FEEDBACK DESIGN METHODS FOR COOPERATIVE AND CONSTRAINED CONTROL PROBLEMS

Theses of Ph.D. dissertation

written by
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Motivation

In the last years the increased computational capabilities of computer systems and the rapid development of software technologies made it possible to execute complex control algorithms in real-time, even on small, low-cost, embedded platforms. This technological improvement gives inspiration to revise the existing control methods and more freely extend them to complex problems, which require higher computational power and were impossible to solve earlier. New directions have been appeared in the research, such as cooperative control, while some classical fields, e.g. constrained control or model predictive control, have gained a particular attention again. Following this trend the dissertation provides novel results in the fields of cooperative control (Thesis 1.), model predictive control (Thesis 2.) and robust constrained control (Thesis 3.).

Basic notions, methods and tools

The thesis and the underlying work is built upon the following methods and tools from systems and control theory.

Linear Parameter-Varying (LPV) system [1],[13]. can be discrete or continuous in time, in the following form:

$$x_+ = A(\rho)x + B(\rho)u \qquad y = C(\rho)x + D(\rho)u$$

where $\rho =\in \mathbb{R}^p$ is a time varying parameter and $()_+$ denotes the 'next value' in discrete-time ($x_+ \doteq x_{k+1}$ at time k) or the time derivative ($\dot{x}$) in continuous time . If the system matrices are *affine* functions of the parameters, and the element-wise bounds for ρ are known and finite, the LPV system can be equivalently rewritten in polytopic form:

$$x_+ = A(t)x + B(t)u \qquad y = C(t)x + D(t)u$$

where $[A(t), B(t)] \in \text{co}\{[A_1, B_1], \ldots, [A_{2^p}, B_{2^p}]\}$ are constant matrices of appropriate dimensions.

$\mathcal{H}_\infty$ **control [19].** An appropriate state or output feedback controller, minimizing the induced $\mathcal{L}_2$ (or in discrete-time ℓ_2) norm between a chosen output and input of a dynamic system (see Fig. 1).

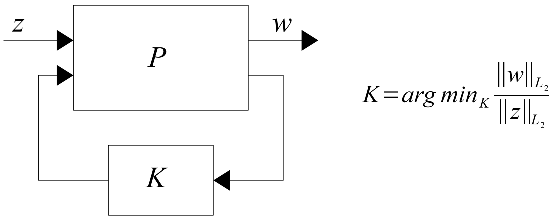

Figure 1: Control loop with plant P and controller K. K is designed to minimize the induced $\mathcal{L}_2$ norm between w and z.

Linear matrix inequality (LMI) [1],[13]. Convex constraint on $x \in \mathbb{R}^m$, expressed in the form $F(x) := F_0 + x_1F_1 + \ldots + x_mF_m > 0$, where $F_0, \ldots, F_m$ are real symmetric matrices.

Passive system [17]. The general state space system $\dot{x} = f(x, u), y = h(x, u)$ is *passive* if there exists a storage function $S : X \to \mathbb{R}^+$, such that the *dissipation inequality* $S(x(t_1)) \le S(x(t_0)) + \int_{t_0}^{t_1} y(t)^T u(t)dt$ holds for all $t_1 \ge t_0$ and all input functions u. It can be checked that passivity involves the stability of the undriven system and the negative feedback interconnection of passive systems results in a passive and therefore stable coupled dynamics.

Dynamic inverse [14],[5]. of a system $\dot{x} = f(x) + g(x)u, \ y = h(x)$ is a dynamical system transforming the original system in a feedback loop into a chain of integrators. Considering now only the SISO case, to obtain the dynamic inverse a nonlinear state transformation $T(x) = [h(x) = y, L_f h(x) = \dot{y}, \ldots, L_f^{r-1}h(x), z(x)]^T$ has to be applied first.

This result in the following dynamics:

$$\begin{aligned}
\dot{\xi}_i &= \xi_{i+1} \quad i = 1 \dots r-1 \\
\dot{\xi}_r &= b(\xi, \eta) + a(\xi, \eta)u \\
\dot{\eta} &= p(\xi, \eta) + q(\xi, \eta)u
\end{aligned}$$

If the dynamics $\dot{\eta} = p(\xi, \eta)$ is stable then the dynamic inverse is given as follows:

$$\begin{aligned}
u &= \frac{-b(\xi, \hat{\eta}) + w}{a(\xi, \hat{\eta})} \\
\dot{\hat{\eta}} &= p(\xi, \hat{\eta}) + q(\xi, \hat{\eta}) \frac{-b(\xi, \hat{\eta}) + w}{a(\xi, \hat{\eta})}
\end{aligned}$$

To apply the dynamic inverse in control one has to ensure the convergence of $\hat{\eta}$ to η as well.

