Connecting reservoirs and water power stations by Markov chains.

"Anybody who can solve the problems of water will be worthy of two Nobel Prizes, one for peace and one for science"

(President J. F. Kennedy)

"Water is a major challenge for the twenty-first century but presents itself as well as a catalyst for peace. Anyone who invests in water, invests in peace."

Water: At the crossroads of peace and science
by András Szőllősi-Nagy, Zaragosa 2004
Nyilatkozat  
(Declaration of authorship) 

Alulírott Gálai Antal kijelentem, hogy ezt a doktori értekezést magam készítettem és abban csak a megadott forrásokat használtam fel. Minden olyan részt, amelyet szó szerint, vagy azonos tartalomban, de átfogalmazva más forrásból átvettem, egyértelműen, a forrás megadásával megjelöltem.

A dolgozat bírálatai és a védőről készült jegyzkönyv a későbbiekben a BME Építőmérnöki Karának dékáni hivatalában lesz elérhető.


Gálai Antal 
(jelölt)
Connecting Reservoirs and Water Power Stations by Markov Chains

Theory, methodology, application

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Connecting Reservoirs and Water Power Stations by Markov Chains

Theory, methodology, application

Thanks is due to my excellent late teachers, professors at BME then JATE, and I express my separate gratitude to my vocational mentors, István Zsuffa and Pál Rózsa - engineers and mathematician scholars - for their cooperation and stimulation spanning over decades, and to my beloved family members who the burdens of my complex vocational and public life/civil manias borne.

PREVIEW

From 1973, based on studies carried out by Roux and Bernie[1], author took part in research activities regarding sizing reservoirs - that time of Fehérvárcsurgó, Hungary - initiated by István Zsuffa. Shortly after the first results we have extended our investigation to cover multipurpose reservoirs, secondary energy production. Based on the interaction between modelling achievements and real word applications, author targeted the core of the method thorough hydrological reformulation of the hydropower model. In this treatise a thorough review - worked out down to the level of engineering applications - of a quing theory model - attributable to a basic equation formulated independently by Australian Moran and Russian Savarensky - and files including Fortran sources and data of 19 examples are found hereby to be published electronically. In the first part original methods developed with my collaboration under recent decades is presented. I outline the construction and sample use of more numerical models implying a number of new scientific elements at the time of formation and still cited until today. In the second part - considered as appendix - a handfull of presented sample applications from three continents covere only a fraction of recent and past uses. Overseas publications and research reports reference to the research - paused for nearly two decades despite my intentions - even with the passing of this much time, quoted as practical one emerging from among the theoretical experiments: in "Comparative Evaluation of Generalized River/Reservoir System Models" by Ralph A. Wurbs at Civil Engineering Department Texas A&M University (Reference and Users Manual for the U.S. Army Corps of Engineers) : "In terms of practical usefulness, the most important storage probability theory models are described as probability matrix methods (McMahon and Mein 1986). Zsuffa and Galai (1987) address probability matrix methods from a practical applications perspective and provide computer programs for implementing the methods. Other methods are of theoretical interest." References to our preinternet publication can be found on URL scholar.google.com.

I spent one and a half decade with the development of the methods of storage, based on engineering knowledge. My role was not restricted to programming, it evolved in a big leap forward regarding the model's development and the examination of the numerical constructions following from. Emerging questions and doubts affecting the essence of the model arise. Responding to questions stemming from these concerns lead to the reformulation and development of both the model and the technical and computational algorithms stemming from it. New ideas, new elements differing from or complementing the international vocational fashions may be born from literary unawareness, as it is shown by the vocational welcome of our comprehensive handbook (Reservoir Sizing by Transition Probabilities, Water Resources Publication (WRP), Littleton, Colorado, USA ISBN - 0-918334-62-4,
In his 2005 book McMahon refers to our book registered in my copyright proportion of 73%, also his publisher recommends ours as a supplementary accessory to McMahon's. In that very same year of 2005 besides our work having been referred to at the Universities of Athens Technology, Texas A&M, and in the Encyclopedia of Water (Wiley, New York), and in a U.S. Army Corps of Engineers research report, the catastrophe analyses made in 1974 and 1982 plus my water power calculations rewarded me as a co-author with the title of "Best Scientific Publication of the 20th century in Mongolia" by the "Great Khural State", the Mongolian State Assembly / Parliament on the recommendation of Scientific Academy of Mongolia. There should be something time resistant. Now, that I settled down to summ up an up to date revision wearing the wind of change and wishing to serve the survival of the method and the knowledge accumulated, my role in reformulating Moran's model will be accentuated in a numbered list bellow following this personal retrospection. In the framework of the dissertation - both for the data increase of the time passed and for the informatics environment totally changed in the meantime - I reviewed the descriptions of the models and examples to prepare reconfiguration of the newer variants of the self-developed softwares for the sake of the next bigger technological migration to meet requirements of up to date conformity regarding networking practice, standard office automation and near future IT development. During gathering, storing, running downstream and putting on power generation, water does not only evaporate, roll stream deposit, but winter by winter due to the considerable cooling of the environment forms ice on the surface and transports it. Most fundamental feature of ice phenomena is ice cover rate, determination of which could happen by digital (web)cameras or mobile phones as well. With this water related computer vision application I enhance my reservoir software package withstanding the test of time as prooved by professional literature. Present publishing of the method especially timely, the population explosion causing bigger water shortage than the climate change and the approaching energy dearth, all secure a more important global role for the storage.

Main objectives and methodology

In terms of practical usefulness, the most important storage probability theory models are described as probability matrix methods. We address probability matrix methods from a practical applications perspective and provide computer programs for implementing the methods. Methods of over sophisticated theoretical interest is not dealt with herein. Goal is matching water supply and water and/or energy demand. Waters, our renewable resource - becoming more and more allocated - fluctuate following the random processes of weather. Water users must adjust themselves to these stochastic processes which comprise a random sequence of highflow and lowflow periods. Based on simple quantitative assumptions and everyday theoretical and numerical basics, a tool for practical engineers and educators aimed to be delivered. This tool is a group of models for performing various analyses for both a single reservoir or some interconnected hydro utilities, even serving multi purpose by attenuating floods, delivering urban, industrial, agricultural supply and/or producing energy. For the selected type of Moran based model, problem identification and model formulation, data preparation and computational details with output presentation and evaluation of results is provided, the latter via examples.
Stochastic storage theory and related models have been addressed extensively in the research literature but applied very little by constructors and operators of reservoir systems. Group of analysis methods is based largely on the theory presented by Moran (1959) and expanded by Gould (1961), Klemes (1981), McMahon and Mein (1986). The objective of this type of stochastic storage theory models is to determine the probability distribution of reservoir storage. Storage probabilities may be computed at steady state or as a time dependent function of the starting conditions. Thus, for a given release policy and initial storage content, the probabilities of the reservoir being at various storage levels at future times during the next several months or several years may be estimated. As the analysis period becomes longer, the storage probabilities at a future time are no longer dependent upon the starting storage contents, a steady state condition is reached which is the base for reliability model for long-term planning of a multiple-purpose reservoir.

The stochastic storage theory models assess system performance based on describing inflows by a probability distribution or stochastic process. The methods typically applied to single reservoirs should also be developed into multiple reservoir analysis procedures. While inflows are assumed independent, through fitting/reading from historical streamflow record - probability distribution of reservoir levels are determined via Markov chain of a stochastic process. Based on this stored volume distributions, the storage versus yield function and corresponding reliability estimators are calculated. Discrete probabilities are used to approximate the continuous distributions of the inflow process. The assumption of first order Markovian processes for representing the inflow process of a reservoir has generally been considered in the literature as adequate for most purposes. Much of the work published in the literature represents modifications or extensions to the basic Moran and Gould models. Moran (1959) presents various procedures for determining storage probabilities. Numerous other authors have presented solutions or extensions to the basic models formulated by Moran.

Like all group of practical procedures our approach treat both time and volume as discrete variables. A reservoir is subdivided into a number of zones and a system of equations developed which approximate occurrence of the possible states of the reservoir storage. Two main assumptions can be made regarding the inflows and outflows, which occur at discrete time intervals. In a mutually exclusive approach, there is a wet period, with all inflows and no outflows, followed by a dry season, with all releases but no inflows. In the more general simultaneous model, inflows and outflows can occur simultaneously. How they change the basic equations and influence the results, how they relate is shown. This also open the way to instationer models applying the different approach within the yearly cycle of time periods based on the nature of inflows and demands of the time interval of a given sequence. The selection from the two approaches is depends on what the demand is for. Estimating energy production should fundamentally be treated in simultaneous model otherwise significant biased output expectations arise.

A nonsteady state analysis can be useful in developing and implementing reservoir operating plans in which allocations of water to alternative users are made at the beginning of each water year, each irrigation season, or other time period of interest, based upon the likelihood of water being available to meet the allocations during the time period. The likelihood of meeting the allocations would be based upon the reservoir storage levels existing at the time the allocations are made. Under this type of operating plan, during drought conditions, as significant reservoir drawdowns occur, the allotment of water to the various users for the upcoming irrigation season or other specified time period is reduced accordingly. Storage probability theory models provide useful information regarding the probabilities of the
reservoir being emptied by the end of the time period given the known present storage level and assuming different alternative withdrawal rates. Steady state probabilities are not dependent upon initial storage levels. In this case, storage probability theory models represent an alternative to regular simulation models, using period-of-record or synthetically generated streamflow sequences, for developing yield versus reliability relationships.

Estimation of water supply and/or power production are discussed below by using queuing theory. The relation between subsequent supply and release systems, and that of simultaneous ones is proven. A minimal approach uses historical yearly or monthly streamflow record and topographical map of the valley. Dam section site selection. Power plants supply a defined energy demand within limits of the turbine capacity. The energy demand driven reservoir model. Expected amount and distribution of shortages. Optimizations. Linear algebraic techniques.

As recent decades proved we have succeeded in giving into the hands of practicing engineers and educators a quite sophisticated versatile tool still having easy to touch relevance to the original physical problem. As in the models based on real technical problems, common sense is needed as the most important necessity, the arising models consist of the coincidence probabilities of independent events describing the phenomenon and quantities examined, resulting from convolution of values of discreet or supposed to be discreet distribution functions. Often plain, but not always obvious transformations of basic relations of the physical model - or algebraic equations stemming from them - turn into starting points of new solutions. At the end I summarize my new scientific results discovered in the course of research in 8 theses.

**Application**

Our research was primarily driven by the needs of practical applications which significantly promoted the prompt utilization of the - by nature - basic investigations. The dissertation's appendix consists of real examples from three continents, case studies illustrate the model suitability for the examination of reservoirs of any size, purpose or operational complexity. Furthermore resulting numerical methods are directly applicable on all such storage tasks where approximational approach of the independence of arriving inflow water quantities in subsequent time units is acceptable.

Hundreds of reservoirs - planned and existing - were investigated and designed by public domain sofware accompanying my publications. There were countless applications even in Hungary without notifying software and modell author. Software pirates also sold rewritten copies, but no modell tuning or development were done later by others except for some sedimentation experiencing (including "discovering" non existent ergodic reservoir state distribution of sedimenting reservoirs:). Extra European applications included a reported master theses in South Korea in late 80s, Nigerian examples in early 1980s, wide ranging Mongolian application of my packages covered from mid 1970s to mid 1980s, while Algerian applications were on top between 1984-1987. By public release of the source package in 1987, wide spread application evolved from then on.
Chapter 1

INTRODUCTION

The population explosion creates new water resources problems, especially in matching water supply and water demand. The world supplies of clean water are becoming more and more allocated for use. Water is a renewable resource. The surface watercourses provide 95 percent of fresh water. However, waters fluctuate following the random processes of weather. Water users must adjust themselves to these stochastic processes which comprise a random sequence of highflow and lowflow periods.

Initially, a simple adaptation to water regimes took place. For settlements man chose locations free from floods and those where drinking water was easily available. It was in the course of demographic explosion during the 18th and 19th centuries that the presently classic water management took its shape by means either of excavation of canals or diking of rivers. Channels of water courses were enlarged to assure a better conveyance of floods. The larger demands for water were met by means of water transfer to greater distances (Fig. 1).

Figure 1. Matching water supply and water demand by using either natural or regulated flows.

Alongside the demographic explosion of the 20th century, the development of water management involving storage has taken place. Instead of adaptation of demand to random processes of water supply, the transformation of water regime by storage has proved to be a reasonable solution. This is the only way to achieve a proper development of limited water resources available on the continents and islands, or the individual regions and countries. However, due consideration must be given to an ever increasing demand, and the control of losses from harmful aspects of waters (Fig. 2).
Water management using storage is an adaptation of random processes of water supply to human needs. Because of this random character, the unavoidable shortages in water supply during dry seasons are covered by stored waters of wet seasons. The storage reservoirs built exclusively for flood control are emptied immediately after the flood has passed in order to enable them to receive the next flood. In reservoirs serving water users, apart from the assurance of a generally low but guaranteed amount of water withdrawal, the water surplus in wet periods are retained for dry periods. While a flood retention reservoir is kept empty most of the time, those serving water users are kept as full as necessary and feasible most of the
time. Because of these contradicting operation strategies, a complex mode of operation must be applied. (Laszloffy, [13]).

The situation is the same in theory and practice with reservoirs for which a distinctly allocated storage space serves the water users. It is kept filled up as needed, while the retention of floods is assured by storage spaces above it. They are kept empty as much and as long as the downstream flooding criteria permit. These patterns of operation are mutually beneficial, affecting one another. A volume of water, retained from large floods and released from a reservoir, shortens the duration of water deficit period. Filling up a storage space for meeting high water demands may require the storage of entire flood hydrograph (Fig. 3). For both storage problems, numerical solutions can only be achieved through the analysis of random processes of water supply and often also of water demand.

Matching the water management requirements with the topographic and hydrologic characteristics of a water course is a problem of searching for the economic optimum. This optimum is the function of natural conditions and economic, social and environmental factors, (see e.g.: Prekopa [19]). While the natural average water supply conditions are stationary in the statistical sense, the social, economic and environmental factors, as well as the cost of construction and investment decisions may undergo an order of magnitude of change with time. Therefore, the optimum may be valid only for a short period of time. A characterization of natural conditions for "storage possibilities" in a region may be economically valid only for a finite, often relatively short period of time.

The content of this publication is the result of work carried out in treating conditions prevailing over a whole country, a region or a river basin. The investment decision maker or the design engineer is given the methods for hydrologic computations which provide the basis for the search of an optimum fitting the particular economic factors at the given time. Therefore, the result of hydrologic computations is not presented in the form of single values (e.g., the storage space necessary to produce the required amount of water release) but a function, in general the yield or release function of a reservoir (Mosonyi, 1948 [18]).

In the case of the reservoir which serves water users this function is given as \( K = f(M) \) where \( K \) stands for the storage capacity to be provided and \( M \) for the water demand to be met. Dealing with a reservoir for flood control, this function is given as \( K = f(Q_{\text{max}}) \), where \( Q_{\text{max}} \) represents the flood flow permitted to be released into the downstream river channel.

Because the stochastic process of water supplies is transformed by the reservoir, the function which represents this transformation can not be deterministic. In fact, in case of the reservoir which serves water users, with a capacity \( K \), the problem is to derive the storage yield function given as

\[
K = f(M, P) \quad (1-1a)
\]

where \( P \) is the probability of meeting water demand \( M \). This assurance relates either to time or to the ratio of delivered to required amounts of water.

For the flood retention reservoir the yield function is defined as

\[
(1-1b) \ K = f(Q_{\text{max}}, P) \quad (1-1b)
\]
Figure 3. Joint operation of reservoirs with relatively high capacity: (a) for various uses, and 
(b) for flood control.

where $P$ is the probability of exceedence of flood peak flow $Q_{\text{max}}$, supposed not to be 
exceeded. Such formulation of yield functions has been developed in the Central European 
practice (Klemes, 1976 [11]).
Both equations 1-1a and 1-1b can be rewritten as a series of conditional distribution functions, namely as

\[ F_1(x \mid y) = p(M < x \mid K = y) \]  \hspace{1cm} (1-2a)

and

\[ F_2(x \mid y) = p(Q_{\text{max}} < x \mid K = y) \]  \hspace{1cm} (1-2b)

Two methods may be followed in presenting the general solutions of the identical equations 1-1a and 1-1b, and 1-2a and 1-2b. For functions of Eqs. 1-1a and 1-1b the yield curves characterized by their different probabilities P, will produce a family of curves presenting the three variable function (P,K,M), with coordinates K, M. The family of curves of Eqs. 1-2a and 1-2b may be obtained by producing the common probability distributions in coordinates M and P, with K as the parameter.

To characterize the multipurpose reservoir cases, families of distribution functions are formulated as:

\[ F(x, y, z) = p(Q_{\text{max}} < x \mid K = y, M = z) \] \hspace{1cm} (1-2c)

where the safety (probability) of water supply, P(K,M), is also given.

In accordance with engineering requirements, the relationships are presented either by a three-variable function, or by co-axially connected graphs. The great number of graphs aid in applying the technique. Graphs are obtained by a computerized procedure assuring clarity of outputs.

After determining a storage capacity K the conditional distribution functions

\[ F(x \mid y) = p(\xi < x \mid M = y) \] \hspace{1cm} (1-3)

are computed for the detailed description of performance of reservoirs.

The examples, provided in the text are the proofs that the random variables investigated provide "description of performance" of reservoirs. They include: the event of depleting storage with the probability P(0)=P(\xi=0); limits in water deliveries \(\mu<M\); the event when a reservoir becomes full with the probability P(n)=P(\xi=K) The events with probability P(0)<P<P(n) give the degree of fullness of the reservoir. To characterize an existing or a planned reservoir with a fixed capacity, the use of Eq. 1-3 or the corresponding graphs is recommended.

By using the function of reservoir performance, detailed investigations may be carried out on evaporation losses from the reservoir, their impact on water yield, the impact of reservoir surface waves on the dam, and the accidental flood mitigation by a reservoir which serves various water users. With the aid of this performance function, the allocation of reservoir water to different users may be made taking into account various safety-of-supply requirements. To make such decisions the distribution function of water restrictions should be used.
Similarly, but based on relationships derived for random water withdrawals of known distribution and the performance function, detailed characteristics can be derived for two or more reservoirs or a chain of reservoirs which are linked either in parallel or in series and operated either free of each other or under the same control. The importance of a well-planned and optimized operational pattern is proportional to the importance of energy output by a reservoir, because the total water-equivalent energy in a reservoir is equal to the product of stored volume, power plant heads, and efficiency of the hydroelectric machinery.

Methods are presented in this text for computing: (1) power of a hydroelectric plant and (2) energy to be produced by a partial use of water surplus. Special minor adjustment makes the model suitable also for cases where a specific program for energy production should be fulfilled by a reservoir, namely the release M is not given as a water volume (with dimension of m$^3$ per year or per month) but as an amount of energy (in kWh per year or per month).

It is a property of the applied numerical method that the continuous processes are discretized. The accuracy will depend on the values of the chosen time interval $\Delta t$ and the water volume interval $\Delta V$.

An extended and in a certain sense generalized variation of the Moran's model is dealt with in this book. It is based on the probability distribution of independent water volumes entering the reservoir. The theoretical justification for the use of this method is that it is an effective and method. It is expected that its usefulness will come from the fact that publications are rare which treat the practical applications of this model. To show this aspect a set of corresponding computer programs and examples are included in the diskette enclosed to the book. These FORTRAN programs, also useable on personal computers, are supplemented by detailed explanations. The text is completed by this diskette including outputs (readouts, printouts) obtained for examples analysed in details, and by graphs plotted using there computer outputs. The examples are given in variations, occasionally in a simplified form. They cover the practical problems encountered by writers while working in Europe, Africa and Asia.

The objective in writing this book is not to cover a number of different methods, (see [16],[22],[30]...). but rather to treat one method in detail from the practical point of view. The applicability is the major concern, with the software needed to assure it in using the computers for computations of various variables related to operation of water storage reservoirs.
Chapter 2

APPLICATION OF THE MORAN’S MODEL

2-1 Basic Assumptions and Relationships

The input discharges \( I(t) \) which enter the reservoir continuously in time are analyzed by discrete series of time intervals \( \Delta t \). The probability distribution of water supply volumes \( I(\Delta t) \) entering during \( \Delta t \) is estimated by the frequency distribution \( R(x) \) derived from observed data. In order to avoid the fit of probability distribution functions, the frequency distribution \( R(x) \) is used, instead. In summary, the stationary probability distribution \( F(x) \) of input discharges of time interval \( \Delta t = 1 \) year is approximated by

\[
R(x) = r(I \leq x) \approx F(x) = p(I \leq x)
\]  

(2-1)

while the nonstationary distributions \( F(x,T) \) of monthly input discharges, or for time intervals a fraction of the year, with \( T \) the sequential number of these intervals within the year, is approximated by

\[
R(x, T) = r(I(T) \leq x) \approx F(x, T) = p(I(T) \leq x)
\]

(2-2)

The following assumptions are made on input and output discharges:

Assumption 1: Inflow water discharges during the successive time intervals \( s-1, s, s+1 \) are independent stochastic variables, defined by

\[
p(I(s\Delta t) = i \mid I((s-1)\Delta t) = j) = p(I(s\Delta t) = i)
\]

(2-3)

Assumption 2: The input discharge into the reservoir within any interval \( \Delta t \) precedes water release.

Under these assumptions the stored water \( \xi(t) \) in the reservoir at the end of the \( s \)-th time interval is given by

\[
\xi(s\Delta t) = \max(\min(\xi((s-1)\Delta t) + I(s\Delta t), K) - M, 0)
\]

(2-4)

\[
\begin{align*}
\xi & \downarrow \\
\xi(s\Delta t) & \downarrow \\
M & \downarrow \\
K & \downarrow \\
I_1 & \downarrow
\end{align*}
\]

and the time series of stored water \( \xi(s\Delta t) \) follows a single-step Markov chain (Moran, 1958 [17]).

In Eq. 2-4, the term

\[
\min(\xi((s-1)\Delta t) + I(s\Delta t), K) = \xi^*(s\Delta t)
\]

(2-5)
implies that at the end of the filling period, figuratively by the end of "spring" in the case of a year, the reservoir impounds either the stored volume at the end of previous year $\xi((s-1)\Delta t)$ plus the inflow volume $I(s\Delta t)$ during the s-th year, or the reservoir is full.

The external term of Eq. 2-4

$$max(\xi^*(s\Delta t) - M, 0) = \xi(s\Delta t)$$

(2-5b)

tells that either the reservoir storage is reduced by the volume $M$ released to meet the water demand during the depletion period, or the reservoir is empty.

Assumption 2 can be replaced by other assumptions. Regardless of the model used, some assumptions must inevitably be made concerning the distribution of inflow and release during the selected $\Delta t$ time interval. In the Rippl's method (Rippl,1883 [20]) both quantities are assumed to be uniformly distributed within the time interval. The reservoir volume thus calculated is consequently smaller than that needed for regulation within the time intervals of inflow and demand fluctuations. The values of the storage characteristic function constructed with the one-year time interval are therefore normally increased by the results of further calculations performed with monthly discharge data of the driest year (Szesztay, 1963, [24]).

Evidently, the Assumption 2 leads to oversizing, while the use of constant values of input and output during $\Delta t$ leads to undersizing of reservoirs. By shortening the time interval, the extent of oversizing or under-sizing can be reduced to below a preset error limit.

The original Moran's assumptions are retained in this text. However, the similar model can be also formulated by using the assumption of uniformly distributed inflow and release over $\Delta t$ (E.g.: Lloyd, [14]). The Moran's assumptions can be justified by inaccuracy being on the "conservative side", whereas the other assumptions may result in errors of the same order of magnitude involving the risk of undersizing.

A solution for this problem was advanced by L. Goda. In constructing the curve for the probability $P$ of storage characteristic curve, the inflow discharges of probability $P$ are deducted from both the water supply and the water demand volumes. The difference between the reservoir capacities thus calculated for different time intervals decreased substantially. The complex computer program provided in this text can be run according to this approach as well (Goda, 1978[8]).

The assumptions underlying the Moran's model and the basic equation (2-4) are illustrated in Fig. 4. The storage capacity $K$ is defined as the net water volume above the lowest reservoir level, or as the total stored water minus the dead storage below the lowest operational reservoir level.
Figure 4. The basic assumptions of the Moran's model and the application of Eq. 2-4: \( Q(t) \) = input, \( M(t) \) = demand, \( W \) = overflow; \( M_r \) = delivered demand; \( S_h \) = shortage, or restriction; \( I \) = water accretion in storage; \( M \) = water depletion in storage; \( \xi \) = state of storage after consumption; \( \xi^* \) = state of storage before consumption; \( K \) = storage capacity.

The objective is to estimate the probability distribution of water volumes \( \xi \) stored in the reservoir. A solution integral equation that can be written on the basis of Eq. 2-4 is possible only in exceptional cases, i.e. with inflow discharges following the special types of distribution. Rather than using such "formal solutions", the model has been formulated with the help of algorithms suited to computer processing. The practical importance of the quest for algorithms free of probability distribution functions has been called to writers' attention by A. Prekopa. Following several studies (Roux, 1965 [21]; Langbein, 1959 [12]), which yielded practical solutions, the continuously varying flows have been replaced by discrete quantities, described by a step distribution function.

The system of algorithms relies fundamentally on entering into the calculation the entire, continuously varying water volume by integer multiples of the same volume unit \( \Delta V \) (Zsuffa, 1969 [25]).
The inflow water volumes \( I \) thus become

\[
I = i\Delta V \quad \text{with} \quad i = 0, 1, \ldots, i_{\text{max}}
\]  

(2-6)

The reservoir capacity \( K \) is then defined as

\[
K = k\Delta V \quad \text{with} \quad k = 2, 3, \ldots, k_{\text{max}}
\]  

(2-7)

The water demand \( M \) is defined as

\[
M = m\Delta V \quad \text{with} \quad m = 1, 2, \ldots, k - 1
\]  

(2-8)

The storage \( \xi \) in the reservoir is

\[
\xi = j\Delta V \quad \text{with} \quad j = 0, 1, \ldots, k - m
\]  

(2-9)

Starting from the frequency distribution \( R(x) \) of inflow discharges the probability of occurrence of any water volume \( I = i\Delta V, (i = 0, 1, \ldots, i_{\text{max}}) \), can be estimated as

\[
p(i) = p(I = i\Delta V) = p((i - 0.5)\Delta V < I < (i + 0.5)\Delta V)
\]

\[
= p(I < (i + 0.5)\Delta V) - p(I < (i - 0.5)\Delta V)
\]

\[
= F((i + 0.5)\Delta V) - F((i - 0.5)\Delta V)
\]

\[
\approx R((i + 0.5)\Delta V) - R((i - 0.5)\Delta V)
\]  

(2-10)

The transition probability

\[
a_{j,i} = p(\xi(S\Delta t) = j\Delta V \mid \xi((S - 1)\Delta t) = i\Delta V)
\]  

(2-11)

is characteristic of Markov chains, i.e., the conditional probability that the water volume \( j\Delta V \) is stored at the end of the \( s \)-th time interval, under the condition that the volume stored in the previous year was \( i\Delta V \), is readily calculated for all the pairs of values \( (i,j) = 0, 1, \ldots, k - m \).

For the reservoir of a given capacity \( K = k\Delta V \) and a given release \( M = m\Delta V \), the following listing provides the series of relations by which the transition probabilities \( A(i,j) \) can be found from the marginal probabilities \( p(i) \) of the inflow discharges, \( I \). The expressions follow directly from the basic equation 2-4:

\[
\begin{bmatrix}
    a_{0,0} = p_0 + p_1 + \cdots + p_m & a_{0,1} = p_0 + p_1 + \cdots + p_{m-1} & \cdots & a_{0,m} = p_0 \\
    a_{1,0} = p_{m+1} & a_{1,1} = p_m & a_{1,2} = p_{m-1} & \cdots & a_{1,m} = p_{m+1-1} \\
    \vdots & \vdots & \ddots & \ddots & \vdots \\
    a_{i,0} = p_{m+i} & a_{i,1} = p_{m+i-1} & a_{i,2} = p_{m+i-2} & \cdots & a_{i,m} = p_{m+i-1} \\
    \vdots & \vdots & \ddots & \ddots & \vdots \\
    a_{k-m,0} = \sum_{i>k} p_i & \cdots & a_{k-m,1} = \sum_{i>k+1} p_i & \cdots & a_{k-m,m} = \sum_{i>k+m} p_i
\end{bmatrix}
\]

The transition probabilities \( A(i,j) \) are summarized as the transition probability matrix \( A \).
In contrast to the use by a majority of mathematicians, the method of compiling the transition probability matrix is different in this text. The subscripts of columns refer to storage during the previous time interval (s-1) and the subscripts of rows to storage volumes during the time interval considered. Each detail of the model to be presented can be developed uniquely also for the image transposed matrix of the matrix used here. The symbol of the transition probability matrix is \( A \). The actual storage volumes pertaining to any fixed state of the previous time interval form a complete set of events, so that the sum of elements in any column is necessarily unity. The transition probability matrix compiled in this way will be referred to as column-stochastic, following a suggestion by P.Rozsa (Galai-Rozsa, 1975 [6]).

2-2. Stationary Annual Inflow Model

The unconditional probabilities

\[
P_j = P(\xi = j\Delta V)
\]

of water stored in reservoir at the end of a time period, \( \xi=j\Delta V \) \( [j=0,1, \ldots, k-m] \), are given by the linear homogeneous set of equations

\[
P_j = \sum_{i=0}^{k-m} \alpha_{j,i} P_i \quad (j = 0, 1, \ldots, k - m)
\]

(2-12)

based on the theorem of complete probabilities. It is readily demonstrated that the set of equations 2-12 can be solved under the condition that

\[
\sum_{j=0}^{k-m} P_j = 1
\]

(2-13)

The basic problem can also be formulated as follows: find the right-hand side eigenvector of eigenvalue \( \lambda=1 \) of transition probability matrix \( A \):

\[
AP = \mathbf{P}
\]

(2-14)

The component \( (k-m+1) \) of eigenvector \( \mathbf{P} \) defines unconditional probabilities of complete depletion, \( P(0) \), or of any water volume \( \xi=i\Delta V \), \( [i=1,2, \ldots, k-m] \) being stored in reservoir. Evidently, the vector \( \mathbf{P} \) is also a stochastic vector.

According to the Markov theorem, in a single-step Markov chain, the \( r \)-step transition probabilities are given by the \( r \)-th power of the single-step transition probability. Unconditional probabilities, which are unaffected by the initial condition or the initial degree of reservoir fullness, and related to the stable so-called ergodic condition, can be obtained also by a sufficiently high power of the transition probability matrix

\[
\lim_{r \to \infty} A^r = \begin{bmatrix} \mathbf{P} & \mathbf{P} & \cdots & \mathbf{P} \end{bmatrix}
\]

(2-15)

The right-hand side of Eq. 2-15 is the method of deriving and checking, the algorithm in practical approximations, namely the computer discontinues multiplying the matrix as soon as
all column vectors of power matrix are identical within a preset deviation $\Delta P$, or all the rows contain identical elements (Savarenkii, 1940 [23]).

Consequently, two alternative ways are available for calculating the desired eigenvector $\mathbf{P}$. Both methods have been incorporated into the computer programs of this text and the preferable one is selected, or both are used depending on the nature of a particular problem. For example of planning the successive development stages of the Kerulen Reservoir in Mongolia, the power model has been used (Goda-Zsuffa, 1978 [9]).

It is thus possible to calculate the eigenvector $\mathbf{P}$ which yields probabilities of water volumes $\xi=i\Delta V$, $[i=0,1,...,k-m]$, stored in the reservoir of given capacity $K$ and given release $M$. The first component of eigenvector is the probability of complete depletion, operational failure, i.e., the risk $P(0)$. For deriving the storage function $K=f[M,P(0)]$ it is necessary to examine the double series of reservoir capacity $K=k\Delta V$ and release $M=m\Delta V$.

