CONTROL DESIGN METHODS FOR OPTIMAL ENERGY CONSUMPTION SYSTEMS

IRÁNYÍTÁS TERVEZÉSI MÓDSZEREK OPTIMÁLIS ENERGIAFOGYASZTÁSÚ RENDSZEREKRE

PhD Thesis

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“Research is to see what everybody else has seen, and to think what nobody else has thought.”

Szent-Györgyi Albert

To my parents.
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Part I. Introduction

“You see things, and you say: ‘Why?’ But I dream things that never were and I say ‘Why not?’” (George Bernard Shaw)

1. Presentation of the Thesis

Sustainable energy solutions are required in every area of our life, from energy generation in power stations through transportation and domestic applications. Much effort is made to ensure the energy needed for humankind, by replacing the use of fossil fuel with alternative ones. This PhD thesis is based on the central idea of optimal energy consumption and management. Split in two, the main part of the thesis deals with energy management solutions and related topics in hybrid electric vehicles, while the second part is oriented to optimal control of hydro-generators.

The current interest in both research topics is reflected by the large amount of publications in the domain: Journals (Automatica, IEEE a.o.), congresses (IFAC Congresses) conferences (Control Design, Conference of Decision and Control, Control Applications of Optimization a.o.), symposia and workshops, research reports, PhD theses.

The present thesis is structured in four parts plus appendices, it has a length of 101 pages plus the appendices, and is based on 180 cited works. From these I contributed to a number of 35 papers, out of which 28 as first or single author. There is a number of 12 papers, enumerated separately, which are not directly linked to the subject of the thesis but are my own work as well. The references are marked as follows: with a general numbering (column 1) from [1] to [180] and referred in each Part under a Part reference number; for example reference [12] is referred in parts I and II under numbers [I-12], [II-78], (see the reference list).

Part I, entitled Introduction describes the motivation and definition of the objectives, followed by an overview of the contributions of the thesis (chapter 2). Part I ends with acknowledgements, my special thanks to the people who contributed most to my success in finishing the PhD.

Part II, entitled Modelling and control of a hybrid vehicle, focuses on control solutions to optimal energy management of a hybrid solar vehicle. The first chapter introduces the topic and makes the reader familiar with some important aspects concerning hybrid electric vehicles.
Chapter 2 is entirely dedicated to the mathematical modelling of the hybrid vehicle. The target is to build mathematical models of an existing prototype of a series structure hybrid solar vehicle. First the modelling concept is presented, and by considering the components relevant from the point of view of fuel consumption optimisation, a non-linear mathematical model is developed. For controller design different ways of obtaining a linearized model are dealt with, namely: linearization around a working point, feedback linearization and finally piecewise linearization, which resulted in a piecewise, bilinear mathematical model. The controllability of the last model was studied using controllability algorithms specific to switching systems. Finally, simulations were run to test the models’ behaviour.

Chapter 3 focuses on control solutions for fuel consumption optimization for the hybrid solar vehicle. Two solutions were approached, Dynamic Programming and Model Predictive Control. Even though Dynamic Programming delivers the global optimum solution to the problem, it cannot be implemented in real-time. Still it is considered a very good reference point to compare it with other sub-optimal results. Model Predictive Control has the advantage of giving a sub-optimal solution to a problem over a finite time horizon, also taking into account the physical constraints of the plant. The control systems were tested through simulations for different values of the controller tuning parameters. As reference signal data from so-called driving cycles were used (driving cycles are pre-defined time-velocity profiles for a driving route).

Chapter 4 focuses entirely on the modelling and control of the electric drive used in the electric vehicle. First the modelling of an electric traction system is presented, followed by the presentation of a new tuning method based on Modulus Optimum conditions, finally cascade control solutions to the electric drive are given, based on the new tuning method. Simulations end the chapter, while the whole part of the thesis is closed with conclusions upon this part and highlighting briefly the contributions.

Part III, entitled Speed control solutions for hydro-generators, deals with a minimax-based cascade control solution for speed control for a hydro-generator with medium water fall. The proposed solution is based on a cascade control structure with an internal Minimax controller (to reject internally located deterministic disturbances) and a main General Predictive Control (GPC) loop (to reject external stochastic disturbances induced by the power system (PS). The first chapter introduces the topic, presents some recent trends in cascade control, the second chapter presents the mathematical model of the hydro-generator used for controller design. Chapter 3 introduces the cascade control used for disturbance rejection enhancement, chapter 4 the application of this structure for the HG, while chapter 5 contains some simulation results. Finally conclusions end part III of the thesis.

Part IV, entitled Contributions, synthesizes the contributions and possible further research directions and topics.

The Appendices treat chapters which are not included in the main parts II and III of the thesis, but are in strong connection with them. Also the used references are listed here.
2. Motivation and definition of objectives, contributions

Nowadays more and more research groups are focusing on topics regarded hybrid electric vehicles, which can be considered as a middle term alternative to conventional ones. The increasing number of conferences and symposia on the topic reflect its interest and actuality. One of the main research interests is to optimize the energy management of hybrid vehicles, to achieve as little fuel consumption as possible.

Fuel consumption optimization must be preceded by a proper mathematical modelling of the vehicle, taking into account the relevant components and dynamics. The literature contains mathematical models of different complexity [I-1], [I-2], [I-3], most of them being based on the power balance of the system components. Different architectures of the hybrid vehicles are used already in practice, such as series, parallel, series-parallel or complex architectures. Each architecture has its advantages and disadvantages, due to their functioning principle. Subsequently, their mathematical models will also be different. The literature does not yet contain mature mathematical models for each architecture, which could be considered even as benchmarks, so there is an urgent need for developing such models. In this context, one target of the thesis is to develop mathematical models of different complexity for a series architecture hybrid solar vehicle, based on an existing prototype. These models would take into account the vehicle dynamics, power balance and some static characteristics [I-4], [I-5], [I-6], [I-30].

In the topic of energy consumption optimisation the literature specifies off-line and on-line solutions. In the first category Dynamic Programming (DP) is one alternative to solve the optimisation problem, resulting in the global optimum [I-7], [I-8], [I-9]. This approach is not feasible since it cannot be computed in real time (it requests a-priori knowledge of the entire reference signal), but it can constitute a very good reference to compare the results with. One frequently appealed feasible solution is Model Predictive Control (MPC) [I-7], [I-10], [I-11], which takes into account not only the reference signal changes (for a given future time horizon), but also the physical constraints of the plant (constraints of the control input signal, controlled output signal). For this reason, the thesis focuses on MPC solutions to the fuel consumption minimization, taking into account constraints of the plant, and the results are compared with DP results. The solution to energy optimization depends very much on the complexity of the mathematical model of the plant, on the definition and tuning of the cost function parameters, on the handling of plant’s physical constraints. Therefore, another target of the PhD thesis is to give MPC solutions to fuel consumption minimization for a hybrid solar vehicle, based on the previously developed mathematical models, taking into account the physical constraints of the plant [I-12], [I-13], [I-14].
Returning to the mathematical modelling, it must be noted that one of the crucial aspects of hybrid electric vehicles is the electric drive itself. During the past years intense research was focused on finding “the best” solution for such applications, each electric motor having its advantages and disadvantages. One popular possibility consists in using brushless DC motors (BLDC-m) [I-15], [I-16], [I-17], [I-18], but also other alternatives are considered, mainly due to their cost. Typical for hybrid electric vehicles is the fact that the electric motor(s) drive the vehicle together with the internal combustion engine, in a combination specific to the given architecture. One important feature of traction motors is that they can function as generators in a so-called “regenerative breaking regime”, producing electrical energy which can be stored in a battery or a super capacitor and used later for various purposes, including driving the electric motor itself. The mathematical modelling and control of traction motors used in hybrid vehicles is important for the model building of the entire vehicle, with emphasis on load disturbance rejection. Therefore, one aim of the thesis is to model and design the control for the traction motor of the hybrid solar vehicle, stressing on the load disturbance rejection properties [I-19], [I-20], [I-21].

Proper disturbance rejection is very important in all control applications, thus in power generation applications (local stabilization of the servo-system, varying load when the power system is connected to the grid). If the plant can be decomposed in two or more functional parts, whose control can be designed in separate loops (cascade control), the disturbance rejection can also be solved in the different control loops (of course only if the nature of the disturbances allows it to). This way not only the local stabilization can be “solved” by cascade control, but also disturbance rejection and the robustness of the system towards disturbances can be improved [I-22], [I-23], [I-24]. In this sense, the actuality of cascade control is sustained by the number and quality of the papers appearing in this topic at various conferences and symposia [I-22] – [I-27]. The thesis introduces a new point of view for disturbance rejection in cascade systems. The disturbance rejection problem was defined for the inner loop in the form of Minimax control, while the outer loop was determined to be Generalized Predictive Control, taking into account the plant’s physical features (fluctuation of the power demand from the grid, which can be predicted to some extent). Based on mathematical models taken from the literature, control design and verification through simulation was performed [I-28], [I-29].

• Contributions of the thesis

An introductory synthesis of the contributions of the thesis is given in table 1.3-1. The contributions are highlighted in more detail at the end of each part of the thesis, and finally, they are again summarized in Part V.
<table>
<thead>
<tr>
<th>Part</th>
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<th>Paragraph</th>
<th>Contributions</th>
<th>Papers</th>
</tr>
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<tr>
<td>II</td>
<td>2.</td>
<td>2.1</td>
<td>Taking into account the components relevant from the point of view of fuel consumption minimization, a non-linear mathematical model of a series architecture hybrid solar vehicle was built.</td>
<td>[I-4], [I-5], [I-6]</td>
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<tr>
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<td>2.</td>
<td>2.1</td>
<td>Linearization of the non-linear vehicle model in one working point. Feedback linearization of the model.</td>
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<td>2.2</td>
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</tr>
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<tr>
<td></td>
<td>3.</td>
<td>3.1</td>
<td>A synthesis of optimal and sub-optimal solutions for fuel consumption optimization: Dynamic Programming and Model Predictive Control.</td>
<td>[I-4], [I-13], [I-14]</td>
</tr>
<tr>
<td></td>
<td>3.</td>
<td>3.2</td>
<td>Application of Model Predictive Control as solution to fuel consumption minimization for a hybrid solar vehicle. Comparison with Dynamic Programming solution (global optimum).</td>
<td>[I-4], [I-13], [I-14]</td>
</tr>
<tr>
<td></td>
<td>4.</td>
<td>4.2</td>
<td>A novel controller design method is defined based on a double parameterization of the optimality conditions specific for the SO method.</td>
<td>[I-20], [I-31], [I-32], [I-33]</td>
</tr>
<tr>
<td></td>
<td>4.</td>
<td>4.3</td>
<td>Two cascade control structures are defined for the electric drive used in the traction system of the hybrid vehicle.</td>
<td>[I-19], [I-21]</td>
</tr>
<tr>
<td></td>
<td>Appendix 2</td>
<td></td>
<td>A Youla parameterization approach of the MO-m, ESO-m and 2p-SO-methods.</td>
<td>[I-31], [I-32]</td>
</tr>
<tr>
<td>III</td>
<td>3.</td>
<td>3.1, 3.2</td>
<td>Definition of the internal disturbance rejection problem for a cascade control structure of a hydro-generator speed control in the form of a minimax optimal problem.</td>
<td>[I-28], [I-29], [I-34]</td>
</tr>
<tr>
<td></td>
<td>3.</td>
<td>3.3</td>
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<td>[I-34], [I-35], [I-36], [I-37]</td>
</tr>
<tr>
<td></td>
<td>4.</td>
<td>4</td>
<td>Application of the proposed control structure to the model of a hydro-generator, simulation results.</td>
<td>[I-28], [I-29], [I-38]</td>
</tr>
</tbody>
</table>
3. Acknowledgements

The present PhD thesis summarises a part of my research results from the period of time 2002-2008 at Budapest University of Technology and Economics (BUTE). During this period I had the chance to be a member of two research groups, at two departments: first as PhD student at the Department of Automation and Applied Informatics, followed by a fruitful period as research assistant at the Department of Control and Transport Automation. I am grateful for having had the chance to gather experience from my colleagues in many different areas on Control Engineering, which had a big influence on my professional development.

First I would like to express my sincere gratitude to my supervisor Professor Dr. József Bokor, member of the Hungarian Academy of Sciences, Head of Department of Control and Transport Automation, BUTE, for supporting and guiding me during the past two years, for sharing precious knowledge in the field of Control Engineering and its modern applications. Without his guidance and support my work would not have been possible.

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Last, but not least, I would like to warmly thank my parents for all.
Part II. Modelling and control of a hybrid solar vehicle

“One man’s “magic” is another man’s engineering”
(Robert A. Heinlein)

1. General aspects
Nowadays available fossil fuel resources become more and more expensive, and their limited reserve is shadowing the energy demand fulfilment of the future generations. Proper alternatives are sought to replace these gradually, for example in the car industry there is a high demand to find alternatives to gasoline and diesel driven vehicles. Electric energy is one possible environmentally friendly and economic alternative for chemical energy, another one could be the use of liquid hydrogen. However, the spreading of both types of vehicles is not expected to happen in the near future, because they need radical changes in the existing infrastructure. Battery or fuel cell equipped vehicles need infrastructure for the regeneration of power sources, while vehicles using liquid hydrogen must be able to refill their hydrogen tank. Moreover, fuel cells and liquid hydrogen tanks need continuous cooling during operation, and batteries or hydrogen tanks need a large amount of space in the vehicle, adding significant weight to it.

A middle term alternative between conventional gasoline or diesel and fully electric or liquid hydrogen driven vehicles can be hybrid vehicles [II-1], [II-2], [II-3], [II-4], [II-5]. Hybrid vehicles differ from conventional vehicles in the number of main energy sources used for driving. While conventional vehicles have only one main energy source (fuel tank with gasoline), hybrid vehicles have multiple main energy sources. The second energy source can be a battery, super capacitor, flywheel or fuel cell. The two different types of energy sources complete each other, and together provide energy for the vehicle. By introducing a second possible energy source can result in a decrease of liquid fuel consumption, this way fuel economy can be increased and environmental pollution decreased. Meanwhile the driveability of the vehicle does not change and only minor changes in infrastructure are needed.

Hybrid vehicles equipped with electric secondary energy sources are called hybrid electric vehicles (HEVs). In countries where the number of hours of sunshine is significant during the whole year, it is worth applying photovoltaic (PV) panels beside the battery. If PV panels are also added to the car structure, hybrid solar vehicles (HSVs) are obtained [II-1], [II-2], [II-6], [II-7].
The main drive train structures for HEVs are as follows [II-8], [II-9], [II-10]: series, parallel, series / parallel and complex hybrids:

1. **Series hybrid**: in a series drive train structure, the ICE only drives a generator that can drive the electric motor (EM) or charge the battery. The car is driven by the EM (or EMs) using the energy from the battery or generator. Regenerative braking and/or PV panels can contribute to the energy efficiency.

2. **Parallel hybrid**: in a parallel structure, the ICE and EM can drive the vehicle together or separately. Of course, regenerative braking and/or PV panels can contribute to the energy efficiency as well. With this type of hybrid vehicle, four working modes are realizable (according to [II-8]).
   a. **Electric mode**: the vehicle is powered by the EM while the ICE is switched off;
   b. **Hybrid mode**: the EM works as motor and assists the ICE in driving the vehicle;
   c. **Recharging mode**: the ICE powers both the vehicle and EM. The EM works as a generator and charges the battery;
   d. **Regenerative braking**: during vehicle deceleration, the EM works as a generator converting kinetic energy into electrical energy.

3. **Series / parallel hybrid**: this structure is a combination of the previous two. The ICE can drive a generator and charge the battery, but can also directly contribute to the driving of the vehicle.

4. **Complex hybrid**: it is a further development of the series / parallel hybrid, which applies parallel driving structure for one axle and series structure for the other axle of the vehicle.

The prototype of the vehicle that is studied has series architecture, so in the followings only this type of drive train structure will be dealt with. A simplified block diagram of a series structure HSV is presented in figure 2.1.1.

![Fig.2.1.1. Basic diagram of a series HSV](image_url)

The main part is the electric motor (EM) which drives the wheels or works as a generator during regenerative braking. The electric generator (EG), the PV panels and the
battery deliver the electrical energy for the EM. The electric generator is in rigid connection with the internal combustion engine (ICE). The Vehicle Management Unit (VMU) coordinates the functioning and management of all these parts; it contains also the supervisory controller, which is in charge of controlling all the units.

During the past years, research in hybrid vehicles has intensified in Europe as well, both within research groups in universities and research institutes, as well as in the car industry. Focusing on the academic area, different universities have built hybrid prototypes, publishing papers based on the latest results. To exemplify, some relevant HEV research centres in Europe are:

- University of Salerno, Italy, where a prototype of the series HSV was built, and which is under continuous development. Papers on mathematical modelling and optimal control of the vehicle energy management have been continuously published [II-8], [II-11], [II-12], [II-13], [II-14], [II-15], [II-16].
  
  o *Note:* The PhD thesis is also focused this prototype, due to a collaboration between the two universities. More technical data on the prototype will be given in the next chapter.

- Different universities in Germany, where Dynamic Programming and Model Predictive Control based algorithms are applied to drive train control [II-17], [II-18], [II-19].

- Istanbul Technical University and also other research institutions from Turkey, where a parallel structure hybrid electric vehicle prototype was constructed, the research focusing on mathematical modelling, simulation and optimal control of the vehicle [II-20], [II-21], [II-22], [II-23], [II-24], [II-25], [II-26].

- Technical University of Eindhoven, the Netherlands, where the research is focused on optimal energy management of HEVs, especially with parallel structure [II-27], [II-28], [II-29], [II-30].

Part II of the thesis approaches two main tasks: first to build mathematical models of a series architecture hybrid solar vehicle, followed by design of optimal control strategies for fuel consumption minimization. Part II is divided in three parts as far as the technical contributions is concerned: chapter 2 presents mathematical models of the hybrid vehicle, chapter 3 focuses on control strategies for fuel consumption optimization, while chapter 4 presents control solutions for the electric drive used for traction in hybrid vehicles.

### 2. Mathematical modelling of a hybrid solar vehicle

As mentioned in the previous chapters, one of the tasks of the thesis is to build mathematical models of a series architecture hybrid solar vehicle (HSV). The prototype on which the modelling is based was built at the University of Salerno, Italy, and the research was carried out in cooperation with our colleagues from there. The modelled prototype is described in detail in [II-11]. The technical data of the series HSV is briefly summarized in table 2.2.1. The vehicle is a Piaggio Micro-Vett Porter, on the roof of which PV panels were fixed, see [II-11].
The main components of the HSV that are taken into account are (based on the relevance to later optimal control design for energy management): the vehicle body, the electric motor (EM), the internal combustion engine (ICE) plus the electric generator (EG) that are integrated in one device, the battery, the photovoltaic (PV) panels.

Table 2.2.1. Technical data of the series HSV.

<table>
<thead>
<tr>
<th>Component</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Vehicle</strong></td>
<td>Piaggio Micro-Vett Porter</td>
</tr>
<tr>
<td>Length</td>
<td>3.37 m</td>
</tr>
<tr>
<td>Width</td>
<td>1.395 m</td>
</tr>
<tr>
<td>Height</td>
<td>1.87 m</td>
</tr>
<tr>
<td>Weight</td>
<td>1620 kg</td>
</tr>
<tr>
<td>Drive ratio</td>
<td>1:4.875</td>
</tr>
<tr>
<td>$C_X$</td>
<td>0.4</td>
</tr>
<tr>
<td><strong>Electric motor</strong></td>
<td>BRUSA MV 200-84V</td>
</tr>
<tr>
<td>Max speed</td>
<td>52 km/h</td>
</tr>
<tr>
<td>Continuous power</td>
<td>9 kW</td>
</tr>
<tr>
<td>Peak power</td>
<td>15 kW</td>
</tr>
<tr>
<td><strong>Batteries</strong></td>
<td>14 Modules Pb-Gel</td>
</tr>
<tr>
<td>Mass</td>
<td>226 kg</td>
</tr>
<tr>
<td>Capacity</td>
<td>130 Ah</td>
</tr>
<tr>
<td><strong>PV panels</strong></td>
<td>Polycrystalline</td>
</tr>
<tr>
<td>Surface</td>
<td>1.44 m$^2$</td>
</tr>
<tr>
<td>Weight</td>
<td>60 kg</td>
</tr>
<tr>
<td>Efficiency</td>
<td>0.13</td>
</tr>
<tr>
<td><strong>Electric generator</strong></td>
<td>Lombardini (505cc engine, 3 phase induction machine)</td>
</tr>
<tr>
<td>Max power</td>
<td>15 kW</td>
</tr>
<tr>
<td>Max efficiency</td>
<td>25% @ 9 kW</td>
</tr>
<tr>
<td>Weight</td>
<td>100 kg</td>
</tr>
</tbody>
</table>

The vehicle dynamics is modelled using the basic dynamical relations for vehicle motion, considering also the rolling resistance, hill climbing and aerodynamic drag. A simple one-dimensional equation was used for this purpose:

$$\omega(t) = \frac{F_d}{w_r} v(t)$$  \hspace{1cm} (2.2.1)

$$M_d(t) = \frac{w_r}{f_r} F_d(t)$$  \hspace{1cm} (2.2.2)

$$F_d(t) = m \cdot v(t) + \frac{1}{2} \rho \cdot v^2(t) \cdot A_d \cdot C_d + m \cdot g \cdot (\cos(\psi(t)) C_r + \sin(\psi(t)))$$  \hspace{1cm} (2.2.3)

where $F_d$ is the drive force, $m$ is the mass of vehicle, $v$ is its velocity, $\rho$ is the air density, $A_d$ is the frontal area of the vehicle, $C_d$ is the air drag coefficient, $C_r$ is the rolling resistance coefficient, $\gamma$ is the road rise angle. $M_d$ is the torque required from the EM, $f_r$ is the final drive ratio and $w_r$ is the wheel radius.

2.1 Simplified non-linear and linearized mathematical model

At a first stage a simplified mathematical model was developed, based on the static characteristics of the components and by adding dynamics to several of them (EM, ICE+EG,
battery) in forms of first order lag elements. The following non-linear mathematical model was built (in block-diagram representation, using Simulink facilities), see figure 2.2.1.

For simulation reasons the input of the system consists in data collected from so-called drive-cycles, which are in fact time-velocity profiles defined for testing purposes. This data is transformed into torque and speed (rpm) demand, which must be fulfilled by the vehicle. Namely, this demand is transmitted to the EM, which “transforms” it into electric power demand $P_e$ to be satisfied by the EG + ICE (resulting in the fuel consumption), and if the PV panels are delivering energy then this can be added to the power demand satisfaction.

For controller design purposes the inputs and outputs of the system must be slightly re-defined, because the controller will decide whether the ICE will be turned on or off (so one control input must be linked to the ICE power). Further details on the system definition will be given when determining the linearized mathematical model.

A. Modelling of components

In what follows the mathematical modelling of the components relevant for fuel consumption optimization is presented. These are: the electric motor EM, the battery, the internal combustion engine plus electric generator (ICE+EG), the photovoltaic (PV) panels. The mathematical modelling of the HSV is dealt with in detail in papers [II-30], [II-31], [II-32], [II-33], [II-34].

- The electric motor:

A detailed description of the mathematical modelling of the EM (in form of a separately excited DC motor) and its cascade control will be presented in chapter 4 of part II.
Nevertheless, a brief overview and a simplified model used for the first non-linear mathematical mode of the vehicle are introduced in what follows.

An attractive solution for electric vehicles and HEV driving systems are Brushless DC machines (BLDC-m) [II-35], [II-36], [II-37]. They can function both in motor and generator regimes. BLDCs are in fact the combination of a permanently excited synchronous motor and a frequency inverter, where the inverter „replaces” the converter of a classical DC motor [II-37], [II-38], [II-39]. From here results also the name Brushless DC motor. BLDCs with inverter are mainly used in high performance electric drives with variable speed, where these values largely outrun the nominal rotation velocity.

The four-quadrant operation mode for the BLDC-machine with control block is presented below in figure 2.2.2, based on [II-40].

![Figure 2.2.2. Operation modes for a BLDC-m](image)

A qualitative modelling is achieved through the presentation of static characteristics, with two possibilities:

- Steady-state torque-speed curves, \( \omega = f(M; U - \text{parameter}) \). The characteristics are based on: \( M = K_t(I - I_0) \) and \( I_0 \approx 0.1 \cdot I_n \Rightarrow M = 0.9K_tI \Rightarrow \omega = \frac{1}{K_e}[U - R_m \frac{1}{K_t} M] \)

(2.2.4)

Where \( M \) is the torque, \( I \) is current, \( U \) is voltage, \( K_t, K_e \) are the electromechanical and the electromagnetic constants of the machine (their values are numerically close).

- Steady-state speed-torque curves \( M = f(\omega; U - \text{parameter}) \); they are obtained by relation:

\[ M = \frac{K_t}{1.1R_m}[U - K_e \omega] \]

(2.2.5)

The characteristic steady-state curves for this latter case are presented in figure 2.2.2 (in normalised values). The diagram is presented in normalized values of the torque and speed, for the first quadrant according to figure 2.2.3. Here \( n_n \) is the nominal speed, in \( P_{el}=P_{max} = \text{constant} \) regime.
In addition, the balance between the electrical and mechanical powers is taken into consideration, according to which $P_{el} = P_m/\eta$. The mathematical model of the BLDC-m is based on the power balance, torque-speed curves; its dynamics is approximated at this stage, for simplicity, with a first order lag term.