Model predictive control (MPC) [8], [9]. The model predictive control is an optimization based method, where assuming discrete-time case, a sequence of possible control moves are designed in each time instant so that they minimize a predefined cost function. The cost depends on the future states of the system, which are predicted by using an appropriate dynamic model of the plant. After having determined the optimal input sequence, its first element is applied to the plant. In the next time instant the procedure is repeated.

Let the system to be controlled be given by $x_{k+1} = F(x_k, u_k)$. Assuming that the state x_k is known, the following optimization problem is solved at each time instant k:

$$\begin{aligned}
&\arg\min_{U,K} J(X_k, U_k) \\
\text{subject to} \quad & x_{k+i+1|k} = F(x_{k+i|k}, u_i) \\
& H(X_k, U) \leq 0 \\
& x_{k|k} = x_k
\end{aligned}$$

where $U_k = u_0, u_1, ..., u_{N-1}$, $X_k = x_{k|k}, x_{k+1|k}, ..., x_{k+M-1|k}$, J is an arbitrary cost function, $x_{k+i|k}$ is the predicted state at time $k+i$ computed from the kth state measurement and N and M denote the control and prediction horizons, respectively. After having determined the optimal value U_k^*, its first element $u_{k|k}^*$ is applied to the plant, i.e. $u_k = u_{k|k}^*$.

Disturbance invariant set [6], [11]. Consider the general, discrete-time system $x_+ = F(x, u, w)$ with constraints $x \in X$ and $u \in U$. Assume that the disturbance w takes its values from the closed set W. Then, a set S is a disturbance invariant set generated by a given controller $u = K(x)$ if $x \in S$ implies $K(x) \in U$, $x \in X$ and $F(x, u, w) \in S$ for all $w \in W$.

THESIS 1
Formation control based on dynamic inversion and passivity

In the last years the increased computational capabilities of computer systems and the rapid development of the communication and sensor technologies have increased the interest in highly automated unmanned vehicles, which are able to cooperate with other vehicles and are able to perform complex tasks beyond the ability of the individual vehicles. Although the application fields of the cooperative control are very different, from unmanned aerial vehicles to mobile sensor networks, in the control design several common points can be found [10],[3],[18].

The control of autonomous vehicle groups is generally hierarchical, where lower level control components are designed to simplify the original vehicle dynamics for the high level components dealing with the cooperative task. The design of the cooperative control becomes, therefore, much easier and can be made independently from the true vehicle dynamics.

Of course, the stability of the entire hierarchically controlled formation is a key issue in the controller design. Despite of this, the literature concentrates mainly on the construction of the high-level controller and does not analyze the dynamic properties of the coupled system. The lack of this comprehensive analysis gave the motivation to construct a control design framework, in which, the the components on the different levels are designed almost independently, but the global stability of the coupled dynamics is guaranteed.

Thesis 1. *A hierarchical, formation stabilization control framework is proposed for autonomous vehicle groups. The controller consists of an artificial potential function based high-level and a dynamic inversion based low-level controllers and applies passivity based external feedback to ensure the existence of a Lyapunov function proving the global stability and the robustness of the entire, interconnected system.*

A dynamic inversion based low-level controller is designed for the road vehicles, which are modeled by the simplified single-track dynamics. It was shown that the zero dynamics of this model is stable, irrespective of the values of the physical parameters of the vehicle.

The linearized vehicle dynamics is then interconnected with a potential function based high-level controller. A passivity-based external feedback is constructed so that an appropriate Lyapunov function can be designed for the entire closed-loop system proving its global stability.

The robustness properties of the proposed control structure is analyzed and a simple method is provided to compute the maximal $\mathcal{L}_2$ gain of the uncertainty model, that is not destabilize the closed loop dynamics.

It is shown by simulation, that the proposed method is able to solve formation reconfiguration problems of a road vehicle group.

Publications related to Thesis 1: [PSBH04],[PB04],[PB05],[PB06], [Pén08a],[Pén08b].

The thesis is described in details in Chapter 2. of the dissertation.

THESIS 2
Robust model predictive control for uncertain LPV systems

The model predictive control is a popular tool for solving constrained control problems. The MPC provides optimal solution (with respect to a prescribed cost function) and handles the constraints in natural way

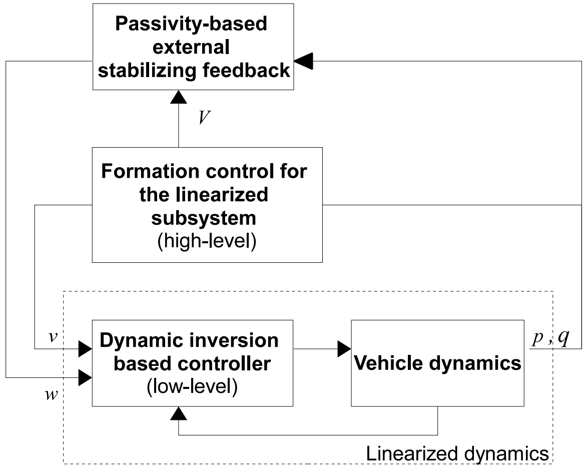

Figure 2: The simplified diagram of the dynamic inversion and passivity based control structure. (p, q are the position and velocity of the vehicles, V denotes the information required from the high-level controllers for the computation of the external feedback.)

as it inserts them directly into the on-line optimization producing at each time instant the actual control input. Like other control methods, MPC has limitations as well.

1. Although the rapid development of computer architectures made it possible to execute complex numerical methods efficiently and fast, the computational time required for the on-line optimization is sometimes still unacceptable high.

2. The power of the MPC can be fully exploited only if the system is linear, piecewise linear or affine.

3. Before the design of the predictive control the originally continuous dynamics have to be discretized. If the system differs from linear time invariant there is no adequate, unique discretization method. Moreover the structure and the stability properties of

the original system may change during discretization [15].

For linear systems with polytopic and norm bounded uncertainty Kothare et. al. proposed in [7] an LMI-based model predictive control. In the thesis this algorithm is revised and improved with the aim of eliminating the three difficulties mentioned above.

Two results are presented, both for discrete-time, parameter varying systems. The first improvement makes the original MPC framework applicable for real-time implementation. The second method extends the MPC framework for a class of uncertain, piecewise polytopic LPV systems. The chosen model structure fits well to the discretization procedure applied for continuous LPV systems depending nonlinearly on the scheduling parameter.

Thesis 2. *a) A linear matrix inequality based model predictive control is elaborated for discrete-time LPV systems with the aim of real-time implementation. The original design procedure is completed with further LMI constraints guaranteeing the stability, feasibility and constraint satisfaction even if the time required for the on-line computation is larger than the sampling time.*

b) Based on the algorithms elaborated so far for uncertain linear time invariant systems the LMI-based model predictive control framework is extended to uncertain, pecewise polytopic LPV models. By following a similar design procedure than the original method, new LMI conditions are derived to ensure the stability, feasibility and constraint satisfaction.

The viability of the proposed algorithms is proved by testing them via simulations on autonomous vehicles and applying them to solve the control problems related to the primary circuit of the Paks Nuclear Power Plant.

Publications related to Thesis 2: [PSB07],[PB07],[PS08]

The thesis is described in details in Chapter 3 of the dissertation.

THESIS 3
Interpolation based constrained $\mathcal{H}_\infty$ control for discrete-time LPV systems

The $\mathcal{H}_\infty$ control is a general, widely used control design approach. Basically it provides disturbance attenuation by minimazing the induced $\mathcal{L}_2$ (or ℓ_2) gain between a predefined output and input. On the other hand, most control problems of interest (such as stabilization, trajectory tracking) can be casted to a $\mathcal{H}_\infty$ problem, even if the system to be controlled is uncertain. This makes the $\mathcal{H}_\infty$ control the most suitable approach for robust control design.

However, the classical $\mathcal{H}_\infty$ theory does not handle constraints, albeit state and control input constraints are always at present in a real control problem. The methods elaborated so far for constrained $\mathcal{H}_\infty$ control of LPV systems suffer from several difficulties: they either be time consuming (MPC), difficult to solve (based on non-convex optimization, nonlinear matrix inequalities) or result in too conservative (LMI based) solutions ([4],[2]).

This gave the motivation to choose a different approach and construct a new - interpolation based - controller structure ([12],[11]), by which the the most difficulties above can be eliminated. The proposed algorithm is suitable for real-time implementation, since it uses control components computed off-line. The main result is the enlargement of the disturbance invariant set ([6]) of the closed loop system. This set is important because it determines the domain of applicability of the controller.

Thesis 3. *An efficient algorithm based on linear programming is proposed for the outer approximation of the disturbance invariant set generated by a discrete-time LPV system under a constant, state feedback controller. Based on this result an interpolation-based constraint $\mathcal{H}_\infty$ state feedback control structure is constructed, which can be applied over a much larger domain than any other single control policy (see Fig. 3).*

The algorithm constructing the invariant set starts from the convex set determined by the prescribed state and input constraints. This set is then tightened till an acceptable approximation of the disturbance invariant set is reached.

For the interpolation a set of unconstrained state feedback controllers are designed first. The set contains many controllers that satisfy the prescribed $\mathcal{H}_\infty$ performance but generate small d-invariant set and contains at least one, which generates large invariant set, but provides poor performance. From the closed-loop systems generated by the controllers an extended system is built. It is proved that the extended system is input-output equivalent to the original system if the latter is controlled by interpolating among the individual control actions. Moreover the extended system has finite ℓ_2 gain, which can be set smaller (better) than the given performance level by choosing the unconstrained controllers with care. This fact verifies the applicability of the interpolating controller. The domain, over which the controller can be actually used is constructed by projection from the disturbance invariant set of the extended system. It is proved that this domain is much larger than that generated by any other constant state feedback controller (see Fig. 3).

The proposed $\mathcal{H}_\infty$ control method is tested on numerical examples and on the control of the pressurizer subsystem working in the primary circuit of the Paks Nuclear Power Plant.

Publications related to Thesis 3: [PKBV08], [PKB09b], [PKB09a].

The thesis is described in details in Chapter 4 of the dissertation.

Future works

The contributions of this thesis relate to three research fields: cooperative control, model predictive control, constrained $\mathcal{H}_\infty$ control.

In the first thesis a passivity based method is proposed as a possible solution to the formation stabilization problem of autonomous vehicles. The control framework was presented for road vehicles with a high level controller using artificial potential functions. The base idea, i.e. using passivity based external feedback to stabilize the hierarchically controlled, coupled dynamics, can be extended to other vehicles as well,

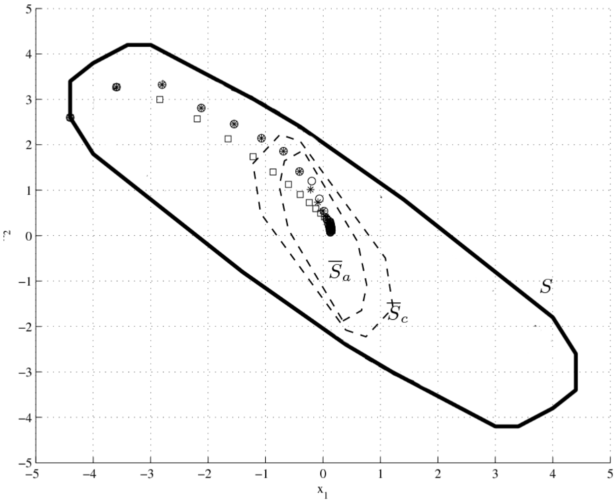

Figure 3: The maximal disturbance invariant sets (domains of applicability) of constant $\mathcal{H}_\infty$ controllers ($\overline{S}_a, \overline{S}_c$) and the disturbance invariant set of the interpolation based controller (S) in a numerical example.

such as mobile robots or even mobile sensor networks. Although it has several advantages, e.g. fully distributed, the use of potential functions in the high-level coordination may limit the applicability of the control framework. In case of more complex cooperative tasks we can get rid of this limitation by partitioning the problem into simpler, intermediate reconfiguration tasks, that can be solved by potential functions. In the future the research can be continued to develop an appropriate 'top-level' controller, which can provide an optimal partitioning and constructs the necessary potential functions for the high-level controller.

The second thesis focuses on the design of model predictive controllers. The problems of real-time implementation and the effects of discretization are investigated. The research can be continued in both directions. The control structure elaborated for real-time implementation can be further improved by taking the properties not only of the computing hardware, but also of the communication network into consideration. In a real networked system one may count on uncertain transmission delays and even data loss. Both can result in performance degradation or loss of stability.

In case of discrete-time controllers the research can be directed towards exploring the properties of the different discretization algorithms and developing methods for incorporating their features in the controller synthesis. The algorithm presented in Thesis 2 and the results of Toth et.al. in LPV discretization ([16], [15]) provide a good starting point to these researches.

The third thesis proposes a practical method for constructing constrained $\mathcal{H}_\infty$ controllers for LPV systems. In contrast to other, existing approaches the interpolation based controller provides more efficiently computable, less conservative solution that moreover works over a much larger domain. In the thesis the controller was applied to disturbance attenuation problems. Our aim is to further test the applicability of the method on more complex $\mathcal{H}_\infty$ problems, e.g. reference tracking, control of uncertain LPV systems.

The presented algorithm currently requires for the entire state to be measurable. In several cases the states of the system are only partially measured, thus the remaining state variables have to be estimated. Augmenting the present controller with state estimation, i.e. designing interpolation-based output feedback $\mathcal{H}_\infty$ controller, would have considerable practical significance. This gives inspiration to continue the researches in this direction as well.

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