Adjusted to computer calculation, the following matrix product has been introduced for deriving the transition probability matrix pertaining to any values $k=2,3,...$ and $m=1,2,...,k-1$ (Zsuffa, 1969 [25,26]).

$$Z_{k,m} \times A_{K=\infty,M=0} = A_{k,m} \quad (2-16)$$

where $Z(k,m)$ is the matrix composed of elements 1 and 0, and of a size depending on numbers $k$ and $m$:

$$Z_{k,m} = \begin{bmatrix} 1,1,1,\cdots 1 & 1,0,0,\cdots 0,0 & 0,0,\cdots \ 
0,0,0,\cdots 0 & 0,1,0,\cdots 0,0 & 0,0,\cdots \ 
0,0,0,\cdots 0 & 0,0,1,\cdots 0,0 & 0,0,\cdots \ 
\vdots & \ddots & \vdots \ 
0,0,0,\cdots 0 & 0,0,0,\cdots 1,0 & 0,0,\cdots \ 
0,0,0,\cdots 0 & 0,0,0,\cdots 0,1 & 1,1,\cdots \ 
\end{bmatrix} \quad (2-17)$$

In other words, the matrix $Z$ is composed of row vectors which partially summarize the matrix $A(\infty,0)$ and the unit matrix, in which the block size depends on the magnitudes of quantities $k$ and $m$.

The matrix $A(\infty,0)$ does not depend on storage capacity $K$, nor on release $M = m\Delta V$ of the reservoir considered. Theoretically, it is transition probability matrix of a reservoir which is infinitely large and from which no water is withdrawn. The inclined rows of the lower triangular matrix of Toeplitz-type are probabilities $p(i) = p(I=i\Delta V)$ of discharges $I=i\Delta V$, $[i=0,1,...]$ which enter the reservoir:
On the basis of Eq. 2-16 the transition probability matrix can be constructed for any reservoir capacity $K$ and any release $M$, or by varying systematically the block size in the matrix $Z(k,m)$, transition probability matrices of all pairs of values $(k,m)$ can be compiled and the desired eigenvector of these matrices calculated. The algorithm described is readily computerized, and for examining capacities $K=k \Delta V$, $[k=2,3,\ldots,k_{\text{max}}]$ and the releases $M=m \Delta V$, $[m=1,2,\ldots,k-1]$, two interconnected computational cycles can be organized.

In practical computations both $Z(k,m)$ and $A(\infty,0)$ are of finite size. When cutting $A(\infty,0)$, the column-stochastic character of the matrix must be ensured by modifying the last row. The cycle computation can be confined to between the limits $K_{\text{min}}$ and $K_{\text{max}}$.

For constructing the storage function the first element $P(0)$ of eigenvector $P(k,m)$ is used only. This is the probability of completely emptying the reservoir of capacity $K$ by release $M$, respectively, with the reliability of meeting a demand of this magnitude being $P=1-P(0)$. It can be readily perceived that a larger reservoir will be capable of meeting the same water demand with a higher reliability, or conversely the reservoir of the same capacity will meet a lower water demand with a higher probability of assurance. The analysis can thus be confined to between the limits $P_{\text{min}}<P(0)<P_{\text{max}}$ (see Fig. 5).

\[
A_{\infty,0} = \begin{bmatrix}
    p_0 & 0 & 0 & \ldots & 0 \\
    p_1 & p_0 & 0 & \ddots & 0 \\
    p_2 & p_1 & p_0 & \ddots & 0 \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    p_i & p_{i-1} & p_{i-2} & \ddots & \vdots \\
    p_{i+1} & p_i & p_{i-1} & \ddots & \vdots \\
    p_{i+2} & p_{i+1} & p_i & \ddots & \vdots \\
    \vdots & \vdots & \vdots & \ddots & \vdots 
\end{bmatrix}
\] (2-18)
2-3 Nonstationary Model with Time Intervals Shorter than the Year

Discharges entering the reservoir during time intervals shorter than the year, such as half-year, season or month, may be distributed differently. However, water volumes entering during successive intervals must be independent from each other. The independence test for the series of deviations from the mean value of a time interval is performed by the computer program using the Wald-Wolfowitz test.

The annual cycle is thus broken into time intervals $\tau$. For half-year $\tau=2$, for month $\tau=12$, etc. From the observational data on water volumes entering during individual time intervals it is possible to develop the transition probability matrix of all time intervals by using Eq. 2-16.

Starting thus from any $s$-th of $\tau$ time intervals of the year, it follows from the Markov theory that the transition probability matrix of the year time interval $A_{year}$ can be found from the matrix product of $\tau$ terms as

$$A_{year} = \prod_{r=1}^{\tau} A_{s+r-1}$$  (2-19)

In Eq. 2-18 $A(s+r)$ is the transition probability matrix of the $r$-th time interval following the starting interval $s$ of $\tau$ intervals of the year:

$$A_{k,m,\Delta t=year/\tau,s+r} = Z_{k,m} * A_{\infty,0,\Delta t,s+r}$$  (2-20)
where $Z(k,m,\Delta t,s+r)$ is the matrix corresponding to Eq. 2-17, which consists of blocks having a size which depends on storage capacity $K$ and the release $M(s+r)$ corresponding to the $(s+r)$-th time interval, while $A(\infty,0,\Delta t,s+r)$ is a Toeplitz-type matrix composed according to Eq. 2-18 of probabilities of arriving water volumes in the $(s+r)$-th time interval.

The right-hand side eigenvector of $\lambda=1$ eigenvalue of matrix product of Eq. 2-19 yields unconditional probabilities $P(s-1)$ of the state of reservoir in the $(s-1)$-th time interval preceding the time interval having serial numbers:

$$P_{s-1,i} = P(\xi_{s-1} = i\Delta V) \quad (2-21)$$

At the end of the $r$-th time interval following the $s$-th time interval, the unconditional probability $P(s+r)$ of reservoir fullness is given by the matrix product

$$\left( \prod_{j=s}^{s+r} A_{j,s} \right) P_{s-1} = P_{s+r} \quad (2-22a)$$

or

$$A_{s+r} * A_{s+r-1} * \cdots * A_{s} * P_{s-1} = P_{s+r} \quad (2-22b)$$

The last term of series is the eigenvector calculated and the probabilities desired are obtained in terms of a vector. The matrices are transition probability matrices formed according to Eq. 2-20, related to corresponding time intervals. In this way, multiplying eigenvector $P(s-1)$ by transition probability matrices $A(s)$, subsequently $A(s+1), A(s+2), \ldots$ vectors are obtained which yield unconditional probabilities of water volumes stored in a reservoir at the end of time intervals $s, s+1, \ldots$ (Harris, 1965 [10]). For practical computation, the transition probability matrix of each time interval must be completed to $k+1$ size, using the values $A(i,j)=0$, $i>k-m$ or $j>k-m$, since $\xi>(k-m)\Delta V$ would be an impossible event according to the basic relationship.

In order to save computer time, the starting time interval $s$ is selected for the highest release $M(s)$. This is the time interval for which transitional probability matrix contains the greatest number of 0 terms.

The analysis is performed by the computer for several time intervals (half-year, season, month, etc.). The storage characteristic function of Eq. 1-3 is plotted for each time interval processed. To respect dimensional uniformity the releases $M(s+r)$ over time intervals shorter than a year must be added up for the year as

$$M_{\text{year}} = \sum_{r=1}^{\tau} M_{s+r} \quad (2-23)$$

Considerable computational and plotting work can be saved by introducing the water volume units for the analysis involving time intervals $\Delta t$ shorter than the year,

$$\Delta V = \frac{\Delta V_{\text{year}}}{\tau} \quad \text{where} \quad \tau = \frac{\text{year}}{\Delta t} \quad (2-24)$$

22
If the time interval selected is the year, then only sections of the storage characteristic curve \( K = f(M, P) \) which fall within the range
\[
m < k - 1 \quad k \leq K_{\max}
\]
(2-25)
can be constructed.

If the time interval selected is \( \Delta t = \text{year}/\tau \) then the analysis can be extended to ranges
\[
m \leq \tau(k - 1) \quad k \leq K_{\max}
\]
(2-26)

Examples of final results produced at the computer are shown in Figs. 6 and 7. Six curves with [P(0)=0.05] constructed for six different time intervals are reproduced in Fig. 6. Capacity differences \( \Delta K \) between curves characterized by the same reliability of supply \( P \), but calculated with different time intervals \( \Delta t \), clearly reflect the fact that a more conservative estimate is arrived at if distributions of inflow and demand within time intervals are taken into consideration. The upper envelope of curves which have the same parameter yields the desired storage characteristic curve \( K=f(M, P) \). It should be noted that capacity \( K \) will be underestimated if the independence assumption is not fully satisfied. The assumption of independence must therefore be checked whenever a new time interval \( \Delta t \) is introduced. In cases for which the single-step autocorrelation in the 2nd-3rd step is higher than 0.2, the analysis will yield unacceptable results for the particular time interval usually short, say 1 - 2 months. The final result, the storage characteristic curve of the Borjad Reservoir is shown in Fig. 7 for the case of storing water for irrigational purposes.

![Figure 6. Combined presentation of storage characteristic curves for P(0)=0.05 of the Borjad Reservoir for six different time intervals.](image-url)
Figure 7. Capacity curves of time Borjad Reservoir with various probabilities of full depletion, with irrigational demand varying during the year.
Chapter 3.

RESERVOIR SIZING WITH STOCHASTIC DEMAND

In addition to assumptions introduced earlier, let us assume that demand \( M(\tau) \) is a random variable. It may assume various values at different calendar time intervals, however, with a known distributions depending on part of the year, so that the probability

\[
f(\tau, j) = p(M(\tau) = j\Delta V) \quad j = 1, 2, \ldots
\]  
(3-1)

is also known for each time interval \( \tau \). Assume also that demands of successive time intervals are independent as well as independent of inflow, or

\[
p(M(\tau) | M(\tau - 1)) = p(M(\tau)) 
\]

and

\[
p(M(\tau) | I(\tau - 1)) = p(M(\tau)) 
\]  
(3-3)

Figure 15. Nonstationary reservoir model for random demand

Demand is a continuous random variable similar to inflow. Its distribution function is replaced by a discrete frequency distribution with increments of the selected water volume unit \( \Delta V \). By using increments or the discrete frequency distribution, the fit of probability distribution functions are avoided and demand process is approximated as close as desired by the size of increments. With these additional assumptions, the reservoir sizing computations are based on Eqs.2-4, 3-12 and 3-20, as described in Chapters 2 and 3. A version of modified model of Fig.4 is shown in Fig.15.

The key to solving this problem is the compilation of transition probability matrix \( A \) of storage conditions. It is then feasible to calculate the right-hand side eigenvector of the matrix pertaining to eigenvalue \( \lambda=1 \). This eigenvector already furnishes the most important results. Using it the performance vector \( B \) can be evaluated for the case of deterministic demand \( M \) as described above, so that the technical indices can be specified for a reservoir meeting demand of random character but of known distribution.

Elements of the transition probability matrix of a reservoir having fixed capacity \( K=k\Delta V \) and meeting demand of the known distribution \( f(i)=p(M=i\Delta V) \) are found in the following ways. Probability \( A(0,0) \) of the event that the reservoir becomes completely empty, under the assumption that it has become already empty in the previous year, equals the probability of demand that exceeds the inflow, or the reservoir full capacity.

In this way

\[
M(\tau) > \min(I(\tau); K) 
\]

Due to independence, the desired transition probability \( A(0,0) \) can be written in the form
Likewise, the probability \( A(0, s) \) of an empty reservoir, with the assumption that storage during the preceding time interval was \( \xi(t-1) = s \Delta V \), is

\[
A_{0, s} = \sum_{i=0}^{k-s} \sum_{j=i+s}^{\infty} p_i f_j + \sum_{i=k-s}^{\infty} \sum_{j=k}^{\infty} p_i f_j = \sum_{i=0}^{k-1} \sum_{j=i}^{\infty} p_i f_j + \sum_{j=1}^{\infty} f_j \tag{3-6}
\]

If the reservoir was filled in the preceding year, the probability of depletion is

\[
A_{0, k} = \sum_{i=0}^{\infty} \sum_{j=k}^{\infty} p_i f_j + \sum_{j=k}^{\infty} f_j \tag{3-7}
\]

The event that reservoir holds exactly one water volume unit, under the condition that it was empty during the preceding year, can occur in two ways: (1) either water demand \( M \) was lower by one unit than the inflow \( I < K - \Delta V \), or capacity, reservoir the than greater was inflow \( (2) > k \), and demand was smaller exactly by one unit than the capacity \( K \). The combined probability of the two events is

\[
A_{1, 0} = \sum_{i=0}^{k-1} p_i f_{i-1} + \sum_{i=k}^{\infty} p_i f_{k-1} \tag{3-8}
\]

Similarly, probability of water volume \( \xi = s \Delta V \) remaining in reservoir after release is

\[
A_{s, 0} = \sum_{i=0}^{k-1} p_i f_{i-s} + \sum_{i=k}^{\infty} p_i f_{k-s} \tag{3-9}
\]

In a more general case where the water volume \( \xi(t-1) = r \Delta V \), with \( r=1,2,...,k-1 \), remained in storage at the end of the preceding time unit, the storage volume \( \xi(t) = s \Delta V \) can occur in the two ways described in detail in the above cases. The combined probability of these two events is

\[
(3-10) A_{s, r} = \sum_{i=0}^{k-r-1} p_i f_{i+r-s} + \sum_{j=k-r}^{\infty} p_i f_{k-s}
\]

Following releases the reservoir can only be full if it was full before the release season and releases were suspended. From this double condition
The full transition probability matrix could be constructed in this way. By summing the elements of columns and remembering that the elementary identities are satisfied, or

$$A_{k,0} = \sum_{i=k}^{\infty} p_i f_0$$  \hspace{1cm} (3-11)

and

$$A_{k,s} = \sum_{i=k-s}^{\infty} p_i f_0$$  \hspace{1cm} (3-12)

further more

$$A_{k,k} = \sum_{i=0}^{\infty} p_i f_0 = f_0$$  \hspace{1cm} (3-13)

The full transition probability matrix as previously given can be called the column stochastic, (Zsuffa, 1978 [28]). It should be noted here that in composing the first version of computer program the transition probability matrix has been formulated for the computer by logical operations corresponding to the basic relationship of Eq. 2-4 instead by an algebraic equation.

By performing the matrix multiplication, it can be demonstrated that the transition probability matrix can be represented by the matrix product

$$Z(k, f(m)) * A(\infty, 0) = A(k, f(m))$$  \hspace{1cm} (3-14)

The product is shown more in detail in Fig.16. Equation 3-14 is a generalized form of Eq. 2-16, which describes the transition probability matrix pertaining to deterministic water demand as considered in Chapter 2. The matrix term $A(\infty, 0)$, composed of inflow volumes, is identical in the two matrix products. Elements of the matrix $Z[k,f(m)]$ are in contrast to the matrix $Z(k,m)$ not 0 or 1 values, but probabilities $f(i)=p(M=i\Delta V)$ of random demands $M=m\Delta V$, with $m=1,2,..$

The matrix $Z[k,f(m)]$ can readily be shown to be column stochastic; the known matrix $A(\infty,0)$ is also column stochastic. Consequently the matrix product of the two column stochastic matrices given transition probability matrix $A(k,f)$ that is also column stochastic.
Figure 16. Matrix product which yields the complete transition probability matrix of random demand releases: $K = \text{fixed}; M = \text{random variable of known distribution}; f_i = p(M = i\Delta V)$ and $p_j = p(I = j\Delta V)$

It is easy to realize that matrix $Z[k,f(m)]$ is a generalization of the matrix $Z(k,m)$ written for the deterministic demands, and consequently the basic relationship of Eq. 3-14 is a generalization of the matrix product of 2-16 written for the deterministic demands.

In case of a fixed demand $Mx = mx\Delta V$, the probability vector pertaining to demands $m = 0, 1, 2, ..., mx, ...$ is

$$ f(i) = \begin{bmatrix} 0 & 1 & \cdots & mx & mx & \cdots & i \\ 0 & 0 & \cdots & 0 & 1 & \cdots & 0 \end{bmatrix} $$

so that if $i = mx$ then $f(i) = 1$ or $f(i) = 0$. In composing the matrix $Z[k,f(m)]$ of this vector $f(i)$, the matrix $Z(k,m)$ given by Eq. 2-17 is obtained.

Re-arranging rows and columns of matrix $Z[k,f(m)]$, the lower triangular matrix $L$ is obtained. Re-arrangement of matrix $A(\infty,0)$ yields the upper triangular matrix $U$. The product of the two matrices $L$ and $U$ is a factorization of matrix $A$, substantiating that rank and order of matrix $A$ are identical. Since rank of matrix remains unchanged if rows and columns are re-arranged, the condition in solving the linear set of equations, i.e., of the basic problem, is satisfied.

The solution shown in Fig. 16 of transition probability matrix provides a rapid algorithm for sizing a reservoir of different capacities $K = k\Delta V$ [$k = 2, 3, ..., k$] intended to meet a stochastic demand of known distribution. In the first step the computer prepares matrix $Z[k_{\max},f(m)]$ for the greatest capacity $K_{\max} = k_{\max}\Delta V$. From matrix $Z[k,f(m)]$, composed for the unspecified reservoir capacity $K = k\Delta V$, matrix $Z[k-1,f(m)]$ of a reservoir having the smaller capacity $(k-1)\Delta V$ is derived by computer in adding the first matrix row to the second row and deleting the first column.

In solving some practical problems, for example the water release for irrigation, statistical investigations have shown that probability of water use $f(i)$ depends on the ratio of water consumed $M(i)$ and demand $M$ (David 1976 [3]), so that
Consequently, by using the probability distribution specified in this manner the consumed part of any water demand $M$ can be described and conventional storage characteristic function

$$K = f(M, P)$$

derived.

Computations by computer techniques are performed first in using the smallest $M_1=m_{\text{min}}\Delta V$ values. Then water demand $M_2=2m_{\text{min}}\Delta V$ is examined. The $f(i)$ values remain unchanged and $\Delta V$ values are doubled. In Eq. 3-16 the quantity $\Delta V$ is no more involved. The changeover to water volume unit $2\Delta V$ entails that pairs of $p(i)$ values related to one unit must be combined, since

$$p(0, 2\Delta V) = p(0) + p(1) = p(I < \Delta V/2) + p(\Delta V/2 < I < 3/2\Delta V) = p(I < 1.5\Delta V)$$
$$p(1, 2\Delta V) = p(2) + p(3) = p(1.5\Delta V < I < 3.5\Delta V)$$
$$p(2, 2\Delta V) = p(4) + p(5) = p(3.5\Delta V < I < 5.5\Delta V)$$
$$p(i, 2\Delta V) = p(2i) + p(2i + 1) = p((2i - 0.5)\Delta V < I < (2i + 1.5)\Delta V)$$

The transition probability matrix of the water demand $M_3=3m_{\text{min}}\Delta V=m_{\text{min}}(3\Delta V)$ or $M_4=m_{\text{min}}(s\Delta V)$, can be compiled in a similar manner by adding three, diagonals of the matrix $A(\infty, 0)$.

The eigenvector of transition probability matrix is suitable for computing technical indices of reservoir performance without changing in any way the above presented method.

This new model is intended for sizing of reservoirs which mainly serve the agricultural demand, where work has been initiated on statistical description of demand (David 1978 [3]). The model has been successfully used for studying water replacement of evaporation losses by storage from the Lake Velence and the Kiskunsagi National Park in Hungary. The model has been also adopted for preparing recommendations for the Kainji Reservoir of $15 \times 10^9$ m$^3$ storage capacity to increase the navigational water levels on the Niger River. The model is, however, a basic tool for sizing of the systems of jointly operated complementing reservoirs.
Chapter 4

ROLE OF RESERVOIRS IN POWER PRODUCTION

By promoting Hydro Power - a sustainable, renewable, environmentally friendly source of energy - worldwide civil engineering society is strongly committed to the sustained protection of the environment, in parallel with economic growth and social progress. The tenet is meeting the needs of the present generation, without compromising the ability of future generations to meet their needs. Hydro Power currently meets about 20 percent of the world's electricity needs. Most medium-term scenarios predict that power generation will primarily be shifted towards new, renewable "fuel" resources reducing the need for decreasing fossil assets. According to current forecasts, awareness of global warming (due to CO₂ emissions from fossil fuel plants and other sources) will lead to a significant political pressure in the next decade if not earlier due to large scale shortages in fossil fuel. As a result, the demand for Hydro Power – the best-proven and most developed form of renewable energy – will grow.

On the other hand liable and provident governments also attach great importance to sustainability. Their sustainability strategies are based on installation of modern, environmentally friendly technologies and processes, and social responsibility by investing in infrastructure where additionally creating new jobs for the local population, subsuppliers, and other industries are created, improving the standard of living in the area.

The Hydro Power advantages

Up to now only about 30% of global Hydro Power resources have been developed. Compared with other energy sources, Hydro Power offers some important advantages:

- Hydro Power is a well-proven form of power generation
- Water is a renewable source of energy
- Hydro Power contributes significantly to the reduction of greenhouse gas emissions
- Hydro Power is a clean form of energy and leaves no environmentally harmful residues
- Hydroelectric power generation is cost efficient and not sensitive to fuel price increases
- In many regions of the world, Hydro Power reservoirs are also vitally important for water supply, irrigation and flood protection
- Civil works construction of Hydro Power plants creates local jobs and supports the regional economies
- Hydro Power conserves fossil fuel resources for the future

Ever increasing role of reservoirs in power production will be modelled in this chapter.

4-1 Power Production is Primary

In the design of hydroelectric power plants, the proper topographical map of valley sections selected for storage reservoirs, with contour lines, as well as data on inflows, must be available. Based on this map, the graphs of reservoir water surface versus water level can be produced for various dam sites. For these dam sites the graphs of volumes of dam material, for selected types of dams, versus the reservoir level can be also produced. More complex graphs are obtained by introducing the distance from the dam sites investigated to the river
mou

The following notation are used in this chapter:

\( F(L,H) = \) Free water surface as function of the distance \( L \) to the mouth of the river and the reservoir height \( H \);

\( T(L,H) = \) Material volume of dams as similarly functions of distance \( L \) and water depth \( H \).

Volumes of dam material are calculated for all the potential dam cross sections and the maximum reservoir operational depths. The reservoir capacities are then determined for given dam sites and reservoir water depths by

\[
V(L, H) = \int_{H_o}^{H} F(L, h) \, dh
\]

(4-1)

where \( V(L,H) \) denotes the storage capacity for given depth \( H \) and distance \( L \) from the river mouth.

In the case of hydroelectric power production, the potential output of energy by power plants situated at the toes of dams is

\[
E(L, H) = \int_{H_o}^{H} (h - H_o) F(L, h) \, dh
\]

(4-2)

with

\[
E(L,H) = \text{Hydroelectric energy potential};
\]

\( H_o \) = Tailrace water level;

\( H-H_o \) = Net head of the power plant; and

\( a \) = The proportionality factor.

Dam volume, storage space, and energy potential change along the watercourse and as a function of reservoir heights. What is needed is to find the optimal location of dam sites. For different reservoir capacities, the optimal location may vary along the valley. For all storage capacities of a river the following optimization problem should be solved

\[
\frac{m \Delta x}{L,H} \frac{E(L, H)}{T(L, H)}
\]

(4-3)

to search for optimal values \( L \) and \( H \) in function of \( V \). In other words, the reservoir sites and their maximal operational levels should be selected along a river with the largest ratio of corresponding energy potential and the total volume of dams to be built. This is an approximation to a more detailed optimization of the benefit/cost ratio of hydroelectric power plants scheme.

In the case of a reservoir to be built for water supply, and flood retention and/or attenuation reservoirs, the optimization problem will take the form:

\[
\frac{m \Delta x}{L,H} \frac{V(L, H)}{T(L, H)}
\]

(4-4)
The problem can be solved in different ways ranging from GIS, DTM back to traditional programming techniques which have successfully been used in several practical cases during the past two decades. However, a computer program developed by writers is also available. This is not presented in this book, however. It was used successfully in several practical cases.

Sizing of a reservoir for hydroelectric power plant follows the selection of dam site or sites. Since the sizing method was originally developed for the case of water supply reservoirs, and described in the previous chapters, the sizing of reservoirs with mixed water uses is presented here first. In practice, a mixed use means that the reservoir is sized as a water supply storage, however, with supply release plus occasionally some overflows which are used for power production. This requires a slight modification only to the basic model, with useful information still provided on energy production.

In the basic model of water supply reservoirs the time interval was divided in two parts: first, reservoir filling and second, reservoir release. This is only a simplification in favor of safety, since a part of inflows, suitable for supply uses, is lost through spilling. In case of energy production this approach may be considered erroneous. When an initial filling before power production occurs, the net head created by storage will be greater than when power production is made simultaneously with filling. Taking this fact into consideration, models presented herein are divided into the two fundamental groups.

The first group of models occurs when temporal sequence is considered, first filling and then release. In this Type 1 model the reservoir is filled by inflow at the first part of the time interval, and then either supply and power or only the power production release takes place. In Type 2 model the power production release takes place simultaneously with inflow. The Type 2 case can be applied to reservoirs which serve either water supply or power production, or both.

The other aspect of classification is whether sizing of the reservoir is made basically for water supply, that is consumption is fixed while power production comes as a secondary use, or sizing is done solely for power production with the power demand fixed. In accordance with this classification, basic equations for four models are presented here.

Let us use the following notations:

\[ \xi_t \] = Water volume stored or the reservoir state at the end of the \( t \)-th time interval;
\[ I_{t+1} \] = Water volume which enters reservoir during the \( (t+1) \)-th time interval;
\[ K \] = Reservoir capacity; and
\[ M_{t+1} \] = Water demand during the \( (t+1) \)-th time interval.
\[ \xi_{t+1} \] = Water volume left, or the reservoir state at the end of the \( (t+1) \)-th time interval.

Then the fixed water demand after filling is

\[ \xi_{t+1} = \max(\min(\xi_t + I_{t+1}, K) - M_{t+1}, 0) \] \hspace{1cm} (4-5)

The fixed water demand which occurs simultaneously with filling is

\[ \xi_{t+1} = \max(\min(\xi_t + I_{t+1} - M_{t+1}, K), 0) \] \hspace{1cm} (4-6)
After a minor transformation of Eq. (4-6), it becomes equivalent to
\[
\xi_{t+1} = max\left(min\left(\xi_t + I_{t+1} - M_{t+1} + M_t, K + M_t\right) - M_{t+1}, 0\right) \tag{4-7}
\]

It can be seen that problem of Eq. (4-6) can be traced back to problem of Eq. (4-5), meaning that if release is simultaneous with inflow, this is equivalent to the case when a water supply reservoir is sized for a capacity which includes the consumptive uses also, in accordance with problem of Eq. (4-5). Therefore, this case can be disregarded.

The basic equation with releases for meeting a fixed power demand after filling has the form:
\[
\xi_{t+1} = max\left(min\left(\xi_t + I_{t+1}, K\right) - M(E_{t+1}), 0\right) \tag{4-8}
\]

\[E_{t+1} = \text{Power demand during the time interval } (t+1)\]
\[M(E_{t+1}) = \text{Water demand during the time interval } (t+1), \text{ which depends on the power demand, as well as on the head, of the preceding state } \xi_t.\]

According to the basic equation, first the volume \(I_{t+1}\) enters reservoir and then, after a potential overflow occurs as determined by the minimum condition, release is made for power production. This hypothesis leads to reservoir undersizing since inflows arriving before release may cause an overflow of the reservoir, so the same volume is considered with a greater head than really exists.

Separating initial filling from succeeding release, heads may result in greater values when compared to actual ones so that a fictitious increase occurs in potential energy. To a certain extent this leads to undersizing. On the other hand, the temporal separation of filling and release, because of potential overflows, goes together with computed water loss (effect of "min" function). The failure of using overflows is equivalent to the oversizing of reservoir capacity.

When releases for fixed power demand are simultaneous with filling, the basic equation takes the form
\[
\xi_{t+1} = max\left(min\left(\xi_t + I_{t+1} - M(E_{t+1})\right), K, 0\right) \tag{4-9}
\]

It can be seen from Eq. (4-9) that when filling and release for power production are simultaneous, it is the decisive case for power production reservoirs, because exaggeration in values of power head is removed.

**4-2 Power Production is Secondary**

This subchapter presents the results obtained in course of sizing of a water supply reservoir, and then applied to the estimation of power production. As it was previously demonstrated, there is no significant difference in modelling between cases when release is simultaneous with filling and when release starts only after the filling is made.

As seen already, the difference between these two cases was, when a simultaneous filling/release case was dealt with, that the capacity could be smaller by the value of water released. The case is considered now when water supply reservoir is sized by using monthly releases. The difference with cases treated in previous chapters is that the tailrace levels...
should be known along with the reservoir levels, this latter also in order to estimate the evaporation losses. In addition, the estimated efficiency of machines and their maximum discharge capacity are required as input data. By changing the maximum discharge flows through turbines for given time intervals, also the annual maintenance time requirement for turbines can be included into the models.

The computer program was prepared by assuming that a part of water release is intended for water supply in case of other purposes, prior to satisfying the power production requirements. In this case the notations used are:

\( H_0 \) = Tailrace level;
\( NOTUR \) = Water supply release that passes by the turbines. (This release has a priority over the power production release, as the decisive mode); and
\( MAXTUR \)= Maximum installed discharge of turbines for given time intervals.

To calculate energy potential of power plants at the dam sites, the subroutine VIZER is used. This subroutine provides a solution to the integral:

\[
E_i = \alpha \int_0^{H_i} (h - H_o) F(h) \, dh
\]  

\( H_i \) = Water level in reservoir if \( i \Delta V \) volume is stored
\( F(H) \) = Free water surface at the elevation \( H \);
\( \alpha \) = Proportionality factor including the conversion into kWh unit, as well as the efficiency of turbines (or all machines); and
\( E_i \) = Energy potential function of the reservoir volume at the dam power plant, as quantity of energy obtained by emptying reservoir through turbines starting with the state \( i \).

The quantity of energy obtained for a release from state \( i \) to a lower state \( j \) is

\[
E_{i,j} = E_i - E_j
\]  

From sizing of water supply reservoir of given capacity \( K \) and demand \( M \), the distribution of reservoir states becomes available. Then, estimation can be made of potential amount of energy.

The release through the power plant is then

\[
M_t = \min(\max(M - NOTUR, 0), MAXTUR)
\]  

\( M_t \) = release through turbines; and \( M \) = demand from the reservoir.

Since distributions of reservoir states and water shortages are known, to obtain an estimation of expected value of annual power production, essentially three cases are distinguished for \( p(x=i) \) is the probability that after release the reservoir storage state is \( i \) units:

(1) Expected value of energy production without empty reservoir states (no shortage):
\[
\sum_{i=1}^{k} P(\xi = i)(E_{i+\min(M;Q_{max})} - E_i)
\]  
(4-13)

since after release a storage volume of \(i\) units is left, \(M\) units can be released through turbines, so that due to power production the reservoir state will change from \((i+M)\) to \(i\).

(2) Expected value of energy with depletion, but without shortage

\[
P(\xi = 0)P(\text{shortage} > 0)(E_{M-i} - E_0)
\]  
(4-14)

where \(p(\xi=0)\) means probability of depletion. This in turn includes probability of all events with shortage. So the probability of depletion without shortage is the difference of the two above events of Eqs. (4-13) and (4-14).

(3) Expected value of energy produced when shortage occurs

\[
\sum_{i=1}^{M} P(\text{shortage} = i)(E_{M-i} - E_0)
\]  
(4-15)

The expected value of energy to be produced by that portion of release which passes through turbines is given by the sum of three energy components given by Eqs.(4-13) through (4-15).

The expected values are computed by the subroutine ENERG. This subroutine should be inserted into the final program of reservoir sizing. The identification name of this program is FHPME.

**4-3 Consideration of Overflow in Case of Power Production being Secondary**

Writers experienced in several practical problems that at least a part of overflow was to be released through turbines. Therefore, the method described above was extended to include the estimation of power production obtained by water diversion from spillover to passing through turbines.

