Development of a numerical model for the prediction of ground borne vibration and noise in buildings

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Summary of PhD dissertation

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1 Introduction and Research Background

Significant vibration in buildings near surface or underground railway tracks or roads is attributed to moving vehicles. Traffic induced vibrations in dense urban environments can cause structural damage in buildings and annoyance to the inhabitants of surrounding buildings in the form of vibrations or re-radiated noise.

This thesis deals with the effect of low frequency and small amplitude vibrations on the building’s inhabitants, and does not concern the effect of vibrations on the structural stability of buildings. The frequency range is limited by an upper frequency value of 200 Hz, and it is assumed that the magnitude of vibrations is in the linear range.

The total vibration path between a moving vibration source – road or railway traffic – and the re-radiated noise in the structure can be divided into three subproblems. The first subproblem is the vibration generation by a moving vibration source. The second problem is the dynamic soil-structure interaction, where the response of a building to the ground vibration excitation is investigated. The third subproblem is the sound radiation by the building’s walls into internal acoustic spaces.

Significant part of the research has been carried out within the framework of the European project Convurt, the acronym is for Control of Noise and Vibration due to Underground Railway Traffic between 2002 and 2005. The objective of the project was to create validated innovative and quantitative modeling tools to enable prediction of locations where ground-borne vibration transmission and thereby noise will occur in metropolitan railway networks. The project aimed to develop and evaluate innovative and cost effective track and tunnel equipment to reduce ground-borne vibration capable of being retrofitted and exported worldwide, and to prepare Good Practice Guides for underground railway operation in order to maintain minimum vibrations for the lifetime of operation.

The project CONVURT succeeded in the development of separate models for the investigation of subproblems of the total vibration chain. On the source side, the main benefits are the Pipe in Pipe (PiP) model developed at the Cambridge University and the coupled periodic FE-BE model developed at the K.U.Leuven. On the receiver side, the Rayleigh integral based approach has been developed at the Budapest University of Technology and Economics. After the project CONVURT finished, the work on model integration has started. One of the most important results of the present work is that the total coupled numerical model has been integrated and a practical application is also presented.

2 Research objectives

In order to be able to handle the problem of traffic induced vibration and noise in dense urban areas, the clear understanding of the vibration generation mechanisms and the phenomena related to vibration propagation in the soil, structures and in the air is essential.
The main objective of the research described in the present thesis is to present a complex numerical model of the whole vibration-chain. The complex model should account for the vibration generation by moving vehicles, vibration propagation in the soil, dynamic soil-structure interaction, vibration propagation in the building and sound radiation into closed rooms.

A reliable model that can take all these effects into account, can be a valuable tool in the hand of the engineer when the vibration isolation of structures settled near railway lines or roads is planned. The model can be used to predict the in-door noise and vibrations before the construction is started, the model can help to define the required amount of vibration or noise isolation, and the model can be used to predict the effect of different noise and vibration isolation techniques, and to optimize the construction costs.

3 Research methodology

In the present thesis a deterministic model of vibration propagation in elastic media and sound propagation in the air is considered. The modeling of these phenomena involves the solution of partial differential equations describing the vibration propagation in linear solids and in the air. The two basic governing equations of the whole problem are the Navier-Cauchy equations and the acoustic wave equation.

The solution methods presented in the thesis are based on Fourier integral transforms, and the solutions are found numerically in the frequency or in the frequency-wavenumber domain. The main numerical methods discussed in the text are the dynamic stiffness matrix method and boundary element methods for the soil domain, a finite element method with modal decomposition for the structural domain and a spectral finite element method for the acoustic domain.

4 New results

The results of my research can be divided into the following three claims or theses:

First, I propose a global numerical model that is capable for the computation of ground-borne noise and vibration in buildings due to surface traffic excitation.

In the second claim I concentrate on the problem of sound radiation into closed rooms by vibrating walls.

In the third claim I concentrate on the application of the boundary element method for dynamic soil-structure interaction problems.

Claim 1 I propose a three-dimensional deterministic integrated coupled numerical model for the computation of ground-borne noise and vibrations in buildings.

1.1 I propose the integration of a source model of Lombaert, a dynamics soil-structure interaction model of Clouteau and Aubry and a newly developed radiation model
for the description of the whole vibration chain. The coupled model accounts for vibration generation by a moving vehicle, vibration propagation in the ground, dynamic soil-structure interaction and sound-radiation into closed rooms.

