COLOUR MOIRÉ PHENOMENON AND ITS APPLICATION FOR TOPOGRAPHICAL MEASUREMENTS

PhD dissertation

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WRITTEN STATEMENT

I below subscribed, Daria Pavelyeva, state that within the same area of science the present PhD dissertation is not in process as well as during the last two years the above mentioned work was not rejected and also unsuccessful presentation of the dissertation or pass of state exams did not take place.

Budapest, 2009. 02. 08.

..........................................

Signature
DEDICATION

I would like to dedicate my dissertation to

my family without whose support

I could not finish my work. Also to my close friend, Evgenij Lomonosov,

whose early death did not allow him to finish the PhD work.
ACKNOWLEDGEMENTS

I would like to express my sincere thanks to my advisor, Professor Klára Wenzel and Consultant Ákos Antal who offered me unlimited advices during the course of my PhD. The knowledge I have gained from them is invaluable.

I thank all employers of the department “Mechatronics, Optics and Engineering Informatics”, who helped me in different situations and were so kind to me, that I have never felt myself like a foreigner during the course of PhD studies in Hungary.

Finally, my sincere gratitude to my bridegroom, Árpád Fodor, for his love and support.
ABSTRACT

A quality demands for 3D (three-dimensional) real-time high-resolution shape measurement are increasing. Numerous topographical tools exist for measurement of surface. Recently in most instances optical methods are preferred over traditional methods due to the opportunity of contactless and whole field measurement data obtaining.

In spite of the fact that moiré phenomenon is often mentioned as an undesirable optical effect in some applications it represents a powerful optical method in metrology. As it is common with most measurement tools, moiré technique has its inherent characteristics that limit its use in certain types of applications. One of the main difficulties faced in moiré pattern analysis is determination of the differences in object height, or as it is called in scientific literature - “hill&valley” problem.

In the dissertation a novel moiré method based on application of special bicolour gratings was theoretically developed for solution of the “hill&valley” problem and practically applied to the real moiré topographical measuring methods.

In the beginning the motivation and objectives of the dissertation are considered. The survey of optical topographical techniques, brief description of traditional moiré effect and its application for measuring technique are presented in the next two chapters. Then the “hill&valley” problem statement and its solution by conventional and colour-based methods earlier presented in scientific literature are considered. In the following chapter the developed colour moiré theory, spectral analysis of colour moiré effect in colour space, methods of digital colour moiré pattern generation and questions of human perception of colour moiré phenomenon are proposed. In the last chapter application of the colour moiré theory for topographical methods is considered by means of the series of surface measurements experiments. Besides it characteristics of colour moiré methods and main concepts of the ambiguities definition in colour moiré patterns are also presented in the work. In conclusion the work summary and the theses are proposed. Results of the tests are attached in the form of PDF file and can be found on the CD-disk.
**List of symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D</td>
<td>three-dimensional</td>
</tr>
<tr>
<td>2D</td>
<td>two-dimensional</td>
</tr>
<tr>
<td>U</td>
<td>plane monochromatic wave</td>
</tr>
<tr>
<td>A</td>
<td>amplitude of the light intensity</td>
</tr>
<tr>
<td>( \omega )</td>
<td>radian frequency</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>phase</td>
</tr>
<tr>
<td>i</td>
<td>imaginary number</td>
</tr>
<tr>
<td>P</td>
<td>transfer function</td>
</tr>
<tr>
<td>p</td>
<td>pitch of the grating</td>
</tr>
<tr>
<td>a</td>
<td>width of the transparent ruling</td>
</tr>
<tr>
<td>F</td>
<td>Fourier series</td>
</tr>
<tr>
<td>I</td>
<td>light intensity</td>
</tr>
<tr>
<td>( I_0 )</td>
<td>background average intensity</td>
</tr>
<tr>
<td>( I_1 )</td>
<td>amplitude intensity of the fundamental harmonic</td>
</tr>
<tr>
<td>N</td>
<td>contour order</td>
</tr>
<tr>
<td>( \theta )</td>
<td>fringe orientation angle</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>grating orientation angle</td>
</tr>
<tr>
<td>c</td>
<td>fringe spacing</td>
</tr>
<tr>
<td>( p' )</td>
<td>average line spacing</td>
</tr>
<tr>
<td>( z_N )</td>
<td>contour plane of order ( N )</td>
</tr>
<tr>
<td>d</td>
<td>distance separating the light source and observer</td>
</tr>
<tr>
<td>l</td>
<td>distance from the light source and observer to the grating plane</td>
</tr>
</tbody>
</table>
\( l_p \) distance from the plane to the centre of the entrance pupils of the projection and observation optics
\( f \) focal length of the projection and observation lenses
SPS spatial phase-shifting
TPS temporal phase-shifting
SCPS spatial-carrier phase-shifting
DYD dark-yellow-dark
CCD charge-coupled device
COL&COL two bicolour gratings with the opposite order of colour bars
COL&COL* two bicolour gratings with the same order of colour bars
BW&COL projecting binary and analyzing bicolour grating
COL&BW projecting bicolour and analyzing binary grating
d width of each colour bar
b width of the opaque bar
n minimal step of sampling
\( \phi_1(\lambda) \) relative colour stimulus function
\( S(\lambda) \) relative spectral power distribution of the illuminant
\( \tau(\lambda) \) spectral transmittance of the colour bar in bicolour grating
\( R(\lambda) \) spectral reflectance factor of the specimen surface.
CIE Commission International d'Eclairage
\( X, Y, Z \) CIE tristimulus values
\( \bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda) \) colour-matching functions of a standard colorimetric observer
\( \lambda \) wavelength
\( \Delta \lambda \) wavelength interval
\( k \) normalizing constant
\( x, y, z \) chromaticity coordinates
CIELUV CIE 1976 \( L^* u^* v^* \) three-dimensional, approximately uniform colour space
JND just noticeably difference
m centre of mass
\( \Delta E_{uv}^* \) difference between two colour stimuli in the CIE 1976 \( (L^* u^* v^*) \) colour space
\( L^* \) luminancy
\( u^*, v^* \) chrominancy
\( Y, u', v' \) colour stimulus
RGB Red Green Blue
CMY Cyan Magenta Yellow
CAS Compact Array Spectrometer
\( v \) spatial frequency of the grating
L, M, S types of human cones
DAC Digital-to-Analog Converter
CRT cathode-ray tube
\( f(\gamma, \xi) \) representation of digital image
\( H \) row of digital image
\( K \) column of digital image
\( W_r, W_g, W_b \) three matrixes of pixel component in digital image
\( r_{K,H}, g_{K,H}, b_{K,H} \) red, green and blue component of the \((K, H)\) pixel for analyzing and deform grating
\( a_{K,H}(r_{K,H}, g_{K,H}, b_{K,H}) \) \((K, H)\) pixel of A matrix
\( e_{K,H}, q_{K,H}, j_{K,H} \) \((K, H)\) pixels of matrixes \(E, Q\) and \(J\) correspondingly
\( E \) matrix representing deform digital grating
\( Q \) matrix representing analyzing digital grating
\( J \) matrix representing resulting digital grating, obtained by overlapping of the analyzing and deform gratings
\( r_1, r_2, g_1, g_2, b_1, b_2 \) red, green and blue components of the \((K, H)\) pixel of analyzing and deform gratings
\( r, g, b \) red, green and blue components of the \((K, H)\) pixel of resulting grating
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1 Introduction

In the chapter motivation and objectives of the dissertation are set. The overview of 3D shape measurement methods is given. Brief synopsis of the existing optic topographical techniques and their comparison is presented. The merits and disadvantages of moiré technique are discussed.

1.1 Motivation

The speed and quality of the measurements is one of the most important criteria for the present-day measuring technique. It is evident that there are many reasons why these parameters are so significant in the modern world.

Moiré fringe technique is an important metrological tool in area of topographical measurements. Development of the new moiré method that allows to get the extra information about the form of measuring surface from the one moiré image offers the great advantage comparing to the traditional moiré technique, where in most cases there is the strong request in several moiré patterns and additional calculations or/and equipment. The novel moiré method will move the moiré technique to the new level of the modern measuring technique.
1.2 Objectives

This dissertation has two main objectives. The first one is development of the moiré method that efficiently and simply allows to determinate the differences in object height from one moiré pattern.

The second objective is validation of the developed moiré method to be capable for effective application in moiré topographical measurements.

1.3 Types of 3D shape measurement methods

High-resolution, real-time 3D shape measurement of permanent or dynamically deformable objects has a huge potential in industry to drive down product cost and increase both productivity and quality. Recently the demand of topographical shape measurement has appeared in a wide variety of applications, including inspection, shape acquisition, reverse engineering, gauging, robot navigation and vision. Many techniques are applied for 3D measurements (Figure 1).

![Figure 1 Block-scheme of 3D shape measurement methods.](image)

Nowadays, in most instances optical techniques are preferred over traditional methods as long as they are typically contactless and provide whole field measurement...
data. Non-invasive, fast, accurate measuring techniques are becoming increasingly important for industrial applications. Optical-based measuring techniques are currently used for many types of measurements including displacement, flow field, temperature, surface roughness and topography, as well as many others. Moiré technique relates to the group of optical topographical methods.

In the next two sections of the chapter the optical methods and the place of moiré technique among them is considered.

### 1.4 Optical topographical techniques

Three-dimensional optical shape measurements deliver the absolute 3D geometry of objects that should be independent from the object’s surface reflectivity, its distance from the sensor, and from illumination conditions. Thus, 3D optical sensors deliver the shape and physical dimensions of object, which are rotation-, translation-, and illumination-invariant [1].

Existing 3D optical measurement techniques can be classified into two major types — active and passive. In active methods some kind of radiation is emitted and than its reflection detected in order to probe an object or environment. In general, active measurement employs below methods:

- time of flight,
- phase measuring,
- triangulation: laser point,
- structured light: fringe projection, moiré topography, laser light plane, coded patterns,
- confocal microscopy,
- interferometry: laser interferometry, holografical interferometry, speckle technique.

The passive techniques rely on detecting reflected ambient radiation; here any kind of radiation is not emitted. To passive measurement technique can be related methods of stereovision and shape-from-shading [2], [3].

Brief review (configuration, principle of operation, advantages and lacks) of optical non-contact methods is presented in Table 1.
## 1 Introduction

<table>
<thead>
<tr>
<th>Method</th>
<th>Typical configuration and principle of operation</th>
<th>Method</th>
<th>Typical configuration and principle of operation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time of light</strong></td>
<td><img src="image" alt="Time of light Diagram" /> The distance is found by measuring the time of short optical pulsedetection sent from laser in direction of object and monitored by detector. The distance is calculated using the velocity of light [2].</td>
<td><strong>Phase measuring</strong></td>
<td><img src="image" alt="Phase measuring Diagram" /> The distance is measured by comparing the phase of signal from the laser with that of the reflected light from the object [2], [3].</td>
</tr>
<tr>
<td><strong>Laser triangulation</strong></td>
<td><img src="image" alt="Laser triangulation Diagram" /> From the deviation of the laser line propagating in the angle onto the objects surface the elevation of the object is determined by the point of incidence [4].</td>
<td><strong>Laser interferometry</strong></td>
<td><img src="image" alt="Laser interferometry Diagram" /> The original wavefront amplitude is split into two parts and each fraction is directed along a different path. As a result the interference pattern appears [5].</td>
</tr>
<tr>
<td><strong>Confocal microscopy</strong></td>
<td><img src="image" alt="Confocal microscopy Diagram" /> Reconstruction of three-dimensional images by using a spatial pinhole to eliminate out-of-focus light or flare in specimens that are thicker than the focal plane [6], [8].</td>
<td><strong>Holographic interferometry</strong></td>
<td><img src="image" alt="Holographic interferometry Diagram" /> The idea of holographic interferometry is based on the simultaneous recording of amplitude and phase of light scattered by an object. The process contains hologram recording and reconstruction [8], [9].</td>
</tr>
</tbody>
</table>
### 1 Introduction

<table>
<thead>
<tr>
<th><strong>Fringe projection</strong></th>
<th><strong>Speckle technique</strong></th>
<th><strong>Moiré topography</strong></th>
<th><strong>Stereovision</strong></th>
<th><strong>Laser light sectioning</strong></th>
<th><strong>Shape-from-shading</strong></th>
<th><strong>Several types of coded patterns</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>The grating is projected onto an object and then formed by the surface shape viewed from another direction [10].</td>
<td>A coherent radiation illuminates an object that scatters light and the scattered light generates a grainy three-dimensional structure in space [11].</td>
<td>The grating placed in front of the object produces a shadow on the object that is viewed from a different direction. Moiré fringe pattern appears by superposition of the grating and its deformed shadow [12].</td>
<td>By seeing the same object with two different 2D views, one is able to reach its third dimension. The technique is based on the phenomenon of human cyclone vision [13].</td>
<td>With projecting the expanded laser line an elevation profile of the object under test is obtained by the camera [14].</td>
<td>Using a single camera, two or more images are taken of an object in a fixed position but under different lighting conditions. By studying the changes in brightness over a surface and employing constraints in the orientation of surfaces, certain depth information may be calculated [16].</td>
<td>Specially coded pattern or a set of patterns are projected onto the measuring scene which is imaged by a single camera or a set of cameras. Every coded pixel has its own codeword, so there is a direct mapping from the codewords to the corresponding coordinates of the pixel in the pattern [15].</td>
</tr>
</tbody>
</table>

| **Table 1** Principles of operation and configurations of optical 3D shape measurement methods. |
1.5 Comparison of optical topographical methods