The power of plants from these releases is already known, so that the rest, up to its maximum discharge capacity, is a portion of overflow. Since distribution of overflows is given (it is computed by programs of sizing of water supply reservoirs) the expected value of energy to be produced by overflows is

\[
\sum_{i>0} P(\text{overflow} = i)\min(Q_{max} - M, i)\Delta V(H_{ep} - H_o)\eta
\]  
(4-16)

where

MAXTUR = Maximum plant discharge capacity for given time interval of the year;
\(M\) = Water volume use from the release, which passes turbines, for the given time interval (both quantities in \(\Delta V\) volumetric units);
\(\Delta V\) = Volumetric unit in the discretization
\(H_{sp}\) = Crest level of spillway; and
$H_0$ = Tailrace level.
$\eta$ = power plant efficiency

The computation of expected supplemental energy, diverted from overflowing water is obtained by the subroutine ENERT.
The identification name of the program applied is FHTFET.

### 4-4 Reservoir Sizing for Given Power Demand

Independently of whether releases through turbines for power production occur simultaneously with or after inflow, data needs are the same for both models and their computer programs. What should be given is the tailrace level, efficiency of turbine and other machines and the maximum installed discharge capacity. In addition, the volume of dead storage space is needed since during water shortage periods a certain minimum volume must be left in the reservoir.

The power demand is determined by its two components: (1) the so-called guaranteed (firm) power, representing the base power of a plant, and (2) the variable power demand, the power above the firm power usually called the secondary power. In this case

$H_0$ = Tailrace water level;
MAXTUR = Maximum installed discharge capacity of turbines; and
KHOLT = Storage space set aside from being the useful storage space which is variable with time intervals within the year.

Sizing of a reservoir of capacity $K$ for power production is conditioned by bivariates ($K$,ERFO). The demand to be met by a power plant is

$$ERFO = ERFIX + m \cdot ERVAR \quad (m = 0, 1, 2, \cdots)$$

(4-17)

where

ERFIX = Firm power demand per time interval; and
ERVAR = Variable power demand, also changing with the time interval.

### 4-5 Case of Power Production After Inflow Occurs

The model developed for this case is similar to model for the secondary power production. In contrast to this secondary power, for which sizing of a water supply reservoir is based on volumetric units of demand, here the power demand over time intervals is considered. While matrices of transition probabilities were produced in case of water supply on the basis of given fixed water releases now the water volumes of release depend on reservoir state and power demand to be met. The basic equation describing the reservoir behavior is modified in the following way:

$$\xi_{t+1} = \max(\max(\min(\xi_t + I_{t+1}, K) - Evaporation, 0) - M(ERFO), 0)$$

(4-18)

As it is seen from Eq. (4-18), inflow is considered first, followed by evaporation and finally power production release. Multiplications of the matrix of transition probabilities are performed by columns. The subroutine for executing multiplications of matrix stores the
transitional probability matrix as an algorithm. By doing so computer storage is saved.

Description of the algorithm is as follows: Since it is assumed that in a given time interval the volume KHOLT is retained in reservoir for future intervals, in the course of power production only states involving larger volumes than KHOLT are transformed. When there is less water, no power is produced. The transformation of inflows and evaporation is quite similar to those applied in the analysis of water supply reservoirs and therefore its description is not repeated here. Then

\[ p(\xi_t = i) \quad i > KHOLT \]  \hspace{1cm} (4-19)

\( \xi_t = \text{Water volume stored in the reservoir before power production release;} \)
\( i = \text{Preceding reservoir state.} \)

The preceding state should be greater than the retained volume KHOLT, meaning that what should be distributed over the smaller volume states are probabilities of states greater than the retained volume. In the course of this distribution, the necessity for matrix multiplication will arise.

\[ p(\xi_{t+1} = j) \quad j < i \]  \hspace{1cm} (4-20)

where \( \xi_{t+1} = \text{Reservoir state after power production has occurred for water volume left in the reservoir.} \)

The essence of this method is to pass through states which are greater than the dead space volume, increase upwards, and to make decisions on which subsequent state should be considered to satisfy a given power demand. Obviously to monotonically increase the preceding states belong to the monotonically increase of subsequent states. In the first step, the state greater than dead space volume is taken as the initial state, while the dead space represents the first final state, with KHOLT+1 = First preceding state (i) and KHOLT = First subsequent state (j).

The energy generated during the transition from the state i to the state j is

\[ E(i, j) = E(i) - E(j) \]  \hspace{1cm} (4-21)

where \( E(i) = \text{Quantity of energy which can be obtained by emptying the reservoir starting with state i.} \) This function is the same as that used in case of the secondary power production. The computer subroutine is also the same.

A test should be made of whether it is sufficient for the storage volume to be reduced from the initial state i to the final state (j+1), meaning that such an ERFO value should be found for which the following inequality holds

\[ E(i, j + 1) < ERFO < E(i, j) \]  \hspace{1cm} (4-22)

Since the purpose of this analysis is to produce probabilities of events which take place in a reservoir which is used for power production, efforts should be directed towards producing probabilities of subsequent events starting from the events with known distribution. E.g., such
a purpose is to produce probabilities of subsequent states $\xi(t+1)$ after power is produced from states $\xi(t)$. In case of a preceding state $\xi(t)=i$, it holds that $j<\xi(t+1)$

Meanwhile a question may arise whether the capacity of turbines was not surpassed by the transformation from the state $i$ to the state $j$. Therefore, the subsequent state $j'$ is modified in the following way

$$j' = i - \min(MAXTUR, i - j) \quad (4-23)$$

If the effect of absorbing capacity MAXTUR occurs, a shortage will arise. Since the loss in power production due to restrictions (depletion, dead space) and the probability of these events are known, besides the state transformation, the distribution of power production can also be calculated and so the distribution of volumes of water passing through the turbines can be computed as well.

Being quite similar to the case of a water supply reservoir, the transformation of inflow and of evaporation is managed in the program by the subroutine MONERO. For state matrices or state distribution vectors, respectively, the same subroutine applies to the transformation of energy production which is executed by the subroutine EROMU. Inside the subroutine EROMU the algorithm outlined above can be easily identified. The identity name of the program used is FHER.

### 4-6 Power Production Simultaneous with Inflow

In contrast to the case of water supply reservoirs, which must meet the demand for irrigation, industrial and drinking water, the power production reservoirs are characterized by both, the water volume and the head. In the design of power plants the decisive factor is power production made simultaneously with the inflows. In this way, reservoir undersizing is avoided. The model described above has been adjusted to this case. As a consequence of assuming a simultaneous filling and emptying of the reservoir for power production, the erroneous consideration of potential accumulated energy during the filling phase is avoided. In conclusion, the equation which describes the reservoir state takes the following modified form

$$\xi(t+1) = \max\left(\min(\max(\xi(t)+I(t+1)-M(ERFO), K)-Evaporation, 0)\right) \quad (4-24)$$

where

- $\xi(t)$ = Water volume left in the reservoir at the end of the preceding time unit;
- $I(t+1)$ = Infow into the reservoir during the time unit considered;
- $ERFO$ = Energy demand, kWh in the time unit considered;
- $M(ERFO)$ = Water released through turbines in the time unit considered, which depends on power demand and reservoir state; and
- $\xi(t+1)$ = Water volume (state) in the reservoir at the end of the time unit considered.

The change which takes place during the simultaneous filling and release of reservoir through the change of state during the period $(t, t+1)$ is considered here as uniform. As a consequence the change in water volume stored will be linear in time. To produce the matrix of transition probability the calculation of water volume $M(ERFO)$ is necessary as
over the time unit considered,

where \( q_0 \) = uniform flow through turbines during the time unit considered.

The quantity of energy produced during the transition from the state i to the state j by a uniform flow \( q_0 \) is

\[
ERFO = A \int_{t_i}^{t_{j+1}} q_0 \cdot (H_t - H_0) \, dt
\]  

(4-26)

with \( H_i < H_t < H_j \), where

- \( H_t \) = Water level in the reservoir at the time \( t \);
- \( H_0 \) = Tailrace water level;
- \( A \) = Proportionality factor;
- \( H_i \) = Water level in the reservoir for the state i; and
- \( H_j \) = Water level in the reservoir for the state j.

Since during the course of transition from the state \( i \) to state \( j \) the change in water volume stored is linear in time, the integrals with respect to volume may be substituted by integrals with respect to time. In that case the lower limit is the initial state (\( i \)) and the upper one is the final state (\( j \)). Thus the integral of energy produced by a uniform \( q_0 \) flow through turbines (ERFO) may be modified in the following way (as \( q_0 \) is constant in time, it may be factored out):

\[
ERFO = q_0 \cdot A' \int_{i}^{j} (H(V) - H_0) \, dV
\]  

(4-27)

where \( H(V) \) = Water level corresponding to volume \( V \) of stored water.

The power production is approximately a linear function of the discharge through turbines.

The energy quantity produced by the unit volume of water during the state transition from \( i \) to \( j \) is

\[
E(i, j) = A' \int_{i}^{j} (H(V) - H_0) \, dV
\]  

(4-28)

Equation (4-28) can be traced back to the integral representing the energy produced by unit volume during emptying of reservoir starting from the state \( i \):
where $HH_i(i) =$ Energy produced by $i$ units of water volume released during a period of the arbitrary length.

If during a given period the reservoir state changes from $i$ to $j$, then the energy produced is

$$E = HH_i(j) - HH_i(i)$$

This quantity is produced, however, when the water volume passing through turbines consists of $(j-i)$ units. Therefore the energy quantity $E(i,j)$ produced by the unit turbine volume becomes

$$E(i,j) = \frac{HH_i(j) - HH_i(i)}{j - i} \quad (4-31)$$

assuming $j > i$. If $i > j$, then

$$E(i,j) = E(j,i) \quad (4-32)$$

because during the transition from $i$ to $j$ the unit turbines volume will come from the same reservoir space (elevation, storage water) and for the same time as in a transition from $j$ to $i$.

For a given river cross section, however, $HH_i$ has to be calculated only once. For different periods and different power demand the calculation of water volumes passing through turbines, related to changes of states, may be performed simply by

$$q_0 = \frac{ERFO}{E(i,j)} \quad (4-33)$$

This volume has an upper boundary, limited by the discharge capacity MAXTUR of turbines.

Special procedure is required in case of the simultaneous filling and power production. No state change takes place when the initial and final states are the same. In this case the energy quantity produced by unit volume becomes

$$E(i,i) = A(H_i - H_0) \quad (4-34)$$

The case of the full subsequent state also needs a special treatment. In this case two events may occur, either the reservoir becomes just full or the overflow occurs. When the reservoir becomes just full, the volume through turbines should be calculated as above.

Consider now the case of overflow, with the overflow event occurring in $f$ units. With the turbine discharge $q_0$, the change of state requires an inflow $(K-i)$. The actual inflow should be greater by the overflow $f$. Since the change of discharge during a time unit is assumed linear, this unit should be divided into two parts: a filling part which is proportional to $(K-i)$ and an
overflow part of \( f \). During the overflow the difference in power head is \( H_K - H_0 \). Since the time unit is to be divided proportionally to the filling and overflow part of the time, the quantity of energy produced by the unit volume is

\[
E_{i,K+f} = \frac{E_{i,K} * (K - i) + E_{K,K} * f}{K - i + f}
\]  (4-35)

So it is seen that in the first part of the time unit the reservoir is filled from the state \( i \) to the state \( K \), with the length of this part

\[
\frac{k - i}{K - i + f} \Delta t
\]  (4-36)

Then, during the second part \( f/(k-i+f) \) of the time unit, which corresponds to the time of overflow, the power head will be that of the full reservoir. For the given power demand, that is with the given changes of the state, the water volume released through turbines is

\[
q_0 = \frac{ERFO}{E(i,j)}
\]  (4-37)

However, in designing the transition matrix, the fact that the final state should be above the dead storage must be considered. Consequently, flow \( q_0 \) through the turbines is equal to 0 if the subsequent state reaches the bottom reservoir storage (KHOLT). The reason is that for this state the withdrawal cannot be made for the power production. The element in the matrix of transition probabilities is then

\[
A_{j,i} = P(\xi_{t+1} = j \mid \xi_t = i) = P(I_{t+1} = j - i + q_0)
\]  (4-38)

(4-31)

Because \( q_0 \) is not an exact multiple of the volume unit \( (\Delta V) \), consequently and in contrast to the programs for sizing water supply reservoirs, no readings can be made from the discrete histograms. For interpolation the cumulative distribution \( p(Q<x) \) is used. Then the element of the transition probability matrix is given by

\[
A_{j,i} = P(\xi_{t+1} < j \mid \xi_t = i) - P(\xi_{t+1} < j - 1 \mid \xi_t = i)
\]  (4-39)

and so

\[
A_{j,i} = P(I_{t+1} < j - i + q_0) - \sum_{h<j} A_{h,i}
\]  (4-40)

For the last row in the transition probability matrix (that is for the transition probabilities belonging to the subsequent state of full reservoir), the deviation of the sum of the other rows from unity may be substituted. Then, in the case of multiplication by the monthly transition probability matrix, what is observed is the change in reservoir states only, so that overflows as special events are not needed.

\[
A_{k,i} = P(\xi_{t+1} = k \mid \xi_t = i) = 1 - P(\xi_{t+1} < k \mid \xi_t = i) = 1 - \sum_{j<k} A_{j,i}
\]  (4-41)
However, after the distribution vector of ergodic state has been produced, and when the indicators of monthly power production are printed, the distribution of overflows also has to be printed and therefore at this time the basis of determining the probability of full state will not be the deviation from unity.

If the state distribution at a given time, say at the end of the time unit \( t \), is known then the change of state developing due to the simultaneous filling and power production in the subsequent time interval is given by

\[
AP_t = P_{t+1} \quad \text{where} \quad P_{t,i} = P(\xi_t = i)
\]  

(4-42)

The different turbine flows, \( q_0 \), and their probabilities can be calculated for each change of state. Therefore, the expected power production and the expected water release can be simply determined. Since transitions from different initial states to all final states are equally possible, the number of shortage events represents a space requirement of \( k^3 \) in computer memory. This is the reason why, although easy to compute, it hasn't been included in the program. The transformation of evaporation is considered after filling and power production (as a subsequent part of the time unit). Apart from this, the model for considering simultaneous evaporation is improved, and with subroutines its traces can be developed. The program used is FHvE.

Further model development

Sedimentation and turbine selection alternatives are also important questions to investigate. Although sedimentation plays less significance in the first part of life cycle of a power plant since it only increases the head by filling up dead space until a later stage when necessary reservoir capacity to save water for subsequent time intervals is running out. An upstream sediment catching auxiliary reservoir is the usual approach in such cases. A sediment research application of quing theory is made by Mosaedi in 1998 [31]. Efficiency rate - yet the only factor of power plant machinery taken into account and considered above as being a constant - depends on turbine characteristics such as functions of head and discharges. This together with production time distributions are considered to be incorporated into the model. Running hydrological sizing of the reservoir is repeated for each selected turbine combination and operational scheme. A handful of such products to chose from is illustrated in the table bellow.

Andritz VA TECH HYDRO is a global supplier of electro-mechanical systems and services ("Water to Wire") for Hydro Power plants. The company is a leader in the world market for hydraulic power generation. Below 15 MW unit output they provide the pre-engineered and modular solutions of COMPACT HYDRO powerplants. Table bellow shows a variety of turbines following modular design for the small hydro power range. The range is comprised of Axial, Francis and Pelton turbines which covers a head range of 1 to 1000 meters. Modular design consists of building a complete unit by a mechanical assembly of standardized components combining the advantages of standardization and the flexibility of tailor-made design remaining highly competitive with respect to high performances, operating reliability, short delivery time and low total investment cost.
<table>
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<th>Andritz Turbine Type</th>
<th>Head H [m]</th>
<th>Flow Q[m3/s]</th>
<th>Output [kW]</th>
<th>Sizes</th>
<th>H*Q</th>
<th>Hmax*Qmax</th>
<th>e%</th>
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<td>600</td>
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<td>2600</td>
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Chapter 5.

MATHEMATICAL ANALYSIS OF RESERVOIRS

Abstract

Reservoir model based on Moran's basic assumption leads to a Markov chain to produce the limiting distributions as eigenvectors of the matrices corresponding to the capacity range investigated. Following proposed transformation the set of matrices are not only of the same structure but holding the same entries as well. The investigation unveiled the formula on the close relation among the matrices thus halving the calculations needed. Further numerical reasoning has led to additional fast algorithms. Besides getting a better insight into the structure of random walk reservoir models, practical numerical results have also been obtained.

Introduction

Both global population and economic explosions create new water resources problems, especially in matching water supply and water demand. The world supplies of clean water are becoming more and more allocated for use. Water is a renewable resource. The surface watercourses provide 95 percent of fresh water. However, waters fluctuate following the random processes of weather. Water users must adjust themselves to these stochastic processes which comprise a random sequence of highflow and lowflow periods. Instead of adaptation of demand to random processes of water supply, the transformation of water regime by storage has proved to be a reasonable solution. This is the only way to achieve a proper development of limited water resources available on the continents and islands, or the individual regions and countries. However, due consideration must be given to an ever increasing demand, and the control of losses from harmful aspects of waters.

Water management using storage is an adaptation of natural random processes of water supply to human needs showing often differently randomized character. Engineers providing solution to basic storage calculations varied from graphical methods via calculus tools to discrete methods of random walks.

Application of Moran's Model

Basic assumptions and relationships on incoming water volume, demand distributions and values regarding storage capacity provides the main equation, inequalities of models like the one Moran and others formulated decades ago. Time and space are discretised by units $\Delta t$ and $\Delta V$ respectively. Under common sense assumptions the stored water $\xi_t$ in the reservoir - upper- limited by reservoir capacity $k$ and under-limited by emptiness $\theta$ - at the end of the $t$-th time interval is given by

$$(5-1) \quad \xi_t = \max \{ \min (\xi_{t-1} + I_t, K) - M, 0 \}$$

where $I$ - denotes natural discharges; $M$ - water demand and $K$ - reservoir capacity and the time series of stored water $\xi_t$ follows a single-step Markov chain (Moran, 1958 [17]). Thus the problem which is continuous by nature is approximated by a discrete model leading to many systems of linear equations. The coefficient matrices of these systems are mutually similar in structure. It will be shown how this property of matrices can simplify the simultaneous
solution of the systems. A significant reduction in calculation is the consequence of some mathematical results concerning the matrix theory.

Let us assume that the distribution of inflow, \( p_i \), and the distribution of consumption, \( f_m \), are known, i.e.:

\[
p_i = P(I_t = i) \\
f_m = P(M_t = m)
\]  
(5-2)

The independence of subsequent inflow volumes was assumed, already in the Moran's reservoir model. Beyond this, we assume the independence of demand between the subsequent time intervals as well. Moreover, the inflow \( I \) of any year is also considered to be independent of the following demand \( M \). From

\[
\xi_{t+1} = \max(\min(\xi_t + I_{t+1}, K) - M_{t+1}, 0)
\]  
(5-3)

it can be seen that two subsequent processes take place in the reservoir, the first one filling and the other one emptying the reservoir. Also, it follows from Eq.(3) that \( K \) is the upper bound for filling of the reservoir and \( 0 \) is the lower bound for emptying it.

Let \( P_{t,i} \) be the probability that the volume of water left in the reservoir at the end of the \( t \)-th year is equal to \( i \), i.e.:

\[
P_{t,i} = P(\xi_t = i)
\]  
(5-4)

Here \( \mathbf{P}_t = [P_{t,0} \ P_{t,1} \ P_{t,2} \ \cdots \ P_{t,k-1} \ P_{t,k}] \) is a time dependent vector series. In order to obtain the distribution vector \( \mathbf{P}_{t+1} \) from \( \mathbf{P}_t \)

\[
P(\xi_{t+1} = i) = \sum_{j=0}^{k} P(\xi_{t+1} = i \mid \xi_t = j) P(\xi_t = j) \\
A_{i,j} = P(\xi_{t+1} = i \mid \xi_t = j)
\]  
(5-5)

The effect of the inflow \( I_{t+1} \) in the \((t+1)\)-th year and that of the demand \( M_{t+1} \) in the same year are to be taken into consideration by premultiplying \( \mathbf{P}_t \) with certain matrices \( \mathbf{A}_{k,j} \) belonging to kapacity \( K \) and consumption following known distribution \( f_m \):

\[
\mathbf{P}_{t+1} = \mathbf{A}_{k,f_m} \mathbf{P}_t
\]  
(5-6)

Its decomposition structure (9) shows that the matrix \( \mathbf{A}_{k,f_m} \) is a primitive irreducible column stochastic matrix. Consequently, the process described by Eq.(3) is ergodic, i.e. it has a limiting distribution satisfying the following equation:

\[
\mathbf{A}_{k,f_m} \mathbf{P} = \mathbf{P}
\]  
(5-7)

The limiting distribution can be obtained from the solution of the system of homogeneous linear equations

\[
(\mathbf{A}_{k,f_m} - \mathbf{I}) \mathbf{P} = \mathbf{0}
\]  
(5-8)

and then normalizing it according to \( \Sigma P_i = 1 \).
Now, let us return to the matrix of transition probabilities. The two factors of product of matrices of Eq 6 can be truncated at the k-th column and k-th row since the last row of the singular matrix $A$ will be replaced:

$$A_{k,f} = F D \quad (5-9)$$

where the truncated matrices $F$ and $D$ are also matrices of transition probabilities. The elements $D_{i,j}$ of the matrix $D$, characterizing the change of the state induced by filling, (but before the consumption) are

$$D_{i,j} = P(\xi^*_t = i \mid \xi_t = j) \quad (5-10)$$

Since the change of state is induced by the inflow to the reservoir, it follows that

$$D_{i,j} = \begin{cases} 0, & \text{if } i < j \\ p_{i-j}, & \text{if } j \leq i < k \end{cases} \quad (5-11)$$

Consequently, and assuming that the reservoir will not be filled, it requires e.g. five units of inflow to get into a state which is five units higher.

In case of the full reservoir

$$D_{k,j} = p(I \geq k - j) = \sum_{h \geq k-j} p_h \quad (5-12)$$

In an analogous way, for the elements $F_{i,j}$ of the matrix of transition probabilities $F$, corresponding to the water demand following the filling,

$$F_{i,j} = \begin{cases} 0, & \text{if } i > j \\ f_{j-i}, & \text{if } j \geq i > 0 \end{cases} \quad (5-13)$$

are obtained. In case of demand, the lower bound is 0. Therefore the 0-th row of the matrix is

$$F_{0,j} = p(M \geq j) = \sum_{h \geq j} f_h \quad (5-14)$$

i.e. starting from the $j$-th state the reservoir will run empty if demand of at least $j$ units is tried to be withdrawn from it.

Based on the foregoing it can be concluded that, apart from its last row, $D$ is a lower triangular matrix of Toeplitz-type while $F$ (disregarding its first row) is of Toeplitz-type as well; it is, however, an upper triangular matrix. Both $D$ and $F$ is column stochastic, sum of elements in each one is equal to one.
On the basis of Eq.(8) the limiting distribution of a reservoir with a fixed storage capacity $K$ can be determined.

The practice of hydrology requires the determination of each limiting distribution belonging to all values of the range of capacities allowed by the technical factors. This would require a large amount of numerical calculations. It was the endeavour for reducing this large amount of numerical calculations and for a better knowledge of the model that led to the analysis presented here.

It follows from equations (11) and (13) that the elements of the matrices $D$ and $F$ are not depending on $K$ ($K$ determines their order only). Further, since they are triangular matrices of Toeplitz type and thus, their structure are the same for various capacities $K$, the question arises whether this property can be utilized in order to reduce the amount of calculations for different capacities $K_{\text{min}} < K < K_{\text{MAX}}$. We are approaching the problem from two directions:

**Problem 1:** Assuming that the limiting distribution which corresponds to the storage capacity $K$ is known, how can the limiting distribution corresponding to the capacity $K+1$, i.e. the eigenvector $P(K+1)$ be determined?

**Problem 2:** On the other hand, in the knowledge of the limiting distribution corresponding to the storage capacity $K$ how can the limiting distribution corresponding to $K-1$, $K-2$, etc. be determined?

The first problem is to be solved as follows. For a fixed capacity $K$ it follows from Eqs.(7) and (9)

$$ F D P = P $$

(5-15)
Premultiplying it with $F^{-1}$, then

$$DP = F^{-1}P$$  \hspace{1cm} (5-16)

i.e.

$$(F^{-1} - D)P = 0$$  \hspace{1cm} (5-17)

is obtained. Singularity of $F$ is discussed later, involving the case of a demand having a non random fixed value i.e. $f(M)=1$ then $F$ - as it is shown later in (32) - is a pure summator in (15) and thus matrix $A$ of (9) behaves like (17) from now on.

As it was mentioned previously, apart from the first row, $F$ is an upper triangular matrix of Toeplitz-type and the elements of the first row can be determined from the fact that for each column of $F$ the sum of elements is 1. It is easy to see that this property of $F$ is inherited by its inverse (see in Chapter's Appendix). For the elements of the inverse let us introduce the following notation:

$$F^{-1}_{i,j} = \begin{cases} 
  g_0, & \text{if } i = j > 0 \\
  g_{j-i}, & \text{if } 0 < i < j \\
  1 - \sum_{h<i} g_h, & \text{if } i = 0 
\end{cases} \hspace{1cm} (5-18)$$

Thus the sum of the elements of each column in the coefficient matrix of Eq.(15) is zero, i.e. there exists a non-trivial solution.

Since the limiting distribution i.e. the solution normalized according to $\sum P_i=1$ is to be found, the system of homogeneous linear equations (17) will be transformed into a system of inhomogeneous linear equations by changing the elements of the last row of the coefficient matrix and the last element of the right hand side into 1-s. The unique solution $P_K=u$ of this inhomogeneous system will serve as a starting vector for the determination of $P_{K+1}=v$.

Making use of Eq.(17) let us write the system of equations for the capacity $K$ in the following way: the vector $u$ should be enlarged by an element 0 and, accordingly, the coefficient matrix should be enlarged by an arbitrary column and by a row with elements equal to zero, apart from the last element. This modification has no effect on the solution of the system; however, it will assure that the coefficient matrix which corresponds to capacity $K$ will differ from the coefficient matrix which corresponds to capacity $(K+1)$ merely by a single dyad. (A one-rank-matrix, i.e. the product of a column and a row vector is called a "dyad").

Thus, the enlarged system of equations corresponding to capacity $K$ has the following form:
The system of equations which corresponds to capacity $K+1$ will be obtained as follows:

$$
\begin{bmatrix}
1 - p_0 & 1 - g_0 & 1 - \sum g_{h<1} & \cdots & 1 - \sum g_{h<K-1} & 1 - \sum g_{h<K} \\
-p_1 & g_0 - p_0 & g_1 & \cdots & g_{K-1} & g_K \\
-p_2 & -p_1 & g_0 - p_0 & g_1 & \cdots & g_{K-2} & g_{K-1} \\
-p_3 & -p_2 & -p_1 & g_0 - p_0 & g_1 & \cdots & g_{K-3} & g_{K-2} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \ddots \\
-p_k & -p_{k-1} & \cdots & \cdots & g_1 & g_2 & \cdots & \cdots \\
0 & \cdots & \cdots & \cdots & 0 & 0 & \cdots & g_0 \\
0 & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2 \\
u_3 \\
\vdots \\
u_k \\
u_{k+1} \\
u_{k+2} \\
0
\end{bmatrix} = \begin{bmatrix}0 \\
0 \\
0 \\
\vdots \\
0 \\
0 \\
0 \\
0 \end{bmatrix} \quad (5-19)
$$

The system of equations which corresponds to capacity $K+1$ will be obtained as follows:

$$
\begin{bmatrix}
1 - p_0 & 1 - g_0 & 1 - \sum g_{h<1} & \cdots & 1 - \sum g_{h<K-1} & 1 - \sum g_{h<K} \\
-p_1 & g_0 - p_0 & g_1 & \cdots & g_{K-1} & g_K \\
-p_2 & -p_1 & g_0 - p_0 & g_1 & \cdots & g_{K-2} & g_{K-1} \\
-p_3 & -p_2 & -p_1 & g_0 - p_0 & g_1 & \cdots & g_{K-3} & g_{K-2} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \ddots \\
-p_k & -p_{k-1} & \cdots & \cdots & g_1 & g_2 & \cdots & \cdots \\
0 & \cdots & \cdots & \cdots & 0 & 0 & \cdots & g_0 \\
0 & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2 \\
v_3 \\
\vdots \\
v_k \\
v_{k+1} \\
v_{k+2} \\
0
\end{bmatrix} = \begin{bmatrix}0 \\
0 \\
0 \\
\vdots \\
0 \\
0 \\
0 \\
0 \end{bmatrix} \quad (5-20)
$$

Here $\mathbf{v}$ is the vector of the limiting distribution corresponding to capacity $K+1$. It is easy to see that the difference of the coefficient matrices in Eqs. (19) and (20), respectively, is one dyad, indeed, in particular:

$$
\begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
1 \\
-1
\end{bmatrix} = \begin{bmatrix}
\sum p_{h>k+1} & \sum p_{h>k} & \cdots & \sum p_{h>1} & 1 - p_0 & 1
\end{bmatrix}
\quad (5-21)
$$

Let us take into consideration that the sum of the elements of each column in the coefficient matrices (19) and (20) is zero i.e. in both cases the left eigenvector corresponding to the zero eigenvalue is a vector with elements 1. Thus the problem is the following. Assuming that the given matrix has a zero eigenvalue and the corresponding right eigenvector is known, how to determine the right eigenvector if the matrix is modified by one dyad in such a way that the zero eigenvalue should be invariant.

To solve this problem the following theorem (Elsner and Rózsa, 1981 [4]) is used. Let $\mathbf{A}$ be a matrix of order $n$ and of rank $(n-1)$, and let $\mathbf{u}$ be the right eigenvector corresponding to the zero eigenvalue i.e.

$$
\mathbf{A} \mathbf{u} = \mathbf{0} \quad (5-22)
$$
Further, let $\mathbf{e} = [1, 1, 1, \ldots, 1]$ be the corresponding left eigenvector with $\mathbf{e}' \mathbf{u} = 0 \quad (5-23)$

Let $\mathbf{A} + q \mathbf{p}'$ be the matrix modified by a single dyad, where $\mathbf{e}' q = 0$ and $\mathbf{v}$ is the eigenvector corresponding to the zero eigenvalue, i.e.:

$$(\mathbf{A} + q \mathbf{p}') \mathbf{v} = 0 \quad (5-24)$$

According to the above cited theorem, if $\mathbf{x}$ denotes a solution of equation

$$\mathbf{A} \mathbf{x} = \mathbf{q} \quad (5-25)$$

then, by means of the formula

$$\mathbf{v} = (\mathbf{1} + q \mathbf{p}') \mathbf{u} - (\mathbf{p}' \mathbf{u}) \mathbf{x} \quad (5-26)$$

the right eigenvector of the modified matrix can be determined.

Let us make use of Eq.(26) for solving problems of Eqs.(19) and (20). The coefficient matrix in (19) corresponds to the matrix $\mathbf{A}$ in (22) while the coefficient matrix modified by a dyad: $\mathbf{A} + q \mathbf{p}'$ [see (24)], can be found in (20) with

$$\mathbf{q} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ -1 \end{bmatrix}, \quad \mathbf{p}' = \begin{bmatrix} \sum p_{h>k+1} \\ \sum p_{h>k} \\ \cdots \\ \sum p_{h>1} \\ 1 - p_0 \\ 1 \end{bmatrix} \quad (5-27)$$

To obtain the vector $\mathbf{v}$ in (26), the solutions of the equations $\mathbf{A} \mathbf{u} = \mathbf{0}$ (22) and $\mathbf{A} \mathbf{x} = \mathbf{q}$ (25) are needed; these solutions can be obtained by solving the simultaneous system of equations

$$\mathbf{A} [\mathbf{u}; \mathbf{x}] = [\mathbf{0}; \mathbf{q}] \quad (5-28)$$

From the last row, which is $(K+2)$-th equation of the system, we get $u_{K+1} = 0$, and $x_{K+1} = -1/g_0$. Thus the last equation can be omitted and the column obtained by the substitution $x_{K+1} = -1/g_0$ is transferred to the right hand side.