1.2 I propose models for the investigation of the effect of dynamic soil-structure interaction on the re-radiated noise in buildings. I show that for the case when the impedance difference between the soil and the structure’s foundation is large, the effect of dynamic soil-structure interaction can be disregarded without significant loss of accuracy in the re-radiated noise in rooms.

1.3 I propose the coupled numerical model as a means for the optimisation of vibration and noise isolation in structures. I show that the model can quantify the effect of typical vibration isolation measures, such as the application of floating floors, box-within-box arrangements or base isolation.

Publications related to this claim are: IJ1, IJ2, IJ3, NJ1, IC2, IC3, IC4, IC6, IC7, IC8, IC9, IC10.

Introduction

In this thesis, I propose a coupled numerical model that is capable for the computation of ground-borne vibration and noise from surface railway traffic. To my knowledge, this is the first application available in the literature, when a coupled numerical model is used to describe the whole vibration chain starting from vibration generation by a moving vehicle and ending with the re-radiated noise in a building.

The total vibration path can be split up into three weakly coupled subproblems. Subproblem 1 is the computation of ground vibrations due to surface or underground railway traffic. Subproblem 2 is the problem of dynamic soil-structure interaction, where the response of a structure to the incident ground vibrations is computed. Subproblem 3 is the acoustic radiation problem, where the internal noise radiated into the rooms of the building by the vibrating walls is predicted.

The incident wave field

The model of a moving vehicle on a longitudinally invariant track is used to compute the incident wave field. The invariance of the track in the longitudinal $y$-direction enables to transform the spatial variable $y$ into the longitudinal wavenumber $k_y$ by means of a Fourier transform and compute the response to moving loads in the frequency-wavenumber domain.

The vehicle is modeled by a set of concentrated masses, representing the train’s axles, connected to the track by Hertzian springs. The invariant track is modeled by a pair of Euler beams representing the rails, distributed spring-dampers representing the rail pads, distributed mass for the sleepers and a spring-damper system for the ballast layer. The track is laying on a horizontally layered soil, characterized by its shear
wave velocity $C_S$, compressional wave velocity $C_P$, material density $\rho$, and hysteretic material damping ratio $\beta$ for each homogeneous layer.

First, the vehicle-track interaction problem is solved in a frame of reference that is moving with the train velocity $v$ along the invariant direction $y$. The compliance matrices of the vehicle $\hat{C}^v(\omega)$ and the track-soil system $\hat{C}^t(\omega)$ are written in the moving frame of reference. These matrices are used to compute the dynamic axle loads $\hat{g}(\omega)$ arising due to the rail unevenness $\hat{u}_{w/r}(\omega)$ as:

$$
\left[ \hat{C}^v(\omega) + \hat{C}^t(\omega) \right] \hat{g}(\omega) = -\hat{u}_{w/r}(\omega)
$$

(1)

The elements of the vector $\hat{u}_{w/r}(\omega)$ are the frequency content of the rail unevenness as experienced by the axles of the vehicle, while the elements of the vector $\hat{g}(\omega)$ are the forces acting on the coupled track-soil model.

The soil’s stiffness, which is incorporated in the track-soil compliance matrix, is computed in the frequency-wavenumber domain by means of a boundary element method using the Green’s functions of a layered half space.

In a second phase, the free field soil vibrations due to a set of moving axle loads $\hat{g}_k(\omega)$ are computed in the frequency-wavenumber domain:

$$
\tilde{u}_{si}(x, k_y, z, \omega) = \sum_{k=1}^{n} \hat{g}_k(\omega - k_y v) \tilde{h}_{zi}(x, k_y, z, \omega) e^{ik_y y_k}
$$

(2)

where $\tilde{h}_{zi}(x, k_y, z, \omega)$ is the transfer function in the frequency-wavenumber domain between a load on the track at position $y = 0$ and a point in the far field at coordinates $\{x, z\}$. The response in the spatial-time domain is then obtained by applying a double inverse Fourier transform.

**Dynamic soil-structure interaction**

A weak coupling between the incident wave field and the structure is assumed, meaning that the presence of the building has no effect on the vibration generation mechanism, and the free field displacements are applied as an excitation on the coupled soil-structure model.