The choice of the appropriate technique is influenced by a number of parameters. Some important features optical measuring methods are compared in Table 2 in order to define the place of moiré technique among the other methods. The Table was composed on the basis of reviewed literature [2] - [15].
<table>
<thead>
<tr>
<th>Optical 3D shape measurement method</th>
<th>Passive</th>
<th>Resolution</th>
<th>Measuring surface</th>
<th>Trans</th>
<th>Colour</th>
<th>Size of measurable object</th>
<th>Complexity of processing</th>
<th>Acquisition time of complex data</th>
<th>Experimental requirements</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time of flight</td>
<td>active</td>
<td>low</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>macro</td>
<td>medium</td>
<td>variable</td>
<td>low</td>
<td>high</td>
</tr>
<tr>
<td>Phase measuring</td>
<td>active</td>
<td>low</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>macro</td>
<td>medium</td>
<td>variable</td>
<td>low</td>
<td>high</td>
</tr>
<tr>
<td>Laser triangulation</td>
<td>active</td>
<td>medium</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>macro-micro</td>
<td>simple</td>
<td>very slow</td>
<td>low</td>
<td>medium</td>
</tr>
<tr>
<td>Fringe projection</td>
<td>active</td>
<td>medium</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>limited appl.</td>
<td>macro-micro</td>
<td>medium</td>
<td>low</td>
<td>low</td>
</tr>
<tr>
<td>Moiré topography</td>
<td>active</td>
<td>medium</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>limited appl.</td>
<td>macro-micro</td>
<td>medium</td>
<td>low</td>
<td>low</td>
</tr>
<tr>
<td>Laser sectioning</td>
<td>active</td>
<td>medium</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>macro-micro</td>
<td>simple</td>
<td>very slow</td>
<td>low</td>
<td>medium</td>
</tr>
<tr>
<td>Coded patterns</td>
<td>active</td>
<td>medium</td>
<td>limited appl.</td>
<td>yes</td>
<td>yes</td>
<td>limited appl.</td>
<td>macro-micro</td>
<td>complex</td>
<td>medium</td>
<td>medium</td>
</tr>
<tr>
<td>Confocal microscopy</td>
<td>active</td>
<td>high</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>nano</td>
<td>complex</td>
<td>medium</td>
<td>high</td>
<td>high</td>
</tr>
<tr>
<td>Laser interferometry</td>
<td>active</td>
<td>high</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>nano</td>
<td>medium</td>
<td>quick</td>
<td>medium</td>
<td>medium</td>
</tr>
<tr>
<td>Holografical interferometry</td>
<td>active</td>
<td>high</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>micro-nano</td>
<td>medium</td>
<td>quick</td>
<td>high</td>
</tr>
<tr>
<td>Speckle technique</td>
<td>active</td>
<td>high</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>nano</td>
<td>medium</td>
<td>quick</td>
<td>high</td>
</tr>
<tr>
<td>Stereovision</td>
<td>passive</td>
<td>medium</td>
<td>limited appl.</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>macro</td>
<td>simple</td>
<td>slow</td>
<td>medium</td>
</tr>
<tr>
<td>Shape-from-shading</td>
<td>passive</td>
<td>medium</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>macro</td>
<td>medium</td>
<td>slow</td>
<td>low</td>
</tr>
</tbody>
</table>

Table 2 Comparison of optical 3D shape measurement methods.
As it is seen from Table 2 each of the methods has its merits and disadvantages. Moiré methods are versatile set of techniques for in-plane and out-of-plane deformation measurement, topographic contouring, slope and curvature measurement and other applications [18]. The main merits of moiré topographical technique [19]:

- measurements cover a whole field, an advantage over point-by-point methods,
- data acquisition and processing is relatively fast,
- profiles are given in the form of 2D contours,
- contouring of specularly and light scattering surfaces is possible,
- the resolution may be varied (from 0.1 mm to 10 mm),
- the method is low-cost, simple and quite fast to operate,
- both static and dynamic events can be studied,
- no special illuminating and observing system is necessary.

As it is common with most measurement techniques, moiré technique also has inherent characteristics that allow or limit its use in certain types of applications. One of the main problems faced in moiré pattern analysis is determination of the differences in object height, also known as “hill&valley” problem [17], [19]. The problem is in appearance of ambiguities by moiré pattern decoding. In detail the problem is considered in Chapter 3.
2 Moiré phenomenon and its application in measuring technique

The chapter presents historical review and general theory of traditional moiré phenomenon. Background of moiré measurement methods and basic methods are viewed. Conventional application of moiré effect in different areas of industry and medicine is considered.

2.1 Historical review

The word moiré means a watered or wavy appearance. The term evolved from the ancient French word *mouaire*, which is itself believed to be derived from the English word *mohair*, the wavy fleece of the Angora goat [20]. It was usually used for the name of the silk fabric which has changing undulating pattern. Later that term appeared in experimental mechanics for the determination of the number of methods, based on the usage of moiré patterns. The word “moiré” seems to be used for the first time in scientific literature by Mulot whose paper concerned stress analysis of loaded mica sheets by moiré fringes [21].

As early as 1859 Foucault proposed a method for testing lenses and optical systems by using low-frequency type gratings. Unfortunately, he did not develop this method because he considered it as less sensitive than the knife-edge method [22].
The phenomenon of moiré fringes was first noted by Lord Rayleigh in 1874. He suggested testing the quality of replicated gratings by superimposing a pair of gratings to produce bar patterns, the geometry of which may be analyzed in terms of the gratings [23].

The idea of Foucault found an application by Ronchi, in 1922, for the optical testing of lenses and mirrors. In 1925 Ronchi studied the case of the moiré patterns created by the superposition of a line and a circular grating [24]. It was shown that the form of the moiré fringes generated by the two overlapping gratings depend on the pitches of the gratings and that the resulting fringes are hyperbolas, parabolas or ellipses.

Raman and Datta treated the phenomenon and gave the parametric equations of the fringes formed by the superposition of two zone gratings with different center-to-center distances. The moiré pattern formed consisted of parallel line fringes or families of circles [25].

Tolenar, in 1945, was the first one who gave an interpretation of the moiré phenomenon based on geometric optics. He described the characteristic properties of moiré fringes formed by coarse gratings of equal or slightly different pitch in terms of their relative rigid-body translation, angular displacement or deformation [26].

In 1952 Kaczer and Kroupa applied the properties of moiré fringes to determine the strain components in a 2D strain field [27].

The phenomena observed by Foucault, Lord Rayleigh, Righi, Ronchi, Raman and Datta did not enjoy the consideration which they merited according to many difficulties encountered in the reproduction of the satisfactory gratings and in the application of the methods.

Only much later the methods of manufacture of diffraction and coarse gratings of high quality were developed. It made their use very efficient for measurements and created new fields of application of moiré fringes in measuring processes [28].

The concept of using the moiré for the topographical measurements was formulated independently and concurrently by Takasaki and Meadows in 1970 [29], [30]. The formation of depth contours by arranging the light source and viewing point at the same distance from the grating is a key feature of moiré topography.

Since that time several hundreds of publications have appeared on moiré proposing novel investigated techniques for new fields of application.

### 2.2 General theory

Moiré patterns are generated by superimposition of two gratings by using an optic system; one of its elements in particular, can be an eye of the experimenter [31]. By the
coincidence of two periodical gratings a third one arises with longer period – moiré fringes (Figure 2).

Moiré image is solely concerned with the intensity variation of light passing successively the two periodical gratings. The grating can be considered as some device, that creates periodical changes in amplitude and (or) phase of the light passing through the grating [32]. Let the plane monochromatic wave falls on the grating, than the wave can be presented as:

$$U_{in} = A \exp(i \omega t + \phi)$$ (1)

where $U_{in}$ - input monochromatic wave, $A$ - amplitude, $\omega$ - radian frequency, $t$ – time, $\phi$ - phase, $i = \sqrt{-1}$. Correspondingly the output wave can be written as:

$$U_{out} = \sum_{n=0}^{\infty} A_n \exp(i \omega t + \phi_n)$$ (2)

where $U_{out}$ - output monochromatic wave.

Then the transmission or transfer function can be found from:

$$P = \frac{U_{out}}{U_{in}}$$ (3)

where $P$ - transfer function.

If the grating influences on the amplitude and the phase simultaneously, then the transfer function contains real and imaginary part. If the grating modifies only the light amplitude then it called amplitude and its transfer function must be real. If only the phase changes, than the grating called a phase one and its transmission function is imaginary.

In most cases amplitude gratings are used. They are usually made on films or glass plates where opaque bars are ruled at regular intervals leaving transparent slits usually of equal width, thus forming an amplitude grating of average 50 per cent transmittance. Satisfactory observation of moiré fringes without special optical equipment is possible if the gratings are of comparatively low frequency; preferably not exceeding 40 lines per mm. In most metrological applications two initially identical rectilinear parallel gratings are used [23]. Thus, let the grating consists of the alternate transparent and opaque rulings of the same width with the pitch $p$ (Figure 3). The pitch...
of the grating is the distance between the edge of a generic opaque bar and the corresponding edge of the next bar.

![Figure 3](image-url)  
**Figure 3** Intensity of light, coming through the grating; \( p \) - pitch of the grating, \( a \) - width of the transparent ruling, \( A \) - amplitude of the light intensity [32].

Then its transfer function \( P \) from Formula (3) for light amplitude can be overwitten as Fourier series:

\[
F(x) = A \left[ \frac{1}{2} + \sum_{n=1}^{\infty} \frac{a_n \sin \frac{2\pi x}{p}}{n} \right] = A \left[ \frac{1}{2} + \frac{2}{\pi} \left( \frac{\sin \frac{2\pi x}{p}}{1} + \frac{1}{3} \frac{\sin \frac{2\pi x}{p}}{1} + \frac{1}{5} \frac{\sin \frac{2\pi x}{p}}{1} + \ldots \right) \right] \quad (4)
\]

Taking a look at this grating through the optical system, then a distorted amplitude distribution of light signal in the image plane will appear. It was experimentally proved that the lens operates as a low pass filter and creates practically sine wave image of originally rectangular object (Figure 4) [32].

![Figure 4](image-url)  
**Figure 4** Scheme illustrating the operation of the optical system as a low pass filter: 1 - light source, 2 - grating, 3 - optical system, 4 - screen, 5 - low pass filter [32].

It is possible to explain suppression of high harmonics by resolution of an objective and photographic materials, as long as from Formula (4) it is seen, that increase of harmonic’s number \( n \) leads to reduction of harmonics’ amplitude \( a_n \) and increase of its spatial frequency \( a/p \). That is why by the practical analysis of the real optical system distribution of light amplitude, passing through the grating, can be taken in the form of two terms of Fourier series:

\[
F(x) \approx C_0 + C_1 \sin \frac{2\pi x}{p} \quad (5)
\]
Let us take two amplitude gratings with the pitches in the direction of $x$ axis are $p_1$ and $p_2$ accordingly, superimposed without gap. Then their transfer functions are next Fourier series:

$$F(x) = A_0 + A_1 \sin \frac{2\pi x}{p_1};$$
$$F_2(x) = B_0 + B_2 \sin \frac{2\pi x}{p_2}. \tag{6}$$

For the resultant transmission function we will have the new Fourier series:

$$F(x) = F_1(x)F_2(x) = A_0B_0 + A_1B_0 \frac{2\pi x}{p_1} + A_0B_1 \frac{2\pi x}{p_2} + A_1B_1 \frac{2\pi x}{p_1} \sin \frac{2\pi x}{p_2} \tag{7}$$

Considering the second and the third term having close frequencies, the expression for distribution of light intensity in image plane can be written in the following way:

$$I(x) = I_0 + I_1 \sin \frac{2\pi x}{p_1} + I_2 \left[ \sin 2\pi \left( \frac{x}{p_1} + \frac{x}{p_2} \right) + \sin 2\pi \left( \frac{x}{p_1} - \frac{x}{p_2} \right) \right] / 2 \tag{8}$$

We will get the equation of amplitude-modulated wave. In our case this phenomenon corresponds to the rise of moiré fringes. For allocation of the useful signal which is an average intensity, it is necessary to suppress the high harmonics corresponding to the lines of primary gratings. Consequently the necessity of using the low pass filter arises. The low pass filter can be realized by reduction of the lens diaphragm, placing at the inexact sharpness by increasing the size of the spot, which magnitude must be several times bigger then the pitch of the grating [32]. Thus, the final formula for distribution of light intensity in moiré image will have the following form:

$$I(x) = I_0 + I_1 \cos 2\pi N(x) \tag{9}$$

where $I(x)$ – light intensity in any point of moiré image, $I_0$ – background average intensity, $I_1$ – amplitude intensity of the fundamental harmonic, $N$ – the contour order.

The orientation and spacing of the moiré fringes for the general case can be determined from the geometry shown in Figure 5 [33].
Figure 5 Geometry used to determine fringe spacing and angle of moiré fringes between two gratings of different frequencies tilted with respect to one another [33].

By the fringe spacing is named the distance between two successive moiré fringes (dark or bright) formed by the superposition of two similar gratings:

\[
c = \frac{p_1 p_2}{\sqrt{p_1^2 \sin^2 2\alpha + (p_2 \cos 2\alpha - p_1)^2}}
\]  \hspace{1cm} (10)

where \(c\) - contour interval or fringe spacing, \(p_1\) and \(p_2\) - pitches of the gratings, \(\alpha\) - the angle between the ruling of the grating and \(y\) axis (grating orientation angle). In the limit \(\alpha = 0^\circ\) and \(p_1 \neq p_2\), the fringe spacing equals:

\[
c = \frac{p_1 p_2}{p_2 - p_1}
\]  \hspace{1cm} (11)

and in the limit \(p_1 = p_2 = p'\) and \(\alpha \neq 0^\circ\), the fringe spacing equals:

\[
c = \frac{p'}{2 \sin \alpha}
\]  \hspace{1cm} (12)

where \(p'\) is the average line spacing [33].

There are three main types of moiré patterns (Figure 6). The first type of the moiré pattern generated by two binary straight-line gratings with the same pitch tilted with respect to one another (Figure 6 a) [32].
2 Moiré phenomenon and its application in measuring technique

The second moiré pattern is obtained when gratings have different frequencies and there is no tilt (Figure 6 b). The third moiré image is generated with gratings of different frequencies which are tilted with respect to each other (Figure 6 c) [32].

2.3 Moiré shape measurement principles

As it has been already mentioned above metrological applications usually utilize two initially identical gratings. One of them follows the deformation of the object or distorted by the specimen in some way. The other grating remains unchanged and is an analyzing one. In the moiré topographical methods it is required to determine the changes in the grating pitch and relate it to the deformation [18]. So, moiré effect is a powerful tool in displaying and measuring the form of objects in three dimensions. The wide availability of digital image processing systems achieved in the 1980’s influenced the development of a great number of fringe pattern analysis methods for shape measurements [19]. However, most of them can be described as variations from one of two basic principles: shadow and projection moiré arrangement.

2.3.1 Shadow moiré

Shadow method is one of the first scientific applications of moiré in measuring technique [30], [34], [35]. The method offers good accuracy and simple arrangement since projected and analyzing gratings are identical: they have the highest degree of binding. The studied object is placed behind a grating, illuminated by a point source and viewed at a certain angle. The distortion of shadow lines is a function of the test surface profile. In practical applications when the light source and the viewing point is positioned at the same distance from the grating then one moiré fringe is a contour line with equal height of the object. Figure 7 shows shadow moiré geometry. The grating is
assumed to be close enough to the object surface so that diffraction effects are negligible [19].

![Schematic arrangement used in shadow moiré topography for the formation plane surface contours. S - light source, O - centre of entrance pupil of the observation optics, G - grating [19].](image)

The field equation for the contour planes $z_N$ can be found:

$$z_N = \frac{Np}{(d - Np)/l}$$

(13)

where $N$ - contour order, $p$ - pitch of the grating, $d$ - distance separating the light source and the observer and $l$ is their distance from the grating plane [17]. Formula (13) indicates that the height is a complex function depending on the position of each object point. Thus, the distance between contour intervals depends on the height of the surface and the number of fringes between the grating and the object. Individual contour lines will no longer be planes of equal height. They are the surfaces of equal height [18].

### 2.3.2 Projection moiré

In the projection moiré method the periodic grating is imaged onto the test surface using projection optics. The observation system images the object surface together with the projected grating on the analyzing grating in the image plane. Moiré fringes are formed in this plane. Schematic representation of the projection moiré system which generates plane surface contours is shown in Figure 8 [19].
If the projection and observations systems have parallel axis than the elevation of the Nth contour can be calculated by [36]:

\[
z_N = \frac{Np l_p (l_p - f)}{df - Np l_p (l_p - f)}
\]

(14)

where \( p \) - pitch of the gratings, \( l_p \) - distance from the plane to the centre of the entrance pupils of the projection and observation optics and \( f \) - focal length of the lenses, \( d \) - distance separating the projection and the observation optics.

### 2.4 Conventional application of moiré topographical methods

Suzuki and Kanaya in 1988 wrote that moiré topography had reached the stage where techniques being developed to make this type of measurement method meet practical requirements [37].

Likewise, Gasvik reported the moiré technique to be a promising method for 3D machine vision in 1989, partly on account of the increasing capacity and decreasing prices of digital image processing for personal computers [38].

The phenomenon of moiré fringes’ formation from the irregularities of the one grating according to its deformation is extensively used in a variety of ways for the displacement measurements:

- sheet metal industry [38],[39],
- measuring of structure deformation under thermal or mechanical loading [40], [41], [42],
- contour inspection of manufactured parts in the production process[43],
- vibrations studying [44], [45],
- road deformation [46],
- plastic surgery, e.g. manufacturing of individual prostheses [47],
- measurement and observation of spinal deformity [48], [49], [50],
- manufacture of dental prostheses (inlays, crowns, bridges etc.) [51], [52],
- shape deformation and volume displacement measurement of tympanic membrane [53],
- foot pressure measurement [54], [55].
3 The problem of ambiguities in moiré patterns and the methods of its solution

In the chapter the problem of ambiguities in moiré patterns is presented and discussed. The brief synopsis of the basic traditional and several methods with application of colour is given.

3.1 Problem statement

Like in any other measuring technique some problems remain to be solved. The reconstruction of the surface is not a trivial problem in moiré methods because of ambiguities arising from the loss of the directional information in the fringe formation process. The ambiguity is whether the fringe lies above or below the neighboring fringe [19]. The concave and convex regions of an object cannot be discriminated from the fringe patterns, and when the experiment is carried out under conditions in which the surface to be measured is out of contact with the grating plane, the fringe index cannot be determined [56]. In Figure 9 the moiré image of the human male back obtained by the traditional moiré method and its graphically drawn contour line system is presented.
Looking at the image it is impossible to say which fringe represents the convex and which one the concave areas (Figure 9 a). In this case the author of the experiment manually determines the type of moiré surfaces (hill or valley) on the back of a living body due to his subjective knowledge (Figure 9 b) [29]. Such a method excludes the process of automatization. Other methods of information extraction from moiré image are discussed below.

### 3.2 Conventional methods

The most popular conventional methods of the fringe pattern analysis are manual, Fourier transformation and phase shift methods. Mostly by manual methods operator informs about the fringe type along the analysis line, so the process can not be automated.

Fourier transformation is based on retrieving the phase function in the spatial frequency plan. The problem of this approach is that it is not capable of providing satisfactory measurements near boundaries and discontinuities. Moreover the Fourier method runs into grave difficulties with edge effects [57].

Phase-shifting techniques may be of spatial (SPS), temporal (TPS) and spatial-carrier (SCPS) types [18]. The methods utilize series of phase-shifted patterns to determine fractional fringe orders. The method is based on harmonic representation of the intensity of fringe patterns. The most common solutions are phase shift by means of: grating or object translation, use of computer generated gratings, application of a titled glass plate or mirror translator. That means impossibility of dynamic event analysis and realization of the measurements in the real time mode [19].


3.3 Methods with application of colour

Besides the traditional fringe analysis techniques the methods with application of colour to moiré contouring were also proposed in the past. Some of them are presented below.

A. Livnat, O. Kafri, and G. Erez analyzed the problem of “hill&valley” [58]. The method applied a double-exposure technique of colour film, the object being illuminated in each exposure by different colours. After the first exposure the object was translated normally to the grating by half of the increment from the first position to the second position. A colour filter was placed in front of the camera lens for the second exposure. The order of the colour sequence in each contour line determined the slope direction.

K. Wenzel in her article proposed to apply bicolour grating [59]. Bicolour grating and binary grating was used for theoretical modeling of the moiré effect. It was hypothesized that by application of bicolour grating in projection method, the moiré patterns of concave and convex objects would differ from each other by the order of colours in moiré fringes. The problem of ambiguities was supposed to be solved from one colour moiré pattern.

In their note authors Keun Cheol Yuk, Jae Heung Jo, and Soo Chang presented a method of determining the absolute order of shadow moiré fringes by using two differently colour light sources [60]. This technique was based on application of the periodic colour structures made by superimposing of two different colour moiré patterns. Red and green light sources were positioned with camera at the same height above the grating plane. The bright and dark fringes were formed by a single red light source, and the periodic colour structures were generated by superimposing of two moiré patterns for both red light and green light source. Special dark-yellow-dark (DYD) structures were generated by superimposing colour structures from the light sources. Identification of the convex and concave areas was made by observing the shift of the DYD structure.

Chang-Hua Hu and Yu-Wen Qin reported on moiré pattern colour-encoding technique that utilizes a colour encoder consisting of digital image-processing system for representation of the fringe orders [61]. The system consisted of projector and CCD camera, connected to a frame grabber. Three black and white fringe images representing height differences had to be obtained. Then they were separately loaded into red, blue and green banks of the frame grabber. After the height difference was obtained by processing of the superimposed images on the true-colour monitor display.

Moon-Sik Jeong and Seung-Woo Kim presented moiré method of colour grating projection, which performs phase-shifting 3D contouring with a single operation of
fringe capturing [62]. Three red, green, and blue pairs of projection and viewing gratings were fabricated on a single glass plate with predetermined lateral offsets. The glass plate was translated along with the colour gratings during measurement at fast speed, so that colour CCD camera simultaneously monitored three separate patterns of moiré fringes with phase offset. The authors stated the time-integral fringe capturing be able to obtain moiré fringes with additional information for phase reconstruction.

The aim of colour application is development of a means of obtaining the information required for solution of the “hill&valley” problem from one moiré image of the test object. Among the above-mentioned methods the idea of Wenzel seemed to me to be the closest to this goal. Artificial colour moiré pattern generated by one bicolour and binary grating was first presented in the article of Wenzel [59]. The hypothesis of obtaining the extra information from one colour moiré image was very attractive. This conception was put at the base of my PhD work.
4 Colour moiré phenomenon

In the chapter the colour moiré phenomenon is discussed from different aspects. The complete work of Wenzel on theme “Colour moiré patterns”, published in Hungarian journal “Kép és Hangtechnika” in 1992 year is also presented.

The next topics are considered here:
- Concept of Wenzel
- General theory of colour moiré effect
- Spectral analysis of colour moiré fringe in colour space
- Methods of digital colour moiré pattern generation
- Human perception of colour moiré phenomenon

Before turning to the next chapters it is significant to mention that colour theoretical description and experiments were made by use of calibrated cathode ray tube (CRT) monitor with RGB colour space [63], [64]. The RGB space is commonly the base colour space for most computer applications, since no transformation is required to display information on the screen.

4.1 Concept of Wenzel

In the traditional moiré phenomenon deformed grating contains the information about the measuring surface, this grating is analyzed by the grating (it means the
sampling of the data from the deformed grating). In this case superposition of the gratings is symmetrical owning to the change of the grey level. For obtaining extra information the phenomenon should be changed to the asymmetrical, for example, by means of bicolour grating application.

Bicolour and binary grating’s interaction was viewed [59]. The width of each colour bar of bicolour grating is equal to \( d \) and the width of each transparent slit of the second grating is equal to \( 2d \); the pitch is \( 4d \). The binary grating has the width of each opaque and transparent bar is equal to \( 2d \); the pitch is \( 4d \) (Figure 10).

![Figure 10 Colour moiré gratings [59].](image)

If the same bicolour grating is applied to projection method for objects examination, moiré patterns of concave and convex should differ from each other by the order of colours (Figure 11).

![Figure 11 Concave and convex surfaces by application of colour moiré phenomenon [59].](image)

### 4.2 General theory of colour moiré effect

#### 4.2.1 Formation of colour moiré pattern

As it was proposed earlier two straight-line periodical parallel gratings are utilized. Then, the colour moiré fringes arise with white light in consequence of the
coincidence of two identical bicolour gratings or one bicolour and one binary grating where moiré fringe appears with a longer period. The binary grating is printed on transparent film where black opaque bars are ruled at regular intervals leaving transparent slits of equal width, thus forming an amplitude grating of approximately 50 per cent transmittance (Figure 12 a). Bicolour grating is printed on transparent film where instead of one black bar two equal colour bars are ruled one after another at regular intervals with transparent slits of equal width (Figure 12 b). By two identical bicolour gratings I assumed the gratings with the bars of the same colours A and B.

![Figure 12](image)

**Figure 12** Binary (a) and bicolour grating (b). A and B—colours of the grating bars; p—pitch of the grating, b—width of the opaque bar, d—width of each colour bar.

The parameters of rectilinear colour and traditional gratings are:

\[
\frac{b}{p} = \frac{1}{2} \quad \text{and} \quad \frac{d}{b} = \frac{1}{2} = \frac{p}{4}
\]

where \( p \) — pitch of grating, \( b \) — width of the opaque bar, \( d \) — width of each colour bar.

The geometrical process of colour fringes formation is the same as in case of the traditional moiré effect (Figure 13).

![Figure 13](image)

**Figure 13** A simplified version of the geometrical process of colour moiré fringes formation.

The next three types of gratings overlapping were taken for research:

- two identical bicolour gratings with the same order of colour bars,
- two identical bicolour gratings with the opposite order of colour bars
- one binary with one bicolour grating.

Selection of above listed types of gratings superposition for moiré effect generation is based on possible combinations that appear in classical moiré measuring technique.
For example, in shadow moiré method one analyzing grating and its shadow are overlapped that talks about only one possible case when two identical bicolour gratings with the same order of colour bars are superimposed. In projection moiré method, where two different gratings can be used all three variants of gratings overlapping are possible.

The principle of colour formation in moiré fringe for three types of gratings superimposition is presented in Figure 14.

Looking at Figure 14 a one can suppose appearance of one-colour moiré fringe due to the symmetrical disposition of the colours regarding to the centre of the bright colour moiré fringe. In Figure 14 b the skew distribution of colours regarding to the centre of the bright colour moiré fringe is evidently seen. As for Figure 14 c it is difficult to make any assumption about the possible distribution of colours in moiré fringe.

For achievement of the moiré fringes with big colour difference by normal light conditions appropriate colours of the bars should be chosen. I supposed to apply red and blue primary colours to the bars of the bicolour grating, because maximum effective sensitivities of the L and S cones are located on the maximum distance from each other and the maximum of L corresponds with monochromatic red colour while the maximum of S – with monochromatic blue colour. Estimates of the effective sensitivities of the L, M, S cone fundamentals (i.e., cones) are shown in Figure 15 [65].
The hypothesis turned to be right during the series of experiments presented later in the work (See Section 4.5).