Since $\mathbf{A}$ is singular and its rank is one less than its order, the $(K+1)$-th equation can be replaced by a row containing merely $1$-s, and hence, from the feasible solutions that one will be chosen, which is normalized according to $\sum u_i = 1$. Due to the change of this row, the corresponding element of the second column on the right hand side can be chosen arbitrarily. Therefor Eq 28. can be written in the following form:
After solving this system, the vectors $u$ and $x$ have to be substituted into (26) and then the vector $v$ should be normalized.

Thus the answer has been found to Problem 1. Making use of the above method, the number of equations in the system to be solved will be halved. Thus, besides getting a better insight into the structure of the model, practical numerical results have also been obtained.

**Problem 2** In the previous case of Problem 1 the steps on the scale of capacity were made upwards, from $k$ to $k+1$. Answering second question the progress is directed from higher capacities towards the lower ones.

As a basis of the analysis the apparent conjecture serves, that the overwhelming majority of elements in the simultaneous systems of equations are the same; thus there are unnecessary repetitions among the operations.

Let us emphasize again that the structure of matrices $D$ and $F$, respectively, for different storage capacities is the same. According to (17) for the elements of the coefficient matrix of the basic problem

$$A_{i,j}(k-h) = A_{i,j}(k)$$

(5-30)

where $A_{i,j}(k)$ denotes the element of transition probability matrix $A$ corresponding to the storage $K=kDV$. (30) holds for arbitrary capacities and for any positive $h$ within the regions $i<k-h$ and $j<k-h$. That means, apart from the last row of the matrix corresponding to the lower capacity, all elements are equal to the elements of the matrices corresponding to the larger reservoir sizes.

Furthermore, since the matrix is singular the last equation of the system can be omitted. As the solution of the system requires non-homogenization, the solution of the basic problem (2) is based on the recognition that it can be done in such a way that (30) holds for the whole matrix, including its last row.

According to this suggestion, let the last row of the coefficient matrices corresponding to any capacity be replaced in the following way:

$$A_{k-h,j}^{'}(k-h) := A_{k-h,j}(k)$$

(5-31)

where $A'$ denotes the matrix corresponding to the modified, non homogeneous system.
Thus the coefficient matrix of the system is non-singular, consequently replacing the last element of the right hand side of (17) to a nonzero one, the problem leads to a non-homogenous linear system of equations with a non-singular coefficient matrix.

From (9) it is obvious that in all technical applications the elements of the main diagonal are dominating, therefore no pivoting is needed. All elements of the right hand sides are equal to zero except of the last ones and therefore they will change at any capacity, in the last step only when using an adequate numerical method for solving the system of equation. In the $i$-th step of the solution of the system we obtain the vector $u$ corresponding to the capacity $k=i$. As a last step the solutions have to be normalized according to the condition $\Sigma u_i=1$ for each capacity i.e. each element of the resulting vectors have to be divided by the sum of their elements.

We can see that determining the limiting distribution vector corresponding to the technically feasible largest capacity, the limiting distribution vectors corresponding to all smaller capacities can be obtained as a by-product as well.

At first let us consider the most frequent case of non random consumption, where, due to $f(M)=1$, the demand has a fixed value: $M$. This is just the classical model of Moran. The form of the corresponding matrix $F$ is:

\[
F_{k,m} = \begin{bmatrix}
1,1,1,\cdots,1, & 1,0,0,\cdots,0, & 0,0,\cdots,0, \\
0,0,0,\cdots,0, & 0,1,0,\cdots,0, & 0,0,\cdots,0, \\
0,0,0,\cdots,0, & 0,0,1,\cdots,0, & 0,0,\cdots,0, \\
\vdots & \vdots & \vdots \\
0,0,0,\cdots,0, & 0,0,0,\cdots,1, & 0,0,\cdots,0, \\
0,0,0,\cdots,0, & 0,0,0,\cdots,0, & 1,1,\cdots,0, \\
\end{bmatrix}
\]

\[(5-32)\]

Since the matrix $F$ is not invertible the previous considerations cannot be applied. In this case the matrix $A$ can also be expressed as a product too:

\[
A_{k,m} = F D = \begin{bmatrix}
\sum_{p_{q\leq m}} & \sum_{p_{q<m}} & \cdots \\
p_{m+1} & p_m & \cdots \\
p_{m+2} & p_{m+1} & \cdots \\
\vdots & \vdots & \vdots \\
p_{k-1} & \sum_{p_{q>k-1}} & \cdots \\
\sum_{p_{q\geq k}} & \sum_{p_{q>k}} & \cdots \\
\end{bmatrix}
\]  
\[(5-33)\]

$F$ shown in (32) is a pure summator and thus matrices $A$ of form (33) belonging to capacities $k$ and $k+1$, similarly to (19-21), differ in one dyad only thus reasoning (19-29) corresponding Problem 1 is valid here as well. As for Problem 2, condition (30) will be satisfied for cases of
matrices of structure (33) of any fixed consumption $M$ and for all possible capacities $K$. Consequently, the Moran-type fixed demand model can be dealt with in a similar way like random consumption's.

Generally speaking the problem at large can be universalized in the following frame: in all cases when - disregarding the full state - the transition probability is independent of the volume of the empty space above the state being examined, the condition (30) is satisfied. As an example of this kind, a power station producing energy during the inflow can be considered under constraint (30) as well. In this case the least restriction is imposed on the elements of the matrix. In this problem even the construction of the matrix elements was a rather time-consuming work, - full marices belonging to all lower capacities shouldn't be neither repeatedly constructed nor eliminated again - thus considerable computing power can be saved.

---

**Appendix to Chapter 5**

**Type invariant inversion of Toeplitz matrices**

It is known from Eq. (13), (14):

$$\begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \end{bmatrix} \mathbf{F} = \begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \end{bmatrix}$$  \hspace{1cm} (5-34)

Multiplying Eq. (34) with $\mathbf{F}^{-1}$ on the right hand side

$$\begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \end{bmatrix} \mathbf{F}^{-1}$$  \hspace{1cm} (5-35)

shows the sum of column elements are 1s. Let us see the diagonal like upper triangle structure of $\mathbf{F}^{-1}$. Partitioning along first row and column, the remaining lower right minor matrix $\mathbf{G}$ should be multiplied by respective part of $\mathbf{F}$ in order to get the identity matrix $\mathbf{I}$ of relevant size:

$$\mathbf{F}^{-1} = \begin{bmatrix} 1 & 1-g_0 & 1-g_0-g_1 & \cdots & 1 - \sum g_{h<k-2} & 1 - \sum g_{h<k-1} \\ 0 & g_0 & g_1 & \ddots & g_{k-1} & g_k \\ 0 & 0 & g_0 & g_1 & \ddots & g_{k-1} \\ \vdots & \ddots & 0 & g_0 & \ddots & g_{k-2} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & \cdots & \cdots & \cdots & \cdots & g_0 \end{bmatrix}$$  \hspace{1cm} (5-36f)
Making use of the diagonal property of these matrices shifted identity matrices are needed. Notice the non-zero superdiagonal in such an example of a 4×4 nilpotent matrix (in fact, matrices of this form are also called shift matrices). The characteristic feature of this matrix is:

\[
G = \begin{bmatrix}
g_0 & g_1 & \cdots & g_{k-1} & g_k \\
0 & g_0 & g_1 & \cdots & g_{k-1} \\
\vdots & 0 & g_0 & \cdots & g_{k-2} \\
0 & \cdots & \cdots & \cdots & 0 \\
0 & \cdots & \cdots & \cdots & g_0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
f_0 & f_1 & \cdots & f_{k-1} & f_k \\
0 & f_0 & f_1 & \cdots & f_{k-1} \\
\vdots & 0 & f_0 & \cdots & f_{k-2} \\
0 & \cdots & \cdots & \cdots & f_0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
g_0 & g_1 & \cdots & g_{k-1} & g_k \\
0 & g_0 & g_1 & \cdots & g_{k-1} \\
\vdots & 0 & g_0 & \cdots & g_{k-2} \\
0 & \cdots & \cdots & \cdots & 0 \\
0 & \cdots & \cdots & \cdots & 0 \\
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & \cdots & 0 & 0 \\
0 & 1 & 0 & \cdots & 0 \\
\vdots & 0 & 1 & \cdots & 0 \\
0 & \cdots & \cdots & \cdots & 0 \\
0 & \cdots & \cdots & 1 & 0 \\
\end{bmatrix}
\]

\[
(1-37)
\]

where super-diagonal keeps 'shifting' diagonally up, until one gets the null matrix, in our case \(N^k = 0\) of order \(k\), thus upper triangle Toeplitz matrices are able to be decomposed into finite matrix polynomials of nilpotent \(N\) of respective size, consequently (37) takes the form of a polynomial product of

\[
(f_0 I + f_1 N + f_2 N^2 + f_3 N^3 + \ldots + f_k N^k)(g_0 I + g_1 N + g_2 N^2 + g_3 N^3 + \ldots + g_k N^k) = I
\]

\[
(5-39)
\]

thus equal coefficients yield:

\[
f_0 g_0 = 1 \quad f_0 g_1 + f_1 g_0 = 0 \quad f_0 g_2 + f_1 g_1 + f_2 g_0 = 0 \quad f_0 g_3 + f_1 g_2 + f_2 g_1 + f_3 g_0 = 0 \quad \ldots
\]

\[
g_0 = \frac{1}{f_0} \quad g_1 = -\frac{f_1}{f_0^2} \quad g_2 = \frac{f_2 - f_0 f_1}{f_0^3} \quad g_3 = \frac{f_3 - 2 f_0 f_1 f_2 + f_0^2 f_3}{f_0^4} \quad \ldots
\]

or in a more general form in order to calculate \(G\) elements recursively:

\[
\sum_{i+j=l} f_i g_j = \begin{cases} 
1 & l = 0 \\
0 & l > 0 
\end{cases}
\]

\[
(5-42)
\]
Chapter 5
ICE INSPECTION - ESTIMATING ICE COVER RATE

During gathering, storing, running downstream and putting on power generation, water does not only evaporate, roll stream deposit, but winter by winter due to the considerable cooling of the environment forms ice on the surface and transports it. Most fundamental feature of ice phenomena is ice cover rate, determination of which happens by no measures but yet - or again - visual survey from river banks even in early 21st century as well. Although in recent decades, something began with manual - more specifically, visual and in one single case analogous - image processing of photography from high banks or tall buildings on the banks, no more serious improvement happened until the appearance of cheap webcams and of mobile phones operating in network picture forwarder mode as well. The basics of webcam river ice inspection is posted in details at vip.water.hu/Galai/Antal/pub/ice/webcam.htm, here and now the summary of my own results affecting ice cover rate is reviewed only.

One of early - ahead of its time - contribution there was the 1982 invention of a simple, low powered shape/pattern recognition algorithm which later developed for ice flow investigation becoming important long time later, just after setting up of web cameras along frozen rivers. (In the meantime page scanners emerged in ice-free, mild years.) First of the application package processing recorded images of ice monitoring webcams determines proportion of ice cover from camera pictures. Next task is observation of moving ice when surface water flow is characterized by estimating rotation and displacement via parsing of individual ice sheets by shape recognition or through statistical analysis of the whole picture as a pattern recognition. Statistical analysis of ice shapes and movement opens the way to simulation of downstream ice flow based on river bed and further available data, to investigations costing a lot less in order of financial magnitude compared to building and observation costs of experimental small scale modells.

Seeing the first TV cam - with rather tiny image then - connected to a small scale "desktop" computer of some 16 kBytes (EMG666 microprogrammed) and being also a civil engineer, the following ideas had come immediately to my mind: To follow ice sheets my basic idea was inspired by bending and twisting of
sticks. A ruler "knows" where to bend and twist easier, bending and rotation follows cross-section's principal axes, and center of gravity, ergo tracing them results in displacement and rotation values, and to top it all, the invariant features - including perimeter, area, moments regarding principal axes - enable recognition of known shapes of previous image after displacements and rotations took place. The innovative idea was that after a short time e.g. some seconds the picture of the next shot will hold generally the same ice particles but in other positions. With regard to well known feature of shapes that the circumference, surface and principal moments of inertia regard to the principal axes originating from centroid are invariant of the coordinate system thus consecutive undistorted pictures provide the same quantitative characteristics for the same ice shape during the move. This 3-4 or more quantitative properties serve as coordinates for each and every ice particle on a picture. Representing a set with these coordinates of this points of each time/picture in the 3-4D space we can catch the equal or nearest points of the different sets thus this could be a base algorithm for a pattern recognition for both ice shapes or OCR. Although lots of other innovative ideas were later developed it is hard to find some among them which is so basic and relies on less preliminary knowledge than this one thus this idea fits the best entry level computer vision introduction.

In the demonstration we calibrate the camera overlooking Danube in Baja. The area of the image to be investigated is the water plane downstream of the bridge, that is a co-planar surface. This constrain on the area of interest and data makes somehow easier both the calibration and the processing of images later as well. My aim is to get a function: the input of that function are the (u,v) pixel coordinates of the camera image and the output provide the (x,y) world coordinates of that point on the water plane. i.e. the function provides a reverse perspectivic and lens distortion for our sample.

In the course of calibration the parameters to be determined are partly internal - called intrinsic - parameters while the rest are external - called extrinsic - parameters depending on camera position relating world coordinate system to camera coordinate system by rotation and translation. Intrinsic parameters are on one hand traditional optical lens characteristics such as focal length, pixelsize while on the other hand the consequences of deficiencies of cost effective production resulting in tangential, pincushion and/or barrel distortion of cheap lenses, light detector CMOS chips and glue causing that camera optical axis and lens optical axis often significantly diverge, thus ignoring cheap cameras' excentricity is significant source of errors. Barrel distortion occures more likely in wide angle cameras while tele-photo produces pincushion effect.
In case of data shortage lens within camera is assumed well positioned and projected image is considered to be undistorted in all directions. Sufficient or abundant data enables lens eccentricity and distortion to be estimated as well.

\[ y = A (Rx + t) \]

\[ R_z = \begin{bmatrix} \cos \varphi & \sin \varphi & 0 \\ -\sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

\[ A = \begin{bmatrix} \alpha & \gamma & u_o \\ 0 & \beta & v_o \\ 0 & 0 & 1 \end{bmatrix} \]

where \( R \) is the rotation matrix and \( t \) is the translation, they are the extrinsic parameters. \( X \) consists of the real word coordinates while \( Y \) yields the image pixel coordinates. \( A \), the camera intrinsic matrix, is given by \((u_0; v_0)\) the coordinates of the principal point, \( \alpha \) and \( \beta \) the scale factors in image for \( u \) and \( v \) axes, and \( \gamma \) the parameter describing the skewness of the two image axes on the light detector CMOS chip. A detailed review can be found at www.water.hu/ice/webcam/index.php?hu#calibration.

The relatively small scale precision - of image processing based on wide spread webcams of low resolution - enables the use of ordinary GPS in collecting calibration data. In hydrology least square and maximum likelihood
estimations have been used for long decades ever since the advent of computers. River ice monitoring does not have fast fundamental changes in the picture and for the sake of comparison along longer time even the camera position is fix as long as possible. The other source of both calibration and processing ease is that the area of interest are co planar planar objects although their plane is changing from day to day but remains parallell with that of the calibration. Since need for and probably use of river monitoring is supposed to be soon wide spread we should focuse on results of research aiming everyday use of computer vision systems targeting at the general public, who are not experts in the field and not willing to invest money for expensive equipment. We should use technique that only requires a camera to observe a known planar shape of the observed river's water plane and 2D metric information will be used. By applying new technique advances 3D computer vision we take the step from laboratory environments to the real world.

To eliminate superposing errors of inprecision of the lens and destorted placement of the light sensor intrinsic and extrinsic parameters of the camera should be separated first. Statistical non linear parameter estimation could further refine the values deriving from the consideration above. Among civil engineers - specially hydrologists - a well known name is Maximum Likelihood Estimation, which could use our previous results as an initial guess. As an iterative approach cyclic iteration could further converge the calibration data.

Estimating Ice Coverage

While processing successive images of river monitoring webcammera, current ice cover rate is to be determined on one hand. On the other hand following ice sheets based on these images an estimation on surface displacement and rotation is possible. Following camera calibration outlined above a vector-vector function describing perspective projection between camera image points and water plain points is available. With the help of this function scanning icy pixels of the screen results in water surface ice coverage rate. Detecting outlines of ice sheets (i.e. continous areas of the image) current and invariant characteristics of the formations could be determined and thus displacement of principal points and rotation of principal axes could be calculated.

The essence of ice cover estimation based on webcammera pictures partly dates back more than 150 years. Being named after mathematician Carl Gustav Jacobi (1804-1851) the Jacobian matrix is the table of all first-order partial derivatives of a vector-valued function in vector calculus. Jacobian is akin to a derivative of a multivariate function and it is denoted by
\[ J = J_F(x_1, \ldots, x_n) = \frac{\partial(y_1, \ldots, y_m)}{\partial(x_1, \ldots, x_n)} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_m}{\partial x_1} & \cdots & \frac{\partial y_m}{\partial x_n} \end{bmatrix} \]

In our case of an ice cover estimating webcam, the projection of planar water surface at a given height into camera pixels is a vector-valued function.

\[
\begin{bmatrix} u \\ v \end{bmatrix} = A(R \begin{bmatrix} x \\ y \\ z = 0 \end{bmatrix} + t) \quad \Rightarrow \quad \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{bmatrix} = R \begin{bmatrix} x \\ y \\ z = 0 \end{bmatrix} + t \quad \Rightarrow \quad \begin{bmatrix} u \\ v \end{bmatrix} = \frac{f}{\tilde{z}} \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix}
\]

Ice cover estimation is an integral over the water domain of the perspective picture in investigation:

\[
\int \int f_{ice}(x, y) \, dx \, dy = \int \int f_{ice}(u, v) \, J(u, v) \, du \, dv \quad \Rightarrow \quad J(u, v) = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix}
\]

The integral is over the water of the camera picture by dx dy. Since we are summing the icy pixels on the picture itself, the integration is done by an integral transformation; it is a change of variables when integrating a function over its domain, here the Jacobian determinant is used. To accommodate for the change of coordinates the Jacobian determinant arises as a multiplicative factor within the integral.

\[
H = AR, \quad p = At \quad \Rightarrow \quad \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{bmatrix} = H \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} + p \quad \Rightarrow \quad \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{bmatrix} = r_{31} x + r_{32} y + t_3 \quad \Rightarrow \quad u = \frac{\tilde{x}}{\tilde{z}}, \quad v = \frac{\tilde{y}}{\tilde{z}}
\]

\[
\begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} (\tilde{x}) & \frac{\partial}{\partial y} (\tilde{x}) \\ \frac{\partial}{\partial x} (\tilde{y}) & \frac{\partial}{\partial y} (\tilde{y}) \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} [h_{11} x + h_{12} y + p_1] \\ \frac{\partial}{\partial x} [r_{31} x + r_{32} y + t_3] \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} [r_{31} x + r_{32} y + t_3] & \frac{\partial}{\partial x} [r_{31} x + r_{32} y + t_3] \end{bmatrix}
\]

\[
= \begin{bmatrix} r_{31} (h_{11} x + h_{12} y + p_1) \quad (r_{31} x + r_{32} y + t_3)^2 \\ r_{31} x + r_{32} y + t_3 \\ r_{31} (h_{22} x + h_{22} y + p_2) \quad (r_{31} x + r_{32} y + t_3)^2 \\ r_{31} x + r_{32} y + t_3 \\ r_{31} (h_{11} x + h_{12} y + p_1) \quad (r_{31} x + r_{32} y + t_3)^2 \\ r_{31} x + r_{32} y + t_3 \\ r_{31} (h_{22} x + h_{22} y + p_2) \quad (r_{31} x + r_{32} y + t_3)^2 \\ r_{31} x + r_{32} y + t_3 \end{bmatrix}
\]

\[
= \begin{bmatrix} h_{11} \quad (h_{11} x + h_{12} y + p_1) \quad (r_{31} x + r_{32} y + t_3)^2 \\ r_{31} x + r_{32} y + t_3 \\ h_{12} \quad (h_{12} x + h_{12} y + p_1) \quad (r_{31} x + r_{32} y + t_3)^2 \\ r_{31} x + r_{32} y + t_3 \\ h_{22} \quad (h_{22} x + h_{22} y + p_2) \quad (r_{31} x + r_{32} y + t_3)^2 \\ r_{31} x + r_{32} y + t_3 \end{bmatrix}
\]
While integrating over pixel areas within the picture, a change of variables is to determine the calculated quantities in the real world. The multiplication factor varies from pixel to pixel. The proportional factor of the linear quantities is roughly the square root of that of the areal one. What we need are the partial derivatives of pixel coordinate functions of horizontal \( u \) and vertical \( v \) with respect to their variables \( x \) and \( y \). On each pixel of the picture the value of the Jacobi determinant yields the proportion between the area of the pixels (\( 1 \times 1 \, \text{dudv} \)) and that of the surface on the water (\( 1 \times 1 \, \text{dxdy} \)) in \( \text{m}^2/\text{pixel}^2 \). As the derivative of inverse function shows, the same Jacobian value applies the other way round but as a division factor.

\[
\begin{bmatrix}
  h_{11} & h_{12} \\
  h_{21} & h_{22}
\end{bmatrix} - \frac{1}{\tilde{z}} \begin{bmatrix}
  r_{31} \tilde{x} & r_{32} \tilde{x} \\
  r_{31} \tilde{y} & r_{32} \tilde{y}
\end{bmatrix} = \frac{1}{\tilde{z}} \mathbf{H} - \frac{1}{\tilde{z}^2} \begin{bmatrix}
  \tilde{x} \\
  \tilde{y}
\end{bmatrix} \begin{bmatrix}
  r_{31} & r_{32}
\end{bmatrix}
\]

Ice cover estimation with no or partial calibration

Under certain circumstances when - e.g. in cases of far away webcams - calibration data for eliminating perspective distortion is not available, more frequent images enable a certain reliability of ice sheet detection and pairing of ice formation of subsequent images by e.g. inertial pattern matching. Stored contour points of no calibration give a chance to process more accurately later. By applying same type of cameras "precalibrated" under office environment described herein, further precision advance could be achieved with the help of the angle between camera direction and water surface independently from water level. Ice cover rate is not depending "so much" on water level since if we move the camera along its axis or the water level changes, all triangles originating from the same pixel points are proportional and thus this angle is enough for determining Jacobian values which are proportionally invariant for pixels referring to mutually covered points of parallel plains. Due to drifting ice, water surface to monitor is easy to limit to the area of e.g. significantly changing domain of the picture. To calculate moving ice cover proportion, the only important part is river surface participating in conveying ice drifts, while backwater/still water behind engineering works does not take part in ice transport.

Angles of camera axis with horizontal and vertical planes can be determined by skyline in the image although subject to radial distortion. View angle of camera could be available from precalibration. Counterballanced contour line of horizon in comparison with image axes and center is enough for
determining the water surface direction which is sufficient for estimating ice cover rate. To obtain true scale data of displacement and rotation, height of camera and water level needed only. Correct geodesic coordinates of computed results are available through a horizontal displacement and a rotation around vertical axis.

Horizon Calibration

Angles between camera axis and horizontal and vertical plains are determinable from radially distorted line of horizon. The relation of the radially undistorted horizon line towards camera axis and the image center is sufficient to estimate water surface plane directions. First radial distortion of pixel line of horizon is removed then the linear equations of the horizon line on the xy coordinates of a group of nearly equidistant representative set of points are set up. From the coefficients of the n equations the values of the two parameters of the best fit line is determined. The method is the same as it is used at removing radial distortion.

\[
\begin{bmatrix}
  x_1 & 1 \\
  \vdots & \vdots \\
  x_n & 1
\end{bmatrix}
\begin{bmatrix}
  a \\
  b
\end{bmatrix}
= \begin{bmatrix}
  y_1 \\
  \vdots \\
  y_n
\end{bmatrix}
\]

Equation of \( ax_i + b = y_i \) horizon setting up from radially undistorted points, two parameters \( a, b \) fitting best to chosen points are searched for. There are more equations than needed, however based on data of uncertain accuracy. Despite one source of inaccuracy lies in the "integerity" of the input pixel coordinates, the results obtained from many less accurate data will statistically be more accurate. The system of inhomogenous linear equations set up from line equations on designated points denoted as \( Dk = d \) earlier above.

\[
k = (D^T D)^{-1} D^T d
\]

Closed form solution of this equation - knowing the current elements of the coefficients in matrices - results in more details as follows:

\[
\begin{bmatrix}
  a \\
  b
\end{bmatrix}
= \left( \begin{bmatrix}
  x_1 & \cdots & x_n \\
  1 & \cdots & 1 \\
  x_n & 1
\end{bmatrix}
\begin{bmatrix}
  x_1 & 1 \\
  \vdots & \vdots \\
  x_n & 1
\end{bmatrix}
\right)^{-1}
\begin{bmatrix}
  x_1 & \cdots & x_n \\
  1 & \cdots & 1 \\
  x_n & 1
\end{bmatrix}
\begin{bmatrix}
  y_1 \\
  \vdots \\
  y_n
\end{bmatrix}
\]

\[
= \left[ \sum x_i^2 - \sum x_i \right]^{-1}
\begin{bmatrix}
  \sum x_i y_i \\
  \sum y_i
\end{bmatrix}
= \frac{1}{\sum x_i - (\sum x_i)^2} \begin{bmatrix}
  \sum x_i y_i \\
  \sum y_i
\end{bmatrix}
\]

From onscreen parameters of horizon the vertical deviation while from distance between horizon line from camera optical center in pixels and focal lenght of optic in pixels the angle between the water surface and camera axis is available.
Chapter 5

Own scientific achievements

Numerical models - set up from basic relationships formulated while solving technical problems - often lead to predictability problems, or resulted in algorithms later proven to be too slow and/or requesting huge memory. The basic fundamentals of the model is often rewritten for the sake of solving or bypassing of these problems, or even merely for the precise problem definition required by coding task. In the course of reformulation newer and newer model modifications arise, though they are often trivial relations, but applied for the first time however; the exact justification of relationships - accepted earlier as experiential ones - appears now and then. Source of new, surprising results could be ignorance, which is not a miracle, young persons draw their new ideas from this just often. This may as well occur to others - though not beginners - being kept at a distance from vocational circulations, compulsion quickly begat some DIY, handymanness promptly produces very many new tools, methods to solve their tasks. At the beginning this was the situation in my case as well, since only the more equal ones or merely geographically more favourable situated ones had their access to the international vocational information through decades in our homeland. Nowadays the quantitative explosion of the informational web may cause a "disadvantage" like this, a lot often faster to code - or say old-fashioned "program" - something, than to sort out the equivalent from among a lot ready to use applications of vague origins put out on www.

The theses listed hereunder are only a mere slice of results - all begat by practice-, and were born during my programming reservoir sizing. Mostly the basic equations and the wishlist was what I received only, and while I was carrying out patching the numerical part I often had to twist the model. Sometimes of that size to wind, that already not too resembled the original one very much. That might have been the reason for acknowledgement I deserved by renowned Australian professor McMahon's book review.

But not only model changes lead to new results, it was often necessary to drastically reduce the quantity of mass calculation and while solving this numerical problems reflexive novelties with a feed back to model sturcture arose sometimes. Taking into account the square growth of consumption field and capacity range to be examined and the examinations belonging to all of the steps over this area - on dots each claiming multiplication/division operation around power of $n+2$ - there are still what to win by conceptual considerations. Suggested by common sense, then justified by my numerical examples, intuitions accelerated the algorithms on a large scale. This is not an exact justification yet on the other hand. Shortly after the questions were put to more university algebrists, - I realized that it is not necessary to march with cannon onto sparrows, a mere sling will do - it turned out that a few problem - being not far from triviality - were solved by myself with plain transformations. This it is not indication of that false mirage how much I superseed my algebrist masters, but, that dispite their limited pure mathematical knowledge now and then application engineers, knowing the model better, get occasionally onto results much more quickly, than theoretical mathematicians got used to more complicated, theoretical problems. Wheel and Rubic's cube are plain comodities, but first somebody had to discover them after all! (the person presumably was rather engineer, than a mathematician... their turn comes later... e.g. at CFD flow fundamentals and numericals? :o)
The majority of the ideas I firstly "discovered" belongs to reservoir modeling, covering various algorithmic extensions of application areas of queueing theory models starting from the deficiency distribution across economic optimization until power generation. As the majority of methods resulting in Markov chains - in our case each single application extension eventually winds up in convolution calculations, what is not a miracle since these are generally interpreted as summation of convolution integrals on discreet distributions according to the model equation, materializing in multiplications of various transition matrices. Taking advantage of their numerical and matrix algebraic characteristics, progress in model building and solution to numerical problems can be made. During my various researches, convolution as functional operator turned up in an extended manner like in computer vision.