The subdomain method proposed by Aubry and Clouteau is used to formulate the dynamic soil-structure interaction problem. The displacement of the ground $\hat{u}^g$ is written as a superposition of two displacement fields, $\hat{u}_0$ and $\hat{u}_{sc}$, where $\hat{u}_0$ is due to the incident wave field satisfying a zero displacement boundary condition on the soil-structure interface $\Sigma$, and $\hat{u}_{sc}$ is the displacement field radiated into the soil by the structural motion $\hat{u}_b$ on the interface.

The finite structure is modeled in the frequency domain by a 3D structural finite element method. The equation of motion of the building is:

$$
\begin{bmatrix}
K_{bb} & K_{bf} \\
K_{fb} & K_{ff} + K_{gf}^g
\end{bmatrix} \dot{\mathbf{u}} + \omega^2 \begin{bmatrix}
M_{bb} & M_{fb} \\
M_{fb} & M_{ff}
\end{bmatrix} \mathbf{u} = \begin{bmatrix}
0 \\
\hat{f}_f
\end{bmatrix}
$$

(3)
Figure 1: The soil displacement field $\hat{u}^g$ (a) is decomposed into (b) a displacement field $\hat{u}_0$ and (c) a radiated field $\hat{u}_{sc}$.

where the structural displacements $\hat{u}^s$ are separated to the displacement DOF of the foundation $\hat{u}_f$ and the remaining DOF of the superstructure $\hat{u}_b$. $M$ and $K$ denote the finite element mass and stiffness matrices, and $\hat{K}^g_{ff}$ stands for the frequency dependent impedance matrix of the soil. The force $\hat{f}_f$ is expressed as the total force due to the wave field $\hat{u}_0$, acting on the clamped interface:

$$\hat{f}_f = \int_{\Sigma} N^T \hat{t}(\hat{u}_0) d\Sigma$$

where $\hat{t}(\hat{u}_0)$ denotes the tractions on the interface due to the wave field $\hat{u}_0$, and $N$ denotes the shape functions.

The soil tractions and the impedance of the soil are computed using a 3D boundary element method in the frequency domain, using the Green’s functions of a layered half-space.

A Craig-Bampton modal decomposition method is used in order to reduce computational costs. The displacement vector of the foundation is written as a superposition of rigid and flexible foundation modes $\Phi_f$, while the displacement vector of the superstructure is decomposed into the quasi-static transmission $\Phi^b_f$ of the foundation modes to the superstructure and the flexible modes $\Phi^s$ of the superstructure with a clamped base:

$$\begin{bmatrix} \hat{\alpha}_b \\ \hat{\alpha}_f \end{bmatrix} = \begin{bmatrix} \Phi_b & \Phi^b_f \\ 0 & \Phi_f \end{bmatrix} \begin{bmatrix} \alpha_b \\ \alpha_f \end{bmatrix}$$
This substructuring method has the advantage that the foundation and the superstructure are decoupled. If modifications are made to the building’s superstructure, the forces resulting from the incident wave field do not have to be recomputed.

**The re-radiated noise in the building’s rooms**

This part of the numerical model is described in details in Claim 3.

**Numerical example**

In a numerical example, I demonstrate the use of the total coupled numerical prediction model. The ground borne noise and vibration in a three-story portal frame office building due to the passage of a high-speed Thalys train is modeled.

I demonstrate the effect of dynamic soil-structure interaction on the in-door noise. The effect of dynamic soil-structure interaction can be disregarded if it is assumed that the structure is much stiffer than the ground and only rigid body displacement modes are excited by the vibrations, or if the structure is much softer than the ground and the structure’s presence does not modify the vibration pattern in the soil. The numerical example discusses the validity of these assumptions. It is shown that, for the case of a relative stiff soil with a shear wave velocity of \( c_S = 250 \text{ m/s} \), the disregarding of dynamic soil-structure interaction effect and a direct excitation of the flexible foundation gives a good approximation of both the structural vibrations levels and the re-radiated sound with reasonable computational effort.

I show that the dominant frequencies of the traffic induced acoustic response are basically determined by the first acoustic resonances of the room.

I investigate the effect of wall absorption on the sound pressure, and show that above the first acoustic resonance, a difference of 5 dB can been found between typical wall absorptions for concrete and carpeted walls.

I show that the room dimensions importantly effect the sound pressure level.