4.2.2 “Effective” and “ineffective” colour moiré effect

The colour moiré patterns obtained by overlapping of the gratings with proposed spectral parameters for the three above mentioned types of superimposition are presented in Figure 16. Spectral generation of the colour moiré patterns shown in this subsection was made in digitally and based on the principle of moiré images formation in reality. This topic will be viewed in detail in Section 4.4.
Two different colour moiré effects appear according to the change of colours in one moiré fringe. Let them call “effective” and “ineffective” colour moiré phenomenon. The “effective” colour moiré effect is the one where fringe has changing colour along the normal to its curve (Figure 16 a, c). Consequently “ineffective” will be that one which fringes are self-coloured, as long as they give no extra information comparing to the traditional moiré pattern (Figure 16 b). From the figure it is clearly visible that “effective” moiré fringes appear in two cases: overlapping of binary and bicolour grating and two identical bicolour gratings with opposite order of colour ruling; “ineffective” effect arises by two identical bicolour gratings with the same order of bars superimposition. Thus, the results of modeling of moiré patterns with colour bars partly conform the assumptions proposed by analysis of colour formation principle in moiré fringes (Figure 14 a, b) and give extra information (Figure 14 c). In detail spectral analysis of colour moiré effect is presented in the next section.

4.2.3 Representation of ambiguity areas on colour moiré patterns

In the general case where the rulings of deformed grating follow curves of any degree the procedure of geometrical and spectral formation of colour moiré fringes follows the same principle as in its simplified version. The geometrical process of colour fringes formation in general case is shown below (Figure 17).
Spectral generation of the colour moiré patterns generated by overlapping of deformed bicolour grating with analyzing binary grating, identical bicolour grating with the opposite order of colour bars and identical bicolour grating with the same order of colour bars is presented in Figure 18.

As it is clearly visible the “effective” and “ineffective” moiré fringes appear in the same cases as for the rectilinear gratings superimposition. The fringes from area I and III on the pattern with “effective” moiré effect have the same order of changing colour and
opposite to the area II along the line. To give an explanation it is enough to have a look separately at the deformed grating (Figure 19 a). The centers of curvature for deformed rulings from areas I and II lie at the same side to normal for each point of grating (Figure 19 b).

![Figure 19 Deformed bicolour grating (a) and disposition of its centers of curvature for I, II and III areas (b) [66], [67].](image)

The similar disposition of centers of curvature for I and II areas talks about the same direction of deformed surface, by other words affiliation to concave or convex type of the surface.

As a result the ambiguities in moiré pattern can be distinguished due to the order of colour in moiré fringes by meeting several requirements (See Section 5.1). Main concepts of the ambiguities definition in colour moiré are presented in Section 5.7.

### 4.3 Spectral analysis of colour moiré fringe in colour space

#### 4.3.1 Theory

While the traditional moiré effect is usually described according to the intensity variation of light passing successively the two periodical structures (See Formula (9)), for colour phenomenon this method is unacceptable. The description of the colour moiré effect is based on the spectral distribution of colours in moiré fringe. One period of colour moiré fringe can be reconstructed in the CIE xy chromaticity diagram if the values and proportions of occurring colours are known. The CIE xy chromaticity diagram was chosen as long as it is the most commonly used.

First for determination of colour values \((x, y)\) chromaticity coordinates) the relative colour stimulus functions of the colours in one period of moiré fringe must be found. For this reason device independent spectral model of colours appearance in one period of colour moiré fringe was developed [68]. By device independent model it was denoted that as analyzer no artificial device was applied but human eye.
Thus, using relative spectral power distribution of the illuminant, spectral reflectance factor of the test object colour and spectral transmittance of the colour bar in the gratings the relative colour stimulus functions of the colours in moiré fringe can be calculated:

\[ \phi_\lambda(\lambda) = S(\lambda) \tau_p(\lambda) \tau_a(\lambda) R(\lambda) \]  

(20)

where \( \phi_\lambda(\lambda) \) - relative colour stimulus function of colour arising in moiré fringe, \( S(\lambda) \) - relative spectral power distribution of the illuminant, \( \tau_p(\lambda) \) - spectral transmittance of the colour bar in the first grating, \( \tau_a(\lambda) \) - spectral transmittance of the colour bar in second grating, \( \tau_a(\lambda) \) and \( \tau_p(\lambda) \) contain the information of the base and colour emulsion, \( R(\lambda) \) - spectral reflectance factor of the specimen surface.

According to the colour formation principle presented in Figure 14 six or four colours can appear in one period of moiré fringe depending on the type of used gratings. The CIE tristimulus values of each colour stimulus can be found from formulas [69]:

\[ X = k \sum_\lambda \phi_\lambda(\lambda) \tilde{x}(\lambda) \Delta \lambda \]  

(16)

\[ Y = k \sum_\lambda \phi_\lambda(\lambda) \tilde{y}(\lambda) \Delta \lambda \]  

(17)

\[ Z = k \sum_\lambda \phi_\lambda(\lambda) \tilde{z}(\lambda) \Delta \lambda \]  

(18)

where \( X, Y, Z \) are the CIE tristimulus values, \( \tilde{x}(\lambda), \tilde{y}(\lambda), \tilde{z}(\lambda) \) - colour-matching functions of a standard colorimetric observer, \( \Delta \lambda \) - wavelength interval, \( k \) - normalizing constant. The chromaticity coordinates \( x, y, z \) are derived from the tristimulus values [69]:

\[ x = \frac{X}{X + Y + Z} \]  

(19)

\[ y = \frac{Y}{X + Y + Z} \]  

(20)

\[ z = \frac{Z}{X + Y + Z} \]  

(21)

When the \( x, y \) chromaticity coordinates are found for each colour, the proportions of the occurring colours must be determined for reconstruction of one period of colour moiré fringe in the CIE \( xy \) chromaticity diagram. It is significant to mention that colour moiré fringe generated by one binary and bicolour grating is the particular case of the two bicolour gratings application for moiré formation. The general model of one period of colour moiré fringe formed by two straight-line bicolour gratings with different pitch and no tilt is shown in Figure 21.
Hence, it is essential to make sampling along the AA line for definition of the proportions of the occurring colours in one period of colour moiré fringe with min step:

$$ n = \left| b_1 - b_2 \right| $$

where $n$ is the minimal step of sampling, $b$ - width of bicolour bar.

Two possible graphs modes can appear. The first one has the form different to line, i.e. colours in one period of moiré fringe do not repeat. The moiré fringe having such a geometrical form of graph in the CIE $xy$ chromaticity diagram conforms to “effective” colour moiré case (Figure 22 a). The second one has the form of line that means colours to be repeatable in one period of moiré fringe. This form of graph indicates “ineffective” colour moiré fringe (Figure 22 b).

Figure 22 Geometrical form of graph by “effective” (a) and “ineffective” (b) colour moiré fringe in the CIE $xy$ chromaticity diagram [68].
Thus, colour moiré fringes can be represented and compared in the CIE xy chromaticity diagram. Moreover by the form of graph in the CIE xy chromaticity diagram “ineffective” cases can be removed.

By application of the proposed method for colour fringe representation an appropriate “effective” case can be chosen among the other ones according to the parameters of measuring. For example, by changing the colour of surface or the type of illuminant the following question can emerge: which colours or type of gratings superimposition should be used (Figure 23)?

![Figure 23 Example of two “effective” colour moiré fringes presentation in the CIE xy chromaticity diagram [68].](image)

The problem can be solved by calculation the colour difference between two colours along one period of moiré fringe: the first colour represents the result of an additive mixture of colours composing one period of moiré fringe and the second colour – separate colours of one period of moiré fringe obtained by sampling with minimum step $n$ (See Formula (22)).

In the CIE xy chromaticity diagram the chromaticity point of two additive mixed colours is located on the line joining the chromaticity points of the two constituent colours. An additive mixture takes an advantage related to the limited resolving power of the eye. Thus, colour moiré phenomenon, composed of colours, appears as a mixture of those colours when the effect is viewed from such a distance that the individual colour stimuli can not be resolved. If two colours are specified of $Y_1, x_1, y_1$ and $Y_2, x_2, y_2$, the luminance and chromaticity of the additive mixture can be commutated by formulae [70]:

$$Y = Y_1 + Y_2$$  \hspace{1cm} (23)
where

\[ m_1 = \frac{Y_1}{y_1} \text{ and } m_2 = \frac{Y_2}{y_2} \]  

The equations for x and y are familiar to the centre of gravity of two masses \( m_1 \) and \( m_2 \) located at \( x_1, y_1 \), and \( x_2, y_2 \), respectively.

Besides the numerous advantages of the CIE xy chromaticity diagram, it lacks one very important characteristic. Namely, if the distance between any arbitrary two points is the same as the distance between another point pair, the perceived distance will not be the same. In order to correct this, researchers are trying to find a perceptually uniform colour space. It has, unfortunately, still not been found. CIE proposed two alternatives as improvements compared with CIE xy space. These are CIE LUV and CIE LAB colour spaces. Although they are referred to as perceptually uniform colour spaces by some authors, they are not. Just for comparison, two perceptually equally distant colour pairs, can differ in the CIE LUV distance as much as 4 times. This is a significant improvement compared to CIE XYZ space, but it is still not perfect [71]. For calculation of colour difference the CIE 1976 L* u* v* (CIE LUV) three-dimensional, approximately uniform colour space was chosen because it is the strict conversion from the CIE XYZ.

The CIE LUV space is widely used in calculation of just noticeably difference (JND) in stimuli, especially with additive colours. In the CIE LUV space products of additive colour mixture fall on a straight line, just as in the CIE xy chromaticity diagram [70]. The colour difference formula \( \Delta E_{uv}^* \) that is simply the Euclidean distance between the two points in CIE LUV space can be calculated from formula [72]:

\[
\Delta E_{uv}^* = \left( (\Delta L^*)^2 + (\Delta u^*)^2 + (\Delta v^*)^2 \right)^{1/2}
\]

\[
L^* = 110 \left( \frac{Y}{Y_n} \right) - 16
\]

\[
u^* = 13L^*(u' - u'_n); \quad v^* = 13L^*(v' - v'_n)
\]

\[
u' = \frac{4x}{-2x + 12y + 3}, \quad v' = \frac{9y}{-2x + 12y + 3}
\]

\[
f \left( \frac{Y}{Y_n} \right) = \left( \frac{Y}{Y_n} \right)^{1/3}, \text{ if } \frac{Y}{Y_n} > 0.00856
\]
\[ f \left( \frac{Y}{Y_n} \right) = 7.787 \frac{Y}{Y_n} + \frac{16}{116}, \text{ if } \frac{Y}{Y_n} \leq 0.00856 \]

where \( \Delta E_{uv}^* \) is the difference between two colour stimuli in the CIE 1976 (L* u* v*) colour space, L* component defines the luminance, u* and v* define chromaticity, \( Y, u', v' \) describe the colour stimulus and \( Y_n, u'_n, v'_n \) describe a specified white object colour stimulus.

Diagram of colour difference \( \Delta E_{uv}^* \) from disposition of colours lying on the line perpendicular to the rulings of the grating in one moiré fringe period is shown below in Figure 24.

By reconstruction of the colour difference \( \Delta E_{uv}^* \) response diagram the better case will be that one, where JND in stimuli over a period of moiré fringe is the highest (Figure 24).

4.3.2 Experiments based on the artificial spectral model

The method, describing the principle of colour moiré fringe representation and its characterization in colour space was tested in the series of experiments. For experiments device independent spectral model was applied. Formation of colours in moiré fringe by overlapping of two identical bicolour gratings was examined [68].

The primary – red, blue, green and secondary – cyan, magenta, yellow colour bars were applied to gratings. The colours on the transparent film were produced by laser (Osé 700) printer. The spectral transmittance curves of the colours were obtained by spectrophotometer (AvaSpec-1024 Fiber Optic Spectrometer) and presented in Figure 25.
A halogen lamp was used as illuminant, its relative spectral power distribution was observed by CCD-based Compact Array Spectrometer (CAS 140B) (Figure 26).

Spectral reflection curve of test surface measured by spectrophotometer (Konica – Minolta CM-3600D) is shown in Figure 27.

The combination of the adjacent colour bars was taken: R-G, G-B, R-B and C-M, M-Y, Y-C. Thus, six cases of two identical bicolour gratings’ superimposition with the same and opposite order of colour bars were taken into consideration.
With known multipliers of the Formula (16) we can calculate six relative colour stimulus functions for each case of two bicolour gratings combination. Then sequentially applying Formulae (16) - (21) the chromaticity coordinates $x$, $y$ of the colours can be found.

By construction of the model of one period of colour moiré fringe as it was shown in Figure 21 and taking into consideration the step of sampling $n$ (Formula (22)) the proportions and sequence of the occurring colours can be determined.

The graphs of one period of moiré fringe in the CIE $xy$ chromaticity diagram generated by overlapping of two bicolour gratings with primary colours are presented in Figure 28.

![Graphs of one period of moiré fringe in the CIE $xy$ chromaticity diagram generated by overlapping of bicolour gratings with primary colours: BR-RB, BG-GB, GR-GR (a) and BR-BR, BG-BG, GR-GR (b) [68].](image)

In Figure 29 the graphs of one period of moiré fringe in the CIE $xy$ chromaticity diagram generated by overlapping of two bicolour gratings with secondary colours are presented.
4 Colour moiré phenomenon

Figure 29 The graphs of one period of moiré fringe in the CIE xy chromaticity diagram generated by overlapping of bicolour gratings with the secondary colours: CM-MC, CY-YC, MY-YM (a) and CM-MC, CY-YC, MY-YM (b) [68].

The selection of the “effective” colour moiré phenomenon is made as it was proposed - by the form of the graphs in the CIE xy chromaticity diagram. By the initial data these are the cases, when the order of colour bars in gratings is the opposite, i.e. BR-RB, BG-GB, GR-RG (Figure 28 a) and CM-MC, CY-YC, MY-YM (Figure 29 a).

As it is good seen the “ineffective” colour moiré phenomenon arises when order of colour bars in gratings is the same, i.e. BR-BR, BG-BG, GR-GR (Figure 28 b) and CM-MC, CY-CY, MY-MY (Figure 29 b).