[1] Duality of transition matrices describing subsequent incoming water and random demands

The input-output symmetry of the Moran storage equation could be described by a pair of transition matrices of the same but transposed structure belonging to input and output subsequent subprocesses respectively. ($\xi_t$ stored water, $I_t$ inflow volume at time $t$, $K$ storage capacity, $M$ consumption volume, $F$ transition matrix based on distribution of random consumption $f_{mt}$, that of inflowing water is $D$, transition in time period is described by matrix $A$.)

\[
\xi_t = \max(\min(\xi_{t-1} + I_t, K) - M, 0) \quad A_{k,f} = FD
\]

Moran published his original model in 1959, time and volume were discretized by units selected expediently, reservoir's examined as a Markov chain based on a basic storage equation. According to this storage model's assumptions, independently incoming water quantities of identical distribution are followed by water consumptions of a given fixed volume. Stochastic variable, water quantity left over in the reservoir is the characteristic to be examined, where the construction of transition probability matrices derived from. Inspired by hints of info on parallel modeling activities István Zsuffa raised attention to the case of random water demands of known distribution. Following combinatoric considerations of probabilities of water quantities of matching supply and demand, while random walk sorted into transitions between given inner states some formulae were formulated for transition probabilities, joined in a matrix, factorized by professor Pál Rózsa for the sake of the numerical examinations. Being present as a kind of go-between, the symmetric regularity of the factors just promptly caught my eye, and the symmetry of the Moran storage equation regarding input and output volumes was immediately hitting my brain as enlightenment that transition matrices of the same but transposed structure should be used for describing both input and output subsequent subprocesses, and with this all the model's calculation, examination, and - what is not a last viewpoint, the education - understanding were greatly simplified.
\[ \xi_t = \max(\min(\xi_{t-1} + I_t, K) - M, 0) \]
\[ p_i = P(I_t = i) \quad f_m = P(M_t = m) \]
\[ P(\xi_{t+1} = i) = \sum_{j=0}^{k} P(\xi_{t+1} = i \mid \xi_t = j)P(\xi_t = j) \quad A_{i,j} = P(\xi_{t+1} = i \mid \xi_t = j) \]

\[ A_{k,f_m} = FD \]

\[
D_{i,j} = \begin{cases} 
0, & \text{if } i < j \\
p_{i-j}, & \text{if } j \leq i < k 
\end{cases} \quad D_{k,j} = p(I \geq k - j) = \sum_{h \geq k-j} p_h
\]

\[
F_{i,j} = \begin{cases} 
0, & \text{if } i > j \\
f_{j-i}, & \text{if } j \geq i > 0 
\end{cases} \quad F_{0,j} = p(M \geq j) = \sum_{h \geq j} f_h
\]

\[
F = \begin{bmatrix}
1 & 1 - f_0 & 1 - f_0 - f_1 & \cdots & 1 - \sum f_{h<k-2} & 1 - \sum f_{h<k-1} \\
0 & f_0 & f_1 & \cdots & f_{k-1} & f_k \\
0 & 0 & f_0 & f_1 & \cdots & f_{k-1} \\
\vdots & \ddots & 0 & f_0 & \cdots & f_{k-2} \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & \cdots & \cdots & \cdots & f_0
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
p_0 & 0 & 0 & \cdots & 0 & 0 \\
p_1 & p_0 & 0 & 0 & \cdots & 0 \\
\vdots & \ddots & p_0 & 0 & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\
p_{k-1} & \cdots & p_{k-2} & \cdots & p_1 & p_0 \\
1 - \sum p_{q<k-1} & 1 - \sum p_{q<k-2} & \cdots & 1 - p_0 - p_1 & 1 - p_0 & 1
\end{bmatrix}
\]

[2] Factorization of yearly product matrix of non stationary model at the period of the largest water demand in year

Size of transition matrices of Markov chains belonging to seasons, or even months - of different non random consumptions - following each other in yearly cycle depends on the number of possible states, their product form yearly transition's matrix which claims minimal time and space when factorized at the time period of largest consumption.
As an example of the public private partnership, decision makers of the district of the planned reservoir should agree on the distribution of the water supply needed. Frequent lack of consumption probabilities disable consideration of random water deliveries. Not to force decision makers, economic and agricultural designers to answer emerging questions regarding fluctuating releases, we reckon with fixed water consumption in the majority of the cases. In a more frequent case however, the proportion rates of the requested water quantity in time - e.g. inside a year - is found desirable to a certain extent. The model represents sequents of events not as consecutive periods of identical feature - that is years -, but repetitive series of seasons, even months, stretching over not necessarily equivalent lengths of time. This time unit refinement could be increased while the consecutive quantities of arriving waters are independent at the required measure of reliability, otherwise we have to resort to other model or numerical methods other than Markov chains. Since water deliveries over periods inside the year follow unknown distributions, the examination is made for the multiple values of consumptions of fixed proportion over seasons/months. Markov chain state tranformation matrices - of every periods following each other cyclically - constituted on the basis of model equation, are of size depending on the number of possible transition states, their matrix product make up the yearly transition matrix, with this multiplication the instacioner model is traced back to stacioner one of presupposing the consecutive periods with an identical feature. Since size of this product matrix depends on which subperiod we start analysing the year, thus I manifested that at the subperiod (season, month) with largest consumption is where the year is worthy to cut into cyclical subperiods. Not merely place is saved this way, but taking into account the square growth of consumption field and capacity range to be examined and the examinations belonging to all of the steps over this area - on dots each claiming multiplication/division operation around power of 5 - there are still what to win by conceptual considerations. Suggested by common sense, then justified by my numerical examples, the cyclic product can be started at arbitrary subperiod, the results do not change, since:

\[
B Ax = x \quad \& \quad Ax = y \implies ABAx = Ax \implies AB y = y
\]

A. B consecutive transition matrices, x and y their consecutive ergodic state distributions.

[3] Equivalency relationship of sequential and simultaneous water consumption models

Basic equations of the two mutually exclusive hypotheses can be transformed into identical forms, so that the two models just lead to identical results at capacities differing from each other exactly with the value of consumption, and therefore they are numerically equivalent.

Frequent objection to this type of storage models is, that in the original basic equation, inflow precedes consumption thus being separated in time results in oversizing compared to the reality. Twisting the basic equation to simultaneous filling and consumption, contrary finding derives. Inflowing water quantities arriving before possible consumptions are taken into account as staying in the reservoir, although some part of them overflows earlier than any supply releases occur in reality thus latently increasing safety of water deliveries which leads to undersizing. This dual approach grasps the boundaries of over- and undersizing and yields a plain solution: adding and subtracting the consumption onto both sides of the equalities of the basic equation derives an identical form of the two equations, capacity is bigger with the
consumption value in the subsequent form of the simultaneous model, otherwise we receive the very same results. Thus I manifested the equivalence between the simultaneous and separated inflow/outflow models. With this finding for the deriving results, I received a field of tolerancy between the boundaries representing over- and undersizing where the reality somewhere between the two could be.

\[
\xi_{t+1} = \max(\min(\xi_t + I_{t+1}, K) - M_{t+1}, 0) \quad \xi_{t+1} = \max(\min(\xi_t + I_{t+1} - M_{t+1}, K), 0)
\]

After a minor transformation second becomes equivalent to first

\[
\xi_{t+1} = \max(\min(\xi_t + I_{t+1} - M_{t+1} + M_{t+1}, K + M_{t+1}) - M_{t+1}, 0)
\]

All above thoughts turn into contrary in the case of energy production, since if water runs on turbines after inflow finishes, then comparing to real circumstances - of constant feeding, continuous power generation - a bigger hydraulic head is taken into consideration resulting in undersizing. Following common sense in calculating power production - Moran type assumption of subsequent supplies and releases should be rejected and - model adjustment now requests a substantial modification.


All volume changes presumed to be simultaneous and uniform in time, the integral by the volume - to get potential energy function - could be transformed by way of substitution into integral by the time and vice versa. The proportion of energy desired and the power generated by change of state shows together with the change in storage volume how much inflow water needed from which the transition probability follows:

\[
p(I_t = \left( \frac{E_t}{\sum_{j} \int_{i}^{j} (H(V) - H_0) dV} - 1 \right)(j - i)\Delta V)
\]

\((E_t \text{ requested energy in time } t, H(V) \text{ inverse function of volume-waterlevel } V(H), H_0 \text{ turbine elevation}, \Delta V \text{ volume unit of discretization, g, } \gamma, \eta \text{ gravitation, water density, rate of efficiency})\)

In our first energy estimations - probabilities of reservoir states, water surface altitude and quantity of fixed consumption served as the computational basis for power production. Excess turbine capacity makes it possible the use of overflow waters too, thus as a result, beyond fixed consumption which basically determines storage operation, I also took into account water quantities partially or fully delivered to turbines after arriving into full reservoir. Diversion of spillway water from maximum hydraulic head onto turbines showed considerable efficiency increase so leaving overflow out of consideration caused significant oversizing in most cases.
To increase computational reality, expected energy values produced by transitions resulting empty reservoir were also considered since water shortages of different magnitude occur. Although calculations of power generation - based on adjusting traditional Moran model to hydroelectric plant - is repaired somewhat this way, but assuming or ignoring of chronologic separation of random inflow and fixed volume water consumption influences the results fundamentally. Furthermore, after accepting condition on temporal separation, role of fluctuation of hydraulic head regarding water level changes within the time unit arises immediately. My finding, the equivalency relationship between sequential and simultaneous water consumption models does not help either, since regarding power generation, results from either simultaneous or separated inflow/outflow differ fundamentally only in near full and overflow situations.

Regarding reality closer cases of transition matrices - formulated according to inflow with simultaneous fixed consumption; - which hydraulic head should be accounted while computing energy potential of water run on turbines is still an open question. There is no doubt this approach shows expected energy between more realistic limits, but due to the magnitude of difference between the energy results of consumptions of simultaneity and sequentiality, upper and lower limits of energy output could be a manifold multiple of each other, thus undermining customer's confidence towards designer's credibility. Serious tasks came up meanwhile, like estimating power production of a reservoir chain on River Niger collecting water from a subcontinent-sized river basin. I could not have followed the usual methods, I must have turned from the regular drive to take the task in a totally new approach; I reached the model's kernel retaining the consistent part, fundamental ideas and the computational manner of the method. What was left over then? Discretization of time and volume variables as well as lower and upper limits: when less water comes reservoir empties, when a lot then overflows.

Aim is the energy - quantity of which fundamentally depends on the neighbourhood morphology of the dam section -, thus formulation of transition matrices of this Markov chain process should be done from potential energy function regarding stored water volume referring to dam section's turbine level, from this special space informatics tool, got by numerical integration of the map. Based on digitized map, this function has already been created while selecting geometrically optimal dam section to be examined as power plant operation site after complemented by hydrographical data, so determining water consumption from potential energy function seemed to be obvious. But how continuously changing hydraulic heads shall be taken into consideration? From what altitude and until how much time the water does arrive onto the turbines? I had to solve countless untold additional emerging questions and doubts by a masterly stroke.

Due to the aspects of power generation all volume changes presumed to be simultaneous and uniform in time, the integral by the volume - to get potential energy function having been used in site selection based on geospatial data - could be transformed by way of substitution into integral by the time and vice versa. This realistic and plausible hypothesis opens the way to model the processes determining transition probabilities. By accepting this reality nearer condition neither time nor altitude data are needed to be dealt with, all such information considering the full morphological data set is already included in potential energy function.

When modifying basic equation for a more accurate estimation of electricity production, it is not the fixed consumption to take into account but the amount of water needed to produce required energy which depends on hydraulic head as well, that is: the energy generated
depends on current amount of water stored in the reservoir; power produced is also subject to inflowing waters arriving at the same time. Presupposing uniformity and simultaneity in time, now we have to take advantage of all topography interpolations to define random walk probabilities in a measure falling nearest to the reality.

Potential energy function of $E(h)$ specifies the energy produced by all water quantity run on turbines during depleting the reservoir from an altitude of $h$. To determine transition probabilities estimation of power generation between inner states is needed. Power generated during transition from $h_1$ to $h_2$ is given as $(E(h_1) - E(h_2))$ which should be less than the value of requested energy to produce otherwise this very transition can not be established because it generates more electricity than the given time period needs, ie, its probability is zero. But if it accurately meets the desired energy to produce, water could not arrive into the depot, because it would leave through the turbines and more energy would be delivered hereby than requested.

If the given change of state from $h_1$ to $h_2$ would not be able to supply the planned energy quantity without additional inflows, a certain calculated amount of arriving water passing through turbines would insure the missing energy then. Since inflow, power generation and change of state happen all at the same time and are uniform throughout the time unit in question (year/season/month), energy produced during the given change of state by a unit water quantity - originating indifferently from either stored water or just flowing in now - is $(E(h_1) - E(h_2))/ (V(h_1) - V(h_2))$ which is the quotient of energy produced by a change of state and the difference in stored water volume. Divide the requested energy quantity by unit water generated electricity in order to find out how much water - following the a known distribution - has to arrive into the reservoir which provides the transition probability in question.

As a general rule, no matter whether the calculated water quantity runing through turbines during either depletion from level $h_j$ to $h_i$ or filling up from level $h_i$ to level $h_j$, due to the uniform turbine discharges the same time is spent on the same interim hydraulic heads both in downwards and upwards water level changes. Calculated water quantity to pass onto turbines equals the sum of arriving water and the change of stored water.

The full subsequent state represents a special case in state transitions, not only the water quantity satisfying energy needs is showered onto the turbines, but diversion of spillway water is put on power generation as well up to the turbine capacity. When overflow occures the pressure on turbines is at maximum, the model is extended with the calculation of energy surplus produced by diverted spillway water. Proportion of energy gained from overflow waters indicates that the given power plant section can energeticswise better be utilized.

In case of hydropower the reservoir is not built exclusively to store randomly available excess water for shortage periods but also to ensure sufficient hydraulic head making power generation possible. The subsequent examinations - being based on transition matrices constructed from claimed energy quantity - may bring to light changes deriving from development or change in operation of existing reservoirs, and a current operation may be tested as a sort of forecast with the examination starting from a known state, and is done as a single matrix multiplication in each time unit. This chapter induced the acknowledgement in McMahon's book review cited above, and if the method survives my personal period, I surely
hope for my name gets reference for this unusual, new approach requiring common wit merely, as the undermentioned compact formula shows transition probabilities concisely:

\[ p \left( I_t = \left( \frac{E_t}{\sum_{j} (H(V) - H_0) dV} - 1 \right) (j - i) \Delta V \right) \]

Presumed uniformity of water volume changes in time of simultaneous inflow and power production is not a necessity, fluctuating demand of electricity could be incorporated as a differentiable function factor tag in energy calculation integral thus accumulating some potential energy surplus according to power request distribution within days and weeks is also possible.

[5] Drastic reduction in calculations regarding random demands on a reservoir capacity range

By computing stationary model’s consecutive capacities for a fixed consumption, the matrix elements belonging to the common non full state transitions are identical. Matrices describe random walk’s target transitions by rows; last row of the minor matrix is the sum of the last two rows belonging to the next bigger capacity. Apart from the last row all numerical operations of the matrices are identical during eliminations along the main diagonal, therefore we can obtain the results belonging to each of the smaller capacities in the course of the elimination of a transition matrix belonging to the largest capacity. With an elegant transformation this algorithmic acceleration is generalized to random consumption as well:

\[ FDP = P \]
\[ DP = F^{-1} P \]
\[ (F^{-1} - D) P = 0 \]

In cases of consumptions and/or capacities being similar to each other, transition matrices formed from Moran type basic assumption perturb slightly. Fortunately in parts of the cases this perturbation makes a difference of a single diad merely. One of the answers to arising double questions serves a practical benefit, the other does as theoretical proof of a conceptual predicate. Question of the practice was how to get the result vector more cheaply - with less numerical operation - from another result vector? The emerging conceptual question responds to that obvious fact, that a smaller reservoir supplies the same consumption with higher risk, that it gets into lower states with a bigger probability, or at last empties with a higher risk. Diad related results stem from the fruitful collaboration of Pál Rózsa who helped to get the distribution of stored water in reservoir of capacity K+1 units from results belonging to capacity of K with less numerical calculations.

From a striking yet trivial transformation of the beginning of above examination demanding mathematical knowledge with yet/already an uncharacteristic depth in engineering practice today, a more universally useful algorithm acceleration resulting in bigger saving crossed my mind which may originate from earlier observations during countless practical examples. The inner elements of the transition matrices belonging to consecutive reservoir capacities were identical. These matrices differ only in size, in last row hereby, that considering their rank was one maller than their order, this last row was foredoomed to be omitted. Since these matrices are filled up with transition probabilities, and we are looking for eigenvectors with sum of 1, by extracting unit matrix from main diagonal which thus turned out to be dominant, not even pivoting is needed. Starting elimination from upper left corner, all respective
elements in every single transition matrix of all capacity belonging to the given consumption distribution and dam section are the same numbers throughout the whole elimination. In other words, while calculating any capacity, the number and operations in the matrices are completely the same, only the smaller capacity matrices run out of last columns and rows earlier, so in the course of calculating the maximum capacity all results of the smaller reservoirs is available as a byproduct. Similarly to the case of fixed consumption, with an elegant transformation this algorithmic acceleration is generalized to random consumption as well:

\[
\mathbf{FDP} = \mathbf{P} \quad \mathbf{DP} = \mathbf{F}^{-1} \mathbf{P} \quad (\mathbf{F}^{-1} - \mathbf{D}) \mathbf{P} = 0
\]

\[
F_{i,j}^{-1} = \begin{cases} 
  g_0, & \text{if } i = j > 0 \\
  g_{j-i}, & \text{if } 0 < i < j \\
  1 - \sum_{h<j} g_h, & \text{if } i = 0 
\end{cases}
\]

\[
\sum_{i+j=1} f_i g_j = \begin{cases} 
  1 & l = 0 \\
  0 & l > 0 
\end{cases}
\]

Necessary exponent of polynomial amount of operation is reduced by one, in case of capacity 50 close to 50 times acceleration is attained as opposed to velocity doubling of diad based speeding up.

The question arose was that this numerical acceleration can be used for the more complicated matrices of power generation just similarly to the case of the fixed and random water volume consumption? Examining inner state transitions of a power station on different capacities, concurrent volume changes presumed to be uniform in time, the process is affected by water(level) between starting and final states only, no matter e.g. how much void empty space is unutilized in the reservoir above. This on the other hand has a consequence, that apart from their overflow affected last rows, transition matrices - of reservoirs with different capacity compiled in a manner not depending on available empty surplus capacity - are identical, that is the method of acceleration is useful here as well. Since transition matrices - due to their identical sums in columns - are singular, a row, in our case the "irregular" last one belonging to full and overflow cases, can simply be omitted.


Just like the subsequent reservoir state distribution is obtained from the distributions of antecedent reservoir states and arriving waters, in that very same manner and resources the shortage volume distribution of \( \chi_{t+1} \) can be computed via a discrete convolutional integral executed by way of multiplication by a transition probability matrix based on the transformed basic equation:

\[
\chi_{t+1} = \max(M_{t+1} - \min(\xi_t + I_{t+1}, K), 0)
\]

The basic deficiency relation does not differ in the case of of random water demands either, only the character of one additional member - value of the planned consumption - turns into a stochastic variable of a known distribution; so an up till now fixed constant turns into a distribution of a single vector containing chances of possible values of consumption, and in the formula providing the probability of the deficiency events an additional sum is found, what means the multiplication with an additional matrix to be constructed adequately. To
compile the transition probabilities the similarity of the undermentioned formulae of \( \xi_{t+1} - \xi_{t+1} \):

\[
\xi_{t+1} = \max(\min(\xi_t + I_{t+1}, K) - M_{t+1}, 0)
\]

\[
\zeta_{t+1} = \max(M_{t+1} - \min(\xi_t + I_{t+1}, K), 0)
\]

\[
\text{FDP} = P - \zeta_{t+1} = \min(\min(\xi_t + I_{t+1}, K) - M_{t+1}, 0) \quad \Phi \text{D} \text{P} = S
\]

At an early stage deficiencies were considered as a slight exaggeration in favour of the safety, depletion of reservoir as risk of malfunction while complemeneter value as operational safety. Similarly to others the expected shortage value were based on that assumptions. Economic calculations started causing problems later (e.g. operational water level regulation of lake Balaton) since there is a real chance of emptying a reservoir in a way that not even a drop is missing from the planned water consumption, and in more serious economic analyses parts of the water delivery claim different safety levels. Shortage distribution was needed for the choice of plantations of different water demand and for expected reservoir operation plans based on the distribution of the safety of water supply. Onsite interactive risk analysis provided ammunition for strategic masterplans, water and electricity of rivers of melted glacier was needed to feed and light the growing population in West Mongolia in the big neighbours' squeeze.

To change the estimate of deficiencies, it was necessary to improve the calculations of model refinement based on assumption closer to the reality. Shortages are just the same type of events like inflow, outflow. The feature to be calculated is the rate of failure of water consumption exactly. I described shortage event just in the same manner as Moran made it with the model's basic equation. Beside the values of consumption and capacity, reservoir state probabilities and distribution of arriving waters served as a basis to the calculation of shortage distribution. The same independence conditions apply to deficiency relations as the hypothesis assumed in the model's basic equation, so shortage distribution can be reckoned via convolution which due to the discretization is a matrix multiplication.

Estimating the distribution of the shortages I had to be aware of that particular fact, that in normal economic environment the damages caused by deficiencies are polyvalent, and only sections by sections can be considered for linear one. A striking example of section linearity is the case of multipurpose reservoir, where communal water shortage causes the largest damage, since to drink is a necessity, and hygiene also requests a prescribed water quantity per capita until a certain level. Second harm grade ensues when industrial water fails to supply factories demanding bigger safety of continous water services than agricultural water utilisation which is characterized with the lower next damage value. Anyway planning of plantations might already take it into consideration to irrigate less water if there is not enough, or to plant not so much from water demanding plants in the precognition of deficiency distribution. In many cases - out of formulae leading generally to matrix multiplications, those calculating expected value of shortage are not enough, - distribution of deficiencies or that of restricted consumption \( M_r \) is needed.

\[
P_r = p(M_r = \mu \Delta V) = \sum_{i=0}^{\mu} p(\xi_{t-1} = i \Delta V) \ast p(I_t = (\mu - i) \Delta V) = \sum_{i=0}^{\mu} P_i P_{\mu-i}
\]
The basic deficiency relation does not differ in the case of random water demands either, only the character of one additional member - value of the planned consumption - turns into a stochastic variable of a known distribution; so an up till now fixed constant turns into a distribution of a single vector containing chances of possible values of consumption, and in the formula providing the probability of the deficiency events an additional sum is found, what means the multiplication with an additional matrix to be constructed adequately. To compile the transition probabilities the similarity of the undermentioned formulae of $\xi_{t+1}$ $\mathcal{X}_{t+1} - \mathcal{X}_{t}$ provide help:

$$\xi_{t+1} = \max(\min(\xi_t + I_{t+1}, K) - M_{t+1}, 0)$$

$$\mathcal{X}_{t+1} = \max(M_{t+1} - \min(\xi_t + I_{t+1}, K), 0)$$

$$\Phi D P = P \quad -\mathcal{X}_{t+1} = \min(\min(\xi_t + I_{t+1}, K) - M_{t+1}, 0) \quad \Phi D P = S$$

Matrix multiplications claim huge operational load however, reducing their number would possibly be beneficial by performing internal subproducts only once and then storing these subproduct matrices. No matter that in case of deficiency the water run out because of water consumption or power production, the reservoir is empty, irrespective of how large a dam built or how much void empty space is currently unused in the reservoir above. The matrices used for the calculations are identical or very similar in their common parts. When computing deficiency the full identity of the matrix elements allows to foreshadow, that although these matrices belonging to different capacities differ structurally in their last row only. Last row - describing overflow, full state - may not have a big effect on the deficiency, since value of consumption should be smaller than capacity, otherwise the reservoir would be empty in all years, and there isn’t a Markov chain and there is nothing to plan.

I introduced the subproduct matrix to generate the shortage distribution belonging to the consumption distribution because of this, the elements of which depend on the distribution of arriving waters and consumption only, it is necessary to create only once and we receive the deficiency distribution of the random consumption as subproduct matrix is multiplied by the vector of state distributions belonging to the different capacities after inflow.

$$p(S_{\text{shortage}}=i) = s_i = \sum_{j=i}^{\infty} f_{j} q_{j-i} = \sum_{j=i}^{\infty} p(M = j) p(\xi_{\text{afterInflow}} = j-i)$$

$$FDP = P \quad \& \quad DP = Q \quad \& \quad \Phi DP = S \quad \Rightarrow \quad \Phi Q = S$$

$$AP = P \quad \& \quad BP = S \quad \Phi_{i,j} = f_{i+j}$$

To find out easily the expected value of the shortages of random consumption, subtract the the expected value of the consumption distribution from the difference between the expected values of water quantity in the reservoir after inflow and after consumption.

[7] Optimising reservoir on a given reliability and/or investment return

During my calculations, isoclines of probability field showed monotony, therefore grid dots of the capacity consumption subfield could be bounded by predefined lower and upper safety. On
the examined dots, cumulative values of expected consumption and loss functions weighted with deficiency distribution were compared with the investment and operational expenses. This type of optimum search is applicable to the case of power generation as well.

Capacity consumption grid of $k^2$ step number magnitude claims the execution of an algorithm of $k^3$ multiplicative operation in each dot. This in total represents $k^5$ order of magnitude operational step. Examination should be carried out over the morphologically appropriate range of reservoir capacities. Not necessary however to do the heavy calculation for all consumption from zero up to the values reaching as far as the order of magnitude of the capacity. Due to the large number of possible states resulting in huge operational load particularly in cases of low consumptions, reservoir empties quasi never and is quasi always full - too big the safety is. In opposite cases of few states belonging to big consumptions on the other hand quickly may we reckon, but the reservoir empties quasi always, and because of this not too much to reserve, the reservoir retain so few water that storage space is only exploited within time unit. Designers would generally like to examine around a prescribed safety, thus to determine optimal reservoir size and operation. This on the other hand presents the necessity to do the heavy calculation along only a line belonging to given water delivery safety on capacity-consumption field and as a result the examination of $k^5$ magnitude turns into one step lower in power number that is $k^4$.

Even more interesting is the optimization - between the claimed operational safety limits - carried out on the basis of economic data. As early as the mid-70s I made and finished at the request of and under the guidance of Dr. Zsuffa operational water level regulation examinations of Lake Balaton with the help of losses (eg in tourism) and (eg, fisheries, irrigation) profit functions and the probability of depletion. To refine the examination I formulated the algorithm determining the distribution of shortages serving as the basis of more sensitive economic investigations.
But unfortunately - similarly to the case of the distribution of water consumption - it is difficult to obtain economic data, and however influencing the sizing significantly, return time and operation expenses often does not play a role in many cases. Though apart from the investment expenditure, with the help of the operational costs and the profit hoped for in the function of variable delivery safety depending on communal, industrial, or agricultural type of consumer, the method may possibly turn into a serious long-term decisions supporting tool.

Though not explicitly related to water, morphological and economical optimization of the reservoir had already roots in the earliest period and more delicate has been finely burnished throughout since then. Difficulties were caused by the lack of digital maps and the unavailability or unreliability of economic data representing expenses and prices. We were ready to go ahead, and this kind of development of the model were blocked by the unpreparedness and/or immaturity of the application environment.

Applying the completed algorithms over capacity-consumption grids the convex surface search was justified. Some cases of multipurpose reservoir investigations included flood reduction as well, in addition to the water and energy utilisation, savings in downstream flood control investment were also taken into consideration as a negative supplementary investment expense in the course of optimum search.

[8] Ice Inspection - Estimating Ice Cover Rate

Ice cover estimation is an integral over the water domain of the perspective picture in investigation. The integral is over the water of the camera picture by dx dy. Since we are summing the icy pixels on the picture itself, the integration is done by an integral
transformation, it is a change of variables when integrating a function over its domain, here the Jacobian determinant is used. To accommodate for the change of coordinates the Jacobian determinant arises as a multiplicative factor within the integral. Following calibration focused on water plane I arrived on the following closed form of the Jacobian:

$$\begin{bmatrix}
\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\
\frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}
\end{bmatrix} = \frac{1}{\tilde{z}} A R - \frac{1}{\tilde{z}^2} \begin{bmatrix}
\tilde{x} \\
\tilde{y}
\end{bmatrix} \begin{bmatrix}
r_{31} & r_{32}
\end{bmatrix}$$

where:

$$[u, v] = A (R \begin{bmatrix} x \\ y \\ z = 0 \end{bmatrix} + t)$$

$$[\tilde{x}, \tilde{y}] = R \begin{bmatrix} x \\ y \\ z = 0 \end{bmatrix} + t$$

$$[u, v] = \frac{1}{\tilde{z}} \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{bmatrix}$$

$$[\tilde{x}, \tilde{y}]$$

$$\int \int f_{ice}(x, y) \ dx \ dy = \int \int f_{ice}(u, v) J(u, v) \ du \ dv$$

$$J(u, v) = \begin{bmatrix}
\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\
\frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}
\end{bmatrix}$$

$$y = A \ (R \mathbf{x} + \mathbf{t})$$

$$R = R_x R_y R_z$$

$$A = \begin{bmatrix}
\alpha & \gamma & u_o \\
0 & \beta & v_o \\
0 & 0 & 1
\end{bmatrix}$$

where $R$ is the rotation matrix and $t$ is the translation, they are the extrinsic parameters. $X$ consists of the real world coordinates while $Y$ yields the image pixel coordinates. $A$, the camera intrinsic matrix, is given by $(u_o, v_o)$ the coordinates of the principal point, $\alpha$ and $\beta$ the scale factors in image for $u$ and $v$ axes, and $\gamma$ the parameter describing the skewness of the two image axes on the light detector CMOS chip.

Ice cover rate on water surface does not depend on distance or unit, it is the function of an angle formed with the plane of the water only. On the other hand, we may obtain this angle from the horizon line shown on the picture to be evaluated. There is no need for difficult on site calibration either. Similarly to stereometry or our eye, the real distances can even be calculated based on pictures taken even with mobile phone from two points of known distances. Based on coordinates exempted from radial distortion of pictures taken by mobile phones or cameras calibrated in office environment to get intrinsic parameters, horizon line equation $a x_i + b = y$ results in closed form:

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix}
\frac{1}{\sum x_i^2 - (\sum x_i)^2} & -\frac{\sum x_i}{\sum x_i^2 - (\sum x_i)^2} \\
\frac{\sum x_i y_i}{\sum x_i^2 - (\sum x_i)^2}
\end{bmatrix} \begin{bmatrix}
\sum x_i y_i \\ \sum y_i
\end{bmatrix}$$

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From parameters of the horizon defined this way the obliquity of the camera - its differing from vertical - could be determined. Based on the pixel distance of the principal point of the camera and the horizon line on the picture, and the focal length of the optics measured in pixel the angle between the camera axis and the water surface can be got as well.

Application

Our research was primarily driven by the needs of practical applications which significantly promoted the prompt utilization of the - by nature - basic investigations. The dissertation's appendix consists of real examples from three continents, case studies illustrate the model suitability for the examination of reservoirs of any size, purpose or operational complexity. Furthermore resulting numerical methods are directly applicable on all such storage tasks where approximational approach of the independence of arriving inflow water quantities in subsequent time units is acceptable.

Hundreds of reservoirs - planned and existing - were investigated and designed by public domain software accompanying my publications. There were countless applications even in Hungary without notifying software and modell author. Software pirates also sold rewritten copies, but no modell tuning or development were done later by others except for some sedimentation experiencing (including "discovering" non existent ergodic reservoir state distribution of sedimenting reservoirs:). Extra European applications included a reported master thesises in South Korea in late 80s, Nigerian examples in early 1980s, wide ranging Mongolian application of my packages covered from mid 1970s to mid 1980s, while Algerian applications were on top between 1984-1987. By public release of the source package in 1987, wide spread application evolved from then on.
BIBLIOGRAPHY


[33] Zhang, Zhengyou/Microsoft: Method and system for calibrating digital cameras, United States Patent 6437823, 2002-08-20
Part II. Applications

INTRODUCTION

The practical execution of the reservoir sizing illustrated by 19 examples follows. In the first example all of the necessary information on how to set up the executable programs, how to run the examples, all the method of the data preparing and the graphical evaluation of the output sheets are detailed. For examples 2-19 only the name of the load module, the name of the data file of the example and the manner of the data preparing are given. The figures of the examples were constructed from the printout of the respective example. The programs and routines were originally written in computer language FORTTRAN 77. They were on two diskettes of 360 KB, enclosed containing all source programs. It enables the reader to examine the program structure and to use them both for education and in their practice. So it is possible for you to repeat the treated examples, and by doing so improve your ability to cope with your own sizing problems.

The authors originally made development under MS-DOS 3.10 on their personal computer using the following software:

\textbf{MS FORTTRAN 77 V3.20 02/84}

\textbf{MS Library manager version 3.0}

\textbf{MS 8086 Object Linker version 3.01.}

Let us see now how to run the programs:

1. Insert the diskette number 1 into the drive A:
2. Enter:

\texttt{RES %1 %2}

where the two parameters %1 and %2 means the name of your hard disc and the path to the necessary softwares respectively. E.g. if you give

\texttt{RES C:C:\FOR\}

then the batch will work on the hard disc called C: and will search for the FOR1, PAS2, FORTTRAN libraries, LIB, and LINK in the subdirectory called FOR on the disc C:

This batch file copies all of the source programs and data files onto the hard disc of your computer making a new sub-directory called WRPRES, translates sources and links them making the load module. To install the package on your PC takes some time depending on your specific hardware environment.

3. Then enter the name of the load module of the program in question and the name of the data file.

E.g. for the first example you have to enter:

\texttt{ERGEV <GALGA11.TXT >GALGA11.PRN}

(for getting the results on your printer of 132 characters monospaced use >PRN as output parameter).

Clicking on file names induce download above. (Especially for TXT and PRN use rightClick "Save Target As" option to avoid opening in browser window. At certain client side settings ERGEV might need " .EXE" extension postfix when intended to run. Using PRN as output filename printout goes to line printer, supposed to set to monospace typing. Omitting output redirection put results onscreen.)

4. For more information see the BAT and FOR files on the disk:

Having an own sizing problem to solve you have to enter the name of your data file instead of GALGA11.

The programs on the originally enclosed diskettes were prepared especially for the original WRP publication and were made to enlighten the text of the book. The programs written primarily for a stone age VIDEOTON R10 computer of 64kB are completed with a lot of comments and the computer supplies also the secondary results to explain and illustrate the details of the applied methods.