The use of the developed methodology is demonstrated by modeling different noise-isolation methods: the base isolation of the building, the application of a floating floor in the rooms of the building and the box-within-box arrangement for noise isolation purposes. The base isolation of the building has been found as the most effective solution for noise isolation.

**Claim 2** I propose the application of the Combined Helmholtz Integral Equation Formalism (CHIEF-method) known from the field of acoustics for the mitigation of fictitious eigenfrequencies in the boundary element method applied to exterior boundary value problems in elastodynamics.

2.1 I show that the material damping of the soil importantly affects the error caused by the fictitious eigenfrequencies. For the case of low material damping, the erroneous effect results in sharp peaks in the frequency content of the impedance.
of the exterior domain. For the case of high material damping, the magnitude of these peaks is significantly attenuated.

2.2 I derive in the thesis that if the representation theorem of elastodynamics written to a surface radiating in the external elastic domain is demanded to hold in an arbitrary point of the interior domain, then the problem of fictitious eigenfrequencies is mitigated.

2.3 I show that the CHIEF method can be effectively used to mitigate the error of fictitious eigenfrequencies when the impedance of cavities embedded into a full space is computed. A convergence study on the number of internal CHIEF points shows that the method is robust, and a relative few additional linear equations are necessary to overcome the problem.

Publications related to this claim are: NJ1, IC1.

Introduction

The problem of fictitious eigenfrequencies is related to the boundary element method used for exterior problems. Exterior problems are the computation of radiated wave fields to exterior unbounded domains by vibrating surfaces; the computation of scattered wave fields from rigid or elastic surfaces; or the determination of the impedance of cavities or foundations embedded into an infinite elastic domain. As the frequency domain boundary element method is applied to these exterior problems, undesired computational errors can occur. These errors appear close to the eigenfrequencies of the interior domain, i.e. the cavity or soil excavation filled with the external material.

When dynamic soil-structure interaction problems are considered in the higher frequency range, where the modal density of the embedded foundations is large, the effect of these fictitious eigenfrequencies becomes very important. Therefore, different upgraded solution techniques mitigating the effect of the fictitious eigenfrequencies need to be developed.

The representation theorem of elastodynamics relates the displacement field \( u_i(\xi, \omega) \) in a closed domain \( \Omega_{\text{int}} \) or in an infinite unbounded exterior domain \( \Omega_{\text{ext}} \) to displacements \( u_j(x, \omega) \) and tractions \( t_j(x, \omega) \) on the domain’s boundary \( \Sigma \).

For the case of exterior problems, where the external wave field radiated by or scattered from the boundary \( \Sigma \) to the infinite external domain \( \Omega_{\text{ext}} \) is searched, the representation theorem can be written in the form

\[
c(\xi)u_i(\xi, \omega) = \int_{\Sigma} t_j^{\text{ext}}(u)(x, \omega)u_j^G(\xi, x, \omega)d\Sigma - \int_{\Sigma} t_j^{\text{ext}}G(\xi, x, \omega)u_j(x, \omega)d\Sigma \quad (6)
\]

where \( x \) denotes points on the boundary, \( \xi \) stands for an arbitrary point in the full space,
\[ u_{ij}^G(\xi, x, \omega) \] and \( t_{ij}^G(\xi, x, \omega) \) denote the Green’s displacement and traction tensors, and

\[
c(\xi) = \begin{cases} 
1 & \text{if } \xi \in \Omega_{\text{ext}} \\
0.5 & \text{if } \xi \in \Sigma \\
0 & \text{if } \xi \in \Omega_{\text{int}} 
\end{cases}
\]  \tag{7}

In equation (6) the surface tractions are related to the external unit normal vector \( n_{\text{ext}}(x) \) on the boundary \( \Sigma \), as shown in figure 2a. This external normal vector points out from the external domain.

For the case of interior problems, where the displacement field radiated into the bounded internal domain \( \Omega_{\text{int}} \) is of interest, the representation theorem can be written as

\[
c(\xi)u_i(\xi, \omega) = \int_{\Sigma} t_{ij}^{\text{int}}(u) (x, \omega) u_{ij}^G(\xi, x, \omega) d\Sigma - \int_{\Sigma} t_{ij}^{\text{int}}G(\xi, x, \omega) u_j(x, \omega) d\Sigma \]  \tag{8}

where

\[
c(\xi) = \begin{cases} 
1 & \text{if } \xi \in \Omega_{\text{int}} \\
0.5 & \text{if } \xi \in \Sigma \\
0 & \text{if } \xi \in \Omega_{\text{ext}} 
\end{cases}
\]  \tag{9}

and the tractions are defined using the internal unit normal \( n_{\text{int}}(x) \).