- **The battery:**

From the various models existing in the literature, a relatively complex one was selected, which models the battery as a real voltage generator considering the change in open circuit voltage when the battery state of charge (SOC) changes. The sketch of this model is presented in figure 2.2.4. The governing equations of this battery model are:

\[
U_{oc} = U_{OC_{\min}} + (U_{OC_{\max}} - U_{OC_{\min}}) \cdot SOC \\
I_b = \frac{U_{OC} - U_{OC}^2 - 4 \cdot (R_{\text{int}} + R_t) \cdot P_b}{2 \cdot (R_{\text{int}} + R_t)} \\
dSOC/dt = \frac{\dot{Q}}{Q_{\text{max}}} = \frac{I_b}{Q_{\text{max}}} 
\]  

(2.2.6)

In this type of formulation positive $P_b$ (battery power) means battery discharge, while negative $P_b$ means battery charge. In [II-28] the efficiency of the battery is also dealt with, modelled with the following expression:

\[
P_b = 1 - \sqrt{1 - 6 \cdot 10^{-5} \cdot P_{bn}} \\
\]

(2.2.7)

Here $P_{bn}$ means nominal battery power. The overall structure of the battery model is presented in figure 2.2.5. The $\eta$ block represents equation (2.2.7), while Battery block replaces equations (2.2.6). The resulting battery model reflects all the important characteristics of a battery. The open circuit voltage decreases when SOC decreases, the battery current calculation in (2.2.6) is asymmetric, which means that higher SOC rate can occur rather in discharging than in charging. Nominal power ($P_{bn}$) losses occur both in charging or discharging mode.
- **Internal combustion engine and electric generator:**

The electric generator and internal combustion engine (ICE) must be fitted to the electric motor and to each other. The EG and ICE have to be fitted to each other using the maximum efficiency region for both of them. This way the EG can be described by a single characteristic curve, between input mechanical and output electrical power as in figure 2.2.6.

The description of ICE is made in a similar way considering the maximum efficiency working line. The fuel map of the proper ICE (which can satisfy the EG input power needs) is depicted in figure 2.2.7.

In the fuel map, the fuel rate values are plotted against ICE torque and angular velocity values. Every combination of torque and angular velocity means a possible output power value for the motor. However, fuel rate is given at every point, from which input power can be calculated using the lower heat value of gasoline.
The ratio of output and input power is the ICE efficiency. The efficiency map can be plotted against torque and angular velocity values (see figure 2.2.8). In points with zero input it can be assumed to be zero.

The determination of optimal working line is possible using a characteristic value mixed from output power and efficiency: \( \text{o}_\text{pt} = M \cdot \omega \cdot \eta \), depicted in figure 2.2.9.

**Photovoltaic panels:**
PV panels are modelled according to [II-41]. This provides a high fidelity PV array model, which in reality is not needed for the supervisory control design of the HSV. In this model for controller design, only the out coming electric power from the PV array is considered, assuming that the array is separately controlled. The PV array output power continuously changes, so it can be considered as a “useful disturbance” on the system.

**B. Linearized model**
First linearization around one working point was performed. The definition of the one working point is crucial and can be subject to debate. The choice here was to consider the point where the ICE+EG works at maximum efficiency.

The inputs, outputs and states of the linearized mathematical model are [II-30], [II-42]:

- **Inputs:**
  - \( u_1 \): ICE power,
  - \( u_2 \): Battery nominal power;

- **State variables:**
  - \( x_1 \): state of dynamics of EM,
  - \( x_2 \): SOC,
  - \( x_3 \): state of dynamics of ICE;

- **Measured disturbance input:**
  - \( d_m \): PV panel power.

- **Outputs:**
  - \( o_1 \): Drive power,
  - \( o_2 \): SOC,
  - \( o_3 \): Fuel rate.
Based on the numerical values of the prototype, and by choosing a sampling time of $T_s=1 \text{ msec}$, the following state-space representation was obtained (which was later used for MPC controller design):

$$
\begin{bmatrix}
  x_1(k+1) \\
  x_2(k+1) \\
  x_3(k+1)
\end{bmatrix} = \begin{bmatrix}
  0.3679 & 0 & 0 \\
  0 & 1 & 0 \\
  0 & 0 & 0.9048
\end{bmatrix} \begin{bmatrix}
  x_1(k) \\
  x_2(k) \\
  x_3(k)
\end{bmatrix} + \\
+ \begin{bmatrix}
  3.78 \cdot 10^{-6} & 6.321 \cdot 10^{-4} \\
  0 & -1.517 \cdot 10^{-11} \\
  2.638 \cdot 10^{-7} & 0
\end{bmatrix} \begin{bmatrix}
  u_1(k) \\
  u_2(k)
\end{bmatrix} + \begin{bmatrix}
  6.321 \cdot 10^{-4} \\
  0 \\
  0
\end{bmatrix} d_w(k) \tag{2.2.8}
$$

$$
\begin{bmatrix}
  y_1(k) \\
  y_2(k) \\
  y_3(k)
\end{bmatrix} = \begin{bmatrix}
  800 & 0 & 0 \\
  0 & 1 & 0 \\
  0 & 0 & 100
\end{bmatrix} \begin{bmatrix}
  x_1(k) \\
  x_2(k) \\
  x_3(k)
\end{bmatrix}
$$

Other possibilities for linearizing the non-linear model are presented in the following two sub-chapters. These, however, are based on a more complex non-linear model.

### 2.2 Complex mathematical model of the hybrid solar vehicle

Based on the technical data of the vehicle and on the previous simple mathematical model, a more complex model was built which takes into account the dynamics of each presented component, and also the static characteristics are more detailed (per sub-component). The aim is to try to model the system in form of piecewise affine (PWA) systems. A brief introduction to PWA systems is given in the following sub-chapter, followed by the presentation of component-modelling in order to obtain such a system representation.

#### A. Piecewise Affine Systems

Piecewise affine (PWA) systems are in fact a modelling framework for a class of hybrid systems. Hybrid systems are processes that evolve according to dynamic equations and logic rules [II-43]. [II-43] names that “a familiar example of hybrid system is a switching system where the dynamic behaviour of the system is described by a finite number of dynamical models, which are typically sets of differential or difference equations, together with a set of rules for switching among these models”. This definition is crucial for the thesis, since a piecewise affine model is sought (in fact, a piecewise bilinear model is obtained), and due to this definition, controllability analysis will be performed, which was proven for switching systems.

The interest in PWA systems has grown exponentially during the past years in the control community [II-44], [II-45], [II-46], [II-47], [II-49], [II-50], [II-51], [II-52]. This is due to the multitude of process families that can be handled this way.
Definition 2.2.1 (PWA systems) [II-43], [II-53]: PWA systems are defined by partitioning the state and input space into polyhedral regions, and for each region an affine state-space equation is associated:

\[
x(t + 1) = A_i x(k) + B_i u(k) + f_i
\]
\[
y(k) = C_i x(k) + D_i u(k) + g_i
\]

if \( [x(k) \quad u(k)] \in D_i \)

where the sub-index \( i \) takes values from 1 to \( N \), and \( N \) is the total number of PWA dynamics defined over a polyhedral partition \( D \).

The dimension of the matrices is the following:

<table>
<thead>
<tr>
<th>Matrix</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_i )</td>
<td>( n_x \times n_x )</td>
</tr>
<tr>
<td>( B_i )</td>
<td>( n_x \times n_u )</td>
</tr>
<tr>
<td>( f_i )</td>
<td>( n_x \times 1 )</td>
</tr>
<tr>
<td>( C_i )</td>
<td>( n_y \times n_x )</td>
</tr>
<tr>
<td>( D_i )</td>
<td>( n_y \times n_u )</td>
</tr>
<tr>
<td>( g_i )</td>
<td>( n_y \times 1 )</td>
</tr>
</tbody>
</table>

Definition 2.2.2 (Polyhedron) [II-43]: A convex set \( Q \in \mathbb{R}^n \) given as an intersection of a finite number of closed half-spaces

\[
Q = \left\{ x \in \mathbb{R}^n \mid \sum_{i=1}^{v_p} \alpha_i V_{P(i)}^p \leq Q \right\}
\]

is called a polyhedron.

Definition 2.2.3 (Polytope) [II-43]: A bounded polyhedron \( P \in \mathbb{R}^n \)

\[
P = \left\{ x \in \mathbb{R}^n \mid P^* x \leq P \right\}
\]

is called a polytope.

A set of properties and operations are defined on polytopes, their presentation is not subject of this thesis. Still, one fundamental property of a polytope is that it can be described also by its vertices [II-53]:

\[
P = \left\{ x \in \mathbb{R}^n \mid x = \sum_{i=1}^{v_p} \alpha_i V_{P(i)}^p, 0 \leq \alpha_i \leq 1, \sum_{i=1}^{v_p} \alpha_i = 1 \right\}
\]

where \( V_{P(i)}^p \) denotes the \( i \)-th vertex of \( P \) and \( v_p \) is the total number of vertices of \( P \).

Returning to the description of PWA systems, the state-space is divided into polyhedral partitions where different dynamics of the systems are valid. In [II-53] the different partitions are delimited by so-called “guard lines”, which are constraints on the states or the input variables. The guard lines delimit, with other words, the region where a given
dynamics \( i \) is active (represented by \([A_i, B_i, f_i, C_i, D_i, g_i]\)). The guard lines are defined as delimiting the following constraints:

\[
G_i^x x(k) + G_i^u u(k) \leq G_i^\text{G}
\]  
(2.2.13)

When the value of a state variable or an input at a certain moment „jumps” into another region (by no longer satisfying the original constraints, but new ones), the dynamics valid in the new region will be active.

Analytical properties of PWA systems (stability, observability, controllability) are subject to current research, many papers appearing in this domain regarding different methods to guarantee or analyse these.

**B. Mathematical modelling of components using piecewise linearization of the static characteristics**

The modelling concept used here is the following [II-122]: the least fuel is consumed if the ICE + EG work at the maximum efficiency point and the EM as well. When the ICE+EG does not operate around the maximum efficiency region, it is optimal to switch it off (of course, this switch does not operate in all cases, since also the transients of switching on and off the motor must be taken into account, when the ICE works with very low efficiency).

The block diagram of HSV with power balances is depicted in figure 2.2.10.

The drive power request \( P_d \) comes from the desire to go through a given path; in this case it will be the urban part of the New European Driving Cycle (NEDC). Further, the EM is modelled separately, taking into account its dynamics, its efficiency and its parameters. For the EM model a cascade control structure was built in order to control the speed and the current (see chapter 4).

The model is based on the following setting of the problem [II-122]:

- Consider the \( \dot{m}_f \) fuel rate as the input \( u \) of the model. This means that the controller decides when to turn on or off the ICE;
- Consider the plant as being on one hand the ICE+EG, on the other the battery (see figure 2.2.11);
Consider the $P_e-P_{PV}$ as disturbances, and the target of the control is to keep the power balance at zero. In this case, the drive cycle to be followed is represented by the $P_e$ of the EM, in form of disturbance input. The power delivered by the photovoltaic panels $P_{PV}$ can contribute to the energy, which will charge the battery.

In figure 2.2.11 the following notations have been used:
- $m_f [g/sec]$ is the fuel rate, which integrated results in the total amount of fuel consumed;
- $m_f [g]$ is total fuel consumed;
- $P_{eg} [W]$ is the power given by the ICE+EG $\geq 0$;
- $P_e [W]$ is the power demand from EM to be satisfied, considered in the model as a measured disturbance;
- $P_{PV} [W]$ is the power from the PV panels, $P_{PV} \geq 0$, it helps charge the battery; $SOC [%]$ is the state of charge of the battery.

The detailed modelling of each component is presented as follows, based on [II-122].

**ICE + EG model**

The non-linear model to be considered for the ICE+EG is depicted in figure 2.2.12.

For the global model, first the different parts are modelled and then concatenated. In what follows, the non-linear static characteristics will be introduced, together with the dynamical parts of each sub-component.
• **Generator load torque:**
Let $M_g$ be the load torque brought by the EG. The static characteristics of $M_g=f(n_g)$ is represented in figure 2.2.13-a.

![Figure 2.2.13-a. The load torque static characteristics, $M_g=f(n_g)$](image1)

![Figure 2.2.14-b. Division and approximation of the non-linear static characteristics with piecewise linear parts (dash – original, line – approximation)](image2)

Taking into account the profile of the graph, this will be divided into 4 linear zones, and for these a following PWL equations result (figure 2.2.14-b):

$$M_g(n) = \alpha_m + \beta_m n$$

$$\epsilon = \frac{1}{J_e} (M_m - M_g) = \frac{1}{J_e} (M_m - (\alpha_m + \beta_m n))$$

(2.2.14)

Let the speed $n \left[1/\text{sec}\right]$ of the EG is one of the state-variables of the system $x_i := n$.

• **Primary motor – own dynamics:**
Let another state variable of the system be the state according to the ICE dynamics. Physical meaning can be that $M$ is the theoretical value, $M_m$ is the realized equivalent value at electrical level (both are torques).

$$M_m(s) = \frac{1}{1 + sT_{jl}}$$

(2.2.15)

$$M(t) = K(n) \cdot \dot{m}_f$$

The static characteristics of the primary motor are depicted in figure 2.2.14-a, followed by the partition into four zones (figure 2.2.14-b).

$$\begin{cases} x_1 = \frac{1}{2\pi J_e} x_2 - \frac{1}{2\pi J_e} (\alpha_m + \beta_m x_1) \\ x_2 = -\frac{1}{T_{jl}} x_1 + \frac{1}{T_{jl}} (K(x_i) \cdot \dot{m}_f) \end{cases}$$

(2.2.16)

Considering the input of system as the fuel rate: $u(t) = \dot{m}_f(t)$ and taking into account the linearization of $K(x_i)$ which is:

$$K(x_i) = \alpha_{ki} + \beta_{ki} x_1$$

(2.2.17)
Defining as one of the outputs of the system the speed \( n \), the state-space equation results as:

\[
\begin{align*}
    \dot{x}_1 &= -\frac{\beta_m}{2\pi J_e} x_1 + \frac{1}{2\pi J_e} x_2 - \frac{1}{2\pi J_e} \alpha_m \\
    \dot{x}_2 &= \frac{1}{T_{fd}} \alpha_x x_1 u - \frac{1}{T_{fd}} x_2 + \frac{1}{T_{fd}} \alpha_k u \\
    y &= x_1
\end{align*}
\]

The regions are delimited by the different domains of \( x_1 \). From the second equation it results that the system is bilinear. On solution to it is to apply feedback linearization.

- **Generator behaviour**
  The generator is behaviour is represented by the \( P_{eg}(n) \) characteristics, given by the producing company. It is depicted in figure 2.2.15-a and 2.2.15-b.

Also, the electric time constant of the ICE+EG couple “acts” here, and it is presented in the form of a first order lag element (see figure 2.2.12), \( P_{EE} \) being the notation of the intermediate signal:
The amplification of the dynamic term was chosen for one, because the actual amplification will be defined by the linearized terms of the static characteristic from figure 6.

\[
P_{EE} = \alpha_{gi} + \beta_{gi} n = \alpha_{gi} + \beta_{gi} x_i
\]

\[
P_{EG} = \frac{1}{1 + sT_{EG}} P_{EE}
\]

(2.2.20)

Defining the electric state as \( x_3 := P_{EG} \), this finally leads to the expression of the third state equation:

\[
\dot{x}_3 = \frac{\beta_{gi}}{T_{EG}} x_i - \frac{I}{T_{EG}} x_3 + \frac{\alpha_{gi}}{T_{EG}}
\]

(2.2.21)

**Battery model**

The battery can be modelled in different ways, the literature gives many alternatives depending on how “easy” or “complicated” the model is chosen to be [II-9], [II-54]. For simplicity, a linear model was derived, based on the technical data of the HSV [II-31], [II-34]:

\[
I_b(P_b) = \alpha_b P_b = 23.333 \cdot 10^{-3} P_b
\]

(2.2.22)

The block diagram of the model is depicted in figure 2.2.16.

![Battery model block diagram](image)

**Figure 2.2.16. Detailed battery structure**

The first state-space equation of the battery:

\[
P_b = \frac{A_{bi}}{1 + sT_{\eta}} P_{bn} \Rightarrow \dot{P}_b = \frac{P_b}{T_{\eta}} + \frac{A_{bi}}{T_{\eta}} P_{bn}
\]

(2.2.23)

The constant \( A_{bi} \) approximates the efficiency, while the first order lag element represents the dynamics of the battery, which was chosen to have \( T_{\eta} = 0.1 \text{ sec} \). The PWL model results from the following equations:

\[
\dot{Q} = -I_b, \quad \frac{dSOC}{dt} = \frac{\dot{Q}}{Q_{\text{max}}} = -\frac{I_b(P_b)}{Q_{\text{max}}}
\]

(2.2.24)

By defining the fourth and fifth state variables as \( x_4 := P_b, x_5 := SOC \), the state-space equation results:

\[
\begin{align*}
\dot{x}_4 &= \frac{A_{bi}}{T_{\eta}} x_3 - \frac{1}{T_{\eta}} x_4 + \frac{A_{bi}}{T_{\eta}} d \\
\dot{x}_5 &= -\frac{\alpha_b}{Q_{\text{max}}} x_4
\end{align*}
\]

(2.2.25)
C. Piecewise bilinear model in continuous time

The final model results as bilinear (the derivatives of the states depend on the product of a state and of the control input) [II-122]. Using the definitions of the input and states as presented above, plus defining the outputs as:

- \( y_1 = n = x_1 \) EG speed;
- \( y_2 = SOC = x_3 \) state of charge;

The global piecewise bilinear model results as (2.2.26), as presented in [II-122].

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5
\end{bmatrix} = \begin{bmatrix}
\frac{-\beta_{mi}}{2\pi J_e} & \frac{1}{2\pi J_e} & 0 & 0 & 0 \\
0 & -\frac{1}{T_{fd}} & 0 & 0 & 0 \\
\frac{\beta_{gi}}{T_{EG}} & 0 & -\frac{1}{T_{EG}} & 0 & 0 \\
0 & 0 & -\frac{A_{bi}}{T_q} & -\frac{1}{T_q} & 0 \\
0 & 0 & 0 & -\frac{a_{bi}}{Q_{max}} & 0
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5
\end{bmatrix}
\]

\[
(2.2.26)
\]

The values of the piecewise linear parameters (\( a_i, \beta_i \)) were determined from the characteristics in figures 2.2.13-b, 2.2.14-b and 2.2.15-b. The numerical values for the four state-space representation will be given in the simulation section.

D. Feedback linearization of the model

The mathematical model of the ICE results as bilinear (see eq.(2.2.26) and figure 2.2.12), since one of the states results as a product of one state and the control input. One solution to handle certain non-linearities (including bilinear systems) is feedback linearization (FL). The idea of FL is to apply a non-linear feedback and a coordinate transformation to the system, so that the resulting closed-loop system is linear and controllable [II-55], [II-56], [II-57].

The closed loop in the case of the ICE model looks as presented in figure 2.2.17.

Defining the system as:

\[
\dot{x} = f(x) + g(x)u \\
y = h(x)
\]

The nonlinear feedback will be:

\[
u = \alpha(x) + \beta(x)v
\]
Figure 2.2.17. Feedback linearization of the bilinear dynamics

Where $v$ is the new input signal to the closed-loop system. The task consists in finding the functions $\alpha(x)$ and $\beta(x)$. Redefining the system (2.2.27) by using simplified notations (only to ease our work at this stage) as:

$$
\begin{align*}
\dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + a_{10} \\
\dot{x}_2 &= a_{22}x_2 + (b_{2x} + b_2)u \\
y &= x_1
\end{align*}
$$

Where

$$
\begin{align*}
b_{2x} &= \frac{I}{T_{fd}}a_{ki}x_i \\
b_2 &= \frac{I}{T_{fd}}a_{ki}
\end{align*}
$$

So, more compactly the state-space equation can be re-written in the form:

$$
\begin{align*}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
y
\end{bmatrix}
&= 
\begin{bmatrix}
a_{11} & a_{12} \\
0 & a_{22}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
b_{2x} + b_2
\end{bmatrix}
u
+ 
\begin{bmatrix}
a_{10} \\
0
\end{bmatrix}

(2.2.30)
\end{align*}
$$

The first step is to determine the relative degree of the system [II-55], [II-57]. Note that the relative degree $r$ is equal to the number of times one has to differentiate the output $y(t)$ at time $t=t_0$ in order to have the value of $u(t_0)$ explicitly appearing in its expression. In this case,

$$
\dot{y}(t) = \ddot{x}_1(t) = a_{11}x_1 + a_{12}x_2 + a_{10}
$$

(2.2.31)

It can be seen that the first derivative of the output does not contain the input $u$ explicitly. So the second derivative must be calculated. Thus:

$$
\begin{align*}
\ddot{y}(t) &= \ddot{x}_1 = a_{11}\dot{x}_1 + a_{12}\dot{x}_2 \\
\ddot{y}(t) &= a_{11}(a_{11}x_1 + a_{12}x_2 + a_{10}) + a_{12}(a_{22}x_2 + (b_{2x} + b_2)u)
\end{align*}
$$

(2.2.32)

As it can noticed, $u$ appears in the expression of $\ddot{y}(t)$, so the relative degree of the system is:

$$
r = 2
$$

(2.2.33)
Stating from eq. (12), this can be further developed as:

\[
\dot{y}(t) = a_{11}x_1 + a_{12}a_{22}x_2 + (a_{12}b_{2x} + a_{11}b_2)u + a_{11}a_{10}
\]  

(2.2.34)

Making the following notations (to simplify the deduction):

\[
\begin{align*}
\delta &= a_{11}x_1 + (a_{12}a_{12} + a_{12}a_{22})x_2 + a_{11}a_{10} \\
\xi &= a_{12}b_{2x} + a_{11}b_2
\end{align*}
\]  

(2.2.35)

The expression of \(u\) can be derived as:

\[
u = \frac{\ddot{y} - \delta}{\xi}
\]  

(2.2.36)

This, finally, results as:

\[
u = \frac{\ddot{y} - [a_{11}x_1 + (a_{12}a_{12} + a_{12}a_{22})x_2 + a_{11}a_{10}]}{a_{12}(b_{2x} + b_2)}
\]  

(2.2.37)

Concluding, the above non-linear feedback represented by eq.(2.2.37) will result into a linearized closed-loop system regarding the states \(x_1\) and \(x_2\). The final closed loop expression results by replacing the expression of \(u\) from (2.2.37) into the original state-space equation (2.2.29), resulting as:

\[
\begin{align*}
x_2 &= a_{22}x_2 + (b_{2x} + b_2) \frac{\ddot{y} - a_{11}x_1 - a_{12}(a_{11} + a_{22})x_2 - a_{11}a_{10}}{a_{12}(b_{2x} + b_2)}
\end{align*}
\]  

(2.2.38)

**Remark 1:** The non-linear feedback contains the states \(x_1\) and \(x_2\), and the derivative of \(y = x_1\). Since \(x_1\) is the speed of the rotor of ICE+EG, it can be measured. The state \(x_2\) is the state according to the mechanical time constant of the ICE+EG, and we consider this one as accessible too (by building a state-estimator, which will not be detailed here).

Solving the replacements, the final and LINEAR state-space representation of the new closed-loop system results as:

\[
\begin{align*}
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} a_{11} & a_{12} \\ -a_{12} & a_{11} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -l/a_{12} \end{bmatrix} v + \begin{bmatrix} 0 \\ a_{10}/a_{12} \end{bmatrix} \\
y &= \begin{bmatrix} l \\ 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}
\end{align*}
\]  

(2.2.39)

It can be observed that the new system is linear regarding to the input \(v\), which is in the order of magnitude of \(\dot{y}(t)\), that is \(\frac{l}{sec^2}\).

**Remark 2:** The solution to the feedback linearization by applying the definition ([II-57]) would be:

\[
u = \frac{1}{L_g L_f^{-1} h(x)} (-L_f h(x) + v)
\]  

(R1)
Applying this to the state-space equation (10), the followings result:

\[
L_j h(x) = \frac{\partial h(x)}{\partial x} f(x) = \begin{bmatrix} 1 & 0 & a_{11}x_1 + a_{12}x_2 + a_{10} \\ 0 & a_{22} \\ a_{22} \\ \end{bmatrix} = a_{11}x_1 + a_{12}x_2 + a_{10} \quad (R2)
\]

\[
L_k L_j h(x) = \frac{\partial (a_{i1}x_1 + a_{i2}x_2 + a_{i0})}{\partial x} g(x) = \begin{bmatrix} a_{i1} \\ a_{i2} \\ \end{bmatrix} \begin{bmatrix} 0 \\ b_{2x} + b_2 \\ \end{bmatrix} = a_{i2}(b_{2x} + b_2) \quad (R3)
\]

\[
L_j h(x) = \begin{bmatrix} a_{11} \\ a_{12} \\ \end{bmatrix} \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + a_{10} \\ a_{22}x_2 \\ \end{bmatrix} = a_{11}x_1 + (a_{11}a_{12} + a_{12}a_{22})x_2 + a_{11}a_{10} \quad (R4)
\]

From which the final form for \( u \) results:

\[
u = \frac{-[a_{11}^2x_1 + (a_{11}a_{12} + a_{12}a_{22})x_2 + a_{11}a_{10}]}{a_{12}(b_{2x} + b_2)} + \frac{v}{a_{12}(b_{2x} + b_2)} \quad (R5)
\]

**Remark 3:** Zero dynamics. Considering the closed loop system (2.2.39), the zero dynamics results, if exists, by setting \( y=0 \). In this case it results as: \( y = 0 \iff x_1 = 0 \)

\[
\begin{align*}
0 &= a_{12}x_2 + a_{10} \\
x_2 &= a_{12}x_2 + \frac{1}{a_{12}}v - \frac{a_{11}a_{10}}{a_{12}} \\
\end{align*}
\quad (R6)
\]

Considering \( v=0 \) and \( x_2 = -\frac{a_{10}}{a_{12}} \) it results: \( \dot{x}_2 = -a_{10}(1 + \frac{a_{11}}{a_{12}}) \), meaning there is no zero dynamics, so the FL can be performed as presented.

**E. Representation as linear parameter varying system**

In recent years, a powerful methodology for nonlinear control and fault detection theory is certainly the Linear Parameter Varying (LPV) formalism for systems in state space [II-58], [II-59], [II-128]-[II-132]. The LPV model class is a specific formulation of the nonlinear systems [II-60], [II-61] using measured, computed or estimate and arbitrary varying time dependent parameters. Parameter dependency is given under the linear form of the time varying coefficient matrices. The LPV description preserves the linear time invariant (LTI) structure. The only difference is at the computation of the coefficients. The parameter vector is a continuously time dependent known function allowing the evaluation of the transformed nonlinear system at every single instant \( t \). It has been shown that nonlinear systems can be cast into an LPV form by several ways [II-61], [II-62].

Define the following continuous time, nonlinear state-space model in LPV form:

\[
\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \end{bmatrix} = \begin{bmatrix} A(\rho(t)) & B(\rho(t)) \\ C(\rho(t)) & D(\rho(t)) \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \quad (2.2.40)
\]

where the parameter \( \rho \in \mathcal{P} \) is the parameter vector over a given compact set \( \mathcal{P} \), while \( A, B, C \)
and $D$ are parameter varying coefficient matrices (in the according dimensions).