An interactive and once upon a time called "user friendly" version of this software for personal computers is also available. A networking WAN version of the package is under preparation. To help standard data input, regular data presentation and in order to work within everyday office environment on a stand alone desk top - a macro driven spreadsheet version like excell sheets of the bellow examples are under consideration. While "Connecting Reservoirs and Water Resources Stations by Markov Chains" connecting past and future of the model applied is also needed. To meet the challenge put by the enormous advance of the software industry regarding both problem solving, artificial intelligence and data presentation and evaluation in the long run, author schedule reformulate the modell in symbol processing and visualizer environment like Mathematica.

Application of the Stationary Model for Investigations under Assumption 1 (Examples 1 to 4)

The application of the stationary model is limited by the long time unit (Δt=1 year) and the basic hypotheses. Examples are presented from the practice on different alternatives of the basic Assumption 2 (see Chapter 2.), to which even this simplest model can be applied. Through four specific examples presented here, a practical method is shown on the different interpretations of Assumption 2, and on its modification which does not affect the computerized execution of calculations. The basic hypotheses related to the four different cases are shown in Figs. A-1 through A-4.

Example 1. It relates to the small Galga Creek (Fig. A-1), a stream located close to Budapest. This creek frequently runs dry in summer. The reservoir is to be used for irrigation. So for its sizing the basic hypothesis can be used without any change.

Example 2. Presented in Fig. A-2, it relates to one of the main rivers of Nigeria, the Benue River. The large flood season on this river, having a tropical flow regime, lasts form May to October. In the dry period from November to April, the river runs practically dry. In order to assure navigation, water stored in the wet season is released in the dry season. However, even in the wet season the reservoir cannot be always filled since water for navigation must be released downstream. However, there is only a small flow requirement since 10 to 20 km downstream the slack water navigation is already assured by the water of tributaries as well as by the Niger River, with abundant water in wet season. From a considerable annual flow, the relatively small flow requirement for navigation is subtracted and the reservoir sizing is made to assure navigation, water stored in the wet season is released in the dry season. However, even in the wet season the reservoir cannot be always filled since water for navigation must be released downstream. However, there is only a small flow requirement since 10 to 20 km downstream the slack water navigation is already assured by the water of tributaries as well as by the Niger River, with abundant water in wet season. From a considerable annual flow, the relatively small flow requirement for navigation is subtracted and the reservoir sizing is made.

Example 3. Presented in Fig. A-3 it relates to the Karasica Creek with a karstic catchment, a stream in Southern Hungary, which doesn't dry out. The reservoir serves irrigation purposes so that water demand occurs in summer only. As to the arriving flows during the summer, it is supposed that they are utilized before reaching the reservoir. Therefore, when examining water demand for an 80 percent reliability of supply, each annual water volume which enters the reservoir is decreased by up to the water volume of 80 percent probability of non-exceedence coming during the irrigation season.

Example 4. Presented in Fig. A-4, it consists of the permanent industrial demand from the reservoir built on the stream referred to in Example 3. Here Assumption 2 is modified. Both the deterministic demand and the random inflow are supposed to be uniform within the time interval. This hypothesis implies that after each demand M the necessary storage space is smaller by that M value, than in the case of Hypothesis 2. This hypothesis (which decreases safety) will be realistic when the cumulative sum of entering flows is greater than the cumulative sum of demand at all moments.
Example 1.
Examination of Reservoir with Large Storage Capacity in Comparison with Mean Annual Inflow. (Temperate zone, streams dry in summer, water demand is for irrigation.)

The classical Moran model can be applied by using the year as the time unit. The reservoir is to be built on the Galga Creek, near the village of Galgamacsa, with a catchment area of 242 sq.km. The flow regime of Galga Creek is shown in Fig. A-5. Two alternatives of the stationary model are used: (1) the model which calculates the ergodic state directly; and (2) the model which starts from a given state and characterizes the subsequent year. Both are computed on the basis of frequency distribution of annual flows. Mean annual flows were computed on a computer by using the water level data stored in the Hungarian Hydrologic Service and the corresponding discharge rating curves. However, the direct load of discharge data is also feasible.

In the first example the sheets of the programs and a few of the outputs are enclosed for help the using. In the other examples only the name of the main program is mentioned with them the adequate program of the enclosed diskettes can be indicated.

Examination of the ergodic state.

Calculations for the ergodic state characteristics were performed by using the programs P1 to P4. The computation is controlled by the main program FWFOP7 (P1) which utilizes the subroutines DATAIN (P2) for loading data, and ERGCON (P4) for the computation itself. The further subroutines called, DISTRB (P3), SORT (standard), and GELG (standard). Data to be loaded are shown in Table A-1.

In order to repeat the computation on your own computer you have to enter

```
TEGRVE <GALGA11.TXT >GALGA11.PRN
```

where ERGEV is the name of the load module and GALGA11.TXT is the name of the data file. If you want to run the program to solve your own problem you have to make a new data file according to the input data format described in Table A-1 below.

Clicking on file names induce download above. (Especially for TXT and PRN use rightClick "Save Target As" option to avoid opening in browser window. At certain client side settings ERGEV might need "EXE" extension postfix when intended to rerun. Using PRN as output
Fig. A-5. The monthly mean discharges(1) and monthly mean discharges of a very dry year (2), of the Galga Creek (Hungary).

Table A-1. Data Used

<table>
<thead>
<tr>
<th>Card</th>
<th>Data loaded in</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Name of reservoir</td>
<td>40X2</td>
</tr>
<tr>
<td>2.</td>
<td>Lower limit of storage capacity, in $\Delta V$</td>
<td>I5</td>
</tr>
<tr>
<td>3.</td>
<td>Upper limit of storage capacity, in $\Delta V$</td>
<td>I5</td>
</tr>
<tr>
<td>4.</td>
<td>Volume unit $\Delta V$ to discretize continuous random variables, (m$^3$)</td>
<td>E10.0</td>
</tr>
<tr>
<td>5.</td>
<td>Lower limit of depletion probability to be examined in percentage</td>
<td>F5.0</td>
</tr>
<tr>
<td>6.</td>
<td>Upper limit of depletion probability to be examined in percentage</td>
<td>F5.0</td>
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<tr>
<td>7.</td>
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GALGA-GALGAMACSA RESERVOIR (ERGEV)

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<tr>
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GALGA-GALGAMACSA YEARLY MEAN DISCHARGES

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Table A-3.

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<td>1.E+06</td>
</tr>
<tr>
<td>0.01</td>
<td></td>
<td>0.3</td>
</tr>
</tbody>
</table>

C-language code:

```c
#include <stdio.h>

int main()
```

#include <stdlib.h>

```c
int main()
```
C                COMPARE THIS WITH DECLARATION LIMITS
C IF(K-M+1.GT.30) GOTO 2
C CALL ERGCON(K,M-1)
C EXAMINE THE PROBABILITY OF DEPLETION AS TO THE UPPER
C LIMIT PHI IS REACHED OR NOT. HIGHER CONSUMPTIONS SHOULD NOT
C BE EXAMINED SINCE DEPLETION WILL TOO OFTEN OCCUR.
C IF(0.0.GT.PI) GOTO 1
C THE MAXIMUM OF M WHERE PI(M).EQ.PLA STILL HOLDS.
C SINCE IT IS SUFFICIENT TO START WITH NEXT CAPACITY FROM HERE
C IF(PLA.GT.PI) M = M +
C INCREASE THE CONSUMPTION
C 2 CONTINUE
C INCREASE OF CAPACITY
C 1 CONTINUE
C C                SEARCH THROUGH SOLVING EQUATION SYSTEM THE EIGENVECTOR
C                (P4)
C                CALL THE SUBROUTINE PRODUCING ERGODIC STATE
C                A(L,L)=1.0
C                P(L)=1.0
C                SEARCH THROUGH SOLVING EQUATION SYSTEM THE EIGENVECTOR
C                COMPARE THIS WITH DECLARATION LIMITS
C                END
C                STOP
C C                INCREASE OF CAPACITY
C C                INCREASE THE CONSUMPTION
C C           SINCE IT IS SUFFICIENT TO START WITH NEXT CAPACITY FROM HERE
C IF(P0.GT.PHI) GOTO 1
C BE EXAMINED SINCE DEPLETION WILL TOO OFTEN OCCUR
C LIMIT PHI IS REACHED OR NOT. HIGHER CONSUMPTIONS SHOULD NOT
C EXAMINE THE PROBABILITY OF DEPLETION AS TO THE UPPER
C HIGHER PROBABILITY OF DEPLETION ARE UNINTERESTING
C CALL ERGCON(A,K-M+1)
C CALL THE SUBROUTINE PRODUCING ERGODIC STATE
C DO 1 I=2,NPI
C          COMPUTE A VECTOR CONTAINING THE PROBABILITIES, ROUNDED IN DV,
C           FROM SPI COMPUTE PI (THAT IS FROM A VECTOR OF NON-EXCEEDANCE
C C                ATTENTION ] IF ANOTHER DISTRIBUTION FUNCTION IS NEEDED
C C                READOUT FROM DISTRIBUTION FUNCTION FITTED TO SAMPLE
C WRITE(*,202)NEV,(Q(I),I=1,N)
C VKD=KD*DV
C VK2=K2*DV
C READ(*,101)NEV,N,(Q(I), I=1,N)
C READ THE SAMPLE OF ANNUAL MEAN DISCHARGES
C READ(*,100)NEVT,K1,K2,KD,DV,PLA,PHI
C TO BE TESTED, VOLUMETRIC UNIT OF DISCRETION AND
C FEEDING THE RESERVOIR
C Q SAMPLE OF ANNUAL MEAN DISCHARGES OF WATERCOURSE
C FEEDING THE RESERVOIR
C INPUT OF THE NUMBER OF YEARS = NUMBER OF ELEMENTS FOR Q
C NAME OF RESERVOIR
C NEV COMMENT VECTOR
C IFPI=64
C READ IN THE NAME OF RESERVOIR, LIMITS FOR CAPACITIES
C TO BE TESTED, VOLUMETRIC UNIT OF DISCRETION AND
C LIMITS FOR DEPLETION PROBABILITIES
C READ(*,100)NEVT,K1,K2,VO,PI,PLA,PHI
C READ THE SAMPLE OF ANNUAL MEAN DISCHARGES
C AFTER THE COMMENT LIMIT, 5 VALUES FOR CARD
C READ(*,101)NEV,(Q(I), I=1,N)
C PRINT OUT INPUT DATA
C V(K)=VO
C VK=K*DV
C WRITE(*,13)NEvt,VO,K1,K2,VO,KD,VO,PI,PLA,PHI
C WRITE(*,13)NEVT,K1,K2,VO,PI,PLA,PHI
C READOUT FROM DISTRIBUTION FUNCTION FITTED TO SAMPLE
C SINCE DISCHARGES ARE IN KI/H UNIT, DV VOLUMETRIC
C UNITS SHOULD BE DIVIDED BY THE NUMBER OF SECONDS
C IN A YEAR = 3.1 536 000
C DISTRIBUTION VECTOR SPI AND P(I) ARE THE RESULTS
CALL DISTR(V,VO,PI,DI,PLA,PI)
C FROM SPI COMPUTE PI (THAT IS FROM A VECTOR OF NON-EXCEEDANCE
C COMPUTE A VECTOR CONTAINING THE PROBABILITIES, ROUNDED IN DV,
C WHICH BELONGS TO THE RELATION OF EQUALITY)
C DO 1 I=1,NEV
C P(I)=SPI(1:PI-I-1)
C P(I)=SPI(1:PI-I-1)
P(I)=SPI(1)
From the output list of data processing performed by using the listed programs on data loaded in the above manner, details of the first sheet of R1, data loaded in can be read. At first the computer prints the results for each M demand, starting from the capacity \( K_{\text{min}} = 10 \times 10^6 \text{m}^3 \) and then it applies an increment to capacity \( K \). In the example this increment is \( 2 \Delta V = 10^6 \text{m}^3 \). To each couple \((K,M)\) the probability of occurrence of stored volume
\[ \xi = 0, 1\Delta V, 2\Delta V, \ldots, (k - m)\Delta V \]

at the year's end is also given by the computer. The temporal safety, related to number of years of satisfying the annual demand \( M = m \Delta V \) from the reservoir, whose capacity is \( K = k \Delta V \), can be characterized by depletion probabilities
\[ P(k,m,0) = \Pr(\xi = 0 \mid K = k, M = m) \]

referred to as the pair of values \( k = k \Delta V, M = m \Delta V \). Using the data in the first three columns (with vertical lines) of the output list, the yield function of storage \( K = [M,P(0)] \), used primarily in Central Europe, can be plotted. (Figures A-6 through A-8), which give the hydrological characterization of feasibility of storage capacity in the ergodic state, for the Galga Creek at Galgamacsza gauging station, Hungary.

<table>
<thead>
<tr>
<th>TIME UNIT</th>
<th>ONE YEAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>VOLUMETRIC UNIT:</td>
<td>1.000E+06 M3 = 1 DV</td>
</tr>
<tr>
<td>CAPACITY OF RESERVOIR</td>
<td>K = 1.000E+07 M3 = 10 DV</td>
</tr>
<tr>
<td>PROBABILITY OF DEPLETION</td>
<td>P(0) = 0.010 &lt;= P0(K,M) &lt;= 0.300</td>
</tr>
<tr>
<td>STATE OF RESERVOIR</td>
<td>X = 0 &lt;= X = K-M</td>
</tr>
<tr>
<td>CONSUMPTION</td>
<td>M = 2.000E+06 M3</td>
</tr>
<tr>
<td>MEAN DISCHARGES (IN M3/SEC)</td>
<td>ANNUAL MEAN DISCHARGES OF WATERCOURSE</td>
</tr>
<tr>
<td>NAME OF RESERVOIR: GALGA-GALGAMACSA RESERVOIR (ERGEV)</td>
<td></td>
</tr>
</tbody>
</table>

\[ \begin{array}{cccccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
1 & 0 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.004 & 0.9952 \\
2 & 0 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.9854 \\
3 & 0.137 & 0.140 & 0.122 & 0.308 & 0.736 & 0.483 & 0.341 & 0.069 & 0.396 & 0.281 \\
4 & 0.136 & 0.324 & 0.421 & 0.345 & 0.607 & 0.212 & 0.176 & 0.405 & 0.645 & 0.841 \\
5 & 0.150 & 0.170 & 0.890 & 0.541 & 0.355 & 0.195 & 0.194 & 0.231 & 0.271 & 0.213 \\
6 & 0.172 & 0.320 & 0.237 & 0.682 & 1.176 & 0.673 & 0.831 & 1.167 & 0.475 & 0.315 \\
7 & 0.1 & 0.013 & 0.004 & 0.005 & 0.010 & 0.011 & 0.018 & 0.033 & 0.069 & 0.096 & 0.8827 \\
8 & 0.047 & 0.019 & 0.067 & 0.037 & 0.8355 \\
9 & 0.139 & 0.074 & 0.078 & 0.7528 \\
10 & 0.121 & 0.130 & 0.237 & 0.682 & 1.176 & 0.673 & 0.831 & 1.167 & 0.475 & 0.315 \\
11 & 0.3872 & 0.624 & 0.5504 \\
12 & 0.001 & 0.011 & 0.004 & 0.013 & 0.013 & 0.018 & 0.019 & 0.0660 \\
13 & 0.013 & 0.011 & 0.013 & 0.004 & 0.018 & 0.0674 & 0.0382 & 0.8314 \\
14 & 0.0561 & 0.02 & 0.0408 & 0.0734 & 0.7400 \\
15 & 0.599 & 0.420 & 0.0742 & 0.0729 & 0.6510 \\
16 & 0.2729 & 0.0766 & 0.0514 & 0.5991 \\
17 & 0.3872 & 0.624 & 0.5504 \\
18 & 0.0003 & 0.0003 & 0.0006 & 0.0013 & 0.0064 & 0.0113 & 0.0133 & 0.0197 \\
19 & 0.0035 & 0.029 & 0.0130 & 0.0175 & 0.0193 & 0.0672 & 0.0848 & 0.8276 \\
20 & 0.0242 & 0.017 & 0.0189 & 0.0262 & 0.0662 & 0.0397 & 0.0722 & 0.7334 \\
21 & 0.0816 & 0.028 & 0.0659 & 0.0446 & 0.0705 & 0.0699 & 0.6385 \\
22 & 0.1923 & 0.0488 & 0.0717 & 0.0684 & 0.0455 & 0.5733 \\
23 & 0.3164 & 0.0737 & 0.0449 & 0.0519 & 0.5132 \\
24 & 0.0110 & 0.0223 & 0.0203 & 0.0028 & 0.0079 & 0.0130 & 0.0175 & 0.0193 & 0.0672 & 0.0380 \\
\end{array} \]
The two problems are:

Problem 1. What size of reservoir has to be built to meet the demand of supplemental irrigation in summer with 80 percent safety for a quantity of 3 \(10^6\) m³ per year (irrigation of 9000 hectares)?

The result is:

\[ K = f(M = 9.149 \times 10^6 \text{m}^3; P = 0.2) = 13.8 = 14.196 \times 10^6 \text{m}^3 \]

(See dotted lines in Figs. A-6 and A-7).

Problem 2. How the storage volume will change with a different demand \( M \)? The answer is given in Figure A-8.

By interchanging parameters the same relationship will be obtained (Fig. A-7) when the series of conditional probability distribution functions \( F(x|y) = P(M < x|K = y) \) are used, which are more common in mathematical statistics. To illustrate the way of drawing this graph, and of its use, the solution for a given point is given in detail is Figs. A-6 through A-8. The family of curves in Fig. A-8 presents the conditional distribution functions of volumes stored in the reservoir of capacity \( K = 14 \times 10^6 \text{m}^3 \), by assuming various demands.

**Examination of operation starting from a given state and continuing into the future years.**

The main program for examination is FWFOY2 (P6) with the subroutines called INDATA (P5), DISTRIB (P5), and YEAR (P7). INDATA (P5) is used to load data in, while the routine DISTRIB (P5) is also utilized with the ergodic examination. The calculation in the form of involution of matrices is performed by routine YEAR (P7).

In order to repeat the computation you have to enter

```
YEAR <GALGA12.TXT >GALGA12.PRN for starting from empty state;
YEAR <GALGA13.TXT >GALGA13.PRN for starting from full state;
YEAR <GALGA14.TXT >GALGA14.PRN for starting from \( \xi = 0 \) or empty state;
```

Posted at http://localhost:8080/nrwpbook.phpExample1 in electronic distribution clicking on file names induce download above. (Especially for TXT and PRN use rightClick “Save Target As” option to avoid opening in browser window. At certain client side settings YEAR might need “.EXE” extention postfix when intended to rerun.

Using PRN as output filename printout goes to line printer, supposed to set to monospace typing. Omitting output redirection put results onscreen.)
C CAPACITY/CONSUMPTION COUPLES WITH LOWER SAFETY (HIGHER PROBABILITY OF DEPLETION) ARE UNINTERESTING.
C PROBABILITY OF DEPLETION FOR COUPLE OF ACTUAL CAPACITY/CONSUMPTION (R, M) (SEE SUBROUTINE EKOREG)
C VOLUMETRIC UNIT TO DISCRETIZE CONTINUOUS RANDOM VARIABLES
C K VALUE OF ACTUAL CAPACITY IN DV
C M VALUE OF ACTUAL CONSUMPTION IN DV
C CYCICAL TEST FOR COUPLES (R, M). STIPULATIONS: K1 <= K <= K2 AND PLA <= P0(K,M) <= PFI
C DO 1 F=1,NPI
C DATA READ-IN FOLLOWS
C CALL ENTRAN
C CYCLES OF CAPACITY AND CONSUMPTION FOLLOW
C XI <= X <= K2, YD <= Y <= M - K1 P0(K,Y) <= PFI
C IF XI = M => XI = M + 1
C CALL ENTRAN
C CALL SUBROUTINE PRODUCING ERODIC STATE
C (EXAMINATION OF CHANGES OF STATE UNTIL CHANGE EXISTS)
C CALL YEAR
C """
C COMMON /DATA/ I0
C COMM/NPI,SPI(40),P(64)
C COMMON /START/I0
C COMMON /CONTR/ I1, I2, K,D, P0, PFI, P0, DV, N, M
C CI1 MINIMUM VALUE OF RESERVOIR CAPACITY TO BE EXAMINED, IN DV
C CI2 MAXIMUM VALUE OF RESERVOIR CAPACITY TO BE EXAMINED, IN DV
C KD STEP INTERVAL OF RESERVOIR CAPACITY IN DV
C PLA LOWER LIMIT OF DEPLETION PROBABILITY
C PHA UPPER LIMIT OF DEPLETION PROBABILITY
C PLA PROBABILITY OF DEPLETION OF COUPLE OF ACTUAL CAPACITY/CONSUMPTION (R, M) (SEE SUBROUTINE EKOREG)
C DV VOLUMETRIC UNIT TO DISCRETIZE CONTINUOUS RANDOM VARIABLES
C M VALUE OF ACTUAL CONSUMPTION IN DV
C COMMON/ENTR/I0,ENTR(40),PI(64)
C SPI VECTOR CONTAINING WITH DV DISCRETION THE NON-EXCEEDANCE DISTRIBUTION OF ANNUAL MEAN DISCHARGES FLOWING IN
C SPI(1) = P ( MEAN DISCHARGE <= (1-I/2) * DV )
C SPI CONTAINING WITH DV DISCRETION EQUALITY PROBABILITIES OF ANNUAL MEAN DISCHARGES FLOWING IN
C PI(1) = P ( MEAN DISCHARGE <= (1+I/2) * DV )
C NPI MAXIMUM NUMBER OF ELEMENTS FOR VECTORS SPI AND PI
Ckeit(1),file="DATA"
C """
The examination relates to a reservoir with capacity $K = 1.4 \times 10^6 m^3$. The output $(R2)$ is the sheet of a complete output list. This output contains description of the problem, input data, and results belonging to the empty state $\xi_0 = 0$ and demand $M = 1 \times 10^6 m^3$.

### Reservoir Sizing

**NAME OF RESERVOIR:** GALGA-GALGAMACSA RESERVOIR (YEAR)

**VOLUMETRIC UNIT:** $1.000E+06 M^3 = 1 DV$

**CAPACITY OF RESERVOIR** $K = 1.400E+07 M^3 = 14DV \leq K \leq 14DV = 1.400E+07 M^3$

**PROBABILITY OF DEPLETION $P_0$** $= 0.010 \leq P_0 (K-M) \leq 0.300$

**STATE OF RESERVOIR** $X$ (IN DV) $0 \leq X \leq K-M$

**CONSUMPTION** $M$ (IN DV) $\text{STEP INTERVAL}$

**INITIAL STATE:** $0$ DV $0 < X \leq K$

**ANNUAL MEAN DISCHARGES OF WATERCOURSE**

<table>
<thead>
<tr>
<th>Comment</th>
<th>GALGA-GALGAMACSA YEARLY MEAN DISCHARGES</th>
<th>MEAN DISCHARGES (IN M3/SEC)</th>
</tr>
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<th>STATES OF RESERVOIR</th>
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<tbody>
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<td>DV</td>
<td>DV</td>
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</tr>
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<td>1</td>
<td>0.000</td>
</tr>
<tr>
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<td>1</td>
<td>0.000</td>
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<td>1</td>
<td>0.000</td>
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<td>1</td>
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</tbody>
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<table>
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<th></th>
<th>(R2)</th>
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Fig. A-9. Evolution of safety of operation with time for storage
capacity \( K=14 \times 10^6 \text{m}^3 \) in case of an empty starting state \( (\xi_0=0) \) and varying demand \( M \), for the Galga Creek at the Galgamacsia gauging station, Hungary.

\[
F(x, s) = p(\xi = 0 \mid M = x; T = t_s + s\Delta t; k = 14; \xi_0 = 0)
\]

Conditional probabilities of depletion (breakdown) in case of demand \( M=x \), storage capacity \( K=14 \times 10^6 \text{m}^3 \) and empty starting state; \( \xi_0=0 \); as a function of time \( T=t_s+s\Delta t \).

The first part of output \( R_3 \) is the fourth sheet of the output list. The curves characterizing the EMPTY INITIAL STATES are shown in Figs A-9 and A-10, plotted by utilizing this sheet. The curve belonging to demand \( M=9 \times 10^6 \text{m}^3 \) was plotted on the basis of data singled out by solid lines in this output sheet. The problem is: Given a reservoir with \( K=14 \times 10^6 \text{m}^3 \); starting from the empty state, and with demand \( M=9 \times 10^6 \text{m}^3 \) per year, what is the probability of deficit in the first, second,... year after the start of operation? The answers are: \( F(9,1)=38 \) percent for the first year; \( F(9,2)=25 \) percent; \( F(9,3)=21 \) percent, \( F(9,4)=19.6 \) percent, and so on. The fourth year gives already a good approximation to the ergodic state.

The second part of output \( R_3 \) is the 32nd sheet of the general output list. This describes the case when the operation starts with a COMPLETELY FULL reservoir, storing a water volume \( \xi_0=K=14 \times 10^6 \text{m}^3 \), after 1, 2, ... 7 years. Data used to plot curves in Figs. A-11 and A-12, which characterize the demand \( M=9 \times 10^6 \text{m}^3 \), are also singled out by solid lines in this output sheet \( R_3 \). The problem is: Given a reservoir with \( K=14 \times 10^6 \text{m}^3 \), starting from the full state and with demand \( M=9 \times 10^6 \text{m}^3 \) per year, what is the probability of deficit in the first, second,... year after the start? The answers are: [see dotted line in Fig.(A-10)] \( F(9,1)=10.5 \) percent (first year); \( F(9,2)=16.6 \) percent; \( F(9,3)=18.4 \) percent, \( F(9,4)=19.0 \) percent; \( F(9,00)=19.2 \) (ergodic state).

In order to test the reservoir structures by the greatest water load, a reservoir in most cases is put into operation immediately after their completion, with the complete full state. Figures A-11 and A-12 show this case, with annual demand \( M=9 \times 10^6 \text{m}^3 \). Regardless whether the operation starts from an empty or a full state, the ergodic state is attained within 5 to 6 years. The graphs in Figs. A-10 and A-12 represent the limiting positions of the ergodic state. These two curves are identical with the curve which is defined by the parameter \( K=14\Delta V \) within the family of curves shown in Fig. A-7. They are computed in another way, namely by using the eigenvector belonging to \( \lambda=1 \) in the matrix of transition probabilities.

Considering now a fixed demand \( M=9 \times 10^6 \text{m}^3 \) and a storage capacity \( K=14 \times 10^6 \text{m}^3 \) in case of a full starting state \( (\xi_0=K) \)
and demand $M$ for the Galga Creek at the Galgamacsa gauging station, Hungary.

$$F(x,s) = p(\xi = 0 \mid M = x; T = t_0 + s\Delta t)$$

where $\xi(t_0) = K = 14 \times 10^6 m^3$.

Conditional probabilities of depletion (breakdown) in case of demand $M$, storage capacity $K = 14 \times 10^6 m^3$ and full starting state, $\xi(t_0) = K = 14 \Delta V$, as a function of time, $t = t_0 + s\Delta t$.

K = 14 $10^6 m^3$, probabilities of depletion (breakdown) anticipated in the future are shown in Figs. A-13 and A-14. In Figure A-14 the probability distributions of annual reservoir states (stored volumes at the end of the year) are also shown.

The problem is:

In the reservoir of capacity $K = 14 \times 10^6 m^3$ a water volume $\xi_0 = 4 \times 10^6 m^3$ is stored. What occurs in the subsequent years with annual demand $M = 9 \times 10^6 m^3$? The answers are $F(4,1) = 13.75$ percent (first year); $F(4,2) = 18.15$ percent; $F(4,3) = 18.91$ percent; $F(4,4) = 19.13$ percent; $F(4,5) = 19.2$ percent (which is already the ergodic state). It can be seen from Fig. A-14 that the conditional distribution functions for a reservoir starting from its full state ($\xi_0 = K$) approach the limiting distribution of ergodic state from the left, or from the low risks, while in the case of starting from its empty state they approach it from the right, or from the high risks.

In the reservoir of capacity $K = 14 \times 10^6 m^3$, fixed demand $M = 9 \times 10^6 m^3$ and varying starting state $\xi(t_0)$ for the Galga Creek at the Galgamacsa gauging station, Hungary.

$$F(x,s) = p(\xi = 0 \mid M = x; T = t_0 + s\Delta t)$$

where $\xi(t_0) = x$, as a function of time $T = t_0 + s\Delta t$.

In the reservoir of capacity $K = 14 \times 10^6 m^3$ a water volume $\xi_0 = 4 \times 10^6 m^3$ is stored. What occurs in the subsequent years with an annual demand $M = 9 \times 10^6 m^3$? The answers are $F(4,1) = 13.75$ percent (first year); $F(4,2) = 18.15$ percent; $F(4,3) = 18.91$ percent; $F(4,4) = 19.13$ percent; $F(4,5) = 19.2$ percent (which is already the ergodic state). It can be seen from Fig. A-14 that the conditional distribution functions for a reservoir starting from its full state ($\xi_0 = K$) approach the limiting distribution of ergodic state from the left, or from the low risks, while in the case of starting from its empty state they approach it from the right, or from the high risks.

In the third part of output sheet (R3) the results for $M = 9 \times 10^6 m^3$ demands and the starting state $\xi_0 = 3 \times 10^6 m^3$ are shown. In Figs. A-13 and A-14 the curve with parameter $\xi_0 = 4 \times 10^6 m^3$ has been plotted by using data with singled-out solid lines of this sheet.

Figure A-15 can be used by the reservoir operator as an aid in decision making. Provided that the actual state of the reservoir is known, here the curve groups are given for different demands and different initial states, supplying the information on the risk incurred in the subsequent first, second, ..., n-th year all as functions of demand $M$. On the basis of initial state, the projection can be made for the risk of...
Example 2.

Case of a Large Reservoir Built on a Watercourse Running Dry in Low Flow Season, With Abundant Water in Wet Season, in Order to Meet Water Demand Throughout the Year.

The classical Moran model can be applied for this case, however the water demand in the wet season should be subtracted from the input. The case reservoir is to be built in Nigeria, at a narrow cross section of the Benue River, a tributary of the Niger, near Makurdi. A special utilization of the storage potential is examined in this example. A certain rate of flows is required by the navigation on the Niger River in dry seasons and the lower stretch of the Benue River has to be assured by utilizing water from the wet season retained in the reservoir. The monthly flow regime of this river is given in Fig. A-16. In the operation of eventual reservoirs in the Niger River system the role of navigation is secondary. This example was prepared exclusively for the purpose of illustrating the methodology presented in this book.

The computerized calculation is carried out in the same way as described in Example 1, namely by using the programs already listed. A modification to Hypothesis 2, namely the subtraction of water demand that occurs in the filling period, is performed in the course of processing the basic data. The computation is carried out in three steps.

The model is used without modification in basic data. This computation produces the yield function of storage as the graphs A and B in Fig. A-17, as conditional distribution functions of the deficit in water supply. Readings in these two graphs lead to the storage space necessary to meet any water demand, as a function of required safety in water supply.

The problem is: How large is the needed storage space to assure a navigation flow of 2500 m$^3$/s with a risk of 5 percent? The answer is: 

\[ M = 2500 \times 1.6 \times 10^9 = 80 \times 10^9 \text{ m}^3/\text{per year} \]

This value is very high since it is due to the demand for which all releases take place in dry season, namely a flow of 5000 m$^3$/s with none in the wet season.

The problem no. 2 is, if the release is only 2500 m$^3$/s in dry season and if the required safety is 99 percent, namely with \( M = 40 \times 10^9 \text{ m}^3/\text{per year} \), what is the needed storage capacity? The answer is 

\[ K = f(M = 8 \Delta V, P_0 = 1) = 66 \times 10^9 \text{ m}^3/\text{per year} \]

This value depends whether releases in the wet season are excluded or not.