In the thesis, based on the work of Schenck related to the acoustic radiation problem, I demonstrate, how the problem of fictitious eigenfrequencies arise in elastodynamics. I show that the exterior surface integral equation with zero surface traction boundary conditions has a non-zero displacement solution at the eigenfrequencies of the internal domain with zero displacement boundary conditions. Therefore, for any traction boundary condition at the eigenfrequency of the internal domain, the solution of the exterior radiation problem is non unique.
Mitigation of the fictitious eigenfrequencies

In order to overcome the problem of fictitious eigenfrequencies in the BEM, several efficient solution methods have been proposed. One of the most fruitful improved integral methods are based on an integral equation that is a combination of the original surface integral and its normal derivative with respect to the surface normal vector. The modified Burton and Miller method, implemented in the software MISS used to compute the soil’s impedance in the practical applications of the thesis, is of this kind.

The modified integral equation can be written as

\[
\frac{1}{2} \left( u_i(\xi, \omega) + \alpha \frac{\partial u_i(\xi, \omega)}{\partial \xi_n} \right) = \int_{\Sigma} t_j(u)(x, \omega) \left( u^G_{ij}(\xi, x, \omega) + \alpha \frac{\partial u^G_{ij}(\xi, x, \omega)}{\partial \xi_n} \right) d\Sigma
- \int_{\Sigma} \left( t^G_{ij}(\xi, x, \omega) + \alpha \frac{\partial t^G_{ij}(\xi, x, \omega)}{\partial \xi_n} \right) u_j(x, \omega) d\Sigma \tag{10}
\]

where \( \alpha \) is the coupling parameter. The fictitious eigenfrequencies of equation the original equation are those of the interior domain clamped at the interface \( \Sigma \), while the critical frequencies of the equation containing the derivatives are the eigenfrequencies of the internal domain with free boundary conditions. As these eigenfrequencies never coincide, the linear combination (10) yields a unique solution for all wavenumbers.

In the implementation method proposed by Clouteau, the normal derivatives are approximated numerically by means of finite difference schemes:

\[
\frac{\partial u_i(\xi, \omega)}{\partial \xi_n} \approx \frac{u_i(\xi, \omega) - u_i(\xi^-, \omega)}{h} \tag{11a}
\]

\[
\frac{\partial u^G_{ij}(\xi, x, \omega)}{\partial \xi_n} \approx \frac{u^G_{ij}(\xi, x, \omega) - u^G_{ij}(\xi^-, x, \omega)}{h} \tag{11b}
\]

\[
\frac{\partial t^G_{ij}(\xi, x, \omega)}{\partial \xi_n} \approx \frac{t^G_{ij}(\xi, x, \omega) - t^G_{ij}(\xi^-, x, \omega)}{h} \tag{11c}
\]

where the points \( \xi^- = \xi + hn^\text{ext} \) are located on the surface \( \Sigma^- \) that is in the internal domain \( \Omega_{\text{int}} \), at a distance \( h \) from the original surface \( \Sigma \).

The discretization of equation 10 with the finite difference approximations results in a system of equations written in the form:

\[
\left( H + \frac{\alpha}{h + \alpha} H^- \right) u = \left( G + \frac{\alpha}{h + \alpha} G^- \right) t \tag{12}
\]

where the \( H \) and \( G \) matrices contain the Green’s functions with the source points \( \xi \) located on the boundary surface \( \Sigma \), while the \( H^- \) and \( G^- \) matrices contain the Green’s functions with the source points \( \xi^- \) located on the internal surface \( \Sigma^- \).
An other family of the improved integral equations is assembled by simultaneously writing the interior and exterior surface integrals or by utilizing the integral equation in points of the interior region. I show in the thesis, how the method of Schenck, originally developed for the acoustic radiation problem, can be used in the field of elastodynamics.