After making calculations by Formulae (23) - (31), the colour difference between two colour stimuli is obtained and the best variant of colours bars for the gratings can be found. Figure 30 represents the response diagrams of colour difference $\Delta E^*_{uv}$ from disposition of colours lying along the line perpendicular to the rulings of the grating in one moiré fringe period for “effective” and “ineffective” colour moiré cases.
Figure 30 The diagrams of colour difference $\Delta E^*$ from disposition of colours lying on the line perpendicular to the rulings of the grating in one moiré fringe period. „Effective” moiré phenomenon presented in case of primary (a) and secondary (b) colours application to gratings. „Ineffective” moiré phenomenon presented in case of primary (c) and secondary (d) colours application to gratings [68].

From Figure 30 it is clearly visible that “effective” moiré fringe appears in case of primary (Figure 30 a) and secondary (Figure 30 b) colours application with the opposite order of bars in gratings. The results of experiment show that the most “effective” colour moiré fringes arise by application of primary red-blue bars combination in bicolour gratings [68].

Figure 30 c, d represents „ineffective” moiré fringe appearance in case of primary and secondary colours application to the bars of gratings. These diagrams (Figure 30 c, d) concern to the “ineffective” cases of the colour moiré effect appearance, as they contain such parts where the colour difference is to small to be observable by the human eye, according to the science literature [73].

Thus, the results of experiments with artificial spectral model confirm the hypothesis of colour moiré fringe description (see Section 4.3). Following statements can be proposed from this section:

1. Geometrical form of graph, that describes the changing of colour in one moiré fringe in the CIE $xy$ chromaticity diagram, allows to define an “effective” colour moiré fringe among and “ineffective” moiré phenomenon:
a) if the graph that describes the changing of colour in one moiré fringe in the CIE $xy$ chromaticity diagram has the form close to line, i.e. the colours recur, then colour moiré fringe is “ineffective”;  
b) if the graph that describes the changing of colour in one moiré fringe in the CIE $xy$ chromaticity diagram has the form different from line, i.e. the colours do not recur, then colour moiré fringe is “effective”;

2. Calculation of the colour difference $\Delta E^*_{uv}$ between two colours in series along one period of colour moiré fringe allows to find the most “effective” colour moiré fringe among the other “effective” colour fringes. Where the first colour represents the result of an additive mixture of colours composing one period of moiré fringe and the second colour is the separate colours sampled in one period of moiré fringe.

### 4.4 Methods of digital colour moiré pattern generation

By application of traditional binary gratings for generation of moiré fringes two modes of superimposition of structures take place. They depend on the type of realization: classical or cooperation of classical method with embedded computer system. The classical mode involves methods where for formation of moiré image only optical equipment is used, for example projection and shadow techniques (See Section 2.3.1 and Section 2.3.2). Moiré contours can be also generated in the memory of computer by the optically obtained snap-shot with deformed grating and digital grating or two digital gratings. These, so known computer–aided moiré methods are actively developing technique essentially enlarging the possibilities in moiré effect application (See Section 5.4).

Methods of digital colour moiré pattern generation were under research. The appearance of the resulting colour moiré pattern depends on the type of superimposition. Knowledge of the relation between the gratings is essential for the correct modeling of colour moiré effect in a definite application.

Three types of superimposition of structures in classical moiré technique were distinguished by Bryngahl: additive, subtractive and multiplicative. He was operating with intensity levels of the structures. In reality moiré image is formed by multiplicative superimposition of the structures in space while additive and subtractive relations were realized by use of special optical equipment [74].

Colour moiré effect can be generated by application of classical moiré equipment in reality by two ways:

1. When the colour bars of the analyzing grating transmit the light, then the colours become different in the places of bars overlapping. The relation between
the gratings describes the appearance of moiré image by traditional moiré technique.

2. When the colour bars of the analyzing grating do not transmit the light in places of stripes overlapping so that the colours of analyzing grating are dominant over the colours of the projecting grating. The relation between gratings arises in classical moiré method if colour bars of analyzing grating are opaque. This is a special case and can be achieved purposely by grating manufacture or by optical mistake in observer equipment.

These two types of gratings overlapping are described below. The gratings were generated in RGB colour space. The RGB colour model is an additive colour model which defines colour using the following components: red (R), green (G), blue (B). The red, green, and blue components are the amounts of red, green, and blue light that RGB colour contains. They are measured in values ranging from 0 to 255 (DAC conversion with 8 bits) [75].

Assume the gratings that are images \( f_1(\gamma, \xi) \) and \( f_2(\gamma, \xi) \) sampled so that the resulting two digital images has \( K \) rows and \( H \) columns. The values of the coordinates \( (\gamma, \xi) \) are discrete quantities. Integer values for these discrete coordinates are set. Thus, the values of the coordinates at the origin are \( (\gamma, \xi) = (0, 0) \). Coordinate convention for digital image representation is presented in Figure 31.

![Figure 31 Coordinate convention for digital image representation [75].](image)

The complete \( K \times H \) digital image of one grating can be written in the following matrix form:

\[
\begin{bmatrix}
f(0,0) & f(0,1) & \ldots & f(0,H-1) \\
f(1,0) & f(1,1) & \ldots & f(1,H-1) \\
\vdots & \vdots & \ddots & \vdots \\
f(K-1,0) & f(K-1,1) & \ldots & f(K-1,H-1)
\end{bmatrix}
\]

(32)

The right side of this equation is by definition a digital image. Each element of this matrix array is a pixel [75]. Using more traditional matrix notation to denote a digital image and its elements we get:
4 Colour moiré phenomenon

While the image represented in the RGB colour model consists of three component images, one for each primary colour, then it can be divided in three matrixes of pixel component:

\[
W_r = \begin{bmatrix}
  r_{0,0} & r_{0,1} & \ldots & r_{0,H-1} \\
  r_{1,0} & r_{1,1} & \ldots & r_{1,H-1} \\
  \vdots & \vdots & \ddots & \vdots \\
  r_{K-1,0} & r_{K-1,1} & \ldots & r_{K-1,H-1}
\end{bmatrix}
\]

\[
W_g = \begin{bmatrix}
  g_{0,0} & g_{0,1} & \ldots & g_{0,H-1} \\
  g_{1,0} & g_{1,1} & \ldots & g_{1,H-1} \\
  \vdots & \vdots & \ddots & \vdots \\
  g_{K-1,0} & g_{K-1,1} & \ldots & g_{K-1,H-1}
\end{bmatrix}
\]

\[
W_b = \begin{bmatrix}
  b_{0,0} & b_{0,1} & \ldots & b_{0,H-1} \\
  b_{1,0} & b_{1,1} & \ldots & b_{1,H-1} \\
  \vdots & \vdots & \ddots & \vdots \\
  b_{K-1,0} & b_{K-1,1} & \ldots & b_{K-1,H-1}
\end{bmatrix}
\]

where \( r_{K,H}, g_{K,H}, b_{K,H} \) - red, green and blue component of the \((K,H)\) pixel for analyzing and deform grating. Then one pixel of the image can be written as \( a_{K,H}(r,g,b) \in A \). Thus, all the operations are realized between the corresponding colour components of the overlapping grating images. Assume the black colour to be absolutely opaque \((0,0,0)\) and white colour \((255,255,255)\) to be transparent.

Analyzing the first type of colour moiré pattern formation, when the colour bars of the analyzing grating transmit the light, then the colours become different in the places of bars overlapping, the next conditions must be met:

1. Overlapping of white with white colour results the white:
   if \( e_{K,H}(255,255,255) \in E \) and \( q_{K,H}(255,255,255) \in Q \) then \( j_{K,H}(255,255,255) \in J \)
   where \( e_{K,H}, q_{K,H}, j_{K,H} \) - \((K,H)\) pixels of matrixes E, Q and J correspondingly.
   \( E \) - matrix represents deform grating, \( Q \) - analyzing grating and \( J \) - resulting grating, obtained by overlapping of the analyzing and deform gratings.

2. Overlapping of black with black results black:
   if \( e_{K,H}(0,0,0) \in E \) and \( q_{K,H}(0,0,0) \in Q \) then \( j_{K,H}(0,0,0) \in J \)

3. Overlapping of white with any other colour results the colour:
   if \( e_{K,H}(255,255,255) \in E \) and \( q_{K,H}(255,0,0) \in Q \) then \( j_{K,H}(255,0,0) \in J \)
   if \( e_{K,H}(255,255,255) \in E \) and \( q_{K,H}(0,255,0) \in Q \) then \( j_{K,H}(0,255,0) \in J \)
   if \( e_{K,H}(255,255,255) \in E \) and \( q_{K,H}(0,0,255) \in Q \) then \( j_{K,H}(0,0,255) \in J \)

4. Overlapping of black with any other colours results black:
   if \( e_{K,H}(0,0,0) \in E \) and \( q_{K,H}(255,0,0) \in Q \) then \( j_{K,H}(0,0,0) \in J \)
   if \( e_{K,H}(0,0,0) \in E \) and \( q_{K,H}(0,255,0) \in Q \) then \( j_{K,H}(0,0,0) \in J \)
   if \( e_{K,H}(0,0,0) \in E \) and \( q_{K,H}(0,0,255) \in Q \) then \( j_{K,H}(0,0,0) \in J \)
5. By change of order of overlapping colours the result colour stays permanent if \( e_{K,H}(0,0,0) \in E \) and \( q_{K,H}(255,255,255) \in Q \) then \( j_{K,H}(0,0,0) \in J \)

if \( e_{K,H}(255,255,255) \in E \) and \( q_{K,H}(0,0,0) \in Q \) then \( j_{K,H}(0,0,0) \in J \)

The next operation satisfies above proposed conditions:

\[
\begin{align*}
    r &= \frac{r_1 \times r_2}{255} ; \quad g = \frac{g_1 \times g_2}{255} ; \quad b = \frac{b_1 \times b_2}{255}
\end{align*}
\]

where \( r_1, r_2, g_1, g_2, b_1, b_2 \) - red, green and blue components of the \((K, H)\) pixel of analyzing and deform gratings; \( r, g, b \) - red, green and blue components of the \((K, H)\) pixel of resulting grating, obtained by overlapping of the analyzing and deform gratings. Let us call the above proposed relation multiplicative.

Analyzing the second type of colour moiré pattern formation, when the opaque colour bars of the analyzing grating do not transmit the light in places of stripes overlapping the colours of analyzing grating are dominant over the colours of the projecting grating. Then, the next conditions must be met:

1. Overlapping of white with white colour results the white.
2. Overlapping of black with black results black.
3. Overlapping of white with any other colour results the colour.
4. Result colour depends on the order of overlapping colours:
   a) overlapping of white coloured pixel belonging to analyzing grating with any coloured pixel of the deform grating results the colour of the deform grating:
      if \( e_{K,H}(255,0,0) \in E \) and \( q_{K,H}(255,255,255) \in Q \) then \( j_{K,H}(255,0,0) \in J \)
      if \( e_{K,H}(0,255,0) \in E \) and \( q_{K,H}(255,255,255) \in Q \) then \( j_{K,H}(0,255,0) \in J \)
      if \( e_{K,H}(0,0,255) \in E \) and \( q_{K,H}(255,255,255) \in Q \) then \( j_{K,H}(0,0,255) \in J \)

   b) overlapping of any coloured (except the white) pixel belonging to analyzing grating with any coloured pixel of the deform grating results the colour of the analyzing grating:
      if \( e_{K,H}(0,0,0) \in E \) and \( q_{K,H}(255,0,0) \in Q \) then \( j_{K,H}(255,0,0) \in J \)
      if \( e_{K,H}(255,0,0) \in E \) and \( q_{K,H}(255,0,0) \in Q \) then \( j_{K,H}(255,0,0) \in J \)
      if \( e_{K,H}(0,255,0) \in E \) and \( q_{K,H}(255,0,0) \in Q \) then \( j_{K,H}(255,0,0) \in J \)
      if \( e_{K,H}(0,0,255) \in E \) and \( q_{K,H}(255,0,0) \in Q \) then \( j_{K,H}(255,0,0) \in J \)

where \( e_{K,H}, q_{K,H}, j_{K,H} \) - \((K, H)\) pixels of matrixes \(E, Q\) and \(J\) correspondingly. \(E\) - matrix representing deform grating, \(Q\) - analyzing grating and \(J\) - resulting grating, obtained by overlapping of the analyzing and deform gratings. Let’s call this relation between gratings dominant.

The operations confirming the above listed conditions can be found and easily generated in several graphical programmes. In CorelDraw the viewed types of colour moiré image formation can be realized by application of “multiplicative” and “normal” (dominant) operations for the first and second types correspondingly:

- By “multiplicative” operation the overlapping colours are multiplied, and then the result is divided by 255. This has a darkening effect. Multiplying black with
any colour results in black. Multiplying white with any colour leaves the colour unchanged.

- By “dominant” operation the transparency colour is applied on top of the base colour.

Four types of gratings overlapping were tested: two identical bicolour gratings with the same order of colour bars, two identical bicolour gratings with the opposite order of colour bars, projecting binary and analyzing bicolour grating, projecting bicolour and analyzing binary grating.

Digital gratings with different frequencies and no tilt were generated in the RGB colour space. The role of the grating was set by selection of its order – in the background for projecting grating and in the foreground – for analyzing grating.

Computer generated models of colour moiré fringes appearance by application of multiplicative and normal relation between the gratings were generated in CorelDraw and presented in Table 3.

<table>
<thead>
<tr>
<th>Relation between gratings</th>
<th>Type of gratings overlapping</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two bicolour gratings with the same order of colour bars</td>
<td>Two bicolour gratings with the opposite order of colour bars</td>
</tr>
</tbody>
</table>

**Table 3** Models of colour moiré fringes appearance by application of multiplicative and dominant relation between the different types of gratings overlapping [76].

**Table 3** was shown on the calibrated CRT monitor to the group of volunteers (50 persons of age 19-65 without colour blindness). The patterns of size 50 mm x 50 mm were generated. During the experiment the following viewing conditions were taken:

- The observer views the screen from a direction normal to the screen; the screen-observer distance is 50 cm that is an optimal condition for working with monitor.
- The surround of the test pattern has the chromaticity of the CRT reference white (colour temperature T= 6500K).
- The room spectral distribution of lighting was as close to D65 light source.

Before the beginning of the experiment brief explanation and corresponding illustration of the of the “effective” and “ineffective” moiré patterns was presented to the group of test participants.
Dependence of the relation type of gratings’ overlapping on the appearance of “effective” colour moiré patterns was examined. The question was: by which combinations of gratings do you see the “effective” colour moiré patterns? The results of tests are shown in Figure 32 and presented in Section 1 of the Appendix (CD-disk).

Let us compare the results of test between dominant and multiplicative relations (Figure 32 a, d). By two types of gratings’ superimposition the results are essentially different. First, one can notice appearance of “effective” colour moiré effect in case of two identical bicolour gratings with the same order of bars superimposition when dominant relation is applied while multiplicative relation does not give the same result. Thus, ineffectiveness of bicolour grating application to classical shadow moiré method can be also supposed because only such combination of bicolour gratings overlapping is possible here. However, appearance of “effective” colour moiré patterns by dominant relation can help to overcome the problem of ineffectiveness in shadow method. Another distinction in results relates to the case when projecting binary grating with bicolour gratings are overlapped - by dominant relation “ineffective” and by multiplicative relation “effective” colour moiré effect appears. The fact of “ineffective” colour moiré image appearance in case when projecting binary and analyzing bicolour grating are overlapped with dominant relation must be marked out according to a possible problem rise in projection moiré method. The problem can be solved by application of multiplicative relation in case when one grating is digitally generated.

Outlining the results of the section, the next has been done:

1. Two methods of digital colour moiré pattern generation were developed:
• multiplicative relation between the gratings gives the possibility of digital colour moiré patterns generation which are similar to the moiré images produced by classical moiré technique,
• dominant relation can be applied for getting “effective” colour moiré patterns by shadow method.

2. Influence of type of gratings overlapping on the colour moiré appearance in human eye was tested.

4.5 Human perception of colour moiré phenomenon

Fifty volunteers (19 – 65 years, without colour blindness) participated in the series of the experiments. The objective of the tests was investigation of colour moiré patterns perception under different conditions by volunteers [77].

For the tests three transparent films with gratings (υ=1 line per mm) of different adjacent colour bars combination were manufactured by means of laser printer (Osé 700). Two of them were with gratings of the same order of colours. In gratings primary (red, green, blue) and secondary (cyan, magenta, yellow) colours were used. The combination of the adjacent colour bars for the first film was: R-G, G-B, R-B, C-M, M-Y, Y-C and for the second one: G-R, B-G, B-R, C-M, M-Y, Y-C (Figure 33 a). All together six bicolour gratings (50 mm x 50 mm) were printed on each transparent film. In addition one binary grating of the same spatial frequency was printed on the third transparent film (Figure 33 b).

![Figure 33](image) Transparent films with printed gratings. The gratings with application of colour (a) and black bars (b).
The spectral transmittance curves of the colour bars of the gratings were taken the same like in Figure 25. The grating patterns were placed on the table, where white list of paper playing the role of background was put. Spectral reflection curve of list of paper was measured by spectrophotometer (Konica – Minolta CM-3600D) (Figure 34).

![Figure 34](image)

**Figure 34** Spectral reflection curve of white list of paper, used as a background during the experiments.

One of the films was fixed on the paper and another one was slightly tilted with respect to the first grating to produce the colour moiré fringes. The scheme of the experiments’ realization is shown in Figure 35.

![Figure 35](image)

**Figure 35** The scheme of the experiments' realization: 1 - overlapped transparent films with gratings, 2 - white list of paper, 3 - fixed transparent film with gratings, 4 - rotating transparent film with gratings.

For the experiment the following viewing conditions were taken:
- The observer views the patterns as it is shown in Figure 35.
- The surround of the test pattern has white colour
- The room spectral distribution of lighting was as close to D<sub>65</sub> light source.

Before the experiment brief explanation and corresponding illustration of the “effective” and “ineffective” moiré patterns was presented to participants.

The participants of the test were asked to sort out:
1. Patterns with “effective” and “ineffective” colour moiré fringes.
2. Combination of gratings that gives "effective" colour moiré fringes with the biggest colour difference.
3. Colour bars combination that gives "effective" colour moiré fringes with the biggest colour difference.

All together three tests were run:
1. Superimposition of two identical bicolour gratings with the same order of colour bars.
2. Superimposition of two identical bicolour gratings with the opposite order of colour bars.
3. Superimposition of binary and bicolour gratings.

The situation of colour moiré pattern appearance by different bicolour gratings was not examined. The results of the tests are presented in Section 2 of the Appendix (CD-disk).

In the first test the volunteers were asked to answer the question: by which combinations of gratings do you see "effective" colour moiré images? According to the results of survey during the tests "effective" colour moiré patterns were observed by most volunteers in two cases of superimposition: binary and bicolour grating (94% of participants) and two identical bicolour gratings with the opposite order of colour rulings (90% of participants) (Figure 38).

![Figure 36 The results of survey on question: by which combinations of gratings do you see “effective” colour moiré images [77]?](image)

The situation with overlapping of two identical bicolour gratings with the same order of colour bars showed its ineffectiveness during the tests – only 4% of participants saw "effective" moiré pattern in this case [77]. An ineffectiveness of colour moiré patterns in case of two bicolour gratings with the same order of colour bars overlapping again gives a reason to suppose impossibility of colour moiré effect application to shadow moiré method by means of traditional technique.
The second survey was on question: which combination of gratings would you chose for formation of the “effective” colour moiré patterns with the biggest colour difference? The result of the experiment was the next: 62% of participants chose colour moiré patterns generated by two identical bicolour gratings with opposite order of colour bars and just 38% - combination of bicolour and binary grating (Figure 37).

Figure 37 The results of survey on question: which combination of gratings would you chose for formation of the “effective” colour moiré patterns with the biggest colour difference [77]?

The result of the experiment shows the preference of two identical bicolour gratings application for “effective” colour moiré pattern generation rather then one binary and bicolour grating, proposed by Wenzel [77].

In Figure 38 the diagram represents the results of the third survey on question: which combination of colour bars in the gratings would you chose for formation of the “effective” colour moiré pattern with the biggest colour difference?

Figure 38 The results of survey on question: which combination of colour bars in the gratings would you chose for formation of the “effective” colour moiré pattern with the biggest colour difference [77]?

From Figure 38 it is seen that combination of R-B (red and blue) colour bars was chosen among the other proposed combinations. The result of the test proves the earlier in the work proposed hypothesis of colours selection for bicolour gratings according to cone fundamentals (See Section 4.2).

Thus, summing up the results of survey with volunteers the following is appropriate to note:
1. “Effective” colour moiré patterns were observed by most volunteers in two cases of superimposition: binary and bicolour grating and two identical bicolour gratings with the opposite order of colour rulings.
2. “Ineffective” colour moiré patterns were observed when two identical bicolour gratings with the same order of colour bars were superimposed during the series of tests.
3. Preference of two identical bicolour gratings application for “effective” colour moiré formation rather than rather use of one binary and bicolour grating.
4. The result of the experiment proves the early proposed hypothesis of colours selection for bicolour gratings according to cone fundamentals.
5. Selection of red and blue bars to bicolour grating for formation of the “effective” colour moiré patterns.

4.6 Summary

The main idea and basic concepts of colour moiré theory were presented in the chapter. Spectral analysis of colour moiré effect in colour space viewed here allowed proposing one of the theses.

The results of experiments with artificial moiré model coincide with the survey of tests with volunteers, namely:

- selection of red and blue primary colour bars for generation of moiré fringes with the biggest colour difference,
- absence of colour change by two identical bicolour gratings combination with the same order of colour bars,
- appearance of “effective” colour moiré fringes by superimposition of binary and bicolour grating and two bicolour gratings with opposite order of colour bars.

Two methods (dominant and multiplicative) of digital colour moiré pattern generation were developed and examined.

The series of tests with volunteers was made to select the best combination of colour bars and gratings for perception of the “effective” colour moiré fringes with the biggest colour difference.
5 Metrological methods based on application of colour moiré effect

In order to validate the effectiveness of colour moiré effect application in measuring technique the series of experiments was made. In the chapter results of colour moiré phenomenon application in shadow, projection, computer-aided and interferometric moiré techniques are presented. Requirements for correct interpretation of the colour information in moiré patterns and limitations of colour moiré use are considered. In the last section main concepts of ambiguities determination in colour moiré pattern are suggested.

During the experiments it was assumed, that camera transforms the colours precisely, colour rulings conform to theoretical colours, and spectrum of projector source of light is permanent. Spectral properties of optical systems were not taken into consideration.

5.1 Requirements for correct decoding of the colour information in moiré pattern

Before the application of the colour moiré effect in the measurements it is essential to mention two parameters that influence on the correct interpretation of colour information coded in moiré pattern.
The first one is observer disposition. According to the traditional moiré measuring methods the observer can be placed in any point of two areas (Figure 39).

As long as deformed grating represents the profile of the object then the disposition of observer in first or second region influences on the angle of inclination of the deformed lines and correspondingly on the way of colours distribution in moiré fringe [78]. For example, the order of colours in moiré fringes (Figure 40 a, b) is opposite while the model of the convex object stays permanent.

The second factor is influence of the order of colour bars in bicolour gratings and the role of gratings in the measurement. By role of grating it is assumed whether the grating is projecting or analyzing. In Figure 41 a, b deformed bicolour grating with different order of colour bars is overlapped with the binary grating. In Figure 41 c, d deformed binary grating is superimposed with the bicolour grating with different order of colour bars.
5 Metrological methods based on application of colour moiré effect

Figure 41 Deformed bicolour gratings and colour moiré pattern. Colour moiré pattern is generated by: analyzing binary and deformed bicolour grating with BR (a) and RB (b) order of colour bars; deformed binary grating and bicolour grating with BR (c) and RB (d) order of colour bars. The models of deformed gratings and corresponding colour moiré patterns are digitally generated by the principle presented in Section 4.4.

It is clearly visible from Figure 41 how the order of colour bars in bicolour grating and the role of grating influence on the order of colours in moiré fringes and consequently on the interpretation of colour moiré pattern.

Thus, two constructive parameters responsible for the correct interpretation of the colour fringes in moiré patterns were developed. The influence of the proposed parameters was approved practically during the series of measurements with several moiré topographical methods (See Sections 5.2, 5.3, 5.4, 5.5).

5.2 Shadow colour moiré method

Earlier in the dissertation the probability of “ineffective” colour moiré images appearance in shadow moiré method has been already mentioned several times. First in Section 4.2, where the principle of colour formation in moiré fringes was viewed (Figure 14 a), then during the series of experiments with artificial model (See Subsection 4.3.2) as well as in test with volunteers (See Section 4.5). The results of the experiments confirmed the fact of impossibility of “effective” colour moiré patterns appearance in traditional shadow moiré technique.

In the section the algorithm of moiré images processing developed for receiving of “effective” colour moiré patterns in shadow method is presented along with the results of its application in measurements.
5.2.1 Method description

The main advantage of the traditional shadow moiré method is its simplicity caused by the fact that projecting and analyzing grating are identical. However in shadow colour moiré method the colour distribution obtained by sampling along a straight line cutting through the moiré fringes is symmetrical, i.e., gives one-coloured or “ineffective” moiré fringes (See Figure 14 a). For obtaining extra information encoded in colour moiré effect should be changed to the asymmetrical, for example, by transformation of colours in analyzing grating.

As it has been already shown in the previous chapter the visual appearance of colour moiré fringes depends on the type of superimposed rulings. Since projecting and analyzing grating are identical, then only superimposition of two identical bicolour gratings with the same order of colour bars is possible in traditional shadow method. At the same time it has been already found that “effective” colour moiré fringes appear when two identical bicolour gratings overlapped with the opposite order of colour bars or if binary and bicolour grating is superimposed. Consequently by changing the order of colour bars in analyzing bicolour grating to the opposite or by its substitution by the binary grating the appearance of “effective” colour moiré pattern can be achieved. The transformation is made digitally on the ready captured colour moiré pattern.

Nowadays digital camera is preferred to the traditional one in many applications, shadow moiré method is not an exception. Then, assume that a digital image of moiré pattern is taken in RGB colour space. The colour formation principle in moiré fringe generated for shadow method is presented below (Figure 42).

![Figure 42](image-url)

**Figure 42** The colour formation principle in moiré fringe generated by superimposition of two bicolour gratings in shadow method. The letters correspond to the colour appearance as a result of two identical bicolour gratings superposition in ideal case [79].

Presence of only 9 different colours in the result shadow colour moiré image refers to the ideal case. For realization of the above proposed colour transformation these nine colours must be sampled in the image. The next step is determination of the analyzing grating frequency. As long as it is already known from the initial conditions the order of colours in analyzing grating, then the colours in analyzing grating \((E, F)\) can be
substituted by $D$ colour and $(G, H)$ colours – by $D$ colour or changed to black (Figure 43 a, b).

![Figure 43](image)

**Figure 43** Transformation of analyzing grating in shadow colour moiré pattern to bicolour (a) and binary grating. The letters in the figure correspond to the different colours [79].

Flow chart of “ineffective” colour shadow moiré pattern processing is presented in Figure 44.

![Flow chart](image)

**Figure 44** Flow chart of “ineffective” colour shadow moiré pattern processing to the “effective” one [79].

Thus, presented method of shadow colour moire patterns processing allows using colour moire effect in shadow moire technique but not when online measurement is required.

### 5.2.2 Experiments

The experiment was made by application of bicolour grating. Experimental setup used for the measurement is presented in Figure 45. The halogen lamp ($T=3200K$) is
placed by 45° to the optical axis of the camera (Canon 350D). The test object is put close to the grating. The moiré fringe pattern is recorded with camera whose optical axis is directed normal to the plane of the grating. The camera and the lamp are placed at 1000 mm from the object.

Relative spectral power distribution of the halogen lamp can be found in Figure 26. The width of each colour bar of the colour grating is equal to 0.25 mm, the width of each transparent slit is 0.5 mm, and the resolution was 1.3 mm. The red and blue colours were taken for the bars of bicolour grating; they were printed by laser printer (Os 700). The spectral transmittance curves of the colours were obtained by AvaSpec-1024 Fiber Optic Spectrometer and were presented in Figure 25a.

The test object and its surface shape is shown in Figure 46. The object was formed from the piece of wood in such a way to contain continuous concave and convex areas. Spectral reflection curve of the test object was already presented in Figure 27.
The captured moiré image is shown in Figure 47 a. Modified moiré patterns obtained after the analyzing grating transformation by the algorithm proposed in the previous subsection are shown in Figure 47 b, c. The algorithm of the moiré image processing was realized with Matrox Inspector 2.1 program.

![Figure 47](image)

**Figure 47** Colour moiré patterns of the object with concave and convex areas and the graphs illustrating the red green blue components of the data that was obtained by sampling along a straight line cutting through the moiré fringes. The captured moiré image (a), modified moiré image by transformation of analyzing grating to the opposite ordered bicolour grating (b), binary grating (c). AB is moiré fringe profile. DAC, digital-to-analog converter, which is the measurement unit of the colour coordinates, that can take any integer value between 0 and 255 [79].

From Figure 47 the effectiveness of proposed shadow moiré image processing is clearly visible. Processed moiré patterns became “effective”, i. e. the moiré fringes have the opposite order of colours according to concave and convex areas of the examined object.

Thus, by processed shadow moiré image processing it becomes possible to get “effective“ colour moiré images by classical shadow technique. The main advantage of the described method is that it allows displaying extra colour information from one shadow moiré image and does not require any special equipment. The experimental
results show that the method of colours’ transformation is effective and useful tool for the shadow colour moiré technique.

5.3 Projection colour moiré method

Effectiveness of bicolour gratings’ application to projection method was examined. The experiments were made within the traditional projection method: no extra equipment was used for generation of colour moiré patterns. The series of experiments was carried out by different factors that influence on colour moiré phenomenon appearance.

5.3.1 Examined factors

In Subsection 5.1 two factors that influence on colour moiré effect appearance have been proposed [78].

The first one is observer and projector disposition (Figure 39). In projection moiré method there are two possibilities of equipment disposition (Figure 48 a, b).

![Figure 48](image)

**Figure 48** Equipment disposition in projection method: projector is on the right and observer – on the left (a), projector is on the left and observer – on the right to equipment axis (b) [78].

The second factor is influence of the order of colour bars in bicolour grating. To this case relates two bicolour gratings application, whether they are superimposed with the same or opposite order of colour rulings and also when bicolour grating is overlapped with binary one.

In addition as the third factor stated for the test was the role of gratings (analyzing or projecting) in case of binary and bicolour gratings application due to the results of test for the hypothesis in Section 4.4.

The influence of the above listed factors on colour moiré image appearance is examined in the next subsection.
5.3.2 Experiments

The experimental setup used for projection colour moiré method is shown in Figure 49. Projector and observer are placed by 45° to each other at the distance of 1000mm to the object. Traditional components were used in projection method: projector (Malisix), two gratings, camera (Canon 350D) and observer.

![Experimental setup](image)

*Figure 49* Photo of the experimental setup for projection colour moiré method used during the experiments.

The observer was developed at our department by Wenzel G. and Antal Á. in 1988-1990 years. The scheme of observer device is shown below in Figure 50.

![Observer device](image)

*Figure 50* Scheme of observer device.

The moiré fringe pattern is recorded with a camera whose optical axis is directed normal to the plane of the analyzing grating. The same object and bicolour grating were applied for examination as in shadow method (See Subsection 5.2.2). The pitch of the binary and bicolour gratings was taken 1 mm, the resolution was 2.6 mm.

Moiré patterns of the test object obtained by two bicolour gratings with the same order of colour bars (Figure 51 a) and with the opposite order of colour bars (Figure 51 b); projecting binary grating with analyzing bicolour grating (Figure 51 c) and projecting bicolour grating with analyzing binary grating (Figure 51 d) are shown below.
5 Metrological methods based on application of colour moiré effect

Figure 51 Colour moiré patterns of the object with concave and convex areas and the graphs illustrating the red, green, blue components of the data that was obtained by sampling along a straight line cutting through the moiré fringe. Moiré patterns obtained by: two bicolour gratings with the same order of colour bars (a), two bicolour gratings with the opposite order of colour bars (b), projecting binary and analyzing bicolour gratings (c), projecting bicolour and analyzing binary gratings (d). AB is moiré fringe profile. DAC, digital-to-analog converter, which is the measurement unit of the colour coordinates, that can take any integer value between 0 and 255.

Analyzing Figure 51 one can notice that experimental results correspond with earlier proposed hypothesis about the influence of some factors on the colour moiré patterns appearance (See Subsection 5.1). For example, equipment disposition influences on the order of colours in moiré fringes. The change of colour bars’ order in the gratings leads to the appearance of colour moiré patterns with contradictory colour information for the same type of surface or rise of “ineffective” moiré fringes. In case of projecting binary and analyzing bicolour grating application “ineffective” moiré fringes (Figure 51 c) appear as was proposed in Section 4.4 when dominant relation was applied; here because of the arising optical mistake in observer by gratings.
superposition. Thus, as a conclusion it can be set that colour moiré phenomenon can be efficiently applied to the classical projection method by meeting several conditions.

### 5.4 Computer-aided colour moiré method

Computer-added moiré methods use the versatility of computer to manipulate with gratings for varying the resolution of the method. Moiré contours can be generated in the memory of computer by two digital gratings or one snap-shot and one digital grating [19].

#### 5.4.1 Method description

The colour moiré phenomenon can be also applied to computer-aided technique. Colour moiré images are supposed to be formed by overlapping of two component structures in computer space. The binary grating is projected obliquely onto the object, and then the snap-shot of the object is made by camera (Figure 52).

![Figure 52 Experimental setup of computer-aided moiré technique. PG - projecting grating; PO - projection optics.](image)

The captured image is stored and superimposed with the digital bicolour grating in the memory of computer [76]. By means of spectral sensitivity parameters of the detector the colour distribution of the snap-shot with deformed stripes can be transformed to the computers’ RGB space.

Colour moiré patterns are obtained by overlapping of the snap-shot of the object with projected binary grating and the digital bicolour grating with appropriate pitch in the memory of computer. Two relations earlier presented in Section 4.4 for generation of colour moiré pattern were applied to the superimposing structures. Effectiveness of the relations’ application was examined.
5.4.2 Experiments

During the experiments red (R:255; G:0; B:0), blue (R:0; G:0; B:255) and white (R:255; G:255; B:255) colours of computer colour space were used for encoding the pixels of the digital grating. As projecting grating was taken a common binary grating with the pitch equal to 0.75 mm, then the resolution is 1.7 mm. In Figure 53 the snapshot of the test object (Figure 46) with deformed grating lines is shown. The same camera and projector were applied for testing as for the previous method. The moiré fringe pattern is recorded with camera whose optical axis is directed normal to the plane of the object. The camera and projector are placed at 1000 mm from the test object. The projector is put by 45° to the optical axis of the camera.

Figure 53 Snap-shot of the test object with deformed grating lines.

Figure 54 presents computer-aided generated colour moiré patterns of the test object with application of dominant (Figure 54 a) and multiplicative (Figure 54 d) relation between analyzing digital bicolour grating and snap-shot of the object with deformed stripes.
5 Metrological methods based on application of colour moiré effect

![Image](87x655 to 269x762)

![Image](296x345 to 591x1120)

![Image](75x525 to 273x641)

**Figure 54** Computer generated colour moiré patterns of the object with convex and concave regions and the graphs illustrating the red, green, blue components of the data that was obtained by sampling along a straight line cutting through the moiré fringe. Colour moiré patterns generated by application of: dominant (a) and multiplicative (b) relations between the gratings. AB is moiré fringe profile. DAC, digital-to-analog converter, which is the measurement unit of the colour coordinates that can take any integer value between 0 and 255 [76].

Results of the practical experiments presented in Figure 54 verify the hypothesis about colour moiré fringe appearance by application of above mentioned relations in computer-aided colour moiré method (See Section 4.4). From Figure 54 a it is good seen that by dominant relation of the structures “ineffective” colour moiré pattern appears while by use of multiplicative relation “effective” colour moiré image is obtained (Figure 54 b).

Thus, colour moiré phenomenon can be efficiently applied to computer-aided method on the assumption if multiplicative operation is used between the superimposed structures.

The main advantages of computer-aided colour moiré method are:

1. Perception of “effective” colour moiré pattern with projecting binary grating and analyzing digital bicolour grating by use of multiplicative relation. It is significant since it was experimentally proved (See Subsection 5.3.2) that it can not be obtained in projection colour moiré method.

2. Surfaces tested by computer-aided color moiré method do not have to be white, due to the possibility of digital post processing.
5 Metrological methods based on application of colour moiré effect

5.5 Computer-aided colour moiré interferometric method

Moiré technique is also used in the interferometric studies of the displacements in objects under deformation. The purpose of applying colour moiré method to interferometry is to display extra information encoded in colour about the object when higher resolution of measurement is required. Since the resolution of the moiré method depends on the pitch of the gratings, the ability to use the interferometer for projecting grating formation and digital analyzing bicolour grating - that is the way to extend the realm of the colour moiré method to the higher level of resolution.

5.5.1 Method description

For realization of computer-aided colour moiré interferometric technique the method described in the previous section was applied. The only difference between two methods was that the projected grating here was generated by special optical system.

Optical system configuration for straight line grating formation and interferogram recording is schematic shown in Figure 55.

The test surface does not have a grating imprinted on its surface. However, the specimen surface is illuminated with a periodic grating that is done by using an optical interference. A coherent laser beam and a beam splitter with one mirror slightly tilted produces straight interference fringes, which fall on the surface to be contoured. Thus the lines projected on the surface are resulting from the interference of two collimated beams. The interferogram is recorded, processed to RGB colour space and stored in the computer memory. The digital bicolour grating with a proper pitch is generated in the memory of the computer. Superimposition of the interferogram and the digital bicolour grating by use of the multiplicative relation (See Table 3) results “effective” colour
moiré image. Then image processing is required for getting the quality colour moiré pattern. As a result "effective" colour moiré pattern of higher resolution is obtained.

5.5.2 Experiments

Photo of the experimental setup used for computer-aided colour moiré interferometric method is shown in Figure 56. During the experiment He–Ne laser ($\lambda = 635$ nm) was used as a coherent source of light. Camera (Motic 1000) was placed by $45^\circ$ to the optical axe of the one mirror and the prism, the resolution was 0.25 mm.

![Experimental setup](image)

**Figure 56** Photo of the experimental setup for colour moiré interferometric method used during the experiments.

As a test object was chosen a white peace of chalk with diameter of 10 mm (Figure 57).

![Test object](image)

**Figure 57** Test object.

Spectral reflection curve of the test surface is shown in Figure 58.
Convex and concave areas of the test object were under research. The specimen \( I \) represents the concave area of the test object and specimen \( II \) – the convex area. Experimental results are presented in Figure 59.
Figure 59 Snap-shot of the specimen I and II of the test object with fringe pattern (a), grey-scale image of the specimen (b), computer generated colour moiré interferogram (c). The graphs (d) illustrate the red, green, blue components of the data that was obtained by sampling along a straight line cutting through the moiré fringe. AB is moiré fringe profile. DAC, digital-to-analog converter, which is the measurement unit of the colour coordinates, that can take any integer value between 0 and 255.

The original image of the specimen with projected straight rulings (Figure 59 a) was first converted to the grey-level image (Figure 59 b) and then overlapped with bicolour grating by use of multiplicative relation that resulted “effective” colour moiré interferogram (Figure 59 c). From Figure 59 c, d it is good seen that moiré images for the
first and second specimens that correspond to concave and convex surfaces correspondingly differ by the order of changing colours in moiré fringes.

Thus, the results of experiment presented in Figure 59 show the effectiveness of computer-aided colour interferometric method application for measurement of small objects, when higher resolution is required.

5.6 Limitations of colour moiré application

Colour of test object. For effective application of colour moiré effect the test object must be diffusive and white. However there is an exception. If computer-aided colour moiré method is used, where the projecting grating is a binary one then the colour of the object can be separated from the grating and changed to white virtually.

Spatial frequency of gratings. For the colour moiré fringe formation an important role plays the right selection of the grating spatial frequency as long as the phenomenon of changing colour in moiré fringe also obligates to the limited resolution of the human eye. Model of one period of colour moiré fringe, generated by overlapping of one bicolour and binary grating, with different spatial frequency of the gratings is presented in Figure 60.

Looking at Figure 60 the upper moiré pattern occurs in eye as a single image with sequence of colours from pale blue to orange while the lower moiré fringe is perceived as two separate gratings. It is the result of the limited resolution of the eye. Thus, by the low spatial frequencies of the superimposing gratings the colour moiré fringe is not generated in the human eye.

Angle of tilt. This limitation is a special case of the limited resolution of the human eye. By increase of the gratings’ tilt angle the perception of change of colours in the “effective” moiré fringe becomes more difficult, until it finally will be indistinguishable.
In Figure 61 colour moiré patterns generated by overlapping of analyzing binary and deformed bicolour grating with different inclination angle are presented.

![Figure 61 Model of the “effective” colour moiré fringe, generated by superimposition of analyzing binary and deformed bicolour grating, with the same pitch and tilted with respect to one another with small (a) and big (b) slope angle.](image)

Looking at Figure 61 the change of colour in moiré fringe of the left pattern is easily observed while recognition of the colours’ change in the fringe shown in the right image causes difficulties.

The series of tests was carried out to determine how the inclination angle and the change of ratio between the frequencies and of overlapping gratings influences on the human perception of moiré effect. Three types of moiré effect were proposed for testing: classical moiré effect, “effective” colour moiré effect generated by binary and bicolour colour gratings and two identical bicolour gratings.

The viewing conditions of the experiment were taken the same as in Section 4.4. The moiré patterns were shown to the volunteers (age 19 – 65 years) on the calibrated CRT monitor. Explanation and corresponding illustration of the “effective” moiré patterns was announced to the participants. The results of the tests are presented in Section 3 of the Appendix (CD-disk).

**Test N 1**: Dependence of moiré fringes perception from the ratio between the frequencies of the overlapping gratings. Moiré patterns of one moiré fringe were generated digitally by superimposition of bicolour and binary grating, two bicolour gratings and two binary gratings (classical moiré effect) with different spatial frequencies. As long as one moiré fringe appears on the length unit when [19]:

\[
|\nu_1 - \nu_2| = 1
\]

where \(\nu\) - grating frequency. Thus, ratio between the gratings’ frequencies was taken the next: 50/49, 49/48…..10/9 pair lines per 100 mm.

The size of the patterns is 100 mm \(\times\) 40 mm. The participants were asked to estimate the moiré pattern on “1” if moiré fringe is seen as a single whole and “0” if not. The results of the test are shown in (Figure 62).
5 Metrological methods based on application of colour moiré effect

The perception of moiré fringe in case of traditional moiré pattern (Figure 62 a) appears already by quite low frequencies of overlapping gratings while in other two cases (Figure 62 b, c) it was fixed only when the frequency of gratings is almost two times higher.

**Test N 2**: Dependence of moiré fringes perception from overlapping gratings’ inclination angle. Thirty digital moiré patterns were generated by superimposition of bicolour and binary grating, two bicolour gratings and two binary gratings (classical moiré effect) of one frequency and different grating orientation angle (0° - 60°). The patterns have the size 100 mm х 40 mm. Forty moiré patterns generated for each type of used gratings with different inclination angle of gratings with step of sampling 2° were shown to the participants on the calibrated CRT monitor. The participants were asked to estimate the moiré pattern on “1” if moiré fringe is seen and “0” if not for the case of classical moiré effect. When moiré patterns are generated with colour gratings then “1” refers to moiré fringe with visible change of colour and “0” if the colours can not be distinguished. The results of the experiment differ for the examined cases (Figure 63).

The fringes of the classical moiré pattern, generated by two binary gratings were percept until the moment when one of the gratings was tilted to another one by 40° (Figure 63 a). The changing colour in fringes of the “effective” colour moiré images generated by two bicolour gratings was already not perceived by the slope angle
exceeding 22° (Figure 63 b). In case of binary and colour gratings overlapping for the “effective” moiré pattern formation the maximum value of the slope angle of the gratings by which changing colour in fringes were still perceived was equal to 17 ° (Figure 63 c).

Presented values of angles and frequencies can not be used as boundary limits for proposed types of gratings according to the dependence on starting conditions of measurement. However, it can be assumed that, by the identical starting conditions of some moiré topographical measurement the area of colour moiré application is smaller comparing with the traditional binary moiré patterns. As long as the appearing low frequency and big inclination angle of the overlapping gratings usually defines the surfaces of great curvature, it means that the colour moiré phenomenon can be applied for measurement of objects with smaller curvature of surface as by case of traditional moiré technique.

5.7 Main concepts of the ambiguities definition in colour moiré patterns

The aim of ambiguities determination is the reconstruction of the continuous phase distribution using colour coded information of the moiré pattern. Sequence of operations for ambiguities determination in the colour moiré images is proposed below:

1. Definition of the closed contours in the moiré pattern.
2. Determination of one point inside of the each closed contour.
3. Definition of the colour distribution in one moiré fringe in the direction from the point that lies on the normal to the fringe.
5. Calculation of \( \Delta E_{uv}^* \) colour difference between the colour of the fringe and one neutral colour.
6. Separation of the fringes to the two groups with a similar \( \Delta E_{uv}^* \) colour difference.
7. Determination of correspondence between \( \Delta E_{uv}^* \) and the type of group (convex or concave area).
8. Relation of the fringes from two groups to convex or concave areas correspondingly.

If the interpretation of the colour moiré pattern does not required to be operated automatically, for example, in medicine by examination of the spine curvature, then for definition of hill and valley areas it is enough to use the Table 4, where by known
equipment disposition, role of gratings and order of bars in gratings one can easily determine convex and concave areas on the colour moiré image due to the order of colours in fringe.

Presented succession of operations is prototype of algorithm for colour moiré image processing. Development of its practical realization is the task of programmers.

### 5.8 Measurement manual for colour moiré patterns decoding

Special measurement manual was composed for simple decoding of the colour information in moiré pattern based on the results of the measurements presented in the dissertation (Table 4). According to the equipment disposition, order of colour bars and role of gratings the sequence of colours in moiré fringe for convex and concave areas can be determined.
<table>
<thead>
<tr>
<th>Observer is on the right to axis</th>
<th>Observer is on the left to axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two bicolour gratings with opposite order of colour bars</td>
<td>Projecting grating with BR order of colour bars, analyzing grating with RB order of colour</td>
</tr>
<tr>
<td>Convex</td>
<td>Concave</td>
</tr>
<tr>
<td><img src="image1.png" alt="Diagram 1" /></td>
<td><img src="image2.png" alt="Diagram 2" /></td>
</tr>
<tr>
<td>Projecting grating with RB order of colour bars, analyzing grating with BR order of colour</td>
<td>Convex</td>
</tr>
<tr>
<td><img src="image5.png" alt="Diagram 5" /></td>
<td><img src="image6.png" alt="Diagram 6" /></td>
</tr>
<tr>
<td>Two bicolour gratings with the same order of colour bars</td>
<td>Convex</td>
</tr>
<tr>
<td><img src="image9.png" alt="Diagram 9" /></td>
<td><img src="image10.png" alt="Diagram 10" /></td>
</tr>
<tr>
<td>Binary and bicolour gratings</td>
<td>Projecting binary grating and analyzing bicolour grating with RB/RB order of colour bars</td>
</tr>
<tr>
<td>Convex</td>
<td>Concave</td>
</tr>
<tr>
<td><img src="image13.png" alt="Diagram 13" /></td>
<td><img src="image14.png" alt="Diagram 14" /></td>
</tr>
<tr>
<td>Projecting bicolour grating with BR order of colour bars and analyzing binary grating</td>
<td>Convex</td>
</tr>
<tr>
<td><img src="image17.png" alt="Diagram 17" /></td>
<td><img src="image18.png" alt="Diagram 18" /></td>
</tr>
<tr>
<td>Projecting bicolour grating with RB order of colour bars and analyzing binary grating</td>
<td>Convex</td>
</tr>
<tr>
<td><img src="image21.png" alt="Diagram 21" /></td>
<td><img src="image22.png" alt="Diagram 22" /></td>
</tr>
</tbody>
</table>

*Table 4* Measurement manual for colour moiré pattern decoding [78].
5.9 Summary

In the chapter requirements of colour moiré fringes’ recognition, limitations of the method were developed. Table 4 can be used as a measurement manual for colour information decoding in moiré images for all presented in the chapter methods as the results of experiments show.

The method of shadow moiré pattern processing allowing to get “effective” colour moiré images from initial “ineffective” patterns was presented. Novel computer-aided colour interferometric moiré method was worked out; its effectiveness was verified during the experiments.

Concepts of colour information processing encoded in moiré images for ambiguities determination were suggested.

Experimental results of the measuring the object with ambiguities by application of different moiré techniques showed the effectiveness of colour moiré phenomenon use in topographical moiré methods.
6 Theses discussion

The next five theses were proposed:

1. **Thesis**
   Application of two bicolour gratings with the opposite order of colour bars for measurement of the object with concave and convex areas allows to resolve the problem of ambiguities due to appearance of the opposite order of colours in moiré fringes representing hill and valley of the object.

This thesis is based on the results of investigation in Sections 4.2, 4.3, 4.5 and practically proved by the moiré topographical measurements (See Sections 5.2, 5.3, 5.4).

2. **Thesis**
   One “effective” colour moiré pattern is sufficient for determination of the differences in object height. One “effective” colour moiré pattern is the one that includes the moiré fringes with the changing colour.

Results of the topographical measurements with use of colour moiré effect showed that it is enough to have only one “effective” colour moiré image of the test object for recognition of the concave and convex areas (See Sections 5.2, 5.3, 5.4, 5.5).

3. **Thesis**
   By representation of colour moiré fringe in CIE xy and CIE Luv colour spaces becomes possible to:
a. State that “effective” colour moiré effect is only that one where along one period of moiré fringe one colour does not appear twice.

b. Distinguish an “effective” and “ineffective” moiré phenomena from the geometrical form of graph, that describes the changing of colour in one period of moiré fringe. “Ineffective” moiré phenomenon is the one which fringes are selfcoloured.

c. Determine the colour moiré effect with the biggest colour difference among the several “effective” moiré phenomena.

The geometrical form of graph, that describes the changing of colour in one period of moiré fringe in appropriate colour system, empowers to define an “effective” and “ineffective” moiré phenomena, in case of several “effective” moiré cases allows to determinate the most contrast one. This thesis was viewed in Subsection 4.3.1 and approved itself in series of experiments (See Subsection 4.3.2).

4. Thesis

Application of bicolour and binary or two bicolour gratings combination to shadow, projection and computer-aided methods allows to get “effective” colour moiré images.

Colour moiré effect was applied to the above listed moiré methods for recognition of the concave and convex areas of the test object. Results of the measurements as well as the description of the methods are presented in Sections 5.2, 5.3, 5.4, 5.5.

5. Thesis

Application of multiplicative relation between two bicolour or one bicolour and binary grating allows to generate the digital colour moiré pattern which is similar to the moiré image produced by the classical moiré technique.

The thesis was based on the results of investigation in Section 4.4 and practically proved by the moiré topographical measurements (See Sections 5.4, 5.5).
7 Conclusions

This chapter summarizes the results of the dissertation work. A review of the publications relating to this research is presented.

7.1 Summary

Moiré topographical methods based on the optical effect of moiré are well-known and time-proved due to their simplicity and cheapness are frequently used in nowadays measurements. As any other measuring tool it has its advantages and lacks. One of its main disadvantages is arising ambiguities in moiré patterns, i.e., impossibility of convex and concave areas determination from one moiré image. Many ideas were proposed for solution of the problem, one of them was application of one bicolour and binary grating for moiré formation. It was hypothesized that such combination of the above mentioned gratings would allowed to recognize the hills and valleys of the object from only one moiré image. However neither theoretical description nor experimental confirmation of the hypothesis was presented.

The dissertation was dedicated to the development of colour moiré theory and examination of its effectiveness in moiré topographical measurements. The colour moiré theory was based on the idea of bicolour grating use.

For obtaining the extra information about colour moiré phenomenon that is required for effective application in moiré technique the effect was investigated from different aspects.
Results of the investigations confirmed the proposed idea: really the superimposition of binary and bicolour gratings gives the moiré pattern with different colored moiré fringes that allowed determining the convex and concave areas. Besides it other variations of gratings overlapping were theoretically described, practically tested and applied for topographical measurements.

The colour moiré effect used in the topographical methods, like the projection moiré, the shadow moiré, the computer-aided moiré method and its interferometric version proved its effectiveness for solution of the “hill and valley” problem. The experiments resulted necessity of “effective” colour moiré image presence for ambiguities determination.

Thus, the moiré topographical methods with application of colour moiré effect undoubtedly have an advantage of the classical moiré techniques provides great opportunities for surface inspection/recognition. However more work is needed on algorithm of colour moiré image processing.

The major contributions made in the dissertation:

1) Developed the main idea and basic concepts of colour moiré theory.
2) Made spectral analysis of colour moiré fringe in colour space.
3) Developed two realtimeions between the gratings for digital colour moiré pattern formation.
4) Investigated the question of human perception of colour moiré patterns.
5) Developed requirements of colour moiré fringes’ recognition, limitations of the method application and conditions of experiment carrying out.
6) Developed the measurement manual for colour information decoding in moiré images according to the parameters of measurement.
7) Developed the method of colour shadow moiré image processing.
8) Examined the effectiveness of colour moiré effect in several moiré topographical methods.
9) Suggested the main concepts of the ambiguities definition in colour moiré pattern.

7.2 Publications

I have started working with the theme of colour moiré effect already by my master degree that was continued through the PhD work. Later during the three years of my studies the next publications appeared:
• A. Antal and D. Paveleva, “Projection method of resolving ambiguities by determining the order of colours in moiré fringes”, Appl. Optics 44, pp. 7709-7713 (2005) [78];
• Antal Á., Paveleva D., “Háromdimenziós felületek azonosítása moiré módszerrel”, Magyar Elektronika, 12, pp. 25-27 (2006) [80];
• Paveleva D., Wenzel K., Antal Á., “A színes moiré jelenség alkalmazásának vizsgálata sikbeli elmozdulások esetén”, Műszaki Szemle 38, pp. 325-328 (2007) [81];

The article “Analysis of Colour Moiré Effect in Colour Space” (Paveleva, D., Wenzel, K.,) was accepted by Periodica Polytechnica journal, its publication is expected in the year 2009 [68]. Besides the above listed articles the methodical textbook “Robot sensors I” (Pavelyeva D., Wenzel K., Antal Á.) was published by TANOK (2004) and applied during the teaching of Russian students under the Russian-Hungarian program[82].
8 References