The difficulty involved with taking a whole year as the time unit can be easily resolved. Inflows which enter the reservoir in wet seasons should be reduced by the flow released for navigation, that is by 2500 m$^3$/s. Since there is practically no flow in the Benue River within dry seasons, the mean annual flows should be reduced by 2500/2=1250 m$^3$/s, a flow reduced to the wet half of the year. Then the calculation should be repeated with flow rates reduced in this manner (proposed by Laszlo Goda, Jr.).
The data reduced for wet season flows served as a basis to produce the family of curves in Fig. A-18 of the yield functions of storage. They indicate that in order to assure the flow requirement of navigation of 2500 m$^3$/s equally in both seasons, a storage capacity of K=80 $10^9$ m$^3$ would be needed if the acceptable risk is one percent.

The flow rate to be released in the wet season from the reservoir is only needed in the stretch immediately downstream of the reservoir since approximately 20 kilometres downstream of the reservoir the safe navigation is already assured by flood flows of tributaries and especially of the Niger River itself. The computation with the lower release of 1000 m$^3$/s in the wet season, meaning that the annual mean flow inputs after the reduction of 1000/2 = 500 m$^3$/s are in accordance with flow requirements of the wet season of 1000 m$^3$/s. The graph (B) of Fig. A-18 is produced on the basis of 500 m$^3$/s instead 1250 m$^3$/s as in graph (A) of Fig. A-18. As a result navigation with a risk of only one percent could be assured by a reservoir capacity of K=33 $10^9$ m$^3$. The navigation between the dam on the Benue River and its confluence with the Niger River could be also realized in wet season by building a navigational weir along the 20 kms stretch, and thus not release in wet seasons even 500 m$^3$/s as in the case of graph (B).

The reservoirs in the Niger-Benue River system are built primarily for energy purposes. However navigation is also improved in dry seasons by a continuous power operation of existing reservoirs. By building an additional reservoir the added benefit will be also the improvement in the safety of navigation.

In order to repeat the computation on your own computer you have to enter

\[
\text{ERGEV <BENUE21.TXT >BENUE21.PRN}
\]

and

\[
\text{ERGEV <BENUE22.TXT >BENUE22.PRN}
\]

where ERGEV is the name of the load module and BENUE21.TXT and BENUE22.TXT are the names of the data files.
Example 3.

Rivers with Highly Fluctuating Flow Regime, Without Drying up, in a Temperate Climate Zone, with Withdrawal for Irrigation.

By using suitable input and the year as the time unit the classical Moran's model can be applied also in this case. The Borjad Dam will be built on the Karasica River near Szederkeny, Hungary. The flow regime of this watercourse, slightly karstic in its nature, is shown in Fig. A-19. The operational mode for this reservoir is opposite to that of the Benue River reservoir presented as Example 2. In the case of the Benue River the flow regime was highly seasonal with a continuous constant demand, while for the Karasica Creek the flow regime is much less seasonal, with the demand for irrigation highly seasonal. However, unlike Example 1, the amount of summer flows should not be neglected.

The computational procedure, proposed by Laszlo Goda Jr., is as follows. The water volume stored in the reservoir is reduced by a flow rate of a given probability \( P \) (as if the actual irrigation takes place) and the storage space is “filled up” by the remaining flows. This storage space is now sized only for demand to be assured with the probability \( P \), meaning that the lower and upper limits of safety are the same.
Only the curve of the yield function of storage, $K=f(M,P)$, with a safety $P$ is plotted. This curve gives flows which can be released from the reservoir with a safety $P$, supplemented by the flow with the same probability $P$, which flow was reduced from the inputs in determining the filling flows.

This calculation is repeated with a number of $P$ risk values (or safety levels) and the curves plotted separately are integrated into a single graph. (Fig.A-20) which represents the yield capacity. Graphs A, B and C of Fig. A-20 are for safety probabilities $P=0.95$, $0.90$ and $0.80$, respectively. Graph D of Fig. A-20 gives the family of curves which are the yield functions, $K=f(M,P)$, under the conditions given above. In order to repeat the computation on your own computer you have to enter

```
ERGEV <BORJAD31.TXT >BORJAD31.PRN
```

These figures were constructed from results of the computation made by the program ERGEV for the data-file BORJAD31.TXT.

Clicking on file names induce download above. (Especially for TXF and PRN use rightClick "Save Target As" option to avoid opening in browser window. At certain client side settings YEAR might need ".EXE" extention postfix when intended to rerun. Using PRN as output filename printout goes to line printer, supposed to set to monospace typing. Omitting output redirection put results onscreen.)

![Graphs A, B, C, and D showing yield functions for different safety probabilities](image)

Fig. A-20. The Borjad Reservoir, Karasica River, at Szederkeny, Hungary, as an example of modification of Assumption 2. The yield curve characterized by safety $P$ is reduced by subtracting the flow of probability from the input (A) with $P=0.95$; (B) with $P=0.90$; (C) with $P=0.80$; and (D) comparison of the resulting three yield functions.

---

**Example 4.**

Rivers with Highly Fluctuating Flow Regime, Without Drying up, in a Temperate Climate Zone, with Continuous Industrial Water Withdrawals.

The procedure is applied for year as the time unit under the assumption of simultaneous uniform filling and demand. The example is the reservoir to be built on the Karasica Creek with flow regime presented in Fig.A-19 (this example is a realistic water supply problem, because one of Hungary’s larger cities, Pecs, could be supplied by water in this way). It can be easily demonstrated that the application of a scheme consisting of uniform continuous filling and simultaneous uniform demand, instead of using the assumption 2, results ultimately in the assurance of any demand $M$ to be met by the storage space reduced exactly by $M$. Under this assumption the yield function of storage $K=f(M,P)$ can be found by a horizontal, linearly increasing changes of the graph constructed under the hypothesis which is based on the annual time unit and on the original two-stage, separate filling and demand schemes. Plotting is based on the computation presented in Example 1, however applied to the Karasica Creek. The case is shown in detail in Fig.A-21. Graph (A) of Fig.A-21 is constructed by using the Moran’s original hypothesis, while graph (B) gives the final result transformed by the modified hypothesis. The graph (B) in Fig.A-21 is constructed by meeting annual water demand of $M=18 \times 10^6 m^3$, with safety of 70 percent, according to the Moran’s hypothesis and a storage space of $K=31 \times 10^6 m^3$. In case of uniform simultaneous filling and release, the needed storage space is $K_r=K_m-M=31-18=13$. The transformation of the graph was carried out in the same way as marked by the dashed line.
Finally, a comparison is made between the functions obtained under three different hypotheses, as shown in Fig. A-22. This figure shows three separate graphs, each containing three curves for $K_f(M)$, corresponding to safety levels of 95, 90 and 80 percent. The curves obtained under the Moran’s hypothesis are drawn as solid lines. The assumption of separate filling and separate demand requires the largest storage space since this is the most unfavorable distribution of flows. Dashed lines are used for curves representing the assumption of uniform simultaneous filling and demand. This assumption requires the smallest storage space since the hypothesized connection between inputs and demand is highly “favorable”, however unrealistic at least to the same extent as the Moran’s hypothesis resulted in a too large storage space. The Goda’s hypothesis leads to a curve which is located between the two extreme curves, drawn with dash-dot lines. This hypothesis results as a realistic approach.

The real situation can be described by the actual hydrograph of flows which enter the reservoir and of real withdrawals from it throughout the year. This is dealt with in the following example.

**Example 5.**

A Relatively Small Reservoir Compared to the Mean Annual Inflow, $K < 0.4V_a$, by Using the Nonstationary Model with Time Units $\Delta t$ Smaller than the Year.
This example is confined to examination of reservoir depletion, or of the volume left in it, respectively. The computations and results cover only the ergodic state. Examinations start from a given state and apply short time intervals, performed in a similar way as with the annual time interval.

The computations characterizing the ergodic state were performed by using the programs attached in the enclosed diskettes. The computation is carried out by the main program FWHFOP4.

In order to repeat the computation on your own computer you have to enter

```
HAVI <BORJAD51.TXT >BORJAD51.PRN
HAVI <BORJAD52.TXT >BORJAD52.PRN
```

where HAVI is the name of the load module and BORJAD51.TXT and BORJAD52.TXT are names of the data files used in the example. Data to be loaded in are shown in Table A-3.

The case of natural flows entering the reservoir, being different during the individual time intervals, is considered. In addition, possible variations in demand within the year are taken into account. Demand for irrigation is shown in Fig. A-23.

### Table A-3. Data for Example 5.

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<th>Card</th>
<th>Data to be loaded in</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Name of the reservoir</td>
<td>4OA2</td>
</tr>
<tr>
<td>2.</td>
<td>Lower limit of storage capacity, in $\Delta V$</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>Upper limit of storage capacity, in $\Delta V$</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>Step interval for storage capacity study in $\Delta V$</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>Number of time intervals within year (max 12, min 1)</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>Rank number of the design period (probability of depletion is studied in this time interval, to control the variations of storage capacity $K$ and demand $M$)</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>Lower limit of depletion probability to be investigated in percentage</td>
<td>F5.1</td>
</tr>
<tr>
<td></td>
<td>Upper limit of depletion probability to be investigated, in percentage</td>
<td>F5.1</td>
</tr>
<tr>
<td></td>
<td>Volume unit $\Delta V$ for discretization of continuous random variables, m$^3$</td>
<td>E10.3</td>
</tr>
</tbody>
</table>

**Table A-3. continued**

<table>
<thead>
<tr>
<th>Card</th>
<th>Data to be loaded in</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.</td>
<td>Fixed demand (industry), in $\Delta V$ (for each time interval)</td>
<td>12I3</td>
</tr>
<tr>
<td>4.</td>
<td>Variable demand (agriculture), in $\Delta V$ (for each time interval)</td>
<td>12I3</td>
</tr>
<tr>
<td>5.</td>
<td>Starting and ultimate month of a partial period</td>
<td>2I3</td>
</tr>
<tr>
<td></td>
<td>Names of partial periods</td>
<td>1OA1</td>
</tr>
<tr>
<td></td>
<td>Occasional reductions within the partial periods (in case of refinement of hypothesis 2), in m$^3$/s</td>
<td>E10.3</td>
</tr>
<tr>
<td>6.</td>
<td>Factor to adjust catchment area</td>
<td>F10.3</td>
</tr>
<tr>
<td>7.</td>
<td>First and last years of flow record</td>
<td>2I5</td>
</tr>
<tr>
<td></td>
<td>Factor to flows (used to multiply the actual data to obtain integers)</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>Comment</td>
<td>3OA2</td>
</tr>
<tr>
<td>8.</td>
<td>First year, monthly mean flows, m$^3$/s</td>
<td>I5, 12I6</td>
</tr>
<tr>
<td>9.</td>
<td>Second year, monthly mean flows, m$^3$/s</td>
<td>I5, 12I6</td>
</tr>
<tr>
<td></td>
<td>First, n-th, last year, monthly mean flows</td>
<td>I5, 12I6</td>
</tr>
</tbody>
</table>

\[ K = f(M, P, \Delta t) \]

The yield curves $K = f(M, P)$ are calculated for five time intervals: year, half-year, quarter-year, two months, one month, as the corresponding families of curves in Fig. A-23 for the above irrigation demand. The example gives the storage space needed for meeting water demand of $6 \times 10^6$ m$^3$ per year, based on different demand for $\Delta t$ time intervals, with the safety of $P = 0.80$. The results for $K$ are:

\[
\begin{array}{cccccc}
\Delta t & 1/12 & 1/6 & 1/4 & 1/2 & 1 \\
\hline
K(M=6; P=0.80; \Delta t) & 10^6 m^3 & - & 5.1 & 5.1 & 3.8
\end{array}
\]

Fig. A-23. The reservoir on the Karasica creek with area $A = 204$ square kilometers, Hungary, with the yield function of a relatively small reservoir, based on computations with different $\Delta t$ time intervals, and demand for irrigation, as

\[ K = f(M, P, \Delta t) \]

The yield curves $K = f(M, P)$ are calculated for five time intervals: year, half-year, quarter-year, two months, one month, as the corresponding families of curves in Fig. A-23 for the above irrigation demand. The example gives the storage space needed for meeting water demand of $6 \times 10^6$ m$^3$ per year, based on different demand for $\Delta t$ time intervals, with the safety of $P = 0.80$. The results for $K$ are:

\[
\begin{array}{cccccc}
\Delta t & 1/12 & 1/6 & 1/4 & 1/2 & 1 \\
\hline
K(M=6; P=0.80; \Delta t) & 10^6 m^3 & - & 5.1 & 5.1 & 3.8
\end{array}
\]

The reservoir on the Karasica creek with area $A = 204$ square kilometers, Hungary, with the yield function of a relatively small reservoir, based on computations with different $\Delta t$ time intervals, and demand for irrigation, as

\[ K = f(M, P, \Delta t) \]

The yield curves $K = f(M, P)$ are calculated for five time intervals: year, half-year, quarter-year, two months, one month, as the corresponding families of curves in Fig. A-23 for the above irrigation demand. The example gives the storage space needed for meeting water demand of $6 \times 10^6$ m$^3$ per year, based on different demand for $\Delta t$ time intervals, with the safety of $P = 0.80$. The results for $K$ are:

\[
\begin{array}{cccccc}
\Delta t & 1/12 & 1/6 & 1/4 & 1/2 & 1 \\
\hline
K(M=6; P=0.80; \Delta t) & 10^6 m^3 & - & 5.1 & 5.1 & 3.8
\end{array}
\]

The case of natural flows entering the reservoir, being different during the individual time intervals, is considered. In addition, possible variations in demand within the year are taken into account. Demand for irrigation is shown in Fig. A-23.
In Fig. A-24 the case is given for a uniform industrial water demand. This is an example of finding the storage space necessary for meeting a water demand of $7.5 \times 10^6 m^3$ per year, based on computation with different $\Delta t$ intervals, and safety $P=95$ percent. The result are:

<table>
<thead>
<tr>
<th>$\Delta t$, in years</th>
<th>1</th>
<th>1/2</th>
<th>1/4</th>
<th>1/6</th>
<th>1/12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K(M=7.5; P=0.95; \Delta t)$ $10^6 m^3$</td>
<td>8.3</td>
<td>6.3</td>
<td>3.7</td>
<td>2.8</td>
<td>1.6</td>
</tr>
</tbody>
</table>

The families of curves may be interpreted and, by drawing envelopes to the corresponding curves, a result valid for the whole region can be obtained (Fig. A-25).

The curves belonging to different time units do not fall together. The reason for these considerable differences is that the hypothesis which separates filling and demand leads to reservoir oversizing. The larger the time interval $\Delta t$ the higher is oversizing, the latter being proportional to the selected time interval.
Fig. A-26. The Karasica Creek Reservoir: Construction of curve characterized as safety with $P=0.95$: (1) uniform simultaneous inflow and demand; (2) reduced inflow; (3) and (4) proposed curve. The annual water quantities are assumed independent, with independence of semi-annual flows doubtful, and shorter periods than 3 month correlated. Irrigation demand.

Separate plotings of yield curves for the safety level $P=0.95$ are plotted in Fig. A-26. This figure also contains curves obtained for the year as time interval, thus applying all the three alternatives of Hypothesis 2. Even more difference appears in calculation covering the uniform industrial demand in the case of the year as time interval (Fig. A-27).

Fig. A-27 The Karasica Creek Reservoir: Construction of curve characterised as safety with $P=0.95$: (1) uniform simultaneous inflow and demand; (2) separated inflow and demand; and (3) proposed curve. Consumption for industry (Uniform consumption).

Attention is drawn here to the fact that the computation performed with the shortest time interval, the month, is not necessarily the most accurate. This is because when the short time intervals (one or two months) are used, Hypothesis 1 of independence not valid. According to a test of independence performed on the mean flows of time intervals, as far as the Karasica Creek is concerned, is at most the semi-annual flows that may be considered independent of one another. The three-month flows, due to the effect of underground storage, are not independent. This is the reason why in the above figures the curves calculated for annual and semi-annual time intervals are drawn as solid lines.

In case of small streams in mountainous regions, with flashy flow regimes, and the impervious terrain, flows of short time intervals such as the three-month and two-month flows, and in certain cases even one-month flows, may be mutually independent.

---

Example 6.

Determination of Yield Functions of Reservoirs
The investigation of yield functions of reservoirs is presented through the analysis of the Borjad Reservoir on the Karasica Creek, already described in examples E-3, E-4 and E-5. From the results obtained in investigation of the ergodic state only the data needed to plot the yield function graphs in this example are printed.

The computation is carried out by the main program FWHTRF1. In order to repeat the computation on your own computer you have to enter

```
HARHT <BORJAD61.TXT >BORJAD61.PRN
```

and

```
HARHT <BORJAD62.TXT >BORJAD62.PRN
```

where HARHT is the name of the load module and BORJAD61.TXT and BORJAD62.TXT are names of the data files used in the example.

Data are loaded in similarly as described in Example 5.

From the results it is shown that the risk of water shortage is lower than the risk of depletion, $P_0$. The difference between them is the probability that the reservoir runs empty (that is $\mu < \Delta V/2$), however, still with no water shortage. Instead of the risk of depletion, the risk of actual deficit (or, respectively, its supplementary value, the safety in terms of time), $P=P_t$, or the safety in terms of quantity, $P=P_v$, is used to draw the yield curve $K=f(M,P)$. The yield function calculated with the temporal safety $P=P_t$ is shown in Fig.A-28 while that with quantitative safety $P=P_v$ is given in Fig.A-29.

![Fig. A-28. The Karasica Creek Reservoir at Borjad, with constant industrial water demand: (A) Construction of the function $K=f(M,P)$ from two families of curves calculated with time intervals of $\Delta t=1$ year and $\Delta t=1/2$ year, with $\Delta V$=1.2 $10^6 m^3$ and $\Delta V=(1/2year)$ $=0.6 10^6 m^3$ and (B) the result: the yield function of the reservoir $K=f(m,P_t)$, with $P_t$=safety in time.](image)

![Fig. A-29. Method of Construction of storage functions (A) and the resulting yield function of the reservoir $K=f(M,P_v)$ (B), of the Borjad Reservoir on the Karasica Creek, for constant industrial water demand with $P_v$=quantitative safety.](image)

Example 7.

Characterization of Behavior of a Reservoir (planned or existing) with Given Storage Capacity.

Time interval $\Delta t=1$ year may be used conveniently to characterize the Borjad Reservoir on the Karasica River with the storage space $K=30 10^6 m^3$. Therefore, the program FWHTF02 was run with time interval of one year.

In order to repeat the computation on your own computer you have to enter

```
HAHT <BORJAD71.TXT >BORJAD71.PRN
```

where HAHT is the name of the load module and BORJAD71.TXT is name of the data file used in the example. Data is loaded in similarly as described in Example 5. The probabilities of occurrence of different levels of fullness, different water shortages representing the restrictions in demand, and different overflows are given in Figure A-30.

Through a summation of probabilities of the discrete values of the computer results, the probability distribution functions of stored water volume (reservoir state), water shortage, and overflow are obtained. If instead of probabilities of non-exceedance of water shortage those of
Example 8.

Computation of Evaporation Losses

The results obtained by the algorithm applied here to the Karasica Reservoir are printed in a concise form, when a detailed printing scheme is chosen, for the series of probabilities of evaporation loss values, computed for various storage capacities, K, and demand, M. The expected value (the mean) of evaporation losses is printed only in this brief output. The simplest way to consider evaporation losses is when in the construction of graphs the demand M is taken into account with a reduced value left after the deduction of the evaporation loss.

The variant containing only the concise data printing is presented in this example. Its investigation is presented in details in Example 9, which includes also the detailed investigation of wave action in the reservoir. The program providing a concise output list is: FWHTPF7.

In order to repeat the computation on your own computer you have to enter

```
HARHTP <BORJAD81.TXT >BORJAD81.PRN
```

where HARHTP is the name of the load module and BORJAD81.TXT is the name of the data file. Data are loaded in similarly as described in Example 5.

In addition to data on inflows which enter the reservoir, data of water surface area for varying water volumes stored in the reservoir, that is the evaporating water surface as a function of reservoir state, are also used.

The time dependent evaporation variable is characterized by the difference between the evaporation from the water surface and the evaporation from the land. In Hungary the former is calculated from monthly data of evaporation pans while the latter is determined by using data series obtained from complex analysis of the hydrological cycle of the catchment area. Based on calculations which cover several regions of the country, further computations were performed to obtain the means and standard deviations of evaporation losses, given as the above differences. Then a map showing the means and standard deviations is produced, from which the point read-out can be made for these values when sizing of reservoirs.

The way of loading data of water surface areas for different reservoir states is as given in the following Table A-4., while loading in mean and standard deviation of evaporation loss is described in Table A-5.

<table>
<thead>
<tr>
<th>Table A-4. Loading in reservoir data for evaporation estimates</th>
<th>Table A-5. Loading in data of mean and standard deviation of evaporation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Card</td>
<td>Data loaded in</td>
</tr>
<tr>
<td>---------------------------------------------------------------</td>
<td>-------------------------------------------------------------------</td>
</tr>
<tr>
<td>1. - Number of corresponding pairs of water depth and water surface</td>
<td>I3</td>
</tr>
<tr>
<td>- Comment</td>
<td>1x, 60AI</td>
</tr>
<tr>
<td>2. - Values of the corresponding pairs</td>
<td></td>
</tr>
</tbody>
</table>
The results are illustrated in Fig. A-31. Family of curves (A) is a variant for the previously discussed yield function, in which the evaporation losses are also taken into account. The calculated evaporation losses were subtracted from the demand $M$. Based on data output sheet, the mean of evaporation loss is presented in the form of a family of contour lines, (B) in Fig. A-31, as functions of capacity $K$ and demand $M$. The sections of the graph belonging to $M=8$ and $K=30$ values, respectively, are also drawn. With constant $M$ and an increased $K$, it can be seen that for the safety of water supply increased, the evaporation loss would increase as well. In turn, for a fixed capacity $K$, although safety decreases with an increase of demand, the evaporation loss will decrease as well.

An alternative to the yield function, characterized by the quantitative safety value, can be drawn up too, Fig. A-32.

**Example 9.**

**Computation of Wave Action**

To determine the crest level of the reservoir dam the knowledge of maximum water level plus the wave action height is needed. This elevation is decided upon after the storage capacity and its operation patterns have been determined. In exceptional cases the decision on the storage capacity $K_{\text{max}}$ also is affected by this level.

This investigation serves as a basis for making a decision on the elevation of dam crest to be applied to a reservoir of the capacity $K=30 \times 10^6 \text{m}^3$, with consideration of the required safety from overtopping, for different demands $M$.

The computation is done by the main program FWHPHF1. In order to repeat the computation on your own computer you have to enter

```
HAHPH <BORJAD91.TXT >BORJAD91.PRN
```

where HAHPTH is the name of the load module and BORJAD91.TXT is the name of the data file. First part of the basic data is loaded in as described in Example 8. The data on wind speed, water depth and fetch, required for the wave action calculations are loaded in as given in Table A-6.
The wind which generates wave action is characterized by the annual maxima of wind speed perpendicular to the dam. Due to theoretical reasons the probability distribution of these values is characterized by the Fisher-Tippet Type II distribution function (Frechet Type p.d.f.). In Hungary, these functions have been fitted to observed data at the gauging stations of the National Meteorological Service, so both parameters are available. Programs to be used in calculating these parameters from direct observations of wind speeds are available (these programs are not discussed here in detail).

In this calculations the probability distributions of values representing the processes in the reservoir are printed. It can be seen from Fig. A-33 that when an increase in demand occurs, the levels drop, meaning that the risk of overtopping the crest by waves decreases. The relationship between the established storage capacity $K$, demand $M$, and crest over height of 1 percent risk is shown in Fig. A-34. The relationship to capacity $K$ depends on the morphological features of the reservoir, because an increase of storage capacity increases the fetch for winds along the reservoir. At the same time, if demand stays the same, the probability of reservoir being full will decrease.

Table A-6 Loading data in for Wave action calculations

<table>
<thead>
<tr>
<th>Card</th>
<th>Data loaded in</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>- Parameter A of the probability distribution of maximum wind speed of a given direction</td>
<td>12F6.4</td>
</tr>
<tr>
<td>2.</td>
<td>- Parameter B of wind speeds</td>
<td>12F6.4</td>
</tr>
<tr>
<td>3.</td>
<td>- Number of data pairs depth/fetch to be fixed on the basis of reservoir layout</td>
<td>I3</td>
</tr>
<tr>
<td></td>
<td>- Step interval of dam crest over heights to be examined, in mm</td>
<td>F5.0</td>
</tr>
<tr>
<td></td>
<td>- Comment</td>
<td>1x, 60A1</td>
</tr>
<tr>
<td>4.</td>
<td>- corresponding pairs of data on water depth (in m) and fetch (in km), five data on each card</td>
<td>5(F7.2, F8.3)</td>
</tr>
<tr>
<td>5.</td>
<td>n</td>
<td>5(F7.2, F8.3)</td>
</tr>
</tbody>
</table>

Fig. A-33. Conditional probability distributions of reservoir water levels increased by the wave height due to wind, for a fixed reservoir capacity and a variable water demand:

$$F(x \mid y, K) = P(H(\xi) + \Delta H > x \mid M=y, K=30 \Delta V),$$

with $\xi=$stored water volume, $H(\xi)=$water level corresponding to $\xi$, $\Delta H=$wave height, $M=$demand, and $K=30 \times 10^6 m^3$ storage capacity.
Example 10.

Computation of Flood Reduction by a Reservoir Serving Water Supply Purposes

The flood reduction effect of a reservoir serving water supply purposes is not characterized by the total overflow volumes. The flood damage is not best described by the surplus volume of long time intervals such as a year or a month, but rather by the peak discharge expressed in m³/s. However, what is retained in a reservoir is not the flow but the volume. To obtain a solution the Marone-Bukovszky algorithm is used. This requires that the probability distribution of the entering flood waves be loaded into the computer. Based on the relationship between the flood peak and the flood volume, probabilities of occurrence of flood peaks $Q_{\text{max}}(i) = f(V=i\Delta V)$, which refer to each flood wave with volume $i\Delta V$ ($i=1,2,...$), have to be available for computations.

The computation is controlled by the main program FWHTFA0. The flood data are loaded in as given in Table A-7. In order to repeat the computation on your own computer you have to enter

```
ARHTH <BORJ101.TXT >BORJ101.PRN
```

where ARHTH is the name of the load module and BORJ101.TXT is the name of the data file.

Table A-7. Data loadings in for computation of reservoir effects on floods

<table>
<thead>
<tr>
<th>Card</th>
<th>Data to be loaded in</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Maximum flood volume, in $\Delta V$</td>
<td>(Number of data-1) Maximum value: 20 I3</td>
</tr>
<tr>
<td></td>
<td>Comment</td>
<td>1X, 10A1</td>
</tr>
<tr>
<td>2.</td>
<td>Probabilities of non-exceedance of peak flows related to the above flood volume</td>
<td>$10F5.0$</td>
</tr>
</tbody>
</table>

Just as with all foregoing examples, this program can run equally with the annual or shorter than the year time intervals. When using a season or month as time intervals, the flood distributions require a special calculation. Preparation of data for computation with annual time interval is shown in Fig. A-35, A-36. Graph A shows the distribution function of annual maximum flood flows on the basis of Gumbel distribution. Graph B represents the relationship between flood volume and flood peak. The course of determination of input data is given in the table under C as marked by arrows.

![Fig. A-35. Preparation of data for the investigation of flood reduction by a water supply reservoir (Karasica Reservoir): (A) Distribution function of annual peaks. (The distribution]](image-url)
function of flood flows was calculated from maxima of 23 years between 1948 and 1970, by using a well-fitted Gumbel distribution.): (B) Relationship of flood peak and flood volume; and (C) Input data

Fig. A-36. Conditional distribution functions of flood flows released downstream, \( F(x|y,K) = p(Q_{\text{max}} < x|M= y,=\Delta V) \), of Karasica Reservoir.

Example 11.

Sizing of Multipurpose Reservoirs

In case of complex water resources utilization, the economic performance of a reservoir can be improved if the flood reduction is combined at the same time with meeting various water supply demands of given safety levels. For determining probabilities of restriction in water supply, a calculation was made for the annual water volumes to be released with the 95 percent and 80 percent safety levels, respectively, starting with the annual input volume of the 50 percent safety level only. These examples are based on the conditions that safety level of water supply for industry should be 95 percent and the safety level for intensive irrigation 80 percent. For meadows and pastures the occasional irrigation may have even a safety level of 50 percent and still result in higher yields that without irrigation.

This Example 11 refers also to the Karasica Reservoir, with its capacity of \( K=20.4 \times 10^6 \text{ m}^3 \). For this reservoir the safety level of meeting a water demand of \( M=15 \times 10^6 \text{ m}^3 \) is 50 percent only. According to Fig. A-37 the annual water supply to be assured for industry with a safety level of 95 percent is \( 8 \times 10^6 \text{ m}^3 \), while that for the intensive irrigation with a safety level of 80 percent requires an additional volume of \( 3 \times 10^6 \text{ m}^3 \). For an extensive irrigation of meadows and pastures an additional volume of \( 4 \times 10^6 \text{ m}^3 \) may be assured but with a 50 percent safety level, meaning that over a long period this demand could be satisfied every second year only, on the average.
The use of graphs in Fig. A-37 may be misleading. The demand of irrigation is concentrated to summer season, while the industrial demand does not change with time. Therefore, the computation has to be repeated with the mixed demand: a constant industrial demand of $M=8 \times 10^6 \text{m}^3$, and a summer demand of $7 \Delta V=8,4 \times 10^6 \text{m}^3$ for the agriculture. By using the first approximation with a mixed demand and the time interval of $\Delta t=0.5$ year, a new distribution is obtained for the restrictions in water supply. From this distribution the value with 95 percent safety level can be determined, with the procedure proceeding as the second approximation. The final result becomes the annual value $9 \Delta V=10.8 \times 10^6 \text{m}^3$ needed for industry and $6 \Delta V$ needed for agriculture. This calculation was done by applying the programs described in Example 7 through the manual control, using four runs executed separately (Fig. A-37). The successive approximations were supplemented by an automatic program. It operates by using the monthly time interval, with the basic condition that for meeting the demand, the demand needs to be guaranteed by a safety level of 95 percent. The program is controlled by the main program FWHMTF. In order to repeat the computation on your own computer you have to enter

\begin{verbatim}
ONTIPA <BORJ111.TXT >BORJ111.PRN
\end{verbatim}

where ONTIPA is the name of the load module and BORJ111.TXT is the name of the data file. The program of data loading in utilizes data prepared as shown in Table A-8.

**Table A-8. Loading of data in for sizing multipurpose reservoirs**

<table>
<thead>
<tr>
<th>Card</th>
<th>Data to be loaded in</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Name of reservoir</td>
<td>40A2</td>
</tr>
<tr>
<td>2.</td>
<td>Lower limit of storage capacity to be investigated, $\Delta V$</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>Upper limit of storage capacity to be investigated, $\Delta V$</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>Number of time intervals within the year (max 12) to be investigated</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>Volume unit $\Delta V$ to discretize continuous random variables, $\text{m}^3$</td>
<td>15X, E10.3</td>
</tr>
<tr>
<td>3.</td>
<td>Min. demand in a time interval (uniform, industrial), $\Delta V$</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>Step interval for the cyclical change of uniform demand, $\Delta V$</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>Prescribed safety for uniform demand</td>
<td>F5.3</td>
</tr>
<tr>
<td></td>
<td>Portion of irrigation water (or of a demand with lower safety than for industry) (max. 12), $\Delta V$</td>
<td>1213</td>
</tr>
<tr>
<td>4.</td>
<td>Starting and ending months of partial intervals</td>
<td>213</td>
</tr>
<tr>
<td></td>
<td>Names of partial intervals</td>
<td>10A1</td>
</tr>
<tr>
<td></td>
<td>Withdrawals in the partial intervals upstream of the reservoir (God's model, including reductions)</td>
<td>E10.3</td>
</tr>
<tr>
<td>5.</td>
<td>Factor to adjust the catchment area (if the flow gauging station is outside the reservoir)</td>
<td>F10.3</td>
</tr>
<tr>
<td>6.</td>
<td>First and last year of flow records used</td>
<td>215</td>
</tr>
<tr>
<td></td>
<td>Factor (to multiply the flow data in order to obtain integers)</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>Comment</td>
<td>30A2</td>
</tr>
<tr>
<td>7.</td>
<td>Calendar year and monthly flow data (as many cards as there are years), $m^3/s$</td>
<td>I5, 1216</td>
</tr>
</tbody>
</table>

For the storage capacity of $20 \times 10^6 \text{m}^3$, in addition to the total annual industrial demand of $12.2 \Delta V=12 \times 10^6 \text{m}^3$ (or monthly values of $2 \Delta V=1 \times 10^6 \text{m}^3$), a total annual demand of $4.5 \times 10^6 \text{m}^3$ can also be met for agricultural uses ($1 \Delta V$ in April, $1 \Delta V$ in May, $2 \Delta V$ in June, $3 \Delta V$ in July and $2 \Delta V$ in August) with a 50 percent safety level, and in a way that industrial delivery does not become less either in August or in September than the restricted water supply $1 \Delta V=0.5 \times 10^6 \text{m}^3$ per month.