The CHIEF method (the name is from Combined Helmholtz Integral Equation Formulation) is based on the fact that if we demand that the solution of the external problem should satisfy the integral equation for $\xi \in \Omega_{\text{int}}$, where $\xi$ is not located on a zero value of an internal displacement mode, then the uniqueness of the solution is ensured.

The combined discretized integral equation takes the form

$$\begin{bmatrix} H_s & H_i \\ G_s & G_i \end{bmatrix} \begin{bmatrix} u \\ t \end{bmatrix} = \begin{bmatrix} G_s & G_i \end{bmatrix} \begin{bmatrix} t \end{bmatrix}$$

(13)

where the matrices $H_s$ and $G_s$ contain the Green’s functions with $\xi \in \Sigma$, and the matrices $H_i$ and $G_i$ contain the Green's functions with $\xi \in \Omega_{\text{int}}$. This system of equations consists of $N_s + N_t$ equations for $N_s$ unknowns, and is therefore solved by means of a least mean squares method.

**Numerical examples**

Based on simple numerical examples, I demonstrate how the proposed CHIEF method mitigates the fictitious eigenfrequencies. In the numerical example, the modal impedance curves of two cavities embedded in a two-dimensional elastic full space are computed with both mitigation methods.

I show that the effect of the fictitious eigenfrequencies is more pronounced if the material damping of the soil is small. Comparing the two mitigation methods, it is shown that the CHIEF method is computationally cheaper at the same accuracy. The convergence study on the number of internal CHIEF points demonstrates that the method is robust, and it works well even with a relative small number of additional linear equations.

**Claim 3** I have developed a spectral finite element formalism for the computation of noise radiation into closed rooms.

3.1 I derived a spectral finite element formalism to describe the sound radiation into closed spaces by vibrating walls and floors. The formalism is derived for the case of rectangular rooms with arbitrary acoustic impedance distribution over the walls.

3.2 I show that the impedance distribution over the walls importantly affects the effectiveness of the proposed spectral finite element method. I show that for the case of low wall absorption or uniform impedance distributions, the system matrix of the method can be truncated to diagonal, what leads to a very effective fast method even for higher frequencies.
3.3 I show that the model is capable to investigate wall openings as absorbing boundary conditions. In this case, the effect of coupling between different acoustic modes plays an important role in the sound radiation, and the application of the fast method is not possible.

Publications related to this claim are: IJ1, IJ2, IJ3, IC6, IC7, IC8, IC9, IC10.

Introduction

Vibrating walls of closed or open rooms radiate sound into the internal space. The computation of the sound radiation involves the solution of the equations of the sound field, taking into account the wall vibrations as boundary conditions. In the last decades, the acoustic finite element and boundary element methods have become the most popular tools for the solution of this problem. Their common drawback is that they divide the room’s volume or boundary surfaces into small elements, interpolate the acoustic variables over the elements with polynomial shape functions and write systems of linear equations containing the field variables in the element nodes. The size of the finite or boundary elements have to be much smaller than the acoustic wavelength, therefore, at high frequencies the application of the method is computationally expensive.

In the recent years, a new method based on the Rayleigh integral has been proposed that avoids the matrix inversion in the solution process, and can yield an efficient tool even for higher frequencies. This method is based on the facts that the room’s shape is rectangular and the walls’ absorption is large enough in order to model the sound reflection from the adjacent walls with a finite series.

In my thesis, I propose a complementary method that can be used very efficiently if the room’s shape is rectangular, and the absorption of the walls is low.

Finite element methods express the internal sound field as a superposition of simple local or global shape functions, spectral finite element methods use exact solutions of the problem as shape functions. The method introduced here writes the internal sound pressure as a superposition of acoustic room modes. These modes are computed analytically, therefore, no mesh is needed for the computations, and the dispersion error is avoided. The introduced methodology is capable to handle the wall absorption in the form of impedance boundary condition. For the case of low wall absorption or uniform impedance distribution on the walls, the method leads to a direct boundary integral representation of the internal sound pressure, so the solution of a linear system of equations is avoided.

The spectral finite element method for rectangular rooms

The equations of the sound field written to a rectangular domain $\Omega$ with a closed boundary $\Gamma$ are solved in the frequency domain:

$$\frac{i \omega}{c^2} \hat{p}(\mathbf{x}, \omega) + \rho_0 \nabla \hat{v}_a(\mathbf{x}, \omega) = 0$$

(14)
and

\[ \rho_0 i \omega \hat{v}_a(x, \omega) + \nabla \hat{p}(x, \omega) = 0 \]  

(15)

where \( x \) denotes the space variable and \( \omega \) denotes the angular frequency. \( p_a \) stands for the sound pressure and \( v_a \) for the particle velocity. \( \rho_0 \) and \( c \) denote the mass density of the air and the speed of sound, respectively.