Quasi-LPV (qLPV) systems are those where among (or all) the scheduling parameters ($\rho$) are state variables as well. Due to this, a qLPV model of a system is not a unique form. The actual value of the state variable is needed for the calculation of the state equation coefficients.

Affinity of the parameter dependent term is:

$$A(\rho(t)) = A_0 + \rho_1(t)A_1 + \cdots + \rho_N(t)A_N$$

$$B(\rho(t)) = B_0 + \rho_1(t)B_1 + \cdots + \rho_N(t)B_N$$

$$C(\rho(t)) = C_0 + \rho_1(t)C_1 + \cdots + \rho_N(t)C_N$$

$$D(\rho(t)) = D_0 + \rho_1(t)D_1 + \cdots + \rho_N(t)D_N$$

(2.2.41)

In the case of the HSV model, $\rho(t)$ contains only the speed of the EG which is $x_1$. The model is reduced to the following form (affine parameter varying form) [II-122]:

$$\begin{bmatrix}
\dot{x}(t) \\
y(t)
\end{bmatrix} =
\begin{bmatrix}
A & B(\rho(t)) \\
C & 0
\end{bmatrix}
\begin{bmatrix}
x(t) \\
u(t)
\end{bmatrix}$$

(2.2.42)

The affinity of the parameter dependent description is given by:

$$B(\rho) = B_{2i} + B_{2xi} = B_{2i} + B_{2xi}(\rho)$$

(2.2.43)

Where the following notations were used, according to the piecewise bilinear model (2.2.26):

$$B_{2i} = \begin{bmatrix}
0 \\
\alpha_{ki}T_{ld} \\
0 \\
0
\end{bmatrix}, \quad B_{2xi} = \begin{bmatrix}
0 \\
\beta_{ki}T_{ld} \\
0 \\
0
\end{bmatrix}, \quad x_1 = \begin{bmatrix}
0 \\
\beta_{ki} \\
0 \\
0
\end{bmatrix}$$

(2.2.44)

These notations will be further used for the controllability tests of the system. Finally, as it can be seen in Figures 2.2.13-b, 2.2.14-b and 2.2.15-b as well, the mathematical model is split into 4 regions according to linear approximations of the static characteristics, regarded to the EG speed.

F. Controllability and stability analysis

One of the most important analytical aspects to be dealt with when designing control systems is the controllability. The continuous HSV model will be examined from different points of view of controllability. First, the theoretical aspects are presented and then the controllability of the HSV system is computed in the simulation section [II-122].

Recalling the bilinear part of the HSV model, one can write:

$$B_{2i}u + B_{2xi}xu = B_{2i}u + B_{2xi}(\rho)u = (B_{2i} + B_{2xi}(\rho))u$$

(2.2.45)

This is indeed an affine LPV form in $B(\rho)$. One takes advantage of the parameter dependent description and concludes that the directions $B_{2i}$ and $B_{2xi}$ span the same subspaces.
in \( \mathcal{X} \), where \( \mathcal{X} \) is the entire state-space. Consequently,

\[
\text{Im}(B_{2i}) = \text{Im}(B_{2xi}) \tag{2.2.46}
\]

The analytical benefit of (2.2.46) is the use of a single dimensional control input, because

\[
\text{Im}(B_{2i}) \oplus \text{Im}(B_{2xi}) = \text{Im}(B_{2i}) = \text{Im}(B_{2xi}) \tag{2.2.47}
\]

Therefore, from a geometric point of view, one can use either \( B_{2i} \) or \( B_{2xi} \) to compute the controllability subspace of the HSV model with the help of [II-63], [II-64]. Moreover, it is possible to use an orthonormal basis spanning the same direction as \( \text{Im}(B_{2i}) \) or \( \text{Im}(B_{2xi}) \) spans. After defining the control input direction as a constant one, the linear piecewise theory (switching theory) for the system analysis can be applied.

The following cases will be studied: local controllability and joint controllability of piecewise systems.

- **Local controllability condition**

  Consider the following theorem by [II-63], which is a sufficient condition for controllability:

  **Theorem 1** ([II-63], Theorem 2): *A controlled switching linear hybrid system with \( m \) modes is controllable, if the controllability matrix \( W_C \) is of full rank.*

  The following notions were introduced: let \( W_i \) denote the controllability matrix of the pair \( (A_i,B_i) \), for \( i=1,...,4 \) in the case of the HSV. \( W_i \) will have the expression:

  \[
  W_i = \begin{bmatrix} B_i & A_i B_i & \cdots & A_i^{n-1} B_i \end{bmatrix} \tag{2.2.48}
  \]

  In addition, define \( W_C \) as:

  \[
  W_C = \begin{bmatrix} W_1 & W_2 & W_3 & W_4 \end{bmatrix} \tag{2.2.49}
  \]

  In the case of the HSV model, each \( B_i \) matrix is constructed as follows:

  \[
  B_i = \begin{bmatrix} B_{2i} & B_{2xi} \end{bmatrix} \tag{2.2.50}
  \]

- **Joint controllability condition**

  Consider the theorem by [II-63], which gives a necessary condition for controllability:

  **Theorem 2** ([II-63], Theorem 7): *If the switching linear hybrid system is controllable, then rank \( W^{kr} = n \).*

  The definition of \( W^{kr} \) (k-th order joint controllability matrix of the system, having \( k_r \) joint controllability coefficient) is defined in detail in [II-63], [II-64].

- **Parameter dependent subspace algorithm**

  [II-64] gives a geometrical interpretation of the switching joint controllability in a compact and recursive form. It is shown that the image of \( W^{kr} \) can be calculated as a result of a subspace algorithm in the case when the matrices \( B_i \) are constant: \( B_1 = \cdots = B_m \).
In this case the $\text{Im}(B_{2i})$ direction coincides with $\text{Im}(B_{2x0})$ which can be described by a single and constant control input map over every region $i=1...4$, resulted from the piecewise linearization.

**Proposition** ([II-64], Proposition 1): Considering a subspace $\mathcal{V}^*$ given by the algorithm below and the $k_r$-th order joint controllability matrix $W^{kr}$. Then

$$\text{Im}W^{kr} = \mathcal{V}^*$$

(2.2.51)

Where:

$$\mathcal{V}_0 = \mathcal{B}, \quad \mathcal{V}^* = \lim_{k \to \infty} \mathcal{V}_k$$

$$\mathcal{V}_{k+1} = \mathcal{B} + \sum_{j=1}^{m} A_j \mathcal{V}_k, \quad k \geq 0$$

And $\mathcal{B} = \text{Im}(B_{2i})$ denotes the image of matrix $B_i$. The value of $m$ is four in this case (number of regions at the HSV piecewise linearization).

- **Stability analysis**

Stability conditions for LPV systems and switched system is a currently researched topic. However, one of the most obvious solutions to characterize the stability aspects is the use of the Lyapunov theorem [II-65]. The stability of the LPV models at a given region can be easily checked by the appropriate dissipative stability condition. Usually, Linear Matrix Inequalities are given to derive the parameter independent or parameter varying solution matrix of the Lyapunov function. On the other hand, switched linear systems can be characterized by a common Lyapunov function as well. Therefore, the quadratic stability condition of the PWLPV system cannot only be derived region by region, but also jointly.

**Definition [quadratic stability]**: The switched continuous time LPV system is jointly quadratically stable with constant solution matrix if there exist a $X = X^T > 0$ so that $A_i(\rho)$ are quadratically stable with the very same $X$:

$$A_i^T(\rho)X - XA_i(\rho) < 0, \quad \forall i \; \text{and} \; \forall \rho \subset P.$$  

(2.2.52)

Since the system preserves the affine dependency, and the problem is convex, it is sufficient to check the above joint stability condition on the vertices of the parameters, i.e. replacing the extreme values of the parameters into (2.2.52).

**G. Sampled system**

In sampling bilinear systems, special attention must be paid to the bilinear term. Considering the model of the HSV as a piecewise bilinear model [122], its discrete form is obtained as follows: sample all 4 models separately according to [II-66], then simply build the global model as composed from the single models. This may be performed because the models are separated from each other by different intervals of one state variable, whose value does not change though sampling. Important thing is that the four existing regions are NOT overlapped, depending only from a single parameter.

Based on recent results from the literature [II-66], one of the best methods to sample a
bilinear system is the following. Consider the original state equation in the following form:

$$\dot{x} = Ax + Bu + B_d d + \sum_{i=1}^{n} N_{i}x_{i}$$  \hspace{1cm} (2.2.53)

where $n$ represents the number of inputs that come into a bilinear expression. In this case $n=1$, since only the control input $u$ builds only one bilinear term with one state variable, the disturbance input $d$ does not produce any bilinear term. Accordingly,

$$\dot{x} = Ax + Bu + B_d d + N_1 xu$$  \hspace{1cm} (2.2.54)

Considering the disturbance term concatenated for simplicity in the input vector, for discretization the formula presented in [II-65] is used, namely:

$$x(k+1) = \Theta x(k) + T_s \Theta Nx(k)u(k) + \Gamma u(k)$$

$$y(k) = Cx(k)$$  \hspace{1cm} (2.2.55)

Where $T_s$ is the sampling time and:

$$Nx(k)u(k) = \sum_{i=1}^{n} N_{i}x_{i}(k)u_{i}(k) = N_1 x(k)u(k)$$

$$\Theta = e^{AT_s}, \quad \Gamma = \int_{0}^{T_s} e^{At} d\tau$$  \hspace{1cm} (2.2.56)

Numerical results will be presented in the simulation section. The constant term of the piecewise bilinear system was taken into account as a fictive input with constant value of one, and so the rules of sampling are applied to it as well.

### 2.3 Simulation of the vehicle models

Two categories of mathematical models for the HSV were presented in this chapter: first a more simple model (non-linear and linearized around one working point), followed by a more complex, piecewise bilinear model, and they are going to be analysed in this order as well. In this sub-chapter first the numerical values for the models are given, followed by different simulation results.

#### A. Simulation of the non-linear HSV model

The reference signal in this case was the New-European Driving Cycle (NEDC) [II-28], [II-30], [II-31]. It consists of four identical urban cycles, followed by an extra-urban cycle. The NEDC is depicted in figure 2.2.18.

Simulations were performed for different insolation values $\lambda$, see [Salerno 06], and the fuel consumption was recorded (table 2.2.2).

<table>
<thead>
<tr>
<th>$\lambda$ [kW/m²]</th>
<th>1</th>
<th>0.8</th>
<th>0.6</th>
<th>0.4</th>
<th>0.2</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOC [SOC]</td>
<td>0.7192</td>
<td>0.7189</td>
<td>0.7186</td>
<td>0.7183</td>
<td>0.7181</td>
<td>0.7178</td>
</tr>
<tr>
<td>total fuel [g]</td>
<td>913.7285</td>
<td>916.0185</td>
<td>918.1688</td>
<td>920.4583</td>
<td>922.613</td>
<td>924.768</td>
</tr>
</tbody>
</table>
It can be seen that the use of PV panels influences the amount of fuel consumed. The trajectory of total fuel consumption for $\lambda=1$ maximal insolation is presented in figure 2.2.19, while the evolution of the state of charge (SOC) is given in figure 2.2.20.

**B. Simulation of the second HSV model**

The non-linear model developed in the second stage, out of which the piecewise bilinear model was developed, was tested for three different versions of the NEDC (shifted in time) [II-31]. The NEDC-based scenarios are depicted in figure 2.2.21.
The three drive cycles have the same velocity profile, but with different maximum values and different durations (810, 1215 or 2000 sec), meaning different accelerations and decelerations. The tracking results are very good, only very small overshoots result, which are considered insignificant in this case. Possible maximum velocity values for NEDCs with different durations and climbing slopes were determined with simulations. The results are summarized in Table 2.2.3.

<table>
<thead>
<tr>
<th>NEDC duration [sec]</th>
<th>Maximum velocity [km/h]</th>
<th>Climbing slope [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>810</td>
<td>40</td>
<td>0</td>
</tr>
<tr>
<td>1215</td>
<td>51</td>
<td>0</td>
</tr>
<tr>
<td>2000</td>
<td>64</td>
<td>0</td>
</tr>
<tr>
<td>810</td>
<td>24</td>
<td>2</td>
</tr>
<tr>
<td>1215</td>
<td>28</td>
<td>2</td>
</tr>
<tr>
<td>2000</td>
<td>32</td>
<td>2</td>
</tr>
<tr>
<td>810</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>1215</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>2000</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

According to the simulation results of this model, the vehicle with these features is capable to climb a 5° slope. The higher the slope the more drive power is required, and the limits of the EM must be taken into account (for this case, as seen in Table I, it is 15kW). On roads with 0° slope the vehicle, modelled as such, it can reach a maximum of 64km/h velocity with a slow NEDC (2000 sec instead of 810 sec).

Simulations with 0° rise slope and partial NEDC were carried out to determine the overall fuel consumption during the drive cycle. Positive electric power requests (from EM) were the input signals and ICE fuel rate was the output. The fuel rate was integrated to get the overall fuel consumption. Regenerative braking and battery power contributions were not considered in this case, so the calculated values represent the worst case, when all the driving power is provided solely by the ICE + EG (Table 2.2.4.):

<table>
<thead>
<tr>
<th>NEDC length [sec]</th>
<th>Maximum velocity [km/h]</th>
<th>Fuel consumption [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>810</td>
<td>40</td>
<td>339,795</td>
</tr>
<tr>
<td>1215</td>
<td>51</td>
<td>512,454</td>
</tr>
<tr>
<td>2000</td>
<td>64</td>
<td>854,05</td>
</tr>
</tbody>
</table>

From Table 2.3.3 results, - as expected - that an increase in the NEDC length results in increased fuel consumption.

- **Piecewise linearization**
  After performing the piecewise linearization of the static characteristics as described previously, the numerical values were derived from the prototype data based on [II-11], [II-31], [II-42]. The range of the EG speed is (0 ... 84 rot/sec), which was decomposed into four
regions. After sampling the piecewise bilinear system with a sampling time of \( T_s = 2 \text{ [msec]} \), the following state-space representation resulted [II-122]:

\[
x(k + 1) = A_{z,i} x(k) + B_{z,i} u(k) + B_{dx,i} d_k + N_i x(k) u(k) + A_{0z,i}
\]

(2.2.57)

The expression of the output is the same for all four regions:

\[
y(k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} u + d
\]

(2.2.58)

The numerical data for (2.2.57) is:

- **I. Region: \( x_i = [0...15] \text{ [rot/sec]} \)**
  \[
  A_{z1} = \begin{bmatrix} 0.097 & 0.021 & 0 & 0 & 0 \\ 0.088 & 0 & 0 & 0 & 0 \\ 0.0 & 0.060 & 0 & 0 & 0 \\ 0.0 & 0 & -0.138 & 0.778 & 0 \\ 0.0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_{0z1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_{z1} = \begin{bmatrix} -0.176 \\ 11.335 \\ 0.127 \\ 0 \end{bmatrix}, \quad B_{dx1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad N_{z1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad N_{z1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
  \]
  \[= \begin{bmatrix} -0.0006 \\ -0.027 \end{bmatrix} \]

(2.2.59)

- **II. Region: \( x_i = [15...25] \text{ [rot/sec]} \)**
  \[
  A_{z2} = \begin{bmatrix} 0.708 & 0.0176 & 0 & 0 & 0 \\ 0.088 & 0 & 0 & 0 & 0 \\ 0 & 0.606 & 0 & 0 & 0 \\ 0 & 0 & -0.138 & 0.778 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_{0z2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_{z2} = \begin{bmatrix} -0.187 \\ 10.814 \\ 0.108 \\ 0 \end{bmatrix}, \quad B_{dx2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad N_{z2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad N_{z2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
  \]
  \[= \begin{bmatrix} -0.003 \\ -0.167 \end{bmatrix} \]

(2.2.60)

- **III. Region: \( x_i = [25...37.5] \text{ [rot/sec]} \)**
  \[
  A_{z3} = \begin{bmatrix} 0.092 & 0.021 & 0 & 0 & 0 \\ 0.088 & 0 & 0 & 0 & 0 \\ 0.0 & 0.606 & 0 & 0 & 0 \\ 0.0 & 0 & -0.137 & 0.778 & 0 \\ 0.0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_{0z3} = \begin{bmatrix} 0.0 \end{bmatrix}, \quad B_{z3} = \begin{bmatrix} 0.1 \\ 10.5 \end{bmatrix}, \quad B_{dx3} = \begin{bmatrix} 0.8 \\ 0 \end{bmatrix}, \quad N_{z3} = \begin{bmatrix} 0.1 \end{bmatrix}, \quad N_{z3} = \begin{bmatrix} 0.2 \end{bmatrix}
  \]
  \[= \begin{bmatrix} -0.005 \\ -0.231 \end{bmatrix} \]

(2.2.61)

- **IV. Region: \( x_i = [37.5...84] \text{ [rot/sec]} \)**
  \[
  A_{z4} = \begin{bmatrix} 1.004 & 0.021 & 0 & 0 & 0 \\ 0.088 & 0 & 0 & 0 & 0 \\ -5.902 & 0.045 & -0.137 & 0.778 & 0 \\ 0.0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_{0z4} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad B_{z4} = \begin{bmatrix} -0.9 \\ 6.2 \end{bmatrix}, \quad B_{dx4} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \quad N_{z4} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad N_{z4} = \begin{bmatrix} 0 \end{bmatrix}
  \]
  \[= \begin{bmatrix} -0.002 \\ -0.077 \end{bmatrix} \]

(2.2.62)
**Controllability analyses**

First the controllability of the system was tested, based on the theoretical aspects from section IV. Regarding the local controllability, the computation of the $W_C$ rank results in a full rank matrix, therefore the switched system is locally controllable.

The joint controllability subspace $\mathcal{V}^*$ has been proven to be full rank over every single region, so $\mathcal{V}^* = X$ using parameter dependent subspace algorithms.

**Stability analysis**

The continuous time stability condition was derived for the HSV model and found as quadratically stabilizing with a constant matrix. Due to the integrating effect in the dynamic matrix, the LMI describing the problem (2.2.52) is not strictly feasible (marginal infeasibility occurs).

**Simulation results**

The simulation results presented here are for two cases, for two different values of the fuel rate. The input $u$ was constant and the disturbance $P_e$ was calculated from the first 500 seconds of the NEDC, based on results from [II-31], [II-33], [II-42], [II-122].

- First case, $u=2$ [g/sec] (total fuel used $m_f=1000$ [g]);
- Second case, $u=0.2$ [g/sec] (total fuel used $m_f=100$ [g]).

Figure 2.2.22 presents the evolution of the battery power $P_{bn}$ versus $P_e$, figure 2.2.23 presents the generator power $P_{EG}$, while figure 2.2.24 depicts the evolution of the state of charge [II-122]. The simulations reflect the expected behaviour of the model: since the input is constant, this fuel rate generates a given power from the EG which remains constant after the transients settle. It contributes to the satisfaction of the $P_e$ power demand. The “rest” of the power is satisfied by the battery (Fig.2.2.22). The fact that in the second simulation considerably less fuel was injected can be seen on the level of the battery power. During the first simulation more power comes from the EG than in the second simulation, so the battery can charge considerably more than in the second case. It must also be mentioned that the simulations clearly indicate the need for an optimal control fuel consumption optimization, this being the next target for development.
To conclude chapter 2, it presents a new mathematical modelling for a series structure hybrid solar vehicle, based on previous versions of the model. The new model treats the non-linear static characteristics by piecewise linearizing them. In addition, a bilinear term appears which is treated as a quasi linear parameter varying system. Aspects of discretization and controllability of the switched system are presented. Finally the sampled state-space model was presented, controllability analyzed and simulations performed. The simulations gave satisfactory results, a further step is to use the mathematical model to design control strategies for fuel consumption optimization.
3. Control solutions for fuel consumption optimization for a hybrid solar vehicle

Hybrid electric vehicles were introduced with the purpose of reducing gasoline consumption by exploring different features of these vehicles, such as energy generation of the electric motors in regenerative braking regime, using the battery as buffer. A complex central management unit is needed to coordinate the energy flow and the resources of the vehicle, and make these as close to optimal as possible. For fuel consumption optimization complex control algorithms must be developed which ensure all these features. The development of such algorithms is in the focus of many research groups, reflected by the increasing amount of papers at dedicated conferences, symposia, workshops, journals.

The palette of control algorithms proposed for energy management in a HEV is very large, but for optimization purposes the focus is mainly on two types of solutions: Dynamic Programming (DP) oriented solutions, which give a global optimum solution to the problem but a-priori knowledge of the whole scenario is required, and Model Predictive Control based solutions. These can be computed for a limited time horizon, delivering a sub-optimal solution, taking into account also constraints of the system (if desired) [II-17], [II-28], [II-67].

The central point in designing control algorithms for energy management is optimality. The global optimal solution of an optimization problem can be reached by means of DP, which requires total a-priori knowledge of the system’s evolution in time. DP solutions to optimal energy management were provided by [II-28], [II-68], and also by [II-8], [II-17], [II-69], [II-70]. [II-28] proposes two solutions to the nonlinear optimization problem (fuel consumption minimization with constraints): the first technique is DP; the second approximates the optimization problem in the form of Quadratic Programming (QP). In addition, as a practical solution, [II-28] introduces Model Predictive Control for the case when the complete driving cycle is not known entirely a-priori, but only for a certain time-horizon (using receding horizon technique). The different control strategies were tested and compared via simulation, for the New European Driving Cycle (NEDC).

Among the advantages of DP, one can name the fact that system constraints and non-linearities can be handled efficiently, but the computational effort increases with the order and complexity of the system. Different adaptations of DP were introduced to overcome its disadvantages, such as in [II-71] a stochastic DP algorithm is developed for energy management, and DP-based Model Predictive Control algorithms are proposed in [II-72], [II-73], [II-74]. Other aspects of Optimal Control were taken into account and used for fuel consumption optimization in [II-27], [II-28], [II-75].

Model Predictive Control (MPC) based algorithms are another option to solve optimal energy management issues in HEVs, having the advantage to be able to implement in real time. Different versions of MPC algorithms with different formulations of the cost function can be found in the literature. Naming some of them, in [II-72] a DP based MPC controller is developed for solving the fuel optimization problem. DaimlerChrysler proposes MPC applied for HEVs several times in cooperation with Karlsruhe University in [II-17], [II-74]. After
building a state-space model of the hybrid vehicle, based on the power balance of the system, the cost criterion and control horizon are defined and predictive control is applied. Simulation results reflect the efficiency of the proposed algorithm.

In [II-73] a differential game theory based solution is proposed for energy optimality, namely: the controller of the car wants to minimize the consumption of the HEV by applying suitable inputs to the desired torques of the motors and to the gearbox. In addition, a second “player” is considered, who wants to maximize the consumption by applying suitable inputs to the inclination angle of the track. This game can be interpreted as a game of the controller against a mostly unknown environment.

The research group from the University of Salerno also focused on mathematical modelling and optimal control of HEVs. [II-76] focuses on simulation and optimal control of parallel hybrid electric vehicles. A dynamic model is used to describe the driver-vehicle interaction with respect to a generic transient and to simulate the vehicle driveline, the internal combustion engine and the electric motor/generator. Neural Network modeling has been proposed for the estimation of future vehicle load aimed at optimizing the supervisory control strategy during a future time horizon. In [II-77] a Genetic Algorithm was implemented to design the rules of a fuzzy logic controller for the optimal management of the energy flow between EM and ICE, accounting for the battery state of charge (SOC) and the route topology. Simulations were performed for the chosen control strategy, resulting in significant improvement of the fuel consumption efficiency.

3.1 Control solution using Dynamic Programming

For energy management optimization two strategies were chosen to be applied. The first one consists in DP, as a solution that delivers the global optimum of the problem, the second strategy is MPC, which delivers a sub-optimal solution which can be applied in real-time and which takes into account the system input and output constraints. Both strategies were tested through simulation, where the NEDC was used as reference signal (see figure 2.2.18).

A. Dynamic Programming solution to fuel consumption minimization

Optimal control of the series HSV was first achieved through DP, which is based on Bellman’s principle saying that: “The parts of an optimal trajectory are all optimal trajectories”. This allows making calculations on a specific problem backward in time, with the assumption of having an optimal trajectory. The result of DP calculations is an optimal input sequence applicable to the system to achieve control goals. DP gives the global optimal solution of the optimization problem. The DP solution to fuel consumption optimization is presented in detail in [II-30].

The drawback of DP solution is that it needs a priori knowledge of the reference signal and disturbances acting on the entire time horizon considered in the calculations. This means that the results of DP can mainly be used just as a reference optimal solution to be compared with other control methods, but cannot be implemented in real time applications. The other drawback of DP is the computational effort needed, which makes it impossible to
apply it in real time solutions. For the used HSV model with NEDC drive cycle, the calculation of the optimal solution on a 1200 sec time horizon needed one hour on a PC with AMD 64 Athlon 3000+ processor and 1 GB DDR 400 RAM.

The minimization of fuel consumption can be achieved by proper switching (balancing) between the energy sources. In a HSV the EM’s power demand can be satisfied from the PV panel, battery and EG. The electric power from the PV panel depends on the sun insolation and cell temperature (see [II-30]). This cannot be controlled; however it can improve the fuel economy of the vehicle.

The system layout used for DP solution is depicted in figure 2.2.10. The fuel consumption optimization can be achieved by the proper use of the EG and the battery, while satisfying the drive power needs and maintaining the battery state of charge (SOC) between certain limits, considering the whole time horizon. The power balance of the system is described by the following equation:

\[ P_e = P_{eg} + P_{bn} + P_{PV} \] (2.3.1)

On the right hand side, \( P_{eg} \) electric generator power and \( P_{bn} \) battery nominal power are the control variables. \( P_e \) electric motor power can be calculated from \( P_d \) drive power need, considering the characteristics of the EM. The controller can influence \( P_{eg} \) and \( P_{bn} \). It can also be noticed that if \( P_{bn} \) is given, \( P_{eg} \) can be determined from (2.3.1). So the optimal solution of the control problem can be generated by the calculation of the \( P_{bn} \) sequence in time. As a constraint, the start and end values of battery SOC are imposed to be the same (“charge sustaining strategy”). The drive cycle for the HSV must be a priori known. The charge sustainability gives limits on battery SOC in time. A diamond shaped limit set can be calculated for every vehicle and drive cycle as it is presented in figure 2.31.

![Figure 2.3.1. Battery SOC bounds with NEDC drive cycle, 1 kW/m² insolation and 25°C cell temperature](image)

The calculations are performed considering the possible SOC values at every time step, which can be achieved according to the constraint, \( SOC(0) = SOC(\text{end}) \), and the minimal and maximal allowed SOC values. The minimal and maximal SOC values are 0.6 and 0.8 respectively, based on [II-67]. Both the upper and lower limits are described with three sections:
1. **Upper:** the maximum possible SOC value which can be achieved from \( SOC(0) \) using maximum battery charge
2. **Upper:** the maximum allowed SOC value
3. **Upper:** the maximum SOC value from which \( SOC(\text{end}) \) can be achieved using maximum battery discharge
1. **Lower:** the minimum possible SOC value which can be achieved from \( SOC(0) \) using maximum battery discharge
2. **Lower:** the minimum allowed SOC value
3. **Lower:** the minimum SOC value from which \( SOC(\text{end}) \) can be achieved using maximum battery charge.

The minimum power (discharge power) is given by the limits of the battery, the maximum power (charge power) is set by the limits of the vehicle according to (2.3.2):

\[
P_{\text{bm\_max}} = P_e - P_{PV} - P_{eg\_max}
\] (2.3.2)

In this case \( P_e \) is positive in EM driving mode and negative in EM braking mode. After calculating the possible battery SOC limits, the solution can be achieved with DP. This starts from \( SOC(\text{end}) \) and \( P_{ef\text{(end)}} \) stepping backward in time. This way in every time step the optimal fuel consumption until the end of drive cycle is calculated. Finally, the minimum fuel path is selected as an optimal solution.

In every step \( k \) the possible battery SOC range has to be considered and compared with the next range (step \( k+1 \)) calculated in the previous step. For every SOC value in range \( k \) all possible SOC trajectories to range \( k+1 \) have to be calculated (limited by maximum battery charge and discharge). This is illustrated schematically in figure 2.3.2.

![Figure 2.3.2. Sketch of DP solution](image)

After determining the possible charge and discharge range, the ICE fuel consumption can be calculated for every trajectory from step \( k \) to \( k+1 \). Adding these values to each total fuel consumption from step \( k+1 \) to \( \text{end} \), the possible total fuel consumptions result from \( k \) to \( \text{end} \) starting from \( SOC(k) \). The minimum of the total fuel consumptions gives the global optimal trajectory from \( SOC(k) \) to \( SOC(\text{end}) \). In step \( k \) these are calculated and stored for every possible \( SOC(k) \) values. After completing this procedure, in step \( SOC(0) \), the global optimal total fuel consumption results. The optimal \( SOC \) trajectory can be determined.
following the minimum fuel path from \( SOC(0) \) to \( SOC(\text{end}) \). This results in the optimal \( P_{bn} \) sequence in time, which can be applied to the input of the vehicle model.

**B. Dynamic Programming applied to the HSV model**

Different calculations and simulations were performed using the mathematical model of the HSV presented in section 2.1. of part II. The NEDC was used as reference, considering the whole range of sun insolation on 25°C cell temperature. Reference results, without controller (but with battery charge with regenerative braking) were generated in [II-30], [II-78]. They are summarized in table 2.3.1:

<table>
<thead>
<tr>
<th>( \lambda ) [kW/m²]</th>
<th>1</th>
<th>0.8</th>
<th>0.6</th>
<th>0.4</th>
<th>0.2</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOC</td>
<td>0.7192</td>
<td>0.7189</td>
<td>0.7186</td>
<td>0.7183</td>
<td>0.7181</td>
<td>0.7178</td>
</tr>
<tr>
<td>total fuel [g]</td>
<td>913.7265</td>
<td>916.015</td>
<td>918.1686</td>
<td>920.4583</td>
<td>922.613</td>
<td>924.768</td>
</tr>
</tbody>
</table>

In every case DP resulted in a lower fuel consumption. Optimal SOC trajectory, fuel consumption and \( P_{bn} \) sequences are presented in figures 2.3.3, 2.3.4 and 2.3.5 for NEDC drive cycle, 1 kW/m² insolation and 25°C cell temperature. The SOC trajectory lies between the prescribed limits in every time step. In fuel consumption (figure 2.3.4) the horizontal sections mean that the ICE was turned off and no fuel was consumed during that period. In the \( P_{bn} \) sequence regenerative braking is used to improve fuel economy.

![Figure 2.3.3. SOC trajectory for the NEDC](image1)

![Figure 2.3.4. Fuel consumption for the NEDC](image2)

![Figure 2.3.5. Optimal \( P_{bn} \) sequence from NEDC drive cycle](image3)
Results from DP calculations are summarized in table 2.3.2, while results from MATLAB-Simulink simulations when the optimal $P_{bn}$ sequence was applied to the vehicle model input are summarized in table 2.3.3 ([II-78]).

### Table 2.3.2 Calculation results from DP

<table>
<thead>
<tr>
<th>$\lambda$ [kW/m²]</th>
<th>1</th>
<th>0.8</th>
<th>0.6</th>
<th>0.4</th>
<th>0.2</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>total fuel [g]</td>
<td>811.6438</td>
<td>814.3697</td>
<td>817.516</td>
<td>820.4546</td>
<td>822.6674</td>
<td>835.5047</td>
</tr>
<tr>
<td>fuel spare [%]</td>
<td>11.172</td>
<td>11.096</td>
<td>10.96</td>
<td>10.865</td>
<td>10.8329</td>
<td>9.6525</td>
</tr>
</tbody>
</table>

### Table 2.3.3 Simulation results using optimal $P_{bn}$ input sequence

<table>
<thead>
<tr>
<th>$\lambda$ [kW/m²]</th>
<th>1</th>
<th>0.8</th>
<th>0.6</th>
<th>0.4</th>
<th>0.2</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOC</td>
<td>0.7004</td>
<td>0.7005</td>
<td>0.7005</td>
<td>0.7005</td>
<td>0.7004</td>
<td>0.7004</td>
</tr>
<tr>
<td>total fuel [g]</td>
<td>855.8369</td>
<td>585.2856</td>
<td>858.8072</td>
<td>860.8495</td>
<td>863.4588</td>
<td>872.5427</td>
</tr>
</tbody>
</table>

As it is presented in table 2.3.2, DP results are almost the same for different insolation values, calculating with the same drive cycle. The fuel spare ranges from 9.7 to 11.2 %. Battery SOC is sustained by DP calculations. Table 2.3.3 shows that in the case of system model simulation with optimal $P_{bn}$ input sequence, lower fuel spare values can be achieved. This is due to the continuous dynamics of the battery, in spite of moving between discrete values of battery charge level as it was in the DP solution, and also to numerical approximations occurring during the simulation. However, overall charge sustainability requirement is satisfied in each case.

- **Fuel consumption equivalent definition**

Before presenting simulation results using MPC control strategies, a so-called “fuel consumption equivalent” value was defined in [II-42]. For fuel consumption minimization different control strategies can be applied, as alternatives to DP, and their performances compared to each other. If after a drive cycle the SOC differs from 0.7, a fuel equivalent can be defined to characterize in terms of fuel needed (or excess) the “distance” from this SOC value. Here the following concept was applied: during the drive cycle the time mean integral of the fuel amount consumed only for charging the battery is calculated.

Starting from equation (2.3.1) and figure 2.2.10, if $P_{bn}<0$ the battery is charged. Suppose the PV panel delivers constant energy, having the same irradiation coefficient all over the drive cycle. In case of battery charging, the sum $P_{bn}+P_{pv}$ defines the power needed from the ICE to charge the battery (naturally through $P_{eg}$). If $P_{bn}+P_{pv}<0$, the $\frac{P_{bn}+P_{pv}}{P_{eg}}$ will represent the ratio of the power that is divided between the battery and the EM. This ratio is an approximation, since the static characteristics of the EG are non-linear.

Define the equivalent fuel rate used for charging the battery as (2.3.3)

$$\dot{m}_{fb} = \frac{P_{bn}+P_{pv}}{P_{eg}} \dot{m}_f$$

(2.3.3)

where $\dot{m}_f$ is the total fuel rate, and $\dot{m}_{fb}$ the equivalent fuel rate for battery charge. The SOC is

42
modified with this value, namely:

\[
\frac{dm_{fb}}{dSOC} = \frac{dm_{fb}}{dt} = \frac{dm_{fb}}{dSOC} \tag{2.3.4}
\]

The time mean value of (2.3.4) is calculated during the simulation, which represents the average amount of consumed fuel per SOC unit:

\[
\overline{\frac{dm}{dSOC}} = \frac{1}{T_0} \int_0^T \frac{dm}{dSOC} dt \tag{2.3.5}
\]

The fuel amount needed for charging the battery (or fuel excess, respectively) is finally defined by:

\[
m_{fb\text{-needed}} = \frac{dm}{dSOC} (0.7 - SOC_{final}) \tag{2.3.6}
\]

where \(SOC_{final}\) represents the SOC level at the end of the simulation. Using this fuel equivalent, some performance indices of a control solution can be evaluated.

### 3.2 Model Predictive Control applied for the hybrid solar vehicle model

The second control strategy that was applied for the series HSV architecture is Model Predictive Control (MPC), as used also for hybrid vehicles in [II-17], [II-72], [II-73], [II-74]. MPC is an advanced control strategy which had spread significantly during the past years in industry as well, due to its increasing popularity [II-79] because of advantages over PID control and abundant possibilities of application. The main advantages of MPC is that the basic formulation is extended to MIMO plants with almost no modification, on the other hand the basic concept of MPC is relatively easy to understand, and it is a powerful tool to cope with constraints effectively [II-80]. Also MPC based strategies can be improved by different means, such as for example measured disturbance estimation [II-133]. Without getting into a detailed presentation of MPC algorithms, the basic “elements” that build the problem formulation are the following:

- Cost function that penalizes the deviations of the predicted outputs from the reference trajectories;
- Internal model of the plant;
- Reference trajectory for the desired closed-loop trajectory;
- Possibility of defining constraints;
- On-line optimization to determine the future control strategy;
- Receding horizon principle.

For design and simulation of the fuel consumption optimization of the series HSV, the MPC Toolbox of Matlab is used. In this sense, the problem formulation follows the steps and form required by this design tool, based on the above presented elements. For a SISO case, the basic idea for designing an application for the MPC Toolbox is depicted in figure 2.3.6, based on [II-81].
The first element to be defined is the plant model that is used in the predictive controller. The model used for MPC control system design is the linearized mathematical model presented in section 2.1. of Part II, with the numerical values given by equation (2.2.8). The system is both controllable and observable.

To recall, the inputs, outputs and states of the plant are enumerated once again, and the discrete state-space equation as well [II-78], [II-42]:

- **Inputs:**  
  - $u_1$: ICE power,  
  - $u_2$: Battery nominal power;

- **State variables:**  
  - $x_1$: state of dynamics of EM,  
  - $x_2$: SOC,  
  - $x_3$: state of dynamics of ICE;

- **Measured disturbance input:** $d_m$: PV panel power.

- **Outputs:**  
  - $o_1$: Drive power,  
  - $o_2$: SOC,  
  - $o_3$: Fuel rate.

And the resulting state-space equation is (2.2.8):

$$
\begin{bmatrix}
    x_1(k+1) \\
    x_2(k+1) \\
    x_3(k+1)
\end{bmatrix} =
\begin{bmatrix}
    0.3679 & 0 & 0 \\
    0 & 1 & 0 \\
    0 & 0 & 0.9048
\end{bmatrix}
\begin{bmatrix}
    x_1(k) \\
    x_2(k) \\
    x_3(k)
\end{bmatrix} +
\begin{bmatrix}
    3.78 \cdot 10^{-6} & 6.321 \cdot 10^{-4} & 0 \\
    0 & -1.517 \cdot 10^{-11} & 0 \\
    2.638 \cdot 10^{-7} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    u_1(k) \\
    u_2(k) \\
    d_m(k)
\end{bmatrix}$$

$$
\begin{bmatrix}
    y_1(k) \\
    y_2(k) \\
    y_3(k)
\end{bmatrix} =
\begin{bmatrix}
    800 & 0 & 0 \\
    0 & 1 & 0 \\
    0 & 0 & 100
\end{bmatrix}
\begin{bmatrix}
    x_1(k) \\
    x_2(k) \\
    x_3(k)
\end{bmatrix}
\tag{2.2.8}
$$

Constraints act upon the system inputs and outputs, enumerated in equations (2.3.7).
The PV power is treated as a measured disturbance, depending on the actual irradiation value.

Further, a quadratic cost function is defined in the form of:

\[
J(k) = \sum_{i=1}^{N} \| \hat{y}(k+i | k) - r(k+i | k) \|^2 + \sum_{i=0}^{N_u} \| \Delta \hat{u}(k+i | k) \|^2 R
\]  

(2.3.8)

Where \( \hat{y}(k+i | k) \) are the predictions at time \( k \) of the output \( y \), \( r(k+i | k) \) is the reference trajectory vector, \( \Delta \hat{u}(k+i | k) \) are the changes of the future input vector, \( N \) is the prediction horizon, \( N_u \) is the control horizon, \( Q \) and \( R \) are weighting matrices.

The choice of the weighting factors, prediction and control horizon is crucial; the aim is to get a balance between good tracking and acceptable control signals [II-82]. As a starting point, it is advisable to normalize all signals in the cost function, and then start systematically tuning each element of the diagonal matrices \( Q \) and \( R \), so that a desired trade-off is achieved.

In what follows, simulation results are presented concerning different tuning parameter values. The reference signals in all cases are: \( r_1 \) – drive power demand calculated from the drive cycle, \( r_2 = 0.7 \) SOC value, \( r_3 = 0 \) for fuel rate. The \( P_d \) drive power demand is calculated from the time-velocity characteristics from figure 2.3.7 (representing the urban part of the NEDC [II-9]), based on the basic dynamical relations of vehicle motion described by equations (2.2.1)-(2.2.3).

![Figure 2.3.7. The urban part of the NEDC (first 800 seconds)](image)

The prediction horizon was \( N=10 \) and control horizon \( N_u=4 \) for all simulation cases. For getting conclusive results, the \( Q \) and \( R \) matrices were fixed for three simulation examples. In what follows, simulation results and calculations are presented for the three cases treated also in [II-42].
The first experiment (figures 2.3.7 to 2.3.10):

\[
Q = \begin{bmatrix}
10^{-2} & 0 & 0 \\
0 & 100 & 0 \\
0 & 0 & 0.001
\end{bmatrix}, \quad R = \begin{bmatrix}
10^{-4} & 0 \\
0 & 10^{-4}
\end{bmatrix}
\]  

(2.3.9)

Figure 2.3.7. Drive power reference tracking

Figure 2.3.8. State of Charge

Figure 2.3.9 Fuel consumption
Synthesising the simulation results, one gets:
- The total fuel consumption is $m_f = 136.4328\, \text{g}$;
- the final SOC is $SOC_{final} = 0.6773$;
- the fuel needed for bringing the SOC to 0.7 following the drive tendency:
  - $m_{fb\text{-needed}} = 79.8137\, \text{g}$;
  - $m_{total} = m_f + m_{fb\text{-needed}} = 216.2465\, \text{g}$.

- The second experiment (figures 2.3.11 to 2.3.14):

  $Q = \begin{bmatrix} 10^{-3} & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 0.001 \end{bmatrix}, \quad R = \begin{bmatrix} 10^{-5} & 0 \\ 0 & 10^{-5} \end{bmatrix}$ \hfill (2.3.10)
Figure 2.3.12. State of Charge

Figure 2.3.13. Fuel consumption

Figure 2.3.14. Control signals $P_{ICE}$ and $P_{bn}$
Synthesising the simulation results, one gets:
- The total fuel consumption is \( m_f = 112.2535 \text{g} \);
- the final SOC is \( SOC_{\text{final}} = 0.6731 \);
- the fuel needed for bringing the SOC to 0.7 following the drive tendency:
  - \( m_{\text{fb-needed}} = 150.8056 \text{g} \);
  - \( m_{\text{total}} = m_f + m_{\text{fb-needed}} = 263.0591 \text{g} \).

- The third experiment (figures 2.3.15 to 2.3.18)

\[
Q = \begin{bmatrix} 10^{-4} & 0 & 0 \\ 0 & 1000 & 0 \\ 0 & 0 & 0.001 \end{bmatrix}, \quad R = \begin{bmatrix} 10^{-5} & 0 \\ 0 & 10^{-5} \end{bmatrix}
\]

(2.3.11)

Figure 2.3.15. Drive power reference tracking

Figure 2.3.16. State of Charge
Synthesising the simulation results, one gets:
- The total fuel consumption is $m_f = 341.3378 \text{g}$;
- the final SOC is $SOC_{final} = 0.6946$;
- the fuel needed for bringing the SOC to 0.7 following the drive tendency:
  - $m_{fb-needed} = 77.7170 \text{g}$;
  - $m_{total} = m_f + m_{fb-needed} = 419.0548 \text{g}$.

One simulation was performed for HSV model without any controller at all, when the ICE delivers all the energy needed for the EM. In this case the total fuel consumption was $m_f = 274.7437 \text{g}$.

Based on the simulation results it can be concluded that for all tuning parameters the total fuel consumed, including the fuel equivalent, gives a value larger than the global optimum. The smallest total fuel is in the case of the first simulation, namely $m_{total} = 216.2465 \text{g}$. This value is smaller than the value without controller. Also in the second
case, the total fuel is smaller than this value: \( m_{\text{total}} = 263.0591 \text{g} \), which reflects also an acceptable result.

In this case, the tracking performances regarding \( P_d \) are good; the overshoots have values between the two other simulations. For the third simulation, the tracking is best of all three, with an exception at the start. However, the fuel consumption is extremely large, compared to the other cases, it is bigger than the value without controller. This case is not acceptable, it is a good counterexample.

The final values of the SOC largely differ in the first two cases from the third one, reflecting the influence of the tuning weights.

The control signals differ slightly in aspect in the first two cases from the third one. It can also be noticed a difference in the fuel consumption, in the first two cases there is a step-wise evolution (meaning there is no fuel consumption at those moments), whereas in the third one there is an almost continuous and gradual increase, ICE functioning at full load. All these differences reflect the importance of proper balancing of the tuning parameters, which defines the switching between the energy sources.

To conclude the chapter, it can be stated that based on the tendencies from the literature regarding hybrid vehicles’ energy management, for the existing linearised model presented in the previous chapter, two control strategies were applied. The first strategy is Dynamic Programming, which delivers the global optimum solution for a constrained optimization problem, but it cannot be implemented in real time. Still it represents a very good comparison for all other sub-optimal solutions. The second control strategy was Model Predictive Control, which delivers a sub-optimal solution to the constrained optimization problem, but it can be applied in real-time successfully. Different simulations were performed and the results are encouraging. Further development of the mathematical model and of the control algorithms used should give even better results, which could be implemented and tested on prototypes.
4. Modelling and speed control of the electric drive

The mathematical modelling and control of the electric drive used in the HSV is presented in detail in [II-34], [II-83]. The electric drive is modelled as a classical DC motor, while for its control two cascade structures were studied. The tuning of the two control loops is based on Modulus Optimum-like criteria, the inner loop is tuned according to Kessler’s Modulus Optimum (MO) criterion, while for the outer loop a double parameterization of the Symmetrical Optimum (SO) method, named 2p-SO-m, was used to correct the controller parameters tuned according to another MO-based method. The author in [II-84], [II-85], [II-86] introduced the 2p-SO-m.

This chapter is structured as follows: first, the mathematical modelling of the electric drive is given, followed by the definition of the control aims and imposed performances upon the traction system. The next part introduces a new tuning method of PID controllers, namely the 2p-SO-m mentioned in the previous paragraph, presenting its theoretical background and some simulation results, preceded by a short introduction to MO-based methods: the Modulus Optimum (MO) method, the Symmetrical Optimum (SO) method, the Extended Symmetrical Optimum (ESO) method. The last part of the chapter presents the application of the 2p-SO-m for parameter tuning of the speed control of the electric drive.

4.1 Mathematical modelling of the electric drive

The traction for an electric vehicle consists in the electric driving system [II-35], [II-37], [II-87]. The energy sources of these vehicles can be different:
- Pure electrical sources, based on batteries;
- Hybrid primary energy sources with different structures and components.

The main part (the drive engine) is an electric motor (EM) driving the wheels, which can be for example a DC-motor (DC-m), with brushes or brushless (with permanent magnets). Both variants are accompanied by dedicated control and power electronics. The electric machine can work as a motor (traction) or as a generator (during the regenerative braking regime).

In case of vehicles with hybrid primary energy sources, the electrical energy for the EM can be delivered (for example) by the battery and by an electric generator (EG) [II-35], [II-34], [II-83], [II-88], [II-89], [II-90]. The functional block diagram of an electrical driving system as part of an electric vehicle is presented in figure 2.4.1. The main components of such a traction system are:
- the EM which drives the wheels and whose control is dealt with in sub-chapter 4.3;
- the energy resources which deliver electrical energy for the EM: - a battery as a buffer for the energy and an electric generator;
- the power electronics and the control unit.

In hybrid applications, the EG is in rigid connection with the internal combustion engine [II-30], [II-31], [II-34], [II-83].
Two aspects are taken into account when modelling the electric drive: on one hand, the equations representing the dynamics of the motor itself, on the other hand the load torque which appears when simulating the electric drive (represented in this case by the vehicle dynamics). The two parts are presented in what follows:

- **Vehicle dynamics.** The basic relations that describe the driven system consist of the simple longitudinal dynamics of the vehicle, presented in chapter 2 of this part, in equations (2.2.1)-(2.2.3).

- **Driving system with DC-m** [II-91], [II-92]. The hypotheses accepted at modelling imply that in normal regimes the DC-m works in the linear domain where the flux (current) is constant in value. An eventual change in the excitation regime will modify the basic model, but a linearization in the new working point results in the basic situation. The basic equations that characterize the functionality of the system are given in (2.4.1):

\[
\begin{align*}
T_A \cdot \dot{u}_a + u_a &= k_A \cdot u_c \\
L_a \cdot \dot{i}_a + R_a \cdot i_a &= u_a - e \\
T_a &= \frac{L_a}{R_a}, \quad e = k_e \omega \\
M_a &= k_m \cdot i_a \\
J_{\text{tot}} \cdot \dot{\omega} &= M_a - M_s - M_f \\
J_{\text{tot}} &= J_m + J_{\text{veh}} + J_w
\end{align*}
\]  

(2.4.1)

where the following notations were used: \(T_A\) – time constant of the actuator (power electronics) [sec], \(u_a\) – armature voltage [V], \(k_a\) – actuator gain, \(u_c\) – command voltage from controller [V], \(L_a\) – inductance [H], \(T_a\) – electrical time constant, \(i_a\) – field current [A], \(e\) – counter electromotive voltage [V], \(k_e\) – coefficient [V/rad/sec], \(\omega\) – rotor speed [rad/sec], \(J_{\text{tot}}\) –
total moment of inertia of the plant \([\text{kg m}^2]\), \(M_a\) – active torque \([\text{Nm}]\), \(M_s\) – load torque \([\text{Nm}]\) (the notation \(M_{\text{load}}\) will be also used), \(M_f\) – friction torque \([\text{Nm}]\), \(J_m\) – moment of inertia of the DC-m \([\text{kg m}^2]\), \(J_{\text{veh}}\) – moment of inertia of the vehicle reduced to the motor axis \([\text{kg m}^2]\), \(J_w\) – moment of inertia of the two driven wheels reduced to the motor axis \([\text{kg m}^2]\).

Accepting that the total inertia of the system can change with maximum 25% regarded to the basic value \(J_{\text{tot}0}\), which corresponds to the vehicle without passengers, it results:

\[
J_{\text{tot}} = J_{\text{tot}0} + \Delta J, \quad \text{with} \quad \Delta J \leq 0.25J_{\text{tot}0}
\]  

Based on equation (2.4.1), a block diagram of system can be built, see figure 2.4.2. The load torque \(M_s\) is generated by the vehicle dynamics.

![Figure 2.4.2. Block diagram of a separately excited DC-motor](image)

Accepting for simplicity that \(T_a\) can be neglected, the derived (linearised) state-space equations of the DC-m are (2.4.3):

\[
\begin{bmatrix}
    i_a' \\
    \omega'
\end{bmatrix} =
\begin{bmatrix}
    -\frac{R_a}{L_a} & -\frac{k_e}{L_a} & \frac{k_m}{J_{\text{tot}}} \\
    k_m & -\frac{k_f}{J_{\text{tot}}} & \frac{1}{J_{\text{tot}}}
\end{bmatrix}
\begin{bmatrix}
    i_a \\
    \omega
\end{bmatrix} +
\begin{bmatrix}
    \frac{k_d}{L_a} & 0 \\
    0 & -\frac{1}{J_{\text{tot}}}
\end{bmatrix}
\begin{bmatrix}
    u_e \\
    0
\end{bmatrix} \tag{2.4.3}
\]

\[
\begin{bmatrix}
    i_a \\
    \omega
\end{bmatrix} =
\begin{bmatrix}
    1 & 0 \\
    0 & 1
\end{bmatrix}
\begin{bmatrix}
    i_a' \\
    \omega'
\end{bmatrix}
\]

Based on the block diagram given in figure 2.4.2, the DC-m's transfer functions can be computed \(\{H_{\omega, u_e}(s), H_{\omega, m}(s), H_{i_a, u_e}(s), H_{i_a, m}(s)\}\). For two particular cases \(k_f \neq 0\) and \(k_f = 0\) (an approximation widely accepted in practice) the transfer functions regarded to the control signal \(H_{\omega, u_e}(s)\) are derived:

- the case \(k_f \neq 0\):
\[ H_{m,ue}(s) = \frac{\omega(s)}{u_e(s)} = k_A \frac{1/k_e}{(1 + R_f k_m k_e) + s[T_a R_f k_m k_e + T_m] + s^2 T_a T_m} \]  
(2.4.4)

where \( T_m = \frac{J_m}{k_m k_e} \) is the mechanical time constant of the plant.

- the case \( k_f = 0 \):

\[ H_{m,ue}(s) = \frac{k_A}{1 + s T_A} \frac{1/k_e}{1 + s T_m + s^2 T_a T_m} \]  
(2.4.5)

In case of electric traction applications \( T_m \gg T_A \), and (2.4.5) can be rewritten as

\[ H_{m,ue}(s) \approx \frac{k_A}{1 + s T_A} \frac{1/k_e}{1 + s T_m (1 + T_m)} \]  
(2.4.6)

This form corresponds to a second order with lag benchmark type model. Numerical values for the application are given in subchapter 4.3.

The speed control for an electric traction system must satisfy multiple requirements:
- Depending on the traffic conditions, the reference of the system is permanently changing (see for example the NEDC);
- The load disturbance is permanently present and changing, depending on the speed, traffic and weather conditions.
- Due to possible changes of the vehicle mass, the equivalent moment of inertia will change and through this the large time constant of the plant.

From these points of view, the control and design method presented chapter 4.2 proves to be of actual interest.

- **The aims of the control system** applied to the DC-m can be grouped as follows:
  - To ensure good reference signal tracking (speed) with small settling time and small overshoot (good transients and zero-steady-state error at \( v = \text{const. velocity} \)).
  - To ensure load disturbance rejection due to modifications in the driving conditions.
  - To show reduced sensitivity to changes in the total inertia of the system:

\[ J_{tot} = J_{r0} + \Delta J, \quad \text{with} \quad \Delta J \leq 0.25 J_{r0} \]  
(2.4.7)

- **Control solutions adopted in the thesis:**

The task of chapter 4 is mainly concentrated around defining the control structures and the appropriate tuning methods for the control of the electric drive. Cascade control systems are among the simplest multivariable control schemes. In spite of their simplicity, they can substantially improve the dynamics and disturbance rejection of the system. Based on this, two cascade control systems are adopted in the thesis, both having two control loops:
  - One internal control loop of the current, consisting in a PI controller and Anti-Windup-Reset (AWR) measure.
  - One external control loop of rotor speed \( \omega [\text{rad/sec}] \) with a PI controller in two variants: a classical one and a more complex control system, where in the outer loop a correction block was added.
4.2 Modulus Optimum based controller tuning methods

Many tuning methods presented in literature are based on “optimization methods”. A short overview allows the following ordering [II-93], [II-94]:
- Minimum optimization methods (different variant);
- LQR optimization of ISE (variants);
- Constrained optimization;
- Optimization in frequency domain;
- Loop shaping methods based on \( L(s) \), \( S(s) \) or \( T(s) \).

In this thesis only optimization in frequency domain based on Modulus Optimum methods is discussed, and as contribution, a new tuning procedure for speed control systems with moment of inertia is introduced.

In order to avoid difficulties due to contradictory results obtained from design according to reference tracking and disturbance rejection, different “optimal” - or in special cases, “optimal-like” - tuning techniques are adopted. Part of them, for example [II-94] – [II-97], are developed in frequency domain and known as Modulus Optimum Methods (MO-m).

A. Optimization in frequency domain

Frequency domain optimization is treated differently in different applications. A first approach appears in papers [II-98], [II-99] (due to C. Kessler) and [II-93], [II-100], where the requirements for an “optimal” named behaviour are formulated through (2.4.8) and (2.4.9), respectively:

\[
M_r(\omega) = |H_r(j\omega)| \approx 1 \quad \text{for values of } \omega \geq 0 \text{ as large as possible,} \quad (a) \quad (2.4.8)
\]

\[
M_{d1,d2}(j\omega) = |H_{d1,d2}(j\omega)| \approx 0 \quad \text{for values of } \omega \geq 0 \text{ as large as possible.} \quad (b)
\]

The so named magnitude optimality conditions were formulated by Whiteley. Based on these, “optimality relations” can be developed between the coefficients of the characteristic equation. Not respecting all the conditions leads to sub-optimal design relations. This aspect is treated further in this part of the thesis.

A second approach is specific to robust control system design [II-102], where the controller is developed ensuring desired maximum value in frequency domain for the sensitivity function \( S(s) \) or for the complementary sensitivity function \( T(s) \):

\[
M_s = \max_{\omega \geq 0} |S(j\omega)| \quad , \quad M_p = \max_{\omega \geq 0} |T(j\omega)| \quad (2.4.9)
\]

and their typical values are within the intervals [II-93], [II-101]:

\[1.2 \leq M_s \leq 2, \quad 1 \leq M_p \leq 1.5\]

For good set point tracking it is necessary that \( M_p \) should be as close to one as possible, and for very good disturbance rejection it is necessary that \( M_s \) should be as small as possible. \( |S(j\omega)| \) and/or \( |T(j\omega)| \) are usually used to express conditions of robust performance [II-103],[II-101], the magnitude function \( |S(j\omega)| \) and/or \( |T(j\omega)| \) can be obtained as function of \( \omega \):

\[|S(j\omega)| = f_1(\omega) \quad , \quad |T(j\omega)| = f_2(\omega) \quad , \quad f_1(\omega), f_2(\omega):[0\rightarrow \infty)\rightarrow R\]
Solving the optimization problem in (2.4.10) means the maximization of the function $f_1(\omega)$ or $f_2(\omega)$ with respect to $\omega$. Such an approach can be also applied to the frequency function of the open loop $L(j\omega) = H_0(j\omega)$.

In field of electric driving system, two of these methods are representative:

- The basic MO-m, shortly presented in paragraph B of this subchapter, using an approach based on papers [II-93], [II-99] and later [II-104]-[II-108].
- The Symmetrical Optimum Method (SO-m) [II-98], [II-99], [II-93], [II-100], [II-107], [II-108] shortly presented in paragraph B.

These methods are based on conditions imposed upon the magnitude-frequency characteristics of the closed loop.

**B. The Modulus Optimum method**

The Modulus Optimum method (abbreviated with MO-m) is based on imposed frequency domain requirements related to relation (2.4.10):

$$H_r(s): \left| H_r(j\omega) \right| = M_r(\omega) = 1 \quad (a)$$

$$H_{d1}(s): \left| H_{d1}(j\omega) \right| = M_{d1}(\omega) = 0 \quad (b)$$

$$H_{d2}(s): \left| H_{d2}(j\omega) \right| = M_{d2}(\omega) = 0 \quad (c)$$

for values of $\omega$ as large as possible. By decomposing the expressions of $M_r(\omega)$, $M_{d1}(\omega)$, $M_{d2}(\omega)$ into Mc-Laurin series, the design conditions could be established if the following requirements were fulfilled ([II-25], [II-9], [II-36]):

$$M_{r}(0) = 1 \quad (1) \quad \text{and} \quad \left. \frac{d^\nu M_r(\omega)}{d\omega^\nu} \right|_{\omega=0} = 0 \quad \text{for} \quad \nu = 1, n \quad (2) \quad (a)(2.4.11)$$

$$M_{d1, d2}(0) = 0 \quad (1) \quad \text{and} \quad \left. \frac{d^\nu M_{d1, d2}(\omega)}{d\omega^\nu} \right|_{\omega=0} = 0 \quad \text{for} \quad \nu = 1, n \quad (2) \quad (b)$$

Condition (1) from expression (2.4.11) can be ensured by poles of $L(s)$ placed in the origin of the $s$ plane. The tuning method based on MO-m tries to fulfil “as good as possible” these requirements [II-99]. The MO-m can be applied in two variants:

- The first variant is based on determining domains of variation of controller parameters that satisfy the imposed requirements, finally determining “the best solution”. The method requires huge amount of calculations [II-109].
- The second variant is based on direct tuning relations. Applying this variant is closer to engineering practice [II-93], [II-99].

In practical applications, different variants and extensions for applying the MO-m related to Kessler’s method are presented in literature [II-93], [II-100], [II-108]-[II-110], [II-111], [II-88]. The MO-m is considered as basic in controller design for electrical driving systems and it will be used as basis for comparison. Often for the plant description low order models (benchmark type models) can be used. $T_2$ is obtained as a small time constant or the
sum of small time constants, (marked in (2.4.12) by \(\tau_i\)), resulting from the “theorem of small
time constants”. The method can also be applied for plants with dead-time \(T_m\):

\[
T_S = \sum_{i=1}^{\nu} \tau_i + T_m
\]  

(2.4.12)

- **Practical situations for optimization based on Kessler’s variant of MO-m. Tuning
relations**

Accepting that the plant models are of low order type with the parameters are (relatively) well
known, for all cases marked of combinations \{plant-controller\} with MO- from Table 2.4.1, it
can be written that:

\[
L(s) = H_c(s)H_p(s) = \frac{k_c k_p}{s(1 + sT_S^c)}
\]  

(2.4.13)

\[
H_p(s) = \frac{k_c k_p}{s(1 + sT_S^c) + k_c k_p} = \frac{a_0}{a_2 s^2 + a_1 s + a_0}, \quad |H_r(j\omega)| = \left[ \frac{a_0^2}{a_0^2 - (2a_0a_2 - a_1^2)\omega^2 + a_2^2\omega^4} \right]^{-1/2}
\]  

(2.4.14)

Table 2.4.1. Practical situations for optimization based on Kessler’s variant of MO-m

<table>
<thead>
<tr>
<th>Cases</th>
<th>(H_p(s))</th>
<th>(H_c(s))</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(\frac{k_p}{1 + sT_S^c})</td>
<td>(\frac{k_c}{s})</td>
<td>MO-1.1</td>
</tr>
<tr>
<td>2.</td>
<td>(\frac{k_p}{(1 + sT_S^c)(1 + sT_1)})</td>
<td>(\frac{k_c}{s(1 + sT_c)})</td>
<td>MO-2.1 and 2p-SO-m</td>
</tr>
<tr>
<td>3.</td>
<td>(\frac{k_p}{(1 + sT_S^c)(1 + sT_1)(1 + sT_2)}) (T_1 &gt; T_2 &gt; T_S)</td>
<td>(\frac{k_c}{s(1 + sT_c)}(1 + sT_c')) (T_c = T_1; T_c' = T_2)</td>
<td>MO-3.1 and 2p-SO-m</td>
</tr>
<tr>
<td>4.</td>
<td>(\frac{k_p}{s(1 + sT_S^c)})</td>
<td>(\frac{k_c}{s(1 + sT_c)})</td>
<td>MO-1.2</td>
</tr>
<tr>
<td>5.</td>
<td>(\frac{k_p}{s(1 + sT_S^c)(1 + sT_1)}) (T_S / T_1 &lt; 0.2)</td>
<td>(\frac{k_c(1 + sT_d)}{1 + sT_f}) (T_d = T_i; T_d / T_f \approx 10)</td>
<td>MO-2.2</td>
</tr>
<tr>
<td></td>
<td>(\frac{k_c}{s(1 + sT_c)(1 + sT_c')}(1 + sT_f)) (T_c' = T_i; T_c' / T_f \approx 10 \ldots 20)</td>
<td>(\frac{k_c(1 + sT_d)(1 + sT_c')}{(1 + sT_f)(1 + sT_f')}) (T_d = T_i; T_d / T_f \approx 10 \ldots 20)</td>
<td>SO-2 (ESO-m)</td>
</tr>
<tr>
<td></td>
<td>(\frac{k_c(1 + sT_c')(1 + sT_d)}{(1 + sT_f')}(1 + sT_f)) (T_c' = T_i; T_c' / T_f \approx 10 \ldots 20)</td>
<td>(\frac{k_c(1 + sT_d)(1 + sT_c')}{(1 + sT_f')}(1 + sT_f)) (T_d = T_i; T_d / T_f \approx 10 \ldots 20)</td>
<td>SO-3 (ESO-m)</td>
</tr>
</tbody>
</table>
The optimality conditions are defined by rel. (2.4.15):

$$2a_0a_2 = a_i^2$$  \hspace{1cm} (2.4.15) 

The use of condition (2.4.15) [II-93] ensures the calculation of controller parameter $k_c$.

$$k_c = \frac{1}{2k_pT_z}$$  \hspace{1cm} (2.4.16) 

The big time constants $T_1$, $T_2$ are compensated based on pole-zero cancellation principle. Correspondingly, the calculation of optimized transfer functions, marked with lower index “0”. Cases 4 and 5 were analysed in detail in [II-112].

$$L_0(s) = \frac{1}{2T_z(1+sT_z)}$$  \hspace{1cm} (2.4.17) 

$$H_{r0}(s) = \frac{\omega_0^2}{\omega_0^2 + 2\zeta\omega_0s + s^2}; H_{r0}(s) = \frac{1}{1+2T_zs+2T_z^2s^2}$$  \hspace{1cm} (2.4.18) 

- **Control system performances in time domain.** Regarding to a step reference the following performances can be underlined:
  - the overshoot, $\sigma_1 = 4.3\%$;
  - the settling time, $t_s = 8.4T_z$;
  - the first settling time, $t_1 = 4.7 \cdot T_z$ .
  - the static coefficient $\gamma_s = y_{\infty} / d_{1,2\times}\bigg|_{r=0, (r=act)}$ depends on the placement of the integral component and type of constant disturbance [II-99].

- **Control system performance in frequency domain:**
  - phase margin (reserve) $\varphi_r = 60^\circ$ ; crossover frequency, $\omega_c \approx 1/2T_z$ ;
  - maximum magnitude of the frequency response: $M_{\text{max}} = \max |T(j\omega)| = 1$ for $(\omega \to 0)$
  - the maximum value of the loop sensitivity function $M_{\text{smax}} = 1.272$ (or its inverse $M_{s0^{-1}} = 0.786$) is in the typically recommended range of $(1.2 < M_s < 2$, [II-93]).

- **External disturbance rejection**
For load disturbances (nted with $d_2$ here) the behaviour of the system is satisfactory only for case MO-1.1. For cases MO-2.1 and MO-3.1, the transients lead to a slow rejection of it:

$$H_{d2o}^{(2,1)}(s) = H_{d2o}^{(1,1)}(s) \frac{1}{(1+sT_1)}$$ \hspace{1cm} (MO-2.1)  \hspace{1cm} (2.4.19) 

$$H_{d2o}^{(3,1)}(s) = H_{d2o}^{(1,1)}(s) \frac{1}{(1+sT_1)(1+sT_2)}$$ \hspace{1cm} (MO-2.2)  \hspace{1cm} (2.4.20) 

where $H_{d2o}^{(1,1)}(s) = \frac{2k_pT_zs}{1+2T_z+2T_z^2s^2}$, in the case of disturbance acting on the output.
The presence of factors \((1+sT_1)^{-1}\) and \([(1+sT_2)(1+sT_3)]^{-1}\) results in a worsening of the response time \(t_{d(2)}\) [II-100], [II-108]. The lower the \(T_2/T_1\) ratio is the bigger the worsening is. In figure 2.4.3 only the cases focused on MO-2.1 case are exemplified, simulated for \(k_p = 1, T_2 = 1\) and different values of \(m=T_2/T_1\), \(m=\{0.05, 0.1, 0.15, 0.2, 0.25, \ldots , 0.5\}\).

![Figure 2.4.3. System response to a step reference input followed by a load disturbance \(m=\{0.05 \ldots 0.5\}\).]

### C. The Symmetrical Optimum method

The basic variant was given by C. Kessler [II-114], as a particular method of the MO-m. The method is presented also in literature under different forms adapted to benchmark type models [II-93], [II-98], [II-100], [II-108], [II-115], [II-116], [II-117]. In [II-107] and [II-115] it is highlighted the fact that SO-m handles well also nonlinearities and time varying parameters. In its practical form, the aim consists in realizing a second order pole in the origin for \(L(s)\) that ensures zero steady state error \((e_\infty = 0)\) for ramp changes of the reference.

A modified version of SO-m was given in [II-93], [II-100], [II-108], [II-111] restricted to benchmark-type plants with integral component (a pole in the origin), having a transfer function \(H_p(s)\) of form (2.4.21) (in Table 2.4.1, the cases marked with SO-1 - SO-3):

\[
H_p(s) = \frac{k_p}{s(1+sT_2)(1+sT_1)(1+sT_3)}
\]  

Using adequate controllers, for all cases \(L(s)\) result with a double pole in the origin:

\[
L(s) = H_c(s)H_p(s) = \frac{k_c k_p (1+sT_c)}{s^2(1+sT_2)} \quad (T_c > T_2)
\]  

Due to this, for \(H_r(s)\) the closed loop transfer function results:

\[
H_r(s) = \frac{k_c k_p T_c s + k_c k_p}{s^3 T_2 + s^2 + k_c k_p T_c s + k_c k_p} = \frac{b_1 s + b_0}{a_3 s^3 + a_2 s^2 + a_1 s + a_0} \quad \text{with} \quad b_0 = a_0, \quad b_1 = a_1
\]

\[
a_0 = b_0 = k_c k_p, \quad a_1 = b_1 = k_c k_p T_c, \quad a_2 = 1, \quad a_3 = T_2.
\]

Based on (2.4.22) the “modulus optimum” conditions result:
\[ 2a_0a_2 = a_1^2 \quad , \quad 2a_1a_3 = a_2^2 \] 

which ensure for \( H_r(s) \) an optimal form “in modulus”, \( H_{r0}(s) \):

\[
|H_{r0}(j\omega)| = \left[ \frac{1 + (a_1/a_0)^2 \omega^2}{1 + (a_2/a_0)^2 \omega^2} \right]^{1/2} \quad \text{(a)}, \quad |H_{r0}(j\omega)| = \left[ \frac{1 + (4Tuzu)^2 \omega^2}{1 + (8Tuzu)^2 \omega^6} \right]^{1/2} \quad \text{(b)} \]

Applying conditions (2.4.25), the controller parameters result in a simple form, easy to be used in practice.

- **SO-1 case:** \( k_c = \frac{1}{2k_pT_\Sigma}, \quad T_i = 4T_\Sigma \) (1) \( k_c = \frac{k_C}{T_i} = -\frac{1}{8k_pT_\Sigma}, \quad T_c = 4T_\Sigma \) (2) \( 2.4.27 \)

- **SO-2 case:** \( k_c = \frac{T_i}{8k_pT_\Sigma}, \quad T_i = T_i + 4T_\Sigma, \quad T_d = \frac{4T_iT_\Sigma}{T_i + 4T_\Sigma} \) (1)

\[
k_c = \frac{1}{8k_pT_\Sigma^2}, \quad T_c = 4T_\Sigma, \quad T_c = T_i \quad (2) \]

In figure 2.4.4 some features of the system performance are presented (for \( k_p=1 \) and \( T_\Sigma=1 \)).

- **Control system performance in time domain:**
  - The overshoot \( \sigma_{r,\sigma} \approx 43.0\% \);
  - The settling time \( t_{r,s} = 16.5T_\Sigma \);
  - The first settling time \( t_{r,fr} = 3.1T_\Sigma \);
  - The steady-state error for step and ramp inputs are zero \((e_\infty=0, \ e_{r,\infty}=0)\).

Figure 2.4.4. Significant diagrams for the SO optimized system
• **Control system performance in frequency domain:**

- The phase margin \( \phi_m \approx 36 \), (maximum value) at the crossover frequency \( \omega_c \approx 1/(2T_\Sigma) \);
- Magnitude plot of the complementary sensitivity function with \( M_{p_{\text{max}}} = 1.682 \) for \( \omega = 0.414/T_\Sigma \);
- The maximum value of sensitivity function: \( M_{s_{\text{max}}} = 1.682 \) for \( \omega \approx 0.6 \) is big. Due to these, the robustness towards plant nonlinearities and time-varying characteristics is reduced.

**D. The Extended Symmetrical Optimum method**

The Extended Symmetrical Optimum Method (abbreviated the ESO-method) was introduced in papers [II-117], [II-118], for low order plants with integral component (see Table 4.2.1). The method consists in a parameterization of the *modulus optimum conditions* under the following form:

\[
\beta^{1/2}a_0a_2 = a_i^2, \quad \beta^{1/2}a_1a_3 = a_i^2
\]

where \( \beta \) is a design parameter which modifies the frequency characteristics from modulus-optimum form. Through this parameterization the method leads to an improvement of the control system performances (for \( \beta = 4 \) all the specific SO-m variants are obtained). Applying the conditions (4.2.21) leads in \( |H_r(j\omega)| \) to the “optimal” form:

\[
|H_r(j\omega)| = \left[ \frac{a_0^2 + a_2^2\omega^2}{a_0^2 - (\beta^{1/2}a_0a_2 - a_i^2)\omega^2 - (\beta^{1/2}a_1a_3 - a_i^2)\omega^2 + a_i^2\omega^4} \right]^{1/2} = \left[ \frac{1 + (a_i^2/a_2^2)\omega^2}{1 + (a_i^2/a_0^2)\omega^2} \right]^{1/2}
\]

(2.4.29)

Following the optimization procedure presented at the MO-m, one gets:

- **Tuning relations**

**Case SO-1:**

\[
k_c = \frac{k_c}{T_i} = \frac{1}{\beta^{1/2}k_pT_\Sigma}, \quad T_c = \beta T_\Sigma = T_i \quad (1) \quad k_c = \frac{1}{\beta^{1/2}k_pT_\Sigma}, \quad T_i = \beta T_\Sigma (2)
\]

(2.4.30)

**Case SO-2:**

\[
k_c = \frac{T_i}{\beta^{1/2}k_pT_\Sigma}, \quad T_c = \beta T_\Sigma, \quad T_i' = T_i \quad (1)
\]

\[T_i = T_i + \beta T_\Sigma \quad T_d = \frac{\beta T_i T_\Sigma}{T_i + \beta T_\Sigma} \quad (2)
\]

(2.4.31)

Applying (2.4.28), for all mentioned cases the transfer functions \( L_0(s) \) and \( H_0(s) \) obtain the same form:

\[
L_0(s) = H_c(s)H_p(s) = \frac{(1 + \beta T_\Sigma s)}{\beta^{3/2}T_\Sigma^2s^2(1 + sT_\Sigma)} \quad \text{with} \quad \frac{1}{\beta^{3/2}T_\Sigma^2} = k_0 = k_c k_p
\]

(2.4.32)

\[
H_0(s) = \frac{1 + \beta T_\Sigma s}{1 + \beta T_\Sigma s + \beta^{3/2}T_\Sigma^3s^2 + \beta^{3/2}T_\Sigma^3s^3} = \frac{1 + \beta T_\Sigma s}{1 + (\beta - \beta^{3/2})T_\Sigma s + \beta T_\Sigma^2s^2}
\]

(2.4.33)
**Control system performances in time domain**

In figure 2.4.5 the performance indices versus $\beta$ are depicted in form of diagrams [II-117] and [II-113]. These diagrams are very useful for controller design by allowing to fix the value of $\beta$ according to the desired performance.

![Figure 2.4.5. Control system performance indices versus $\beta$.](image)

**In Frequency domain**

- **The phase margin** $\phi_m$ versus $\beta$ is given in figure 2.4.5 in a graphical form.

- **Sensitivity function analysis.** Based on the relation of $S_0(s)$ the maximum sensitivity value $M_{s0}$ and its inverse $M_{s0}^{-1}$ were calculated. Figure 2.4.6 (a), (b), (c) presents the Nyquist diagrams calculated for $\beta = 4, 9, 16$; the $M_{s0}=f(\beta)$ circles and the values of $M_{s0}^{-1}$ are also marked. The curves point out the increase of robustness when the value of $\beta$ is increased.

- **Magnitude plot of the complementary sensitivity function.** $M_p(\omega, \beta)=|H_0(j\omega, \beta)|$ and its maximal value $M_{p_{\text{max}}}$ were calculated and the dependences depicted in Figure 2.4.7 (a)-(c): $M_{p_{\text{max}}} (\beta=4) \approx 1.6823$, $M_{p_{\text{max}}} (\beta=9) \approx 1.2990$, $M_{p_{\text{max}}} (\beta=16) \approx 1.1978$.

![Figure 2.4.6. Nyquist curves and $M_{s0}^{-1}$ circles for $\beta = 4, 9, 16$ and the $M_{s0}^{-1}=f(\beta)$ circles](image)

![Figure 2.4.7. Magnitude plot of the $M_p(\omega, \beta)$ for $\beta=4, 9, 16$](image)
• External disturbance rejection: The $\gamma_n$ static coefficient is always zero. Simulation results for $k_p = 1$, $T_\Sigma = 1$, $T_1 = 10$ and $T_2 = 4$ illustrate the situation for $d_2$-type disturbance, figure 2.4.8.

![Figure 2.4.8](image)

(a) Output response (shifted with steady state value 1), (b) control signal, and (c) error for cases SO-1, -2, -3 and different values of $\beta$.

The increase of the phase margin (accompanied with the decreasing of $\omega_c$) leads also to an increase of the settling time. The sensitivity decreases while the phase margin increases. Good tracking performances are ensured. These effects are highlighted in figures 4.2.6 (b), (c). The reference behaviours can be corrected using adequate reference filters.

4.3 A double parameterization of the Symmetrical Optimum method

Based on the positive results given in [II-100], [II-118] and [II-117] a new tuning method is proposed, called a “Double parameterization of the Symmetrical Optimum method (2p-SO-m)”, which was introduced by the author in papers [II-114], [II-84], [II-115]. The aim is to fulfill good tracking performances and efficient disturbance-rejection for a special case of applications for plants without integrating components characterised by $T_i > T_2 >> T_\Sigma$, with the possibility of adjusting the $\phi_{m}$ phase-margin.

For these requirements the MO-m does not give satisfaction. The double parameterization ensures the satisfaction of both requirements. The transfer function of the plant corresponds to the initial approach of SO-m, (2.3-1), for plants with transfer function.
\[
\dot{H}_p(s) = H_p(s)e^{-sT_a}
\]

\[
H_p(s) = \frac{k_p}{(1 + sT_1)(1 + sT_2)(1 + s\tau_1)(1 + s\tau_2)\ldots(1 + s\tau_k)}
\]

(2.4.34)

Applying the theorem of small time constants, (4.2.27) can be rewritten as:

\[
H_p(s) = \frac{k_p}{(1 + sT_Z)(1 + sT_1)(1 + sT_2)} , \quad T_1 > T_2 >> T_Z , \quad T_Z = \sum_{i=1}^{k} \tau_i + T_m
\]

(2.4.35)

For this class of plants with dominant time-constant(s) a double parameterization (marked 2p-) is proposed. The resulted tuning technology was applied later in detail in [II-113], [II-34], [II-83] and verified through simulation.

The double parameterization is based on the followings:

1. First, with the condition that \( \frac{T_Z}{T_1} << 1 \), the parameter \( m \) is defined:

\[
m = \frac{T_Z}{T_1}
\]

(2.4.36)

2. Second, the use of the optimization relations (2.4.28) specific for the case of plants with integral component:

\[
\beta^{1/2} a_0 a_2 = a_i^2, \quad \beta^{1/2} a_i a_3 = a_2^2
\]

(4.2.30)

The method was called Extension through a double parameterization of the Symmetrical (Optimum) method and is marked with 2p-SO-m. Since the tuning relations do not satisfy the “optimum” conditions (2.4.25) the term “Optimum” could be omitted. The controller can ensure:

- Use of pre-calculated (crisp) tuning relations;
- The possibility of improving the control system’s phase margin;
- The possibility of improving good reference signal tracking by using reference filters with parameters that can be easily fixed.
- The possibility of improving load disturbance rejection for some specific cases.

It must be mentioned that only a minority of the tuning methods presented in the literature deals with load disturbance rejection (for example [II-121], [II-123], [II-124], [II-125], [II-127]), even if - in most cases - the control systems operate with constant reference and are subject to disturbances. Because of this, an efficient rejection of the effect of load disturbance becomes often dominant.

- **Tuning relations of controller parameters.**

In accordance with the conditions imposed by Kessler, the situations of interest are characterised by values of \( m<(<<)1 \). Replacing the closed loop relations into the second parameterization (same procedure described like at the MO-m), it results:

\[
\beta^{1/2} k_c k_p (T_1 + T_Z) = (1 + k_c k_p T_c)^2 (a)
\]

(2.4.38)

\[
\beta^{1/2} (1 + k_c k_p T_c) T_1 T_Z = (T_1 + T_Z)^2 (b)
\]

The tuning relation of \( k_c \) given in a double-parameterised form, is:

\[
k_c = \frac{(1 + m)^2}{\beta^{1/2} k_p T_1 T_Z} (1 + m) \quad \text{or, with} \quad T_Z' = T_Z \left(1 + \frac{T_Z}{1+m}\right)
\]

(2.4.39)
\[ k_c = \frac{(1+m)^2}{m} \beta^{3/2} k_p T_{\Sigma} = \frac{(1+m)^3}{m} \beta^{3/2} k_p T_{\Sigma} \]  
(2.4.40)

\( T_c \) can be determined by replacing \( k_c \) into (2.4.38):

\[ T_c = \beta T_{\Sigma} \left[ 1 + \frac{(2 - \beta^{3/2})m + m^2}{m} \right] \]  \( \text{or} \quad T_c = \beta T_{\Sigma m} \)  
(2.4.41)

\[ \Delta_m(m) = [1 + (2 - \beta^{3/2})m + m^2] \quad \text{and} \quad T_{\Sigma m} \Delta_m(m) = \frac{T_{\Sigma} \Delta_m(m)}{(1+m)^2} \]  
(2.4.42)

For the particular values \( \beta=4, 9, 16 \) (that give integer square roots) the controller parameters \( \{k_c, T_c\} \) get more compact forms, Table 2.4.2.

**Table 2.4.2. The controller parameters \( \{k_c, T_c\} \) for particular values \( \beta=4, 9, 16 \)**

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( k_c = \frac{(1+m)^2}{\beta^{3/2} k_p T_{\Sigma} m} )</th>
<th>( \Delta_m(m) )</th>
<th>( T_c = \beta T_{\Sigma} \left[ 1 + \frac{(2 - \beta^{3/2})m + m^2}{m} \right] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>( k_c = \frac{(1+m)^2}{8k_p T_{\Sigma} m} )</td>
<td>( (1+m)^2 )</td>
<td>( T_c = 4T_{\Sigma} \frac{(1+m^2)}{(1+m)^2} )</td>
</tr>
<tr>
<td>9</td>
<td>( k_c = \frac{(1+m)^2}{27k_p T_{\Sigma} m} )</td>
<td>( (1-m+m^2) )</td>
<td>( T_c = 9T_{\Sigma} \frac{(1-m^2)}{(1+m)^2} )</td>
</tr>
<tr>
<td>16</td>
<td>( k_c = \frac{(1+m)^2}{64k_p T_{\Sigma} m} )</td>
<td>( (1-m) )</td>
<td>( T_c = 16T_{\Sigma} \frac{(1-m^2)}{(1+m)^2} )</td>
</tr>
</tbody>
</table>

The transfer function regarding load disturbance \( H_{d20}(s) \) depends essentially on the transfer function of the plant:

- **Case 1:** \( H_p(s) = \frac{k_p}{(1+T_{\Sigma} s)(1+sT_1)} \) and using a PI controller:

\[ H_{d201}(s) = \frac{H_p(s)}{1+L_0(s)} = \frac{\beta^{3/2} k_p T_{\Sigma} m}{(1+m)^2} \frac{s}{T_{\Sigma} s^3 + \beta^{3/2} T_{\Sigma} s^2 + \beta T_{\Sigma} s + 1} \]  
(2.4.43)

- **Case 2:** \( H_p(s) = \frac{k_p}{(1+sT_{\Sigma})(1+sT_1)(1+sT)} \) and using a PID controller

\[ H_{d202}(s) = \frac{H_{d201}(s)}{1} \frac{1}{(1+sT_{\Sigma})} \]  
(2.4.44)

**Discussions upon the particular case \( T_1 >> T_{\Sigma} \):** For \( T_1 >> T_{\Sigma} \), by accepting Kessler’s simplifying conditions, it can be written:

\[ \frac{k_p}{1+sT_1} \approx \frac{k_p}{sT_1} = k_p' \quad \text{and} \quad H_p(s) = \frac{k_p'}{s(1+sT_{\Sigma})} \]  
(2.4.45)
• Control system performances

○ System performance regarding the reference input. Figures 2.4.9 (a), (b), (c) and (d) highlight the unit step reference response, \( y(t) \), for a second order lag benchmark type plant model, with \( k_p = 1, T_\Sigma = 1, T_1 = 10, T_2 = 4 \). As comparison, the case when the controller was designed according to MO-m was taken (2p-SO- solid line, MO-m- dashed line, both tuned according to the relations presented before). The MO-m is frequently used for PI(D) controller tuning in driving systems. Some conclusions are available:

- For small values of \( m \) (0.05, 0.1) the first settling time proves to be convenient even if the overshoot and the settling time are bigger. By increasing the value of \( \beta \) and \( m \), this advantage disappears.

- For the same \( m \) value, by increasing the value of \( \beta \), the overshoot \( \sigma \), decreases and the oscillations are diminished; for values of \( \beta \) between 4 … 6 the settling time decreases, then it increases by further increasing \( \beta \).

- By increasing the value of \( m \), the overshoot decreases.

- In case of a ramp input the steady state error is non-zero, having the value:

\[
e_{r_{ss}} = \lim_{t \to \infty} e_r(t) = \lim_{s \to 0} s \cdot S_0(s) \frac{1}{s^2} = \lim_{s \to 0} \frac{\beta^{3/2} T_\Sigma^2 m}{(1 + m)^2} s(1 + sT_1)(1 + sT_\Sigma)
\]

\[
= \frac{\beta^{3/2} T_\Sigma^2 m}{(1 + m)^2} \left( \frac{\beta^{3/2} T_\Sigma^3 s^3 + \beta^{3/2} T_\Sigma^2 s^2 + \beta T_\Sigma s + 1}{s^2} \right) = \beta^{3/2} T_\Sigma^2 m
\]

or, with \( T_\Sigma = T_1 / (1 + m) \) results:

\[
e_{r_{ss}} = \beta^{3/2} T_\Sigma^2 m
\]

(2.4.46)

For values of \( \beta \) bigger than 9, the transients are slower if \( m \) is increased, the trend being to become a-periodic. The performance indices depicted in Figure 2.4.9 are based on simulation results and interpolation; the curves have \( m \) as parameter. Because of pole-zero cancellation the values for the performance indices for MO-m are independent from \( m \).

Figure 2.4.9. System performances regarding the reference input
Figure 2.4.10. Unit step reference response (2p-SO – solid line, MO – dashed line)
System performance regarding the load disturbance

The duration of the load disturbance response and the maximum deviation of the output are of main interest [II-124], [II-125]. Figures 2.4.12 (a),(b),(c) and (d) highlight the step load disturbance response, \( y(t) \), for the same plant and controller parameters as in the previous paragraph. Also as comparison, the case when the controller was designed according to MO-m was taken (2p-SO- solid line, MO-m- dashed line). From Figures 2.4.12 the following conclusions can be drawn:

- For the same value of \( m = T_{2}/T_{1} \), by increasing the value of \( \beta \), the overshoot increases;
- By increasing the value of \( m \) the overshoot increases.
- Compared with MO-m performances, for \( m < 0.15 \) (also depending on \( \beta \), recommended domain \( 4 < \beta \leq 9 \)) the effect of load disturbance is faster rejected. The smaller \( m \) is \( (T_{1} >> T_{2}) \) the more favourable this property is.

It can be observed that for an increase of \( \beta \) over the value of 9, the use 2p-SO-m does not offer benefits. Also for MO-m differences in transients for disturbance rejection occur.

Figure 2.4.11 presents the performance indices regarding the load disturbance with \( m \) on the horizontal axis and \( \beta \) parameter.

Figure 2.4.11. System performances regarding the load disturbance \( \sigma_{d2}, t_{d2} = f(\beta), m \)-parameter
Figure 2.4.12. 2p-SO-m Unit step load disturbance response, with $\beta$ and $m$ – parameters (2p-SO – solid line, MO – dashed line)
• Analysis in frequency domain.
  o Sensitivity function analysis. To characterise the sensitivity of the control system, for \( \beta = 4, 5, 6, 7, 8, 9, 12, 16 \) and \( m = \{0.05, 0.10, 0.15, 0.20\} \) the maximum sensitivity value \( M_{s0} \) and its inverse \( M_{s0}^{-1} \) are calculated, Table 2.4.3. The values of \( \omega_c \) and \( \phi_m \) are calculated and represented in Table 2.4.4 when the maximum magnitude of the frequency response is \( M_p = \max |T(j\omega)| = 1 \) for \( (\omega \to 0) \). In both cases the dashed values are in the recommended domain, or strictly near it.

<table>
<thead>
<tr>
<th>( m )</th>
<th>( \beta )</th>
<th>( M_{s0} / M_{s0}^{-1} )</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>12</th>
<th>16</th>
</tr>
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<tbody>
<tr>
<td>0.05</td>
<td></td>
<td></td>
<td>1.602</td>
<td>1.45</td>
<td>1.36</td>
<td>1.303</td>
<td>1.263</td>
<td>1.235</td>
<td>1.180</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( M_{s0}^{-1} )</td>
<td>0.624</td>
<td>0.690</td>
<td>0.735</td>
<td>0.767</td>
<td>0.792</td>
<td>0.810</td>
<td>0.847</td>
<td>0.876</td>
</tr>
<tr>
<td>0.10</td>
<td></td>
<td></td>
<td>1.529</td>
<td>1.385</td>
<td>1.302</td>
<td>1.248</td>
<td>1.212</td>
<td>1.185</td>
<td>1.136</td>
<td>1.103</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( M_{s0}^{-1} )</td>
<td>0.654</td>
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<td>0.768</td>
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<td>1.255</td>
<td>1.206</td>
<td>1.172</td>
<td>1.149</td>
<td>1.106</td>
<td>1.076</td>
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<tr>
<td></td>
<td></td>
<td>( M_{s0}^{-1} )</td>
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<td>0.853</td>
<td>0.870</td>
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</tr>
<tr>
<td>0.20</td>
<td></td>
<td></td>
<td>1.406</td>
<td>1.285</td>
<td>1.217</td>
<td>1.172</td>
<td>1.143</td>
<td>1.122</td>
<td>1.083</td>
<td>1.058</td>
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<td></td>
<td></td>
<td>( M_{s0}^{-1} )</td>
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<table>
<thead>
<tr>
<th>( m )</th>
<th>( \beta )</th>
<th>( \omega_c )</th>
<th>( \phi_m )</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>( \omega_c )</td>
<td>0.461</td>
<td>0.406</td>
<td>0.365</td>
<td>0.334</td>
<td>0.308</td>
<td>0.287</td>
<td>0.241</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \phi_m )</td>
<td>39.4</td>
<td>45.0</td>
<td>49.4</td>
<td>53.0</td>
<td>56.1</td>
<td>58.7</td>
<td>64.9</td>
<td></td>
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</tr>
<tr>
<td>0.10</td>
<td>( \omega_c )</td>
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<td>0.371</td>
<td>0.328</td>
<td>0.295</td>
<td>0.268</td>
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<td></td>
<td>( \phi_m )</td>
<td>42.4</td>
<td>48.7</td>
<td>53.8</td>
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<td>( \omega_c )</td>
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<td></td>
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<tr>
<td></td>
<td>( \phi_m )</td>
<td>45.8</td>
<td>52.8</td>
<td>58.5</td>
<td>63.3</td>
<td>67.5</td>
<td>71.1</td>
<td>79.7</td>
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<tr>
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<td>( \omega_c )</td>
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<td>0.312</td>
<td>0.265</td>
<td>0.228</td>
<td>0.199</td>
<td>0.174</td>
<td>0.122</td>
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<tr>
<td></td>
<td>( \phi_m )</td>
<td>49.6</td>
<td>57.2</td>
<td>63.4</td>
<td>68.5</td>
<td>72.7</td>
<td>76.4</td>
<td>84.1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 2.4.13 (a),(b),(c),(d) presents the calculated Nyquist plots where the \( M_{s0}^{-1} \) circles are marked for the values with bold. The curves point out for each \( m \) the increase of robustness when the value of \( \beta \) is increased.
Figure 2.4.13. Nyquist curves and $M_{S0}^{-1}$ circles for different $m$ and $\beta$ and the $M_{S0}^{-1}=f(\beta)$ circles

- **Magnitude plot of the complementary sensitivity function.** For $m$ and $\beta$ - parameters, the graphics of $M_p(\omega)=|H_p(j\omega)|$ are calculated and depicted in Figure 2.4.14; its maximal value $M_{p\text{max}}$ is synthesized in Table 2.4.5, where the dashed values represent the recommended domain or strictly near it. By increasing the value of $\beta$ the value of $M_{p\text{max}}$ decreases, the system becomes less and less oscillatory.

<table>
<thead>
<tr>
<th>$m$</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<tbody>
<tr>
<td>0.05</td>
<td>1.573</td>
<td>1.415</td>
<td>1.321</td>
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<td>1.102</td>
<td>1.067</td>
<td>1.008</td>
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<td>1.199</td>
<td>1.114</td>
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</tr>
<tr>
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<td>1.241</td>
<td>1.113</td>
<td>1.042</td>
<td>1.006</td>
<td>0.999</td>
<td>0.998</td>
<td>0.997</td>
</tr>
</tbody>
</table>

Figure 2.4.14. Magnitude plot of the sensitivity and complementary sensitivity function
• **Design methodology and steps**
Considering the choice of the 2p-SO-m design method justified, the design steps are:

- Starting from the approximated form of the plant transfer function $H_p(s)$, one of the variants of the 2p-SO-m is chosen and the value for $m$ is determined ($m_0$);
- Possible modifications of $m$ are estimated, due to modification of the dominant time constant;
- The value of $\beta$ is determined which satisfies the imposed performances for the nominal value $m_0$ and the parameter changes are evaluated for changes of $m$;
- The controller $H_c(s)$ is chosen and its parameters calculated. If necessary, a reference filter $F_r(s)$ is added;
- The control structure is extended with supplementary functions:
  - Limitations of the control signal, AWR measure,
  - Limitations of the actuator output,
  - Feed-forward filters;
- Taking into account the neglected aspects of the plant (or design), further corrections and fine-tuning can be expected;
- The solutions are verified step by step.

A Youla-parameterization approach of the MO-m, ESO-m and 2p-SO-m is presented in Appendix 2.

### 4.4 Cascade control of the electric drive

- The numerical data for the EM for the considered application are defined in [II-83], [II-34], and presented in Table 2.4.6.

**Table 2.4.6. Numerical values for the DC-m**

<table>
<thead>
<tr>
<th>Torque</th>
<th>Rotation</th>
<th>Useful power</th>
<th>Voltage</th>
<th>Current</th>
<th>Absorbed Power</th>
<th>Efficiency</th>
<th>Electrical time const</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Nm]</td>
<td>[rot/min]</td>
<td>[kw]</td>
<td>[V]</td>
<td>[A]</td>
<td>[kw]</td>
<td>[%]</td>
<td>[sec]</td>
</tr>
<tr>
<td>50.16</td>
<td>1605</td>
<td>8.43</td>
<td>77.6</td>
<td>126</td>
<td>9.78</td>
<td>86.18</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Other electrical data:
- $R_a \approx 0.1 \Omega$ - estimated value from the car builder,
- Gain and time constant of actuator: $k_A=1.33 V/V$, $T_A=0.02$ sec;
- Gains for current and speed sensors: $k_{M_i}=0.0238 V/A$, $k_{M_\omega}=0.0178 V/(rad/sec)$.

- Numerical values regarded to the vehicle ([II-83]):
  - Total mass of vehicle, including an 80kg heavy driver: $m_{tot}=1860$ kg;
  - Frontal area of vehicle: $A_f=2.4 m^2$;
  - Air drag coefficient: $C_d=0.4$;
  - Air density: $\rho=1.225$ kg/m$^3$;
  - Rolling resistance coefficient: $C_r=0.015$;
  - Wheel radius: $w_r=0.3 m$;
  - Final drive ratio: $f_c=4.875 Nm/(rad/sec)$. 

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Starting from the energy conservation principle, relations (2.4.47)-(2.4.49) were deduced, the numerical value is:

\[
J_{veh} = 1860 \cdot \frac{0.3^2}{4.875^2} = 7.04 \text{ kg m}^2
\]  
(2.4.47)

Considering the moment of inertia of the wheels and electric motor having the value of:

\[
J_{w} = 1.56 \text{ kg m}^2
\]  
(2.4.48)

The total inertia results as:

\[
J_{tot} = J_{veh} + J_{w} = 7.04 + 1.56 = 8.6 \text{ kg m}^2
\]  
(2.4.49)

which results in a mechanical time constant of \( T_m = 5.43 \text{ sec} \).

The two time constants are \( T_m = 5.43 \text{ sec} \) and \( T_a = 0.1 \text{ sec} \) resulting in a value of \( m: m \approx 0.02 \).

A. Control structures, controller design

As mentioned in the introductory part of the chapter, the aims of the control structures applied to the DC-m are grouped as follows:

- To ensure good reference signal tracking (speed) with small settling time and small overshoot. The steady state variable reference (ramp) error should be as small as possible.
- To ensure load disturbance rejection due to modifications in the driving conditions.
- To show reduced sensitivity [II-34] to changes in the total inertia of the system.

Due to previous experiences in the domain, two classical solutions were adopted, both having two control loops in a cascade control structure [II-83], [II-34]:

- One internal control loop of the current, consisting in a PI controller extended with an AWR measure [II-126].
- One external control loop of speed \( \omega \) [rad/sec] (motor speed) with a PI controller.

The two controllers are designed separately. The first control solution is a classical one, depicted in Figure 2.4.15. The second control structure, see Figure 2.4.16, differs from the first through the outer loop, in which a forcing block was added to correct the current reference for the inner loop. This can decrease the response time of the system.

Figure 2.4.15. First cascade control structure for the DC-m (Simulink diagram)
The inner loop: the current control loop

The inner loop is identical in both cases, and it consists of a PI controller with AWR measure [II-126]. The parameters of the current controller were based on the MO-m having the design relation (2.4.16) and Table 2.4.1:

\[ H_{ci}(s) = \frac{k_{ci}}{s(1+sT_{ci})}, \quad k_{ci} = \frac{1}{2k_{pi}T_{\Sigma i}}, \quad T_{ci} = T_a \]  

(2.4.50)

where \( k_{pi} \) – the gain of the inner part of the plant, containing the actuator, electric circuit and current sensor), \( T_a \) – electric time constant, \( T_{\Sigma i} \) – equivalent of small (parasitic) time constants (resulting from the power electronics time constant, the sensor and filter time constants), with \( T_a > T_{\Sigma i} \). For the given application the plant transfer function results as:

\[ H_{pc1}(s) \rightarrow \tilde{H}_{pc1}(s) = \frac{k_{p1}}{(1+sT_{\Sigma i})(1+sT_a)} \]  

(2.4.51)

where \( k_{pci} = k_{pi} = A \frac{1}{R_a} k_{Mf} k_{fi} \approx 1.75 \) and \( T_{\Sigma i} = T_A + T_a + T_{\Sigma i} \approx 0.04 \) sec. and the controller parameters results as \( k_{ci} = 7.0, \quad T_{ci} = 0.1 \) sec.

The AWR measure was introduced to attenuate the effects of going into limitation of the controller and realized according to [II-126] having the value of \( T_l = 0.005 \). Other methods for handling constraints of the control signal can also be used, for example a solution where the controller itself is by a dynamic feedback of a static saturation element. The inner optimized control loop can be approximated as follows:

\[ H_{ci}(s) = \frac{1}{k_{M1}} \frac{1}{1 + 2T_{\Sigma i} s}, \quad k_{M1} = 0.0238 \text{ V/A}, \quad 2T_{\Sigma i} \approx 0.08 \text{ sec.} \]  

(2.4.52)

The external loop: the speed control loop

Figure 2.4.16. Second cascade control structure for the DC-m: speed control with reference forcing block (Simulink diagram)
As for the outer loop, it consists of a PI controller in two variants for implementation: one homogenous variant, and one case when a forcing reference filter was introduced. Neglecting the friction coefficient, $k_f$, for the simplified design method ($m \approx 0.02$) the plant transfer function used for speed controller design can be considered:

$$ H_p(s) = \frac{k_p}{s(1+sT_{eo})} \quad (2.4.53) $$

where $T_{eo} \approx 2T_{xi} + T_{io}$ stands for the current loop and parasitic time constants ($T_{io} \approx 0.05$), $k_p$ characterizes the dynamics of the mechanical part of the driving system ($J_{tot}$), the inverse of the current sensor $k_{io}^{-1}$ and the speed sensor $k_{Mo}$.  

The speed controller is of PI type having the transfer function:

$$ H_{Co}(s) = k_{Co} \left( 1 + \frac{1}{sT_{Co}} \right) = \frac{k_{Co}}{s} \left( 1 + sT_{Co} \right) \quad (2.4.54) $$

Based on the desired control performances the parameter $\beta$ is chosen for $\beta \approx 16$. A larger value of $\beta$ ensures less oscillating transients and a bigger phase margin. Due to the fact that in this case $m$ was very small, no corrections were applied at this stage. Finally the parameters of the speed controller result as: $k_{Co} \approx 35.0$ and $T_{Co} = 1.75$.

For the second case the controller is the same and the feed forward correction term is a Derivative with first order with lag type filter with the transfer function:

$$ H_{ff}(s) = \frac{56.0s}{1+s} \quad (2.4.55) $$

### A. Simulation results

The simulation scenarios are the following: the first control structure is simulated, followed by the second cascade structure simulations (comparison of the currents and dynamics), ended by simulations for the first case regarding sensitivity aspects for a change in the mass of the plant. The reference signal is the same for all three cases, consisting in an acceleration part, a part with constant velocity and a part of deceleration until a stop is reached. The load of the system is taken into account as in [II-83], [II-84]. The registered variables are: the velocity (speed), the current and the electromagnetic torque $M_a$ vs. disturbance torque $M_d$.

- **Case of the simple cascade structure**

The simulation results are depicted in figures 2.4.17, 2.4.18 and 2.4.19.

![Figure 2.4.17. Speed reference tracking](image-url)
**Case of cascade structure with correction of the current reference**

In this case the differences in the current behaviour are depicted, together with the active power (dashed line – simple cascade structure, solid line – structure with current correction). The differences in the speed dynamics are insignificant, the active power differences are proportional with the current, figures 2.4.20 and 2.4.21.
Figure 2.4.21. Comparison of the active powers

- **Simple cascade structure with modified load**

The simulations are presented in figures 2.4.22 and 2.4.23 (for the first cascade structure): the mass of the vehicle is changed with +25% of it (solid line – original load, dashed line – increased load): $m_{veh}=m_{veh0}+\Delta m=1860+0.25\times 1860=2332\text{kg}$.

Figure 2.4.22. Behaviour of the current

Figure 2.4.23. Active torque $M_a$ vs. disturbance torque $M_s$
The speed was not presented since almost the same behaviour resulted. But in order to achieve this performance, the current is higher (since it needs more power to carry the increased weight). Still the current does not reach its maximal admissible value (accepted here for 4 times the nominal value of $126 \, A$).

The active power is higher ($12 \, kW$ compared to $9kW$ at starting), but without exceeding the maximal power of $15 \, kW$ of the machine. Both the active torque and the load torque are higher, as expected. In the simulations non-linear phenomena induced by the limitations did not occur.

To conclude the chapter, an efficient cascade control structure for electrical drives used for electrical traction vehicle in two variants – without and with a forcing feed forward term for the current reference. The numerical data regarded to the application is based on a real application of a HSV.

In order to ensure superior performances, for controller design different variants of the modulus optimum tuning method were used: the MO-method for the inner loop, and for the outer loop the Extended Symmetrical Optimum method (ESO-m) and a correction of it based on the tuning method named a Double Parameterization of the ESO method (2p-ESO-m) was used.

Simulations were performed using the Matlab/Simulink environment, for a reference drive cycle, derived from the NEDC Cycle. The simulated cases reflect a very good behaviour of the system both regarding reference tracking and also sensitivity to parameter changes.

5. Part conclusions and contributions

Part II of the thesis focuses on three main chapters, dealing in with the following topics:

- Chapter 2 is dedicated to the mathematical modelling of a series hybrid solar vehicle (HSV). The work is divided into three main parts: first a simplified non-linear and linearized mathematical model is built, based on the modelling of the vehicle components relevant from the point of view of fuel consumption optimization. The second part concentrates on a more complex mathematical model, namely to model the non-linear system as PieceWise Affine (PWA) systems. One way to treat non-linearities is to apply feedback linearization, presented in this chapter for the HSV. Regarding PWA systems, chapter 2 presents a complex Piecewise Bilinear model of the vehicle, both in continuous and in discrete time, where also controllability and stability of the system were studied. Finally, the third part of the chapter presents numerical data and simulation results of both the simplified and the complex mathematical model.

- Chapter 3 focuses on optimal control solutions for fuel consumption minimization of the HSV. After a brief literature review, which presents the latest trend in this topic, two solutions are approached. First a Dynamic Programming (DP) solution is given, which delivers the global optimum for the optimization problem, but it is not feasible
in real-time. Another solution applicable also in practice is Model Predictive Control (MPC), which also takes into account the physical constraints of the plant. After defining the cost function, simulations were run for different values of the tuning factors Q and R. Three simulation results were presented, considered as representative since they refer to significantly different cases, conclusions are drawn based on these results. The important remark must be made that for energy optimization MPC solutions are a viable alternative, so further research and improvement is of high actuality.

- Chapter 4 introduces an efficient cascade control structure for electrical drives used for electrical traction vehicle in two variants – without and with a forcing feed forward term for the current reference. The numerical data regarded to the application is based on a real application of a HSV. In order to ensure superior performances, for controller design different variants of the modulus optimum tuning method were used: the MO-method for the inner loop, and for the outer loop the Extended Symmetrical Optimum method (ESO-m) and a correction of it based on the tuning method named a Double Parameterization of the ESO method (2p-ESO-m) was used. Simulations were performed using the Matlab/Simulink environment, for a reference drive cycle, derived from the NEDC Cycle. The simulated cases reflect a very good behaviour of the system both regarding reference tracking and also sensitivity to parameter changes.
Part III. Speed control solutions for hydro-generators

“And what is a man without energy? Nothing – nothing at all”

(Mark Twain)

1. Introduction. Cascade control structures

Part III of the thesis presents a cascade control solution dedicated to the speed control for hydro-generators (HGs). When controlling complex plants, it is often needed to use advanced control structures, which ensure good performance of the system, and also robustness to parameter changes and to acting disturbances. In order to choose an adequate control solution, the mathematical modelling of the plant is necessary. For control system design of the HG a benchmark type mathematical model was used.

Part III is structured as follows: chapter 2 presents in detail the mathematical model used for the control of the HG, chapter 3 focuses on a cascade control design for the speed control of the HG, based on fact that the plant can be separated into two decoupled parts. The mixed design technique of hydro-turbine application (speed control) was first introduced in papers [III-1], [III-2], focusing on a cascade control structure with an internal minimax controller, to reject internally located deterministic disturbances and a main Generalized Predictive Controller (GPC) loop, to reject external (stochastic) disturbances induced by the power system. The external loop consists of a GPC structure [III-3] used in its polynomial representation, which was transformed into an Internal Model Control (IMC) structure. The disturbance acting on this part of the plant is the active power induced by the power system and considered as a stochastic one. Under these conditions, the combined cascade control structure can be an attractive solution to ensure the control system performance enhancement. Finally, the design steps are presented. The solution was tested through simulation for a numerical case study regarding to a real application. Chapter 4 presents simulation results based on numerical data, followed by chapter 5 which concludes Part IV of the thesis, underlining the contributions of the author.

The review on classical design solutions in speed control of HG highlights the fact that most of the practical solutions are based on a local stabilization of the servo system and the use of a PI (PID) controller in the main loop. The main PI (PID) controllers are tuned according to a large variety of methods. In the case of the classical design methods, the speed controller is designed for a nominal or a specific regime based on the robustness of the controller. The
solutions are verified for different values of the parameters as well, accepting the fact that in a real power system, at normal functioning regime, these modifications are not significant.

Cascade control is a widespread control technique for industrial applications. Its use is motivated by its advantages and also by the necessity to use in some cases such multiple loop structures due to the physical features of the plant. The objective of cascade control is to split a control process into two (or even more) parts, where the outer controller generates on its output the reference signal for the secondary, inner controller.

State feedback can be considered as a generalization of cascade control, where not only some, but all state variables are fed back. In addition, inner variables can be kept limited using limitation elements inserted in the cascade loop. If the inner loop is unstable, a local stabilization can be performed, and thus the primary controller can handle the stabilized part. The main advantages of using cascade structures can be briefly summarized in the following:

- Improved disturbance rejection which acts in a distributed way on the plants, by allowing a faster rejection by the secondary, inner controller. In this case the choice of the secondary controller is very important, so that it can allow a fast recovery from the effect of disturbances.
- A better control in the primary loop and improved dynamical performance through optimizing the inner loop.

There are also disadvantages of applying cascade controllers, which usually come from the nature of the plant (see [III-4]). The main interests in using cascade control structures are:

- Development of new design methods based on combining different structures and design methods, by which the particularities of the given plant can be taken into account;
- Spreading of the application areas to new domains of automatic control.

In this context, the papers [III-5] – [III-12] are mentioned in a form of a survey. [III-12] presents an excitation controller for a single generator based on modern multi-loop design methodology. The proposed controller consists of two-loops: a stabilizing (damping injection) loop and a voltage-regulating loop. The task of the stabilizing loop is to add damping in the face of voltage oscillations. In [III-6] the design and experimental testing of a prototype cascade feedback control system is treated. The overall controller architecture consist of three control sub-tasks. In [III-7] the objective of the application is a cascade model-based predictive control technology for increasing boiler efficiency.

In [III-8] two new two-degree-of-freedom control structures are proposed for cascade control systems, both of which are identical in the controller design procedure. The primary controller used for set point tracking is derived in terms of $H_2$ optimal performance objective. The secondary controller is responsible for rejecting load disturbances that act into the intermediate process and therefore called load disturbance estimator, which is inversely figured out by proposing the desired complementary sensitivity function of the inner loop for disturbance rejection.

In [III-9] based on experience and engineering insight, a systematic procedure for cascade control structure design for complete chemical plants (plant wide control) is presented. In [III-10] the aim is to improve control performances using cascade control and
Smith predictor. In [III-11] a new simultaneous online automatic tuning method for cascade control using a relay feedback approach is presented. Departing from the traditional approach towards tuning of cascade control systems where the secondary and primary loops are tuned in strict sequence, the proposed idea is to carry out the entire tuning process in one experiment. For ease of practical applications, the entire procedure of controller design may be automated and carried out online. A direct controller tuning approach to tune the controllers is proposed here.

Other interesting aspects regarded to cascade control structures are presented in papers [III-13] (a nuclear reactor application), [III-14] (dual RST-control of an Inverted Pendulum with Simulink S-functions Implementation), [III-15] (Robust GPC-QFT approach), [III-16] (AWR applications in cascade control) (a.o., see for example the Proceedings of the European Control Conference 2007 Kos, Greece, July 2-5, 2007). An interesting Robust servo control approach for mechanical systems is presented in [III-39].

2. Mathematical modelling of a hydro-generator

The plant consisting in a hydro-power plant (power generating system) [III-17], [III-18], [III-19] is presented in figure 3.2.1 (a) (using [III-20], with the authors’ acceptance). The plant consist in a hydroelectric dam, a penstock system, the servo system which controls the water flow through the turbine (acting the wicket gate), the turbine and the synchronous generator connected to the power system (PS). The plant is a MIMO system with interconnections between each input and each output, figure 3.2.1 (b).

![Figure 3.2.1. Block scheme of a power generating system](image-url)
In normal functioning regime the plant can be considered with minor (or without) interconnections (or decoupled), so each transfer channel can be modelled independently [III-20]. A simple formula for approximating electric power produced in a hydroelectric plant is:

\[ P = \eta \cdot \rho \cdot g \cdot H \cdot Q \]  

(3.2.1)

where \( P \) [W] is the power in watts, \( \eta \) the efficiency, \( \rho \) [kg/m\(^3\)] the water density [kg/m\(^3\)], \( H \) [m] the water-fall (the height), \( Q \) [m\(^3\)/sec] the flow rate, \( g = 9.81 \) [m/sec\(^2\)] the gravity.

The two basic control systems acting upon the HG (figure 3.2.1 (b)) are:

1. The speed control of the HG which ensures the active power transfer \((p_G)\) from the HG to the PS controlling the frequency (speed of the Synchronous Generator – SG) in the Power System – PS, [III-21], channel \( u_{C_D} \rightarrow \omega \); in frame of this PhD thesis mainly this system will be approached. \( p_G \) represents the disturbance.
2. The voltage control of SG which ensures the reactive power \((q_G)\) transfer between the SG and PS, [III-21], channel \( u_{C_E} \rightarrow u_{G} \); \( q_G \) represents the disturbance.

- **Modelling of the power generating systems’ blocks**

The part of the process which will be used for modelling is presented in figure 3.2.2 [III-17], [III-18], [III-19], [III-22]. Under simplifying conditions [III-17], [III-19] the diagram can be divided into separate subsystems. The assumption that the angular speed \( \omega \) represents the controlled output is valid in the conditions of a large power SG connected to a weak PS or of a SG connected to a local load (insulated regime) [III-19].

The active power plays the role of load disturbance:

\[ P(t) = m_s(t) \cdot \omega(t) \]  

(3.2..2)

and is presented in figure 3.2.2 by the load torque \( m_s(t) \). The changes in water flow through the turbine represent a second category of disturbances which must be taken into account when designing the control system.
For the development phase of the control system, the modelling of the subsystems and plant is treated thoroughly, from simple to very detailed models [III-17] – [III-24] and [III-20], [III-25] and [III-5] (for the actuator). Based on papers [III-17] – [III-25], table 3.2.1 presents a synthesis of the most frequently used transfer functions:

- the hydraulic subsystem,
- the synchronous generator and the load subsystem (particularly the power system).
- the actuator (electro hydraulic servosystem).

A. **The hydraulic subsystem**

The subsystem - known also as *dam construction (including the penstock) and turbine* – converts the energy of a hydronenergetic reservoir in mechanical energy necessary for acting the SG, and it is characterized by (3.2.7) or (3.2.8), where $T_w$ - represent the water time constant, $T_l$ - the reflexion time constant. In [III-19], [III-22] these parameters appear with values depending on the steady state operating point and on the characteristics of the hydraulic turbine.

B. **The synchronous generator coupled to the power system**

The detailed mathematical model of the synchronous generator (SG) used in power system (PS) transient regime studies can be replaced also by reduced order models [III-17]- [III-24]. A linearised model in form of first order with lag (2.1.9) is mainly accepted for the development of the speed controller: $T_m$ - represents the mechanical time constant of the HG, $\alpha_m$ - is the network self-control coefficient and represents a measure of the degree of connection of the SG to the PS. The value of $\alpha_m$ depends of the steady-state operating point, usually $0 \leq \alpha_m \leq 1.3$

- $\alpha_m = 0$ for idle running of the HG (pre-synchronizing regime);
- $0 < \alpha_m \leq 1.3$ for the HG connected to the PS; small values of $\alpha_m$ correspond to a HG operating in insulated regime on a local idle.

The value of $\alpha_m$ increases when the degree of connection to the PS increases.

C. **Mathematical modeling of the servosystem (the actuator)**

Based on [III-25] and neglecting the minor nonlinearities in the servosystem, a linearised block diagram can be constructed, see figure 3.2.3. Using the notations from [III-20], the following state model is presented:

\[
\begin{align*}
\dot{x}_1 &= \frac{k_\nu g_0}{T_1} e_c \\
\dot{x}_2 &= \frac{1}{T_2} x_1 \\
\Delta y &= x_2, \quad x_{m1} = k_{m1} x_1, \quad x_{m2} = k_{m2} x_2
\end{align*}
\]  

\[(3.2.3)\]
The servosystem can be stabilized using different control design methods: pole placement, modern control systems which take into account also the disturbance acting on this level. The disturbances acting on the plant are located in two places [III-22]:
- disturbance acting upon the actuator due to the water flow [III-17];
- load disturbance due to the changes in power demand; they have an oscillatory non-persistent behaviour.

Table 3.2.1. Most frequently used transfer functions for the plant subsystems

<table>
<thead>
<tr>
<th>No</th>
<th>Block</th>
<th>Mathematical Model</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The servosystem</td>
<td>[ H_{\Delta m \Delta y}(s) = \frac{\Delta y(s)}{\Delta u_c(s)} ]</td>
<td>[ \dot{x}_1 = \frac{k_s g_0}{T_1} e_c, \dot{x}_2 = \frac{1}{T_2} x_1, \Delta y = x_2 ] (3.2.4) Model for the unstabilised servosystem</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[ H_s(s) = \frac{k_s}{1 + s T_s} ] (3.2.5)</td>
<td>Model for the stabilized servosystem</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[ H_s(s) = \frac{k_s}{1 + 2 \zeta T_s s + T_s^2 s^2} ] (3.2.6)</td>
<td>Model for the stabilized servosystem</td>
</tr>
<tr>
<td>2</td>
<td>Penstock-turbine subsystem (hydraulic part)</td>
<td>[ H_{\Delta m \Delta y}(s) = \frac{\Delta m(s)}{\Delta y(s)} ]</td>
<td>[ \frac{1 - s T_w}{1 + s T_w/2} ] (3.2.7) First order nonminimum-phase model</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[ 1 - s T_w + \frac{T_w^2 s^2}{1 + s T_w/2 + 2 T_w^2 s^2} ] (a) or [ 1 - s T_w + \frac{(2 T_L / \pi)^2 s^2}{1 + s T_w/2 + (2 T_L / \pi)^2 s^2} ] (b) (3.2.8) Second order nonminimum-phase model</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>[ T_L ] - the reflection time constant [III-22]</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Turbine-SG</td>
<td>[ H_{\Delta m \Delta y}(s) = \frac{\Delta \omega(s)}{\Delta m(s)} ]</td>
<td>[ \frac{1}{\alpha_m + s T_m} ] (3.2.9) First order with lag type model [ 0.3 \leq \alpha_m \leq 1.3 ]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[ \alpha_m ] - network self-control coefficient</td>
<td></td>
</tr>
</tbody>
</table>
3. Disturbance rejection enhancement through Minimax-GPC cascade control

3.1. Proposed cascade control structure

Based on [III-1], [III-2], the chapter presents a new two-stage control solution which deals separately with the disturbances acting on the plant. The particular structure is applied to speed control of a HG (the main application), but it can be applied also in other practical applications, where the plant can be separated into two parts.

The advantages of classical cascade control systems are well known. In the proposed structure the internal loop solves the rejection of the deterministic part of the disturbance using minimax control for worst case of the inner disturbance. Further, Generalized Predictive Control (GPC) is used in its polynomial representation for the outer loop, which was transformed into an IMC structure. The disturbances acting on this part of the plant are considered stochastic ones.

Remark: The design method based on the Minimax criterion is presented in continuous time because of the simplicity in understanding this version. The discrete-time equivalent can be easily deduced based on [III-26], [III-27].

Many practical applications are dealing with plants that can be decomposed into sub-processes $P_1, P_2$, figure 3.3.1.

![Figure 3.3.1. Decomposition of the plant](image)

The independently acting disturbances $d_1$ and $d_2$ can be un-measurable (but estimable) or, in some cases, measurable. It is assumed that $d_1(t), d_2(t) \in L_2$, i.e.

$$\int_0^\infty d_1^T(t)d_1(t)dt < \infty, \quad \int_0^\infty d_2^T(t)d_2(t)dt < \infty \quad (3.3.1)$$

The nature of these disturbances is considered to be different and the simultaneous rejection of both disturbances by a single controller can become a difficult task. In the
proposed control structure their rejection becomes an independent task for two controllers connected in a cascade loop.

The efficient rejection of internal disturbances $d_1$ upon the output can be ensured through a local loop, which is optimized both as far as the control signal $u$ and the disturbance is concerned. This way the structure will have a favourable behaviour regarding both transfer channels, $u \rightarrow y_1$ and $d_1 \rightarrow y_1$. Figure 3.3.2 presents the cascade control structure.

Figure 3.3.2. Cascade GPC control structure

3.2. Optimal controller design for disturbance rejection based on minimax criterion

As introduced in the previous section, the design of the “inner” control loop, which is affected by $d_1$ disturbances, can represent a linear quadratic minimax problem, where the controller minimizes a given cost function when the disturbances maximize this cost function [III-28], [III-29], [III-30]. A state feedback controller is used (Figure 3.3.2) for which a minimax gain is calculated, which is able to reject disturbances up to the value of the worst case disturbance in $H_\infty$ sense (also determined in the algorithm) [III-28], [III-29].

Consider the $P_1$ plant (figure 3.3.1) as a linear system, its dynamics described as:

$$\dot{x}(t) = Ax(t) + Bu(t) + Ld_1(t)$$
$$y_1(t) = Cx(t)$$

$y_1(t)$ representing the inner disturbance. For the linear system a quadratic cost function is defined in order to obtain the optimal control signal, and the effect of the disturbance is included explicitly with a weighted quadratic term of $d_1$:

$$J(\dot{u}, d_1) = \frac{1}{2} \int_0^\infty \left[ y_1^T(t) y_1(t) + \rho^2 u^T(t) u(t) - \gamma^2 d_1^T(t) d_1(t) \right] dt$$

(3.3.3)

where $\rho$ is a design parameter and $\gamma$ is a free parameter which has to be chosen so that it is in line with the Riccati equation to be solved ([III-28], [III-29]). The disturbance term is with negative sign due to the solution possibilities of the Control Algebraic Riccati Equation (see details in [III-28], [III-29].

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The target is to minimize the cost function and obtain the worst case disturbance rejection state-feedback (minimax gain), under which conditions disturbances up to the worst case disturbance are rejected. Reformulating the task, a differential-game problem results:

$$\min_{u(t)} \left\{ \max_{d(t)} [J(u, d_1)] \right\}$$  \hspace{1cm} (3.3.4)$$

Supplementing the cost function with a co-state vector and solving the Euler-Lagrange equation, then deriving the Hamiltonian matrix, finally the optimal control signal \(u^*\) and the worst case disturbance \(d_1^*\) are obtained:

$$u^*(t) = -\frac{1}{\rho^2} B^T P x^*(t) = -K_u x^* \quad , \quad d_1^*(t) = \frac{1}{\gamma^2} L^T P x^*(t) = K_d x^*$$  \hspace{1cm} (3.3.5)$$

Here \(x^*\) stands for the optimal state resulting from the Modified Control Algebraic Equation (MCARE) (3.3.6), while \(P\) represents a positive definite and symmetric \((P = P^T > 0)\) solution to the MCARE:

$$PA + A^T P + C^T C - P \left( \frac{1}{\rho^2} BB^T - \frac{1}{\gamma^2} L L^T \right) P = 0$$  \hspace{1cm} (3.3.6)$$

Finally the feedback gain matrix \(K\) is given by its two components \(K_u\) and \(K_d\), where \(K_u\) is the gain component related to the optimal control and \(K_d\) is the gain component related to the maximal disturbance. This is emphasized in figure 3.3.3. In this figure \(d^*\) represents the worst case disturbance (in the sense of \(H_\infty\) norm) which can still be successfully rejected by the control system. Disturbance \(d^*\) is computed only for simulation purposes, to emphasize the actual value of the worst case disturbance. In reality this \(d_1\) is the existing disturbance which must be rejected, so for real cases \(d^*\) does not exist, but disturbance \(d_1\).

![Minimax control loop for theoretical representation](image)

Figure 3.3.3. Minimax control loop for theoretical representation

For the optimized system which is physically implemented the closed loop transfer functions (for the SISO case) regarding the disturbance – the sensitivity function \(S_1(s)\) - and regarding the reference - the complementary sensitivity function \(T_1(s)\) - is:
This also simplifies the calculus of the external control loop. With the help of a Matlab program the optimal solution is obtained in an efficient way [III-31].

3.3. GPC controller design in IMC representation

The main controller is designed on basis of GPC algorithm. There exists a polynomial form of GPC in case if there are no constraints acting on the plant taken into account. The plant model (linear or linearized) can be described by the following CARIMA model:

\[ A(q^{-1})y(t) = q^{-D_r}B(q^{-1})u(t-1) + \frac{C(q^{-1})}{\Delta}d_z(t) \]  

(3.3.8)

\( u(t) \) is the control sequence, \( y(t) \) the output sequence, \( d_z(t) \) is a zero mean white noise, \( D_r \) is the physical dead time. Polynomials \( A, B \) and \( C \) are described in the backward shift operator \( q^{\alpha} \), \( \Delta = 1 - q^{-1} \). The \( C(q^{-1}) \) polynomial is chosen for 1, for simplicity. The minimized cost function, in order to obtain the control, sequence is:

\[ J = \sum_{j=N^1}^{N^2} \delta(j)[\hat{y}(t+j|t)-r(t+j)]^2 + \sum_{j=1}^{N^i} \lambda_j[j]\Delta u(t+j-1)]^2 \]  

(3.3.9)

where \( N^1 \) and \( N^2 \) are the limits of the prediction horizon, \( N_u \) is the control horizon, \( \hat{y}(t+j|t) \) is the \( j \)-step ahead prediction of the output, \( r(t+j) \) is the future reference trajectory and \( \delta(j) \) and \( \lambda(j) \) are weighting sequences.

After the minimization the control law is obtained:

\[ \Delta u(t) = W(r-f) = \sum_{i=N^1}^{N^2} k_i[r(t+i)-f(t+i)] \]  

(3.3.10)

\( W \) is the first row of the matrix \( (G^T G + \lambda I)^{-1}G^T \), \( f \) is the free response, \( r \) is the reference signal and \( G \) is a matrix containing elements of the plant’s unit step response. If there are no constraints, the GPC algorithm can be transformed into a two-degree-of-freedom (2DOF) polynomial structure, (see figure 3.3.4):

\[ R(q^{-1})\Delta u(t) = T(q^{-1})r(t) - S(q^{-1})y(t) \]  

(3.3.11-a)

where \( R, S, T \) are polynomials in the backward shift operator. The \( R(q^{-1}) \) and \( S(q^{-1}) \) polynomials can be calculated and the result will be:

\[ R(q^{-1}) = \frac{T(q^{-1}) + q^{-1} \sum_{i=N^1}^{N^2} k_i I_i}{\sum_{i=N^1}^{N^2} k_i} \]  

\[ S(q^{-1}) = \sum_{i=N^1}^{N^2} k_i F_i \]  

(3.3.11-b)

where the polynomial \( T(q^{-1}) \) is a free parameter (often chosen to be 1).
The main idea for IMC structures is to include the model of the process in the control loop, and the serially connected controller is chosen to the best realizable inverse of it, [III-2], [III-32]. In figure 3.3.5 the IMC structure is presented, completed with two filters $F_r$ and $F_y$ and a constraint handling block. Of course, this is valid only with stable plants.

Making the two structures equivalent (the RST and the IMC), it results that:

$$F_r = T, \quad F_y = S, \quad C_{IMC} = \frac{A}{R\Delta A + BSz^{-d}} \quad (3.3.12)$$
4. Cascade control of the hydro-generator

Based on [III-1] and [III-2], the next section presents a new concept in speed control of HGs, a Cascade GPC with Minimax Optimal Inner Control Loop, combining the advantages of LQ design technique for the inner loop and the GPC algorithm for the main loop. The plant can be separated into two decoupled parts. The mathematical models for the subsystems were presented in chapter 2 of this part, table 3.2.1. The proposed solution is based on a cascade control structure and design steps are presented.

The internal loop consists in a double integrating servo-system, which must be stabilized; the loop solves the rejection of inner deterministic disturbance. The solution is based on a minimax control for worst case of the inner disturbance. The external loop consists in a GPC structure used in its polynomial representation, which was transformed into an IMC structure. The disturbance acting on this part of the plant is the active power induced by the power system and considered as a stochastic one. The solution was tested through simulation for a numerical case study similar to a real application [III-19].

- The plant and its mathematical model.

The plant is decomposed into sub-processes figure 3.3.1 (P1, P2), the structure of the plant is: 
\[ u(t) \] the control signal, \[ y(t) \] the controlled output. Different types of disturbances, \[ d_1(t) \] and \[ d_2(t) \], act on the plant, which must be particularized as function of the involved application. Local loops can be used for achieving good performances. The transfer function regarding the \[ u \rightarrow y \] is of form

\[
H_p(s) = P(s) = P_1(s) \cdot P_2(s)
\]  
(3.4.1)

In the considered application, the inner part P1 of the plant is the unstabilised hydraulic servo-system, which represents the actuator (the positioning system), Figure 3.4.1:

![Figure 3.4.1. Simplified structure of unstabilised servo-system](image)

where EHC – the electro-hydraulic converter; SVD – the slide-valve distributor; MSM – the main servo-motor; \[ u \] - the control signal, \[ y_1 \] – the gate position of the turbine; \[ x_1 \] and \[ x_2 \] – the state variables associated with the SVD and MSM, respectively.

The local stabilization of the servosystem is solved practically based on a feedback loop [III-25], the stabilising algorithms can be different. For example, the classical
Mannesmann-Rexroth solution [III-33] is based on a pole-allocation technique, resulting a second order with lag mathematical model, which can be approximated with a first order system.

The external part corresponds to the hydraulic subsystem and the synchronous generator coupled to the power system (chapter 2). It is accepted that the servo-system can be stabilized in the form of $P_1(s)$:

$$P_1(s) = \frac{k_s}{1 + 2\zeta T_s s + T_s^2 s^2}$$

(3.4.2)

The load-type disturbance $d_1(t)$ acts at this level: the whole water column (having the height of the dam) [III-17], [III-22] is weighing on the system and modelled as a deterministic disturbance. The worst case is at 10% open blades of the turbine, the best case is when the blades are completely open. Suppose that the modifications of \(d_1(t) = x_3(t)\) are given by:

$$d_1^0(t) = \text{constant}. \quad \text{So, the state-space mathematical model of } P_1 \text{ can be expressed as:}$$

$$\dot{x}(t) = A x(t) + b u(t) + L d_1^0(t)$$

$$y_1(t) = c^T x(t)$$

(3.4.4)

in the case of an additive to $x_1(t)$ - type disturbance $d_1(t)$:

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1/T_{i1} & 0 & 1/T_{i2} \\ 0 & 0 & -1/T_c \end{bmatrix}, \quad b = \begin{bmatrix} g_0/T_{i1} \\ 0 \\ 0 \end{bmatrix}, \quad c^T = [0 \ 1 \ 0], \quad L = \begin{bmatrix} 0 \\ 0 \\ 1/T_c \end{bmatrix}$$

(3.4.5)

The $P_2$ part of the plant corresponds to the hydro-turbine HT (T in figure), the penstock system (PsS) and the SG connected to the PS. For normal operating regimes the simplified linearized models were synthesized in chapter 2 table 3.2.1. It is assumed that the turbine is ideal, without losses and at full load, and $P_2$ is given by [III-17] – [III-24]:

$$P_2^*(s) = k_{p2}^* \frac{1-sT_w}{1+sT_w/2}$$

(3.4.6)

where $k_{p2}^* = k_w$ is the plant gain, $T_w$ represents the water starting time. The HG (including the turbine) connected to the PS can be approximated with the transfer function $P_2^{**}(s)$:

$$P_2^{**}(s) = \frac{k_{p2}^{**}}{\alpha_m + sT_m}$$

(3.4.7)

where $T_m$ stands for the mechanical time constant of the HG-SG depending on the rotating part’s inertia. The parameter $\alpha_m$ has the values between $0.3 \leq \alpha_m \leq 1.3$ with its extreme values $\alpha_m = 0$ (before synchronization of the SG with the PS); the largest value for $\alpha_m$ is for SGs operating connected to the PS with infinite load. In the design phase of the controller, values of $\alpha_m = 1$ are considered. In many situations the extreme case of $\alpha_m = 0$ needs an alternative controller.

Connecting these subsystems, the resulting transfer function for the plant results in $P_2(s)$:
Load disturbances \( d_2(t) \) are induced by the power system and characterise the active power request. The transfer function of the plant controlled by the GPC is

\[
P_2(s) = \frac{k_{p2}(1-sT_w)}{(1+sT_w/2)(\alpha_m+sT_m)}
\]

(3.4.8)

In accordance with IEEE Committee reports [III-23], [III-24], the resulted model (3.4.9) is widely used in studies regarded to modelling speed control systems of hydrogenerators, and current approaches with this respect include suboptimal control problems solved by various control techniques. The simultaneous rejection of the independently acting disturbances \( d_1(t) \) and \( d_2(t) \) by a single controller can become a difficult task. This is the reason why in the presented control structure their rejection becomes an independent task for two cascade controllers.

- **Disturbance rejection in the cascade control structure**

Under these conditions the combined cascade control structure can be an attractive solution to ensure the control system performance enhancement. The proposed cascade control structure is presented in Figure 3.3.2. The structure contains two controllers:

- the inner, a state-feedback controller that realizes the stabilized electro-hydraulic system (EHS) and insures adequate transients and behaviour regarding disturbance \( d_1(t) \),
- the main GPC controller, which deals with reference tracking and rejection of disturbance \( d_2(t) \).

The stabilized EHS must ensure an aperiodic response or a slightly oscillating one, \((0<<\zeta<1)\). The dynamics is imposed by the hydraulic part of the system. In steady-state regime \( d_1(t) \) can be considered almost constant or slowly variable.

In case of disadvantageous behaviour of the system (see for example [III-25], [III-35], [III-36], [III-37]) the random disturbance on the loop, \( d_2(t) \), induced by the electrical PS can be characterized as an oscillating disturbance with a relatively small value for \( \zeta \), which can be considered as the output of a second order lag filter with the transfer function [III-38]:

\[
F_{dist}(s) = \frac{\omega_0^2}{\omega_0^2 + 2\zeta\omega_0 s + s^2}
\]

and

\[
d_2(s) = F_{dist}(s)v(s)
\]

(3.4.10)

\( v(s) \) - step or impulse. \( \omega_0 \) characterizes the frequency, specific for the power system in different functioning regime. Generally the value of \( \omega_0 \) is not constant, but in a given point of the power system its variation range is not too large. In discrete form, they can be approximated also as random variations, described by discrete relation given in [III-38]:

\[
d_2(t) = \frac{C(q^{-1})}{D(q^{-1})}\xi(t)
\]

(3.4.11)

where \( \xi(t) \) is a zero mean white noise.
Using spectral analysis techniques it is possible to detect the frequency around which the main disturbance is located. In the literature it is mentioned that such a disturbance filter can be tuned according to [III-38]:

\[
\frac{C(q^{-1})}{D(q^{-1})} = \frac{1}{(1-q^{-1})(1-a e^{j\alpha} q^{-1})(1+a e^{j\alpha} q^{-1})}
\]

(3.4.12)

where \(\alpha=2\pi f_0 h\) (\(h\) the sampling period, \(f_0\) – a characteristic frequency of power system [III-35]) and \(0 << a < 1\).

The validation of the proposed control solution. Simulation results

The case study is dedicated to the speed control of a real-like application. The parameters of the plant in (3.4.8) and (3.4.9) according to [III-19], [III-33] are: \(g_0 = 0.0625, T_{i1} = 0.001872\), \(T_{i2} = 0.0756\), \(k_{wo} = 1, T_w = 2.2\) sec, \(\alpha_m = 1\) (the SG is accepted to operate connected to the power system with infinite load), \(T_m = 6.8\) sec.

The gain components for minimax control are calculated as presented in section 3.2. For this calculus a Matlab program was written. As a first step, the value of the parameter \(\gamma\) was established using interval halving (the parameter \(\rho\) is chosen to be \(\rho=0.1\)) applied to the CARE, and the final minimal value is \(\gamma=\gamma_{min}=0.028\).

Solution to the CARE leads finally to the min-max gain:

\[
K_u = \begin{bmatrix} 43.706 & 99.311 & 40.879 \end{bmatrix}, \quad K_d = \begin{bmatrix} 149.58 & 345.35 & 140.65 \end{bmatrix}
\]

(3.4.13)

Frequency domain analysis can be performed in order to check the performance of the control loop. Here only the closed-loop transfer function \(P_1(s)\) is presented which is further needed for the cascade control structure:

\[
P_{Cl}(s) = \frac{441.6}{s^2 + 1459s + 4.86 \cdot 10^4}
\]

(3.4.14)

In its simplified form the stabilized EHS can be modelled according to (4.1.1). Accepting the first-order Pade approximation for the dead-time element, the sub-plant \(P_2\) can be characterized by a transfer function:

\[
P_2(s) = \frac{1 + 2.2s}{(1 + 1.1s)(1 + 6.8s)} e^{-4.4s}
\]

(3.4.15)

Since the stabilized EHS has a very small (neglectable) time constant compared to the rest of the process, and it disappears at sampling, at the design phase of the GPC outer loop only the steady-state value of \(P_1(s)\) is taken into consideration, so the composed process transfer function is

\[
P_T(s) = 0.0101 \cdot P_2(s).
\]

The second step is to apply GPC to the process, having the following parameters:

\[
N_1 = 1, \quad N_2 = 70, \quad N_u = 1, \quad \delta = 1, \quad \lambda_u = 0.1
\]

(3.4.16)
Filter $T(q^{-1})$ was chosen for simplicity to be $T = 1$. A sampling time of $h = 1.45$ sec was chosen (so the discrete dead time results as $T_D = 3$). The $R$ and $S$ polynomials are computed also with a Matlab program, and the results in this case are:

\[
R(z^{-1}) = 0.183 + 0.0093 z^{-1} + 0.0094 z^{-2} + 0.0095 z^{-3} + 0.0041 z^{-4}
\]
\[
S(z^{-1}) = 11.1385 - 23.3389 z^{-1} + 7.2004 z^{-2}
\]

The IMC structure parameters result:

\[
F_r(z^{-1}) = T(z^{-1}) , \quad F_w(z^{-1}) = T(z^{-1})
\]

\[
C(z^{-1}) = \frac{5.465 - 10.05z^{-1} + 5.84z^{-2} - 1.09z^{-3}}{1 - 2.788z^{-1} + 2.815z^{-2} - 1.214z^{-3} + 0.1896z^{-4}}
\]

The testing through simulation of the proposed cascade control structure was performed in two different ways.

- **First the inner loop** was simulated as a continuous subsystem (this simulation is sustained by the real implementation solutions – analogue electronics, local microcontroller). A comparative test was made:
  - the control structure using minimax controller,
  - an LQ controller having tuning parameters $Q=1$ and $R=0.01$.

Zero reference signal was supposed, and the initial conditions for the state variables were chosen for:

\[
x_1(0) = 1 , \quad x_2(0) = 2 , \quad x_3(0) = 1
\]

The figure 3.4.2 presents the output signal’s and the states’ evolution without disturbance. An exponentially decreasing disturbance of type $d_1$ is applied to both systems, acting as in (3.4.10). Its time evolution is depicted in figure 3.4.3. Applying this disturbance, the states and outputs are presented for both minimax and LQ control (Figure 3.4.4). It can be noticed that rejection of the disturbance is better in the case of the minimax control.

Figure 3.4.2. Outputs and states of minimax and LQ control
Secondly, the GPC structure was tested through simulation. The following scenario was chosen: a step reference followed by two non-simultaneous disturbances, \( d_1 \) – type step disturbance (acting at time moment 100 sec) having the amplitude of -1, then \( d_2 \) – type disturbance (amplitude -0.05), according to the specific application and modelled with a second-order filter having the transfer function:

\[
F_{\text{dist}}(s) = \frac{1}{1 + 0.5s + 0.4s^2} \tag{3.4.20}
\]

The amplitude of the step is -0.05, acting at time moment of 170 sec. (this is similar to the one presented for example in [III-34], [III-35], Figure 3.4.5 emphasizes the simulation results.

From the analysis of the simulation it results a good behavior of the system both regarding reference tracking and disturbance rejection.
5. Part conclusions and contributions

Part III of the thesis presents a cascade control solution dedicated to the speed control of hydro-generators (HGs). Preceded by a detailed presentation of mathematical modelling aspects of HGs, a cascade control structure is presented for speed control and internal stabilization of the servo part. A two-stage cascade control structure with an internal minimax state controller dedicated for rejecting internally located deterministic disturbances and a main GPC loop is introduced. In case of many applications the maximal value of the inner disturbance can be estimated or calculated. Since the design effort in case of the minimax controller is increased, a computer aided design is used both in this case and for the GPC controller as well. An IMC form for the RST polynomial representation was deduced. The use of the GPC controller under IMC representation based on the RST structure has the advantage of easy implementation.

Simulations give good results and sustain the efficiency of both the inner loop and of the GPC controller, regarding disturbances that are specific for the aimed applications. Then, the control solution is applied to the speed control of hydro turbine generators. The solution involves a cascade control structure with an internal minimax state-feedback controller to reject internally located deterministic disturbances and a main GPC loop. The proposed approach is justified because in case of many applications (e.g. the presented one) the maximal value of the inner disturbance can be estimated or calculated.

Since the design effort in case of the minimax controller is increased, a computer-aided design is used both in this case and for GPC in IMC structure representation. Digital simulation results of the case study show that the control system ensures good performances. The results validate this control structure and its design method.
Part IV. Contributions

“Things should be made as simple as possible, but not any simpler”

(Albert Einstein)

1. Contributions of the PhD thesis

The last part of the thesis, synthesizes the contributions of the thesis, the main conclusions that can be drawn, and enumerates some possible further research directions. A short overview of the contributions was introduced in the first part of the thesis, in chapter 3.

The central point of the thesis is energy generation and energy management. Starting from this idea, two applications are presented, and the according problems which should be solved. The first application, dominating the thesis, is a series architecture hybrid solar vehicle. The main task is to solve the optimal energy management (and so achieve minimal fuel consumption). Part II is divided into three main chapters, each dealing with a different task: mathematical modelling of the hybrid vehicle, control strategies for fuel consumption optimization, mathematical modelling and control of the electric drive. The second application is oriented towards the power generation sector, and deals with the mathematical modelling and control of a hydro-generator. In what follows, the contributions of each part are presented in more detail.

A. Contributions in Part II

Part II of the thesis is oriented towards energy management optimization aspects in a series structure hybrid solar vehicle. Part II is structured on three main parts: mathematical modelling of the vehicle, fuel consumption optimization strategies and control of the electric drive.

- Chapter 2: is dedicated to the mathematical modelling of a series hybrid solar vehicle (HSV). The work is divided into three main parts: first a simplified non-linear and linearized mathematical model is built, based on the modelling of the vehicle components relevant from the point of view of fuel consumption optimization. The second part concentrates on a more complex mathematical model, namely to model the non-linear system as PieceWise Affine (PWA) systems. One way to treat non-linearities is to apply feedback linearization, presented in this chapter for the HSV. Regarding PWA systems, chapter 2 presents a complex Piecewise Bilinear model of the vehicle, both in continuous and in discrete time, where also controllability and
stability of the system were studied. Finally, the third part of the chapter presents numerical data and simulation results of both the simplified and the complex mathematical model.

- **Chapter 3**: focuses on optimal control solutions for fuel consumption minimization of the HSV. After a brief literature review, which presents the latest trend in this topic, two solutions are approached. First a Dynamic Programming (DP) solution is given, which delivers the global optimum for the optimization problem, but it is not feasible in real-time. Another solution applicable also in practice is Model Predictive Control (MPC), which also takes into account the physical constraints of the plant. After defining the cost function, simulations were run for different values of the tuning factors Q and R. Three simulation results were presented, considered as representative since they refer to significantly different cases, conclusions are drawn based on these results. The important remark must be made that for energy optimization MPC solutions are a viable alternative, so further research and improvement is of high actuality.

- **Chapter 4**: introduces an efficient cascade control structure for electrical drives used for electrical traction vehicle in two variants – without and with a forcing feed forward term for the current reference. The numerical data regarded to the application is based on a real application of a HSV. In order to ensure superior performances, for controller design different variants of the modulus optimum tuning method were used: the MO-method for the inner loop, and for the outer loop the Extended Symmetrical Optimum method (ESO-m) and a correction of it based on the tuning method named a Double Parameterization of the ESO method (2p-ESO-m) was used. Simulations were performed using the Matlab/Simulink environment, for a reference drive cycle, derived from the NEDC Cycle. The simulated cases reflect a very good behaviour of the system both regarding reference tracking and also sensitivity to parameter changes.

- **Appendix 2** presents a Youla parameterization approach of the MO-m, ESO-m and 2p-SO-methods.

**B. Contributions in Part III**

Part III of the thesis deals with a control application from the power generation sector, namely cascade control of a hydro-generator.

- **Chapter 2**: Presents a detailed description of mathematical modelling aspects of HGs.
- **Chapter 3**: Introduces a cascade control structure for speed control and internal stabilization of the servo part. A two-stage cascade control structure with an internal minimax state controller dedicated for rejecting internally located deterministic disturbances and a main GPC loop is introduced. In case of many applications the maximal value of the inner disturbance can be estimated or calculated. Since the design effort in case of the minimax controller is increased, a computer aided design is used both in this case and for the GPC controller as well. An IMC form for the RST polynomial representation was deduced. The use of the GPC controller under IMC representation based on the RST structure has the advantage of easy implementation.
• **Chapter 4:** Presents the application of the Minimax-GPC cascade control structure to the hydro-generation application. The solution involves a cascade control structure with an internal minimax state-feedback controller to reject internally located deterministic disturbances and a main GPC loop. The proposed approach is justified because in case of many applications (e.g. the presented one) the maximal value of the inner disturbance can be estimated or calculated. Simulations give good results and sustain the efficiency of both the inner loop and of the GPC controller, regarding disturbances that are specific for the aimed applications. The results validate this control structure and its design method.

2. **Possible further research directions**

The approach control topics and given solutions can support further research topics. Some of them would be:

- Develop piecewise affine and LPV mathematical models for other structures of hybrid electric vehicles as well;
- Implement and perform real experiments of control systems and development methods on the hybrid electric vehicle (see Part II);
- Develop new “auto-calibration” methods for PI and PID controllers based on ESO-m and 2p-SO-m (see Part II);
- Developing Combined Control strategies in speed control of HG and try to implement them on real applications (see part III);
- Handle real applications in constraint cases.

The relevance of the presented design methods and according results is sustained by different papers published during the past years.
Appendix I. Youla parameterization approach of the MO, ESO and 2p-SO methods

1. General aspects

The Youla parameterization (called as Q-parameterization) is a design method applied for both stable and unstable plants [A1-1]-[A1-6]. The Youla parameterization requires polynomials relative to the system’s properties. The disadvantage of the method consists in fact that in case of high order, non-minimum phase or unstable plants the controller results as a non-conventional one.

In this chapter the controller design method based on MO-m, ESO-m and 2p-SO-m [A1-6] are transposed in a Youla-parameterization form. The study is justified by the possibility of imposing favorable forms on the expressions of $S(s)$ and $T(s)$ that ensure the desired system performances.

2. Co-prime factorization

If $G(s)$ is a bounded rational form $|G(j\omega)| < \infty$, with real coefficients there exists a co-prime factorization over the set of all bounded rational forms with real coefficients

$$G(s) = \frac{N(s)}{M(s)} \quad \text{with} \quad G(s) \in \varphi \quad (A1-1)$$

$$N(s)X(s) + M(s)Y(s) = 1 \quad \text{(Bezout’s Identity)} \quad (A1-2)$$

$$N(s), X(s), M(s), Y(s) \in \varphi \quad , \quad \varphi - \text{the set of all bounded rational forms with real coefficients.}$$

The set of all stabilising controllers for a plant given by in form of (A1-1) is defined as:

$$C(s) = \frac{X(s) + M(s)Q(s)}{Y(s) - N(s)Q(s)} \quad \text{where} \quad Q(s) \in \varphi \quad (A1-3)$$

$Q(s)$ - represents the so-called e Youla parameterization polynomial.

If $P(s)$ is stable, the co-prime factorization (A1-1) and (A1-2) can be particularised as:

$$N(s) = P(s) \quad , \quad M(s) = 1 \quad , \quad X(s) = 0 \quad , \quad Y(s) = 1 \quad (A1-4)$$

Accordingly, the controller (A1-3) is determined as:
\[ C(s) = \frac{Q(s)}{1 - P(s)Q(s)} \quad , \]

Figures A1-1 present the basic block diagram (a) and the restructured block diagram (b) regarding the Youla parameterization of controller design.

\[ S(s) = 1 - P(s)Q(s) \quad . \]

It results that \( S(s) \) and \( C(s) \) depend only on \( Q(s) \) and \( P(s) \), and in controller design the following steps are taken [A1-6]:

*Step (1):* For the given plant \( P(s) \), calculation of \( C(s) \) is performed with \( Q(s) \) as parameter. Calculations of \( S(s) \) and \( T(s) \) follow.

*Step (2):* Establishing of a \( Q(s) \) through which the imposed performances for \( S(s) \) or \( T(s) \) are ensured.

*Step (3):* Establishing the controller \( C(s) \) that fulfills the imposed requirements.

*Step (4):* Verification of desired performances.

3. **Youla parameterization of the MO-m**

The plant transfer function is considered to be of one of the following two forms:

\[ P(s) = \frac{k_p}{(1+sT_2)(1+sT_1)} \quad (1) \quad \text{or} \quad P(s) = \frac{k_p}{(1+sT_2)(1+sT_1)(1+sT_3)} \quad (2) \quad (A1-7) \]

The design is exemplified only for (A1-7) (1), the case (2) being solved similarly.

*(1) Calculation of \( C(s) \) with \( Q(s) \) parameter.* Replacing (A1-7) into (A1-5), results:

\[ C(s) = Q(s) \frac{(l+sT_2)(l+sT_1)}{(l+sT_2)(l+sT_1) - k_pQ(s)} \quad (A1-8) \]

with the realizable expression for \( S(s) \) and \( T(s) \):

\[ S(s) = \frac{(l+sT_2)(l+sT_1) - k_pQ(s)}{(l+sT_2)(l+sT_1)} \quad (A1-9) \quad T(s) = \frac{k_pQ(s)}{(l+sT_2)(l+sT_1)} \quad (A1-10) \]

*(2) Establish the stable \( Q(s) \).* Using (A1-9) or (A1-10) results:
Using $T(s)$ with its specific MO-m form and replacing into (A1-10) results $Q(s)$:

$$Q(s) = \frac{1}{k_p} \frac{T(s)(1+sT_\zeta)(1+sT_\iota)}{1+2T_\zeta s+2T_\zeta^2 s^2} \quad \text{(A1-12)}$$

$$Q(s) = \frac{1}{k_p} \frac{(1+sT_\zeta)(1+sT_\iota)}{1+2T_\zeta s+2T_\zeta^2 s^2} \quad \text{(A1-13)}$$

**Remarks.**

1. If the design requirement is imposed in form of zero steady-state error then:

$$S(s) \big|_{s=0} = 0 \Leftrightarrow \frac{(1+sT_\zeta)(1+sT_\iota) - k_p Q(s)}{(1+sT_\zeta)(1+sT_\iota)} \bigg|_{s=0} = 0,$$

and results: $Q(0) = \frac{1}{k_p}$ and a PID-controller is obtained [A1-6]. The controller structure becomes more complicated if the design requirement refers to a very restrictive transfer function $H_{d2\zeta}(s)$.

### 4. Youla parameterization of the ESO-m

The transfer function of the plant is given as:

$$P(s) = \frac{k_p}{s(1+sT_\zeta)} \quad \text{(a)} \quad \text{or} \quad P(s) = \frac{k_p}{s(1+sT_\zeta)(1+sT_\iota)} \quad \text{(b)} \quad \text{(A1-16)}$$

The design is exemplified only for transfer function (A1-16) (a).

1. **Calculation of $C(s)$ with $Q(s)$ parameter.** Replacing, results $C(s)$, $S(s)$ and $T(s)$

$$C(s) = \frac{Q(s) s(1+sT_\zeta)}{s(1+sT_\zeta)(1+sT_\iota)} \quad \text{(A1-17)}$$

$$S(s) = \frac{s(1+sT_\zeta) - k_p Q(s)}{s(1+sT_\zeta)} \quad \text{(A1-18)}$$

$$T(s) = \frac{k_p Q(s)}{s(1+sT_\zeta)} \quad \text{(A1-19)}$$

2. **Establish a stable $Q(s)$**:

$$Q(s) = \frac{1}{k_p} T(s)s(1+sT_\zeta) \quad \text{or} \quad Q(s) = \frac{1}{k_p} (1-S(s))(1+sT_\zeta) \quad \text{(A1-20)}$$

The system performances are imposed through $T(s)$, which is specific for ESO-m:

$$T(s) = \frac{I + \beta T_\zeta}{I + \beta T_\zeta s + \beta^{3/2} T_\zeta^2 s^2 + \beta^{3/2} T_\zeta^3 s^3} \quad \text{(A1-21)}$$

3. **Establish the controller $C(s)$**. The resulting controller is a PI type controller

$$C(s) = \frac{I}{\beta^{3/2} k_p T_\zeta^2 s} (I + \beta T_\zeta s), \quad k_c = \frac{1}{\beta^{3/2} k_p T_\zeta^2} , \quad T_c = \beta T_\zeta \quad \text{and} \quad T_c' = T_2 \quad \text{(A1-22), (A1-23), (A1-24)}$$

### 5. Youla parameterization of the 2p-SO-m

The plant transfer function is (A1-7) (a) or (b), the design is exemplified only for (A1-7) (a).
(1) Calculation of $C(s)$ with $Q(s)$ parameter. Replacing, results $C(s)$, $S(s)$ and $T(s)$

\[
C(s) = Q(s) \frac{(1+sT_x)}{(1+sT_x)(1+sT_y)} \quad (A1-25)
\]

\[
S(s) = \frac{(1+sT_j)(1+sT_x) - k_p Q(s)}{(1+sT_y)(1+sT_z)} \quad (A1-26)
\]

\[
T(s) = \frac{k_p Q(s)}{(1+sT_j)(1+sT_z)} \quad (A1-27)
\]

(2) Establish a stable $Q(s)$:

\[
Q(s) = \frac{1}{k_p}T(s)(1+sT_j)(1+sT_x) \quad \text{or} \quad Q(s) = \frac{1}{k_p}(1-S(s))(1+sT_j)(1+sT_x) \quad (A1-28)
\]

The system performances are imposed through the choice of $T(s)$ as a proportional-derivative-with $3^{rd}$ order lag model. Accepting the use of a PI controller:

\[
T(s) = \frac{1+sT_c}{a_3 s^3 + a_2 s^2 + a_1 s + 1} \quad (A1-29)
\]

where

\[
a_3 = \frac{T_x T_j}{k_p k_c}, \quad a_2 = \frac{T_x + T_j}{k_p k_c}, \quad a_1 = \frac{1 + k_p k_c T_c}{k_p k_c}, \quad a_0 = 1 \quad (A1-30)
\]

(3) Establish the controller $C(s)$.

\[
C(s) = \frac{1}{k_p} \frac{(1+sT_j)(1+sT_x)(1+sT_y)}{(a_3 - T_c s^2 + a_2 - T_c s + 1)} \quad (A1-31)
\]

Using successive replacements, one gets

\[
a_3 = \frac{T_x^3}{(1+m)^2} \beta^{3/2} = T_x^3 \beta^{3/2}, \quad a_2 = \frac{T_x^2}{(1+m)^2} \beta^{3/2} = T_x \beta^{3/2}, \quad a_1 = T_x \beta, \quad a_0 = 1 \quad (A1-32)
\]

\[
Q(s) = \frac{1}{k_p} \frac{(1+sT_j)(1+sT_x)(1+\beta T_{\Sigma m} s)}{\beta^{3/2} T_x^3 s^3 + \beta^{3/2} T_x^2 s^2 + \beta T_x s + 1} \quad (A2-33)
\]

For the second case the supplementary time constant is $T_2^\prime = T_2$.

In this case, for Youla parameterization design the following choice can be made:

- Place the zero $z_3 = -\frac{1}{\beta T_{\Sigma m}}$ as a function of the values of $\{\beta, T_2, m = T_2/T_1\}$;

- The poles of the system are function of $\{\beta, T_2, m\}$:

\[
\Delta(s) = \beta^{3/2} T_x^3 s^3 + \beta^{3/2} T_x^2 s^2 + \beta T_x s + 1 \quad (A1-34)
\]

\[
p_1 = \frac{1}{\beta T_x}, \quad p_{2,3} = -\left(\beta - \beta^{1/2} T_x \pm \left[\left(\beta - \beta^{1/2} T_x^2 - 4 \beta T_x^2\right) T_x^2 - 4 \beta T_x^2\right]^{1/2}\right) \quad (A1-35)
\]

The poles’ placement given by (A1-35) leads to a $Q(s)$ of form (A1-33), and finally in step (3), the controller is expressed as a PI controller.
6. Conclusions

Appendix I. presents in detail the way of transposing the positive results gained from classical design methods based on modulus conditions (MO-m, SO-m) or conditions derived from these (ESO-m and 2E-SO-m) into a Youla-parameterization formulation. If the imposed conditions are adequately chosen, the controller is easy to implement. If the conditions are inadequate then the controller structure results as more difficult to comprehend and the solutions are less accepted in the practice.
Appendix II. References

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