When the industrial water demand becomes $1 \Delta V$ per month, or $6 \times 10^3 \text{m}^3$ per year, then $1 \Delta V$ in April, $1 \Delta V$ in May, $2 \Delta V$ in June, $4 \Delta V$ in July...
and \(2\Delta V\) in August, or \(10 \times 10^6\) m\(^3\) per year, can even be assured for agricultural uses with a high 95 percent safety level, increasing the agricultural delivery to \(2\Delta V\) in April, \(4\Delta V\) in May, \(4\Delta V\) in June, \(8\Delta V\) in July and \(1\Delta V\) in August. The industrial consumption is not endangered. Further increasing of the agricultural demand delivery causes a total disruption in the industrial water delivery.

**Example 12.**

**Sizing of Simultaneously Operated Pairs of Reservoirs**

As example, the small river Bodva in northeastern Hungary, is chosen. There is a potential site at the Szalonna cross section of Bodva for a reservoir with capacity 100 \(10^6\) m\(^3\). A gauge exists at this section. Downstream of the planned reservoir enters the Rakaca Creek which brings an additional 10 to 15 percent of water to the river. Another potential site for water storage is located downstream of its confluence, with the second reservoir to be operated simultaneously with the upstream reservoir. However, the upper reservoir may be operated according to an operational rule of its own, by disregarding the operation rule of the downstream reservoir. To size the downstream reservoir, the outflow of the upstream reservoir plus the natural flow of the Rakaca Creek should be considered jointly as its input. Computation is carried out under the control of the main program FLPHF, to which data are loaded in similarly as given in Example 8. Since it is program FLPHF that is also used when a couple of reservoirs, operated in interactive manner, have to be sized, it is suitable to consider a random demand also.

In order to repeat the computation on your own computer you have to enter

```
TLHTP <BODVA121.TXT >BODVA121.PRN
```

where TLHTP is the name of the load module and BODVA121.TXT is the name of the data file.

Data to be loaded in are given in Table A-9. Always the data of the upstream reservoir are read first.

**Table A-9. Loading of data for sizing of simultaneously Operated Pairs of reservoirs**

<table>
<thead>
<tr>
<th>Card</th>
<th>Data to be loaded in</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>- Name of reservoir</td>
<td>40A2</td>
</tr>
<tr>
<td>2.</td>
<td>- Storage capacity to be investigated, in (\Delta V)</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>- Not used</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>- Number of time interval within the year (max12)</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>- Not used</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>- Volume unit (\Delta V) to discretize continuous random variables, m(^3)</td>
<td>E10.3</td>
</tr>
<tr>
<td>3.</td>
<td>- First and last months of partial periods</td>
<td>2I3</td>
</tr>
<tr>
<td></td>
<td>- Name of partial period</td>
<td>10A1</td>
</tr>
<tr>
<td></td>
<td>- Withdrawal in the partial period upstream of the reservoir, m(^3)/s</td>
<td>E10.3</td>
</tr>
<tr>
<td>4.</td>
<td>- factor to adjust catchment area</td>
<td>F10.3</td>
</tr>
<tr>
<td>5.</td>
<td>- First year of flow record</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>- Factor for flow data (in order to obtain integers)</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>- Comment</td>
<td>30A2</td>
</tr>
<tr>
<td>6.</td>
<td>- Calendar year</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>- Monthly flow data (one card per year)</td>
<td>12I6</td>
</tr>
<tr>
<td>7.</td>
<td>- Probabilities for random demand for each (\Delta V) and time interval</td>
<td>I3</td>
</tr>
<tr>
<td></td>
<td>- Number of demands investigated</td>
<td>F6.0</td>
</tr>
<tr>
<td></td>
<td>- Name of time interval</td>
<td>1x,30A2</td>
</tr>
<tr>
<td>8.</td>
<td>- Probabilities according to discretization by (\Delta V)</td>
<td>10F5.0</td>
</tr>
</tbody>
</table>

The manner to be used for reading in the reservoir water levels and the reservoir evaporation data were described in Example 8, while the data required for calculation of wave action in Example 9. To solve this problem a more complex program is used for random demand. If fixed demand is used, it should be specified by the probability 1.0, that is

\[
P(M = m(f)\Delta V) = 1
\]

and for all demand for which \(m>m(f)\), zero probability should be loaded in by using cards Nos.7 and 8 of the loading. In this example, the reservoir of study is the upstream one, that at Szalonna, with storage capacity \(K_1=100 \times 10^6\) m\(^3\) and annual demand \(M_1=90 \times 10^6\) m\(^3\) (Fig.A-38).

![Fig. A-38. Computation for the two simultaneously operated reservoirs on the Bodva River: (A) Reservoirs; (B) Yield functions of upstream](image-url)
reservoir with safety in time $P_t$ and quantitative safety $P_v$; (C) Distributions of inflows into the downstream reservoir; and (D) Yield functions of the downstream reservoir for assumed a capacity $K_1=100 \times 10^6 \text{m}^3$ and demand $M_1=90 \times 10^6 \text{m}^3$ per year for the upstream reservoir.

Figure A-38 gives in graph (A) the position of reservoirs, in graph (B) two families of curves as yield functions of the upstream reservoir. The curves were plotted on the basis of concise output sheets presented in Example 6. This figure shows the points of corresponding values of $K_1=100 \times 10^6 \text{m}^3$ and $M=90 \times 10^6 \text{m}^3$ per year, the safety in time is 91.8 percent and the safety in quantity is 97 percent. The distribution of outflows from the upstream reservoir is shown in graph (C), with the probability distribution of annual mean flows of the tributary entering the main stream between the two reservoirs. The downstream reservoir is sized by using the distribution given by the convolution of the two distribution functions. The conditional yield function of the downstream reservoir is given in graph (D) for the storage capacities from $K_{2\text{min}}=20 \times 10^6 \text{m}^3$ to $K_{2\text{max}}=70 \times 10^6 \text{m}^3$ with the annual random demand of $10 \times 10^6 \text{m}^3 < M < K_2$.

Remark: On your diskette only the $M=3\Delta V=30 \times 10^6 \text{m}^3$ water demand is given. If you are interested for other water demands change the data in the input file.

Example 13.

Computation in Case of Random Demand

The Karasica Reservoir serves here also as the example for investigation of the case of random demand. The experience indicates that the actual delivery of water to irrigation systems follows a random fluctuation. According to certain statistical surveys in Hungary, within the total water demand $M$, which has been planned and fixed in legal terms, some users often do not require water (about 10 percent of the total planned water demand), some withdraw only half of the planned quantity (30 percent of users), some use all the planned quantity (40 percent of users), while some require 1.5 times (10 percent of users) and 2.0 times (10 percent of users) of their planned quantity. That is, if the probability of demand is denoted by $f$ and the demand is $M = 2\Delta V$, it can be written that

\[

t(0) = p(M(d) = 0) = 0.10 \\
 t(0.5) = p(M(d) = 1\Delta V) = 0.30 \\
 t(1.0) = p(M(d) = 2\Delta V) = 0.40 \\
 t(1.5) = p(M(d) = 3\Delta V) = 0.10 \\
 t(2.0) = p(M(d) = 4\Delta V) = 0.10
\]

For computerized calculation the most complex main program, FWVPHF3 is used. In order to repeat the computation on your own computer you have to enter

```
HVTHPH <BORJ131.TXT >BORJ131.PRN
```

where HVTHPH is the name of the load module and BORJ131.TXT is the name of the data file. Data to be loaded in are shown in Table A-10.

<table>
<thead>
<tr>
<th>Card</th>
<th>Data to be read in</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>- Name of reservoir</td>
<td>40A2</td>
</tr>
<tr>
<td>2.</td>
<td>- Lower limit of storage capacity to be investigated, in $\Delta V$</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>- Upper limit of storage capacity to be investigated, in $\Delta V$</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>- Step interval for increments in the storage capacities, in $\Delta V$</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>- Number of time intervals within the year</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>- Rank number of the design time interval (This is the time interval for which the depletion probability is investigated, for given capacity and demand)</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>- Lower limit of depletion probability to be investigated, in percentage</td>
<td>F5.1</td>
</tr>
<tr>
<td></td>
<td>- Upper limit of depletion probability to be investigated, in percentage</td>
<td>F5.1</td>
</tr>
<tr>
<td></td>
<td>- Volume unit $\Delta V$ to discretize continuous random variables, $\text{m}^3$</td>
<td>E10.3</td>
</tr>
<tr>
<td>3.</td>
<td>- First and last months of partial periods</td>
<td>2I3</td>
</tr>
<tr>
<td></td>
<td>- Name of partial period</td>
<td>10A1</td>
</tr>
<tr>
<td></td>
<td>- Withdrawal in the partial period upstream of the reservoir, $\text{m}^3/s$</td>
<td>E10.3</td>
</tr>
<tr>
<td>4.</td>
<td>- Distribution of random demand for each time interval</td>
<td>I3</td>
</tr>
<tr>
<td></td>
<td>- Number of demands investigated</td>
<td>I3</td>
</tr>
<tr>
<td></td>
<td>- Sum of probabilities (1 to 100 per cent, 1000 per mil, etc.)</td>
<td>F6.0</td>
</tr>
<tr>
<td></td>
<td>- Comment</td>
<td>1x, 30A2</td>
</tr>
<tr>
<td>5.</td>
<td>- Probabilities according to discretization by $\Delta V$</td>
<td>10F5.0</td>
</tr>
<tr>
<td>6.</td>
<td>- Factor to adjust catchment area</td>
<td>F10.3</td>
</tr>
<tr>
<td>7.</td>
<td>- First and last year of record</td>
<td>2I5</td>
</tr>
<tr>
<td></td>
<td>- Factor to multiply flow data</td>
<td></td>
</tr>
</tbody>
</table>
According to experience, when choosing the unit of calculation, $\Delta V$, the simplest solution is to take half of the actual demand having a 40 percent probability. In case of high water demand the level of accuracy can be improved by choosing smaller $\Delta V$. In such cases the probabilities of demand are also set smaller proportionally. The procedure is presented here together with the supplementary calculations (evaporation, wave action, etc.).

The storage considered in the investigation $16 \times 10^6 \text{ m}^3$. The fixed water demand studied is $8 \times 10^6 \text{ m}^3$ per year. In the first run the unit used was $\Delta V=4 \times 10^6 \text{ m}^3$. As a result, with a demand of given distribution the reservoir will be empty, on the average, 5.9 percent of time over a long period, and the random demand will be met 97 percent of time.

The development of smaller water demands for the same random conditions was also studied. Runs with step intervals $\Delta V=1 \times 10^6 \text{ m}^3$ and $\Delta V=2 \times 10^6 \text{ m}^3$ were repeated. The result of computation is given in Fig. A-39. This figure gives both variants of yield functions for given $K=f(M,P)$: (A) demand characteristics; (B) the reservoir output capacity in terms of safety in time, $P_t$; and (C) the reservoir output in terms of quantitative safety, $P_v$. In (B) and (C), the vertical ordinates are water changing demand. This is called the nominal water demand and in accordance with the basic condition, the probability attached to its value is 0.4, that of its half is 0.3. To 1.5 or 2.0 times demand, the probability is 0.1, as is also 0.1 probability of zero demand. Therefore, the simplest algorithm to construct the graph is to alternate the volume unit $\Delta V$. Therefore, the results mean that the nominal water demand of $8.0 \times 10^6 \text{ m}^3$ per year, with a use characterized by the given probability, could be met by 82.2 percent safety level in time. This means that on the average, over a long period of time, a deficit may arise in every $100/17.8=5.6$ years. Taking into account both, the years with deficits and the probability distribution of withdrawals, the multiannual mean of actual water releases available is $0.858.8 \times 10^6 \text{ m}^3$ per year = $6.9 \times 10^6 \text{ m}^3$ per year.

Example 14.

Computation of Pairs of Jointly Operated Reservoirs in Parallel

This discussion of jointly operating systems begins with the examination of the simplest case, being a pair of reservoirs in parallel. The example is a small Kapos River, Hungary. Its valley is densely populated, with new water demands to be only met by constructing reservoirs on tributaries. The first reservoir has already been completed on the Deseda Creek at Toponar. This reservoir alone is not able to meet water demand with sufficient safety level. To cover water shortages which may arise when only this basic reservoir is used, another reservoir could be built on an other tributary, the Orci Creek. This other reservoir should be sized for a random demand created by water shortage in the Toponar Reservoir. Probabilities of various shortage occurrences can be then calculated. Computations are executed by using the method and program already known (see Fig. A/4).

First, the behavior of the existing reservoir has to be analysed in detail. This computation is carried out by the program HAVI for the input file TOPON141.TXT as it was discussed in detail in Example 7. The first run related to the Toponar Reservoir with capacity $K_1=15 \times 10^6 \text{ m}^3$ considered the annual demand $M=14.5 \times 10^6 \text{ m}^3$. According to results the probability of deficit is 0.57. The probability that water demand $1\times 10^6 \text{ m}^3$, will be satisfied, $P(M=1\Delta V)=0.027$; for demand $2\Delta V=1.0 \times 10^6 \text{ m}^3$ it is equal to 0.025, and so on. Then the potential auxiliary reservoir to be located on the Orci Creek was sized for this given distribution in order to meet the random demand.

The program HVTHPH discussed in detail in Example 13 was run for the input file ORCI142.TXT.

Fig. A-40. Study of the jointly operated Toponar - Orci Reservoirs in parallel: (A) River location; (B) Schemes of reservoirs and gauging stations, and (C) Water storage in the main reservoir; with requirement from the auxiliary reservoir.
For the sake of simplicity, the program sections computing evaporation and wave action were omitted. In accordance with the topographic conditions, the storage capacity values investigated were between $4 \times 10^6 \text{m}^3$ and $10 \times 10^6 \text{m}^3$.

Fig. A-41. Yield capacity of the secondary reservoir, by characterizing the safety of the system of Toponar - Orci reservoirs in parallel.

The conditional probabilities of deficit (that is safety levels) in the jointly operated system depend on the capacity of the second reservoir, as shown in Fig. A-41 by a heavy solid line.

According to Fig. A-41 the required safety $P_t=0.9$ can be assured by a storage capacity as small as $K=8 \times 10^6 \text{m}^3$. The additional opportunities offered by a topographically favorable storage capacity of $10 \times 10^6 \text{m}^3$ were investigated by using in addition to random demand also a number of various fixed demands, $M_{\text{fix}}$. It means that the way for considering a demand $M_{\text{fix}}=1 \Delta V=0.5 \times 10^6 \text{m}^3$ per year is to connect probabilities of random demands to demands higher by one unit. The same could be to apply for a demand $M_{\text{fix}}=2 \Delta V$ by a shift of two units in probabilities, and so on.

Example 15. Investigation of Jointly Operated Reservoirs in Series

The problem is presented for the case of reservoirs, an existing and a planned reservoir, in the system of Gaja Creek. For successive approximations the known programs are used in an interactive mode.

The practical problem is to meet, with a small risk, the maximum water demands now stretching to the maximum the reservoir built at Fehervarszrogo, with the capacity $K=25 \times 10^6 \text{m}^3$. This reservoir cannot be enlarged economically, but at an upstream site, near Bakonycseryne, an excellent potential for a reservoir exists for a storage space of about $K=15 \times 10^6 \text{m}^3$.

Computations begin with the study of the existing reservoir. It was run the program HARHT for the input file GAJA151.TXT in the way presented in Example 6. For example, a demand $M=23 \times 10^6 \text{m}^3$ per year can be met with a risk of 50 percent, and the average shortage becomes $3.6 \times 10^6 \text{m}^3$ per year. To avoid the shortages, the second reservoir is investigated.

As the second step, an independent study is performed for an upstream, to-be-built reservoir. Running the program TLHTP for the input file GAJB152.TXT. At the first trial, an upstream reservoir with the capacity $K_1=15 \times 10^6 \text{m}^3$ and demand $M=5 \times 10^6 \text{m}^3$ per year was considered, to which the downstream reservoir is connected by assuming the simultaneous operation as presented in Example 12.

Because of water retention in the upstream reservoir, the operational safety of the downstream reservoir decreased, so that, the probability of deficit from the intermediate flow into the downstream reservoir rose from 0.504 to 0.690, and the expected value of water shortage increased to $5.4 \times 10^6 \text{m}^3$ per year.

Probabilities of water shortage to occur in the reservoir by a simultaneously operated pair of reservoirs are given in the results of the previous run. These will constitute the probability distribution of random demand satisfied by the upstream reservoir, in the first step of successive approximations. Since the mean water shortage in the downstream reservoir of $5.4 \times 10^6 \text{m}^3$ per year $= 5 \Delta V$, is just the same as the fixed demand obtained in the previous (initial) run, the fixed demand on the upstream reservoir in the second trial was zero. In the first sheet of output of this second run, among the input data to the upstream reservoir, probabilities of demand are given by the second data block. It can be seen, indeed, that these probabilities are the same as those of water shortage in the downstream reservoir in the previous run. However, the results related to the downstream reservoir in the second trial differ considerably from those obtained in the first run. The
expected value of shortage is $6 \times 10^6 m^3 = 6 \Delta V$ and the distribution of shortages is also characterized by the other probabilities. Accordingly, for the third run the vector of probabilities characterizing the water demand to be met by the upstream reservoir is given by water shortages belonging to the downstream reservoir, calculated in the second run.

The results obtained in a subsequent trial, as compared to those given in the previous run, differ less and less.

The results given here, related to the upstream reservoir, may be characterized as those of jointly operated system. So the water demand given as $M = 23 \times 10^6 m^3$ per year, stretching the reservoir built at Fehervarsurgo beyond its capacity of $K = 25 \times 10^6 m^3$, can be met by a 88.1 percent safety in terms of time, if a reservoir is built upstream at Bakonycsernye with the capacity $K = 15 \times 10^6 m^3$. To be justified by an economic evaluation, the computation was repeated by including with a smaller upstream reservoir of $K = 10 \times 10^6 m^3$. According to the results of the last step, with an upstream reservoir with the capacity of $K = 10 \times 10^6 m^3$, the system can be operated with a safety of 79.4 percent. The results of interactive computations performed for different upstream storage capacities are shown in Fig. A-42. From this figure it can be seen, for example, that with a safety of $P = 0.8$ the upstream reservoir should be built with a capacity of $K = 11 \times 10^6 m^3$. The interactive mode is suitable for automatic work as well.

---

**Example 16.**

**Computation of Secondary Energy Production.**

(Overflow Producing No Energy.)

The results obtained by the algorithm applied here to the Kainji Reservoir on River Niger are printed in a concise form. The basic input of the computation is the annual water volumes of River Niger 1954-1969, and the curve of reservoir area and elevation. The name of the main source program to be used is FHPME. To do the computation you have to enter:

```
HPME <KAIN161.TXT >KAIN161.PRN
```

<table>
<thead>
<tr>
<th>Card</th>
<th>Data loaded in</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Number of corresponding pairs of water depth and water surface</td>
<td>I3</td>
</tr>
<tr>
<td></td>
<td>Name of reservoir</td>
<td>1x,60A1</td>
</tr>
<tr>
<td>2.</td>
<td>Values of the corresponding pairs of water depth and water surface, in meters and square kilometers, respectively; five data on each card</td>
<td>5(F7.2,F8.3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n+1</td>
<td>Level of dead storage space, m</td>
<td>F7.2</td>
</tr>
</tbody>
</table>

The data concerning the turbines:

<table>
<thead>
<tr>
<th>Card</th>
<th>Data loaded in</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Level of turbines, m</td>
<td>F7.2</td>
</tr>
<tr>
<td></td>
<td>Efficiency of turbines, percent</td>
<td>F4.0</td>
</tr>
<tr>
<td></td>
<td>Comment</td>
<td>1x,60A1</td>
</tr>
<tr>
<td>2.</td>
<td>Discharge capacity of turbines per time unit in $\Delta V$</td>
<td>12I3</td>
</tr>
<tr>
<td>3.</td>
<td>Consumption per time unit passing by turbines in $\Delta V$</td>
<td>12I3</td>
</tr>
</tbody>
</table>

---

**Fig. A-42.** Investigation of jointly operated reservoirs in series of the Gaja Creek: (A) Reservoirs in series; (B) Safety in time $P_t$ as function of capacity of the upstream reservoir.
where HPME is the name of the load module and KAIN161.TXT is the name of the input data file. The first part of the data are loaded in as it is described in Example 5. The data to be loaded in for the energy computation is shown in the following A-11 table.

Finally the data for computation of the evaporation losses are loaded in as it is shown in Example 6. The reservoir capacity is increased from $2.5 \times 10^9$ to $10 \times 10^9$ m$^3$ by $\Delta V=0.5 \times 10^9$ m$^3$ steps. The constant consumption is also increased by the same steps. The discharge capacity of turbines (8$\Delta V$) does not limit the consumption. The results are printed out in concise form. They are printed out for all examined K capacity and M consumption for each month the expected value of consumption and the probability of disturbance and depletion and the values concerning on the reservoir states, overflows and evaporation losses. The expected values of the monthly and yearly energy production are shown in the last column of the table.

The relation of the expected value of energy production and the reservoir capacity in the function of consumption is represented on the figure A-43. The range of $0.95<P<0.995$ security is shown.

Fig. A-43. Kainji Reservoir on the Niger River. Secondary energy production with the constant water consumption M, characterised by the safety in time.

**Example 17.**

Computation of Secondary Energy Production. (Overflow Producing Energy.)

This example uses the same data as the previous one. River Niger, reservoir Kainji Dam. Since the dam serves navigation purposes on the river as well, we consider the consumption (water volume from the reservoir) as the need for water on the tail water. The whole input and output has the same form as described in the previous example. The name of the main source program to be used is FHTFET. In order to repeat the computation you have to enter:

```
HTFET <KAIN171.TXT >KAIN171.PRN
```

where HTFET is the name of the load module and KAIN171.TXT is the name of the input data file.

The capacity of the reservoir is in the range of $2.5 \times 10^9$ to $10 \times 10^9$ m$^3$. Unit volume is $0.5 \times 10^9$ m$^3$. The water released for both navigation and energy production purposes is varying by the same unit. The level of dead space is investigated from 122.0 to 132.0m above the sea level by step of 2m. The discharge capacity of turbines are considered to be larger than the released water for navigation in order to maintain a surplus to drain the overflow through turbines. Thus the turbine capacity used in the example is larger than the navigation demand by $1 \times 10^9$, $2 \times 10^9$, ... $5 \times 10^9$ m$^3$/month. The computer output is illustrated in Fig. A-44. and A-45.

![Kainji Reservoir on the River Niger. Secondary energy production with the constant consumption M and with the overflow. The mean energy production as function of reservoir capacity K, the constant consumption M, and the level of dead space H, with the safety in time P. The capacity of turbines: C=4.5 $10^9$ m$^3$/month.](image-url)
Figure A-44 shows the effect of different dead space levels for consumptions $M = 2, 3, 4 \times 10^9 \, \text{m}^3/\text{month}$. The higher the dead space level is the more energy obtained from the reservoir. Since the turbines receive water from the spillway as well, the more energy can be obtained the less water is released as ordinary consumption. The safety of navigation is illustrated on the charts by dotted lines. Figure A-45 shows the effect of different turbine discharge capacity.

![Figure A-44](image1)

![Figure A-45](image2)

**Example 18.**

**Sizing of Reservoirs for Primary Power Production**

Practical example: Erdeneburen Reservoir, water system Kohdo (Mongolia).

This example illustrates the difference between the results of model for simultaneous repletion and consumption (energy-production) and the classical model that is the model for consumption after repletion. (Note that in the case of a water-supply-reservoir the classical model assures the acceptable result.)

The results of two kinds of computer programs are compared here.

(a) The energy production after repletion is analysed by the first program.

The example was elaborated by the program giving concise output form. The name of the main source program to be used is FHERR. In order to repeat the computation you have to enter:

```
FHERR <ERDEN181.TXT >ERDEN181.PRN
```

where HERR is the name of the load module and ERDEN181.TXT is the name of the input data file. The data to be loaded in for the energy computation is shown in Table A-12, while data of the water surfaces of the reservoir and the characteristics of the turbines are loaded in as it is shown by the following table A-13:

<table>
<thead>
<tr>
<th>Card</th>
<th>Data to be loaded in</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Name of the reservoir</td>
<td>40A2</td>
</tr>
<tr>
<td>2.</td>
<td>Lower limit of storage capacity, in $\Delta V$</td>
<td>I5</td>
</tr>
<tr>
<td>3.</td>
<td>Rank number of the design period (probability of depletion is studied in this time interval, to control the variations of storage capacity $K$ and demand $M$)</td>
<td>I5</td>
</tr>
<tr>
<td>4.</td>
<td>Lower limit of depletion probability to be investigated in percent</td>
<td>F5.1</td>
</tr>
<tr>
<td>5.</td>
<td>Upper limit of depletion probability to be investigated, in percent</td>
<td>F5.1</td>
</tr>
<tr>
<td>6.</td>
<td>Volume unit $\Delta V$ to discretize the continuous random variables</td>
<td>E10.3</td>
</tr>
<tr>
<td>7.</td>
<td>Starting and ultimate month of a partial period</td>
<td>2I3</td>
</tr>
<tr>
<td>8.</td>
<td>Names of partial periods</td>
<td>10A1</td>
</tr>
<tr>
<td>9.</td>
<td>Occasional reductions within the partial periods (in case of refinement of hypothesis 2), in $\text{m}^3/\text{s}$</td>
<td>E10.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Card</th>
<th>Data loaded in</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Number of corresponding pairs of water depth and water surface</td>
<td>I3</td>
</tr>
<tr>
<td>2.</td>
<td>Values of the corresponding pairs of water depth and water surface, in meters and square kilometers, respectively; five data on each card</td>
<td>5(F7.2,F8.3)</td>
</tr>
<tr>
<td>3.</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>n+1</td>
<td>Level of dead storage space, $\text{m}$</td>
<td>F7.2</td>
</tr>
</tbody>
</table>

The data concerning the turbines:

<table>
<thead>
<tr>
<th>Card</th>
<th>Data loaded in</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Level of turbines, $\text{m}$</td>
<td>F7.2</td>
</tr>
<tr>
<td>2.</td>
<td>Efficiency of turbines, percent</td>
<td>F4.0</td>
</tr>
<tr>
<td>3.</td>
<td>Comment</td>
<td>1x, 60A1</td>
</tr>
<tr>
<td>4.</td>
<td>Discharge capacity of turbines per time unit in $\Delta V$</td>
<td>12I3</td>
</tr>
<tr>
<td>5.</td>
<td>Minimum energy demand per time unit in KMY</td>
<td>12E6.0</td>
</tr>
</tbody>
</table>

Fig. A-45. Kainji Reservoir on the River Niger. Secondary energy production
Finally the data for computation of the evaporation losses are loaded in as it is shown in Example 6.

(b) Now the model of the simultaneous repletion-energy production follows. The main source program is FHVER. In order to repeat the computation you have to enter:

```
  HVER <ERDEN182.TXT >ERDEN182.PRN
```

where HVER is the name of the load module and ERDEN182.TXT is the name of the input data file. The input data and their formats are the same as it was described for the program HERR. The data of water surfaces, turbines and evaporation are the same as well.

The results of the two programmes concerning a same level of crest ($H_0=1300m$) are compared grafically. The upper graph of the Fig. A-46 shows the mean energy-production as the function of the demand for different K capacity. The mean of the productable energy remains the same as the demand till a certain value, that is in this domain the security of the production is 1.0. After this limit the mean of the energy-production is less than the demand but still rising and reaches a maximal value. After this maximum the mean value of the energy-production decreases with the rising of the demand the point of the maximum is always the upper boundary of the economic energy-production.

![Fig. A-46. Erdeneburen Reservoir, on River Kobdo, Mongolia. Primary energy production. Real energy production $E$ as the function of demand $E$ and reservoir capacity $K$ at level of crest $H=1300m$. (A) with energy production after filling, (B) with simultaneous energy production.](image)

Fig. A-46. Erdeneburen Reservoir, on River Kobdo, Mongolia. Primary energy production. Real energy production $E$ as the function of demand $E$ and reservoir capacity $K$ at level of crest $H=1300m$. (A) with energy production after filling, (B) with simultaneous energy production.

After the computation by the two different models, the model of simultaneous filling-release mode gives the larger value of the mean energy-production. The reason is that the simultaneous operation decreases the losses by overflows. The mode of release after filling gives more energy, because the released water volumes utilize greater energy heads. Consequently, and for the example of the Erdeneburen reservoir, the standard Moran's model, namely release after filling, gives a greater safety but less mean energy production. This relationship depends on the topographic and hydrologic conditions of the analyzed reservoir. The computation by both models is recommended.
This example deals with an algorithm, described in Chapter 8, for increasing the efficiency by a different mathematical approach based on new theoretical results.

To illustrate the power of the modified algorithm we suggest to run the data of Example 1. We get the same results, with manyfold increase in computing speed, however. So to do the computation you have to enter:

```
ERGEVSP <GALGA191.TXT >GALGA191.PRN
```

where ERGEVSP is the name of the load module originating from main source FSPEEDX.FOR. GALGA191.TXT is the name of the input data file, which is the same data of Example 1 with little changing in formats according to Table A-14 below:

If your consumption is not a prescribed constant value, but follows a given distribution you may use the other modified program. In order to compare the results with that of the previous examples you have to run the program with data file GALGA192.TXT. By doing so you get the same results as if you run the program described in Example 13 with GALGA192.TXT. You should realise all the modified programs running at increased speed use one year as time interval.

To do the computation you have to enter:

```
ERGVEPS <GALGA192.TXT >GALGA192.PRN
```

where ERGVEPS is the name of the load module originating from main source FSPEEDY.FOR. GALGA192.TXT is the name of the input data file following format description of Table A-15:

<table>
<thead>
<tr>
<th>Table A-14. Data Used</th>
<th>Table A-15. Data Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Card</td>
<td>Data loaded in</td>
</tr>
<tr>
<td>1.</td>
<td>- Name of reservoir</td>
</tr>
<tr>
<td>2.</td>
<td>- Max. value of storage capacity, in $\Delta V$</td>
</tr>
<tr>
<td></td>
<td>- Lower limit of consumption, in $\Delta V$</td>
</tr>
<tr>
<td></td>
<td>- Upper limit of consumption, in $\Delta V$</td>
</tr>
<tr>
<td></td>
<td>- Step interval for consumption, in $\Delta V$</td>
</tr>
<tr>
<td></td>
<td>- Volume unit $\Delta V$ to discretize continuous random variables, ($m^3$)</td>
</tr>
<tr>
<td>3.</td>
<td>- Comment</td>
</tr>
<tr>
<td>4.</td>
<td>- Size of sample (= number of years)</td>
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<td></td>
<td>(m$^3$/s)</td>
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<tr>
<td>5.</td>
<td>- Annual mean flows (10 data per card)</td>
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<td>(m$^3$/s)</td>
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<tr>
<td>7.</td>
<td>- Probabilities according to discretization by $\Delta V$</td>
</tr>
</tbody>
</table>