The boundary condition of the partial differential equations can be written as

\[ \hat{p}(x, \omega) = z_a(x, \omega)(\hat{v}_b(x, \omega) - \hat{v}_a(x, \omega)) n(x) \quad x \in \Gamma_a \]  

(16)

where \( z_a \) is the acoustic impedance of the walls, \( v_b \) denotes the structural velocity and \( n \) stands for the internal normal of the boundary.

In the spectral finite element method, we assume that the pressure \( \hat{p}(x, \omega) \) can be written as a superposition of modes in the domain \( \Omega_a \):

\[ \hat{p}(x, \omega) = \sum_n \Psi_n(x) \hat{Q}_n(\omega) \]  

(17)

where \( \Psi_n(x) \) denotes a pressure mode of the internal domain with a corresponding eigenfrequency \( \omega_n \), and \( \hat{Q}_n(\omega) \) denotes the mode’s participation factor or modal coordinate. Introducing this assumption into the weak form of the wave equation, we get the basic matrix equation of the spectral finite element method:

\[ (\Lambda + i k D - k^2 I) Q = F \]  

(18)

where the column vector \( Q \) contains the modal coordinates, \( \Lambda = \text{diag} \{ \omega_n^2 / c^2 \} \) is the diagonal generalized acoustic stiffness matrix containing the modal wave numbers, \( D \) is the generalized damping matrix, \( I \) denotes the unit matrix and \( F \) stands for the generalized acoustic load vector. The \( n \)-th element \( F_n \) of this vector is given by

\[ F_n = i k z_0 \int_{\Gamma_a} \Psi_n(x) \hat{v}_a(x, \omega) n(x) d\Gamma \]  

(19)

and the elements of the damping matrix \( D \) are given as

\[ D_{nm}(\omega) = \int_{\Gamma_a} \frac{\Psi_n(x) \Psi_m(x)}{\bar{z}_a(x, \omega)} d\Gamma \]  

(20)

where

\[ \bar{z}_a(x, \omega) = \frac{z_a(x, \omega)}{z_0} = \frac{z_a(x, \omega)}{\rho_0 c} \]  

(21)

is the normalized acoustic impedance.

The spectral finite element method provides a computationally efficient way for solving the acoustic wave equation in a closed acoustic domain if the modes \( \Psi_n \) of the domain are a-priori known.
For a general problem with arbitrary geometry, this is unfortunately not the case. However, for practical applications for the computation of ground borne noise in buildings with conventional shoe-box shaped rooms, the method is a very efficient way of noise prediction.

For a rectangular room domain defined by the coordinates \( 0 < x < L_x, 0 < y < L_y \) and \( 0 < z < L_z \), the pressure modes \( \Psi_n(x) \) with rigid boundary conditions can be written in the form

\[
\Psi_n(x) = \Psi_n(x, y, z) = B_n \cos \left( \frac{\pi l_n x}{L_x} \right) \cos \left( \frac{\pi l_n y}{L_y} \right) \cos \left( \frac{\pi l_n z}{L_z} \right)
\]

where \( B_n \) stands for the modal amplitude and the \( l_{nx}, l_{ny}, l_{nz} \) dimensionless numbers give the number of half wavelengths along the three coordinate axes.

**Simplifications of the formalism**

Considering that the matrices \( \Lambda \) and \( I \) of equation (18) are diagonal, the structure of the damping matrix \( D \) very importantly influences the effectivity of the method.

I show in a numerical example that the damping matrix can be truncated to diagonal under some circumstances.

At relative low frequencies, the acoustic impedance can be computed from the walls’ acoustic absorption coefficient \( \alpha \), which gives the ratio of the absorbed and the incident acoustic energy when a normal incident acoustic plane wave is reflected from the surface. The relation between the acoustic absorption coefficient and the wall’s impedance can be approximated as

\[
Z_a = \rho_0 c \frac{1 + \sqrt{1 - \alpha}}{1 - \sqrt{1 - