Numerical Renormalization Group Study of the Two-Channel Kondo Model

Ph.D. Thesis

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Research Preliminaries

The study of strongly correlated electron systems is one of the central themes in condensed matter physics. Special types of low-dimensional systems that show strong correlations are called quantum impurity models. These models describe interactions between local degrees of freedom – like a spin – and a continuum of extended states – as e.g. a conduction electron band. Their prototypes are the Anderson and the Kondo models. These models account for several experimental findings like the resistance minimum and power law-like behavior observed in non-magnetic metals in the presence of magnetic impurities. More recent experiments exploring quantum impurity physics take place in mesoscopic systems with reduced dimensionality such as thin films and wires of dilute magnetic alloys, metallic point contacts, nanoscopic electronic devices like quantum dots, etc. Furthermore the investigation of these models is the starting point to understand more complex strongly correlated systems in the framework of dynamical mean field theory.

Since their creation, our understanding of these models has evolved considerably due to the powerful theoretical tools available for their study. The Numerical Renormalization Group (NRG) method, devised by Wilson in the seventies, is still possibly the most reliable and versatile from among them.

Scientific Goals

With this dissertation I have aimed to contribute to the efforts improving the accuracy of the spectral function calculations for quantum impurity models using the recently developed Density Matrix Numerical Renormalization Group (DM-NRG) method. These developments have been timely e.g. because of the possible applications of mesoscopic devices in quantum computing. My DM-NRG studies concentrated on the two-channel Kondo (2CK) model which is the simplest quantum impurity model exhibiting non-Fermi liquid properties. It has possible relevance in the description of heavy fermion systems. Moreover, the existence of the two-channel Kondo state has recently been justified using a double quantum dot device. Even though the 2CK model became paradigm for describing non-Fermi liquid physics, so far little has been published about its dynamical properties. Up to this date, these properties can be reliably computed for the whole frequency range only with NRG. Nevertheless, in the vicinity of the 2CK fixed point a conformal field theory based scaling approach can be used to predict the analytic properties of the various Green's functions. Another aim of this work was to carry out a detailed analysis of the results obtained using the two approaches.

Thesis Points

1. I have worked out a flexible NRG procedure which is capable of handling an arbitrary number of compact Lie group symmetries that a quantum impurity model possesses [1, 2].

2. I have implemented the above procedure for spin- and charge-SU(2) and U(1) symmetries in structure oriented C++ pro-

gramming language by writing a flexible NRG code [3, 2] which can be downloaded from the site http://www.phy.bme.hu/∼dmnrg/. I have demonstrated that the use of non-Abelian symmetries is advantageous for reliability and performance reasons.

3. Using NRG, in the presence of channel anisotropy I have calculated the frequency-dependent linear conductance of the double quantum dot device that has recently been built to justify the feasibility of the 2CK states. I have computed the universal conductance scaling curves that connect the one- and two-channel Kondo fixed points of the two-channel Kondo model that describes the system [4]. I remark that presently NRG is the only method to calculate these scaling curves.

4. Based on group theoretical considerations, I have shown that in the recently devised DM-NRG method the reduced density matrix preserves its diagonal form even in such cases when an arbitrary number of compact non-Abelian Lie group symmetries of the quantum impurity model are considered [2].

5. I have implemented the DM-NRG procedure for an arbitrary number of spin- and charge-SU(2) and U(1) symmetries into the NRG code mentioned above. This way it became possible to calculate very accurate, spectral sum rule preserving spectral functions [3, 2].

6. Based on conformal field theoretical results, I have classified the highest-weight fields of the electron-hole symmetrical two-channel Kondo model at the two-channel Kondo fixed point according to the group $\mathrm{SU}_{C1}(2) \times \mathrm{SU}_{C2}(2) \times \mathrm{SU}_S(2)$,

and determined the relevant and leading irrelevant perturbations to the 2CK fixed point Hamiltonian [5]. Using DM-NRG, I have computed the retarded Green's functions of the highest-weight fields at zero temperature at the two-channel Kondo fixed point and in the presence of relevant perturbations such as the channel anisotropy, or the magnetic field using the groups $\mathrm{SU}_{C1}(2) \times \mathrm{SU}_{C2}(2) \times \mathrm{SU}_S(2)$ and $\mathrm{SU}_{C1}(2) \times \mathrm{SU}_{C2}(2) \times \mathrm{U}_S(1)$, respectively. I have also performed spectral function calculations for fermionic operators in the presence of marginal potential scattering with the use of the symmetry $\mathrm{U}_{C1}(1) \times \mathrm{U}_{C2}(1) \times \mathrm{SU}_S(2)$ [5].

7. Based on simple scaling arguments and conformal field theoretical considerations, I have expanded the highest-weight fields at the two-channel Kondo fixed point in terms of the operators of the free theory. I have determined the analytic form of the universal scaling curves in the asymptotic regions and their numerical form for the whole frequency range. In all cases my DM-NRG calculations confirmed the analytic expectations [5].

8. I found that the boundaries of the various 2CK scaling regimes depend not only on the type of the perturbation but also on the operator investigated. In a small magnetic field, I observed a universal resonance in the local fermions' spectral function and that the dominant superconducting instability arises in the composite superconducting channel [5].

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