Spatial chaos in discrete mechanical systems: elastic linkage and elastic web of links

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Summary and Theses of PhD dissertation

Summary

Postcritical behaviour of discrete and continuous bar structures is relevant in case of imperfection sensitivity in engineering practice, but it also plays an important role in biology when spatial configurations of macromolecules are investigated. Biological filaments, like DNA, (bio)polymers, or tendrils, and engineering structures, like marine cables and drill strings may exhibit complicated spatial patterns. For the investigation and characterization of such spatially complex configurations, *chaos theory* can provide useful tools. This phenomenon of spatially complex behaviour of structures was lately called *spatial chaos*, although it still does not have a general definition.

Here we derive and organize equilibrium configurations of simple, one and two dimensional discrete mechanical models considering large displacements. The aim of the dissertation is to characterize and more deeply understand spatial chaotic behaviour. The main goal is to give such a definition of spatial chaos which can be fairly easily used for boundary value problems defined on one or more dimensional finite domain.

We start with discussing the buckling problem of a simply supported elastic linkage loaded by a follower load. It is shown, that the related initial value problem is *area-preserving*. Then we investigate a clamped non-linear elastic linkage under general loading and prove, that the related initial value problem is non-dissipative in every case. We prove, that every equilibrium configuration of the clamped non-linear elastic linkage under general loading is uniquely connected to a periodic orbit of the corresponding dynamical system. Based on this observation, we suggest, that a boundary value problem should be called chaotic, if the number of solutions depends exponentially on the extent of the domain, and the exponent—the topological entropy of the corresponding dynamical system—is positive. Bifurcation analysis of the elastic linkage is carried out for the clamped case under various loads and for the simply supported case loaded by a follower force.

The buckling problem of a two dimensional structure, the elastic web of links is studied in the second part of the dissertation. The web is supported by a fixed hinge in one case and by a fix hinge and a roller in another case. In both cases, the web is loaded in one direction. The equilibrium configurations are obtained using the simplex algorithm based on piecewise linearisation. Bifurcations of the trivial equilibrium path is analyzed. Finally, the similar behaviour of the elastic web of links and the unbendable, unstretchable rod is pointed out for infinitesimally small displacements.

Theses

Thesis 1. *The buckling problem of the simply supported elastic linkage under a non-conservative loading, the follower force was shown to correspond to an area preserving initial value problem. The bifurcation diagram was pointed out to be topologically equivalent to the one related to the discrete model of the Euler-buckling.*

Thesis 2. *The buckling problem of a clamped, non-linear elastic linkage under a general loading was proven to lead to an area preserving mapping independent of the loading being potential or not and of the non-linear material behaviour. The boundary value problem of the discrete mechanical model always corresponds to a non-dissipative initial value problem with discrete time variable.*

Thesis 3. *Every equilibrium configuration of an N element, clamped, non-linear elastic linkage under a general loading is uniquely associated with a periodic orbit of length $4N + 2$ of the corresponding initial value problem. We suggest that a boundary value problem should be identified as spatially chaotic if the number of solutions depends exponentially on the extent(s) of the domain and the exponent is positive. This definition was applied for the boundary value problems discussed here. In case of the elastic linkages the exponent was found to be proportional to the topological entropy of the corresponding initial value problem, it is the natural logarithm of the load parameter.*

Thesis 4. *The number of bifurcation points on the trivial equilibrium path of an N element, initially straight, horizontal elastic linkage*

- *clamped at one end*
  - *loaded by a horizontal force at the free end is exactly N under compression,*
  - *loaded by equal horizontal forces at the hinges is at most N under compression,*
  - *loaded by horizontal distributed load on the elements is zero,*
- *with a fix support and a roller at the ends loaded by a horizontal force at the roller is exactly N under compression.*

Thesis 5. *In case of the vertically loaded N storey elastic web of links*

- *supported by a fix hinge and a fixed hinge and a roller, a global permutation symmerty was found and utilised for the numerical computation of the equilibrium configurations,*
- *supported by a fix hinge, the number of equilibrium configurations was numerically measured to depend exponentially on N, but to be independent of the other extent of the web,*
- *supported either by a fixed hinge or by a fixed hinge and a roller, there exists an $N-2$-tuple cups catastrophy at the single non-trivial bifurcation point of the trivial path,*
- *the analogy with the unbendable, unstretchable elastic rod was shown in the infinitesimally small neighbourhood of the trivial equilibrium state.*