} + 2(T_k + T_{k+1})p_{k+1}^{'} + T_k p_{k+2}^{'} = 3[\frac{T_k}{T_{k+1}}(p_{k+2} - p_{k+1}) + \frac{T_{k+1}}{T_k}.(p_{k+1} - p_k)]$$

where $k=\{1,..,n-2\}$, but the number of unknown coefficients is: $i=\{1,..,n\}$. On the other hand, if we can state the starting and docking velocity of the curve ($p_0^{'}(t_0) = v_{start}$, $p_{n-1}^{'}(t_n) = v_{end}$), then the number of unknown coefficients will decrease by 2, that is the linear equation system will be determined. By solving this linear equation given in matrix form, the unknown derivatives $[p_0^{'},...,p_n^{'}]$ can be determined and substituted back, to define the segments and consequently the composite function $p(t)$.

$$\begin{bmatrix} 1 & 0 & .. & & & \\ T_1 & 2(T_0+T_1) & T_0 & 0 & .. & \\ 0 & T_2 & 2(T_1+T_2) & T_1 & 0 & .. \\ . & & & & & \\ ... & 0 & T_{n-1} & 2(T_{n-1}+T_{n-2}) & T_{n-2} & \\ ... & & & ... & .0 & 1 \end{bmatrix} \cdot \begin{bmatrix} p_0^{'} \\ p_1^{'} \\ p_2^{'} \\ . \\ p_{n-1}^{'} \\ p_n^{'} \end{bmatrix} = $$

$$\begin{bmatrix} v_{start} \\ 3[T_0/T_1(p_2-p_1) + T_1/T_0(p_1-p_0)] \\ 3[T_1/T_2(p_3-p_2) + T_2/T_1(p_2-p_1)] \\ . \\ 3[T_{n-2}/T_{n-1}(p_n-p_{n-1}) + T_{n-1}/T_{n-2}(p_{n-1}-p_{n-2})] \\ v_{end} \end{bmatrix}$$

Selected publications linked to the thesis 3

[T3-1] I. Nagy: "*Off-line Mobile Robot Path Planning Process with Given MLE and Constructing the Safety Final Route*", Proc. of the INES2005, IEEE Int. Conf. on Intelligent Engineering Systems, ISBN 0-7803-9474-7 (+CD), pp: 279-282, Greece, (Athen) 2005.

[T3-2] I. Nagy, A.L. Bencsik: "*A Global Path Planning Algorithm, Based on Maximal Localization Error*", /Proc. Jubilee Conf., Science in Engineering, Economics and Education, (Budapest) Hungary 2004.

[T3-3] I.Nagy: "*Genetic Algorithms Applied for Potential Field Building in Multi-Agent Robotic System*", Proc. of the IEEE Int. Conf. on Computational Cybernetics, ICCC2003, ISBN 963-7154 18 4 (+CD), pp: 105-108, Hungary (Siófok) 2003.

[T3-4] I. Nagy, A.L. Bencsik: „*A Simulation System for Behavior-based Potential Field Building in Multi-Agent Mobile Robot System*", Proc. of the 3rd IAESTED International Conference on Computational Intelligence, ISBN: 978-0-88986-672-0, (+CD), pp: 7-12, Canada (Banff-Alberta) 2007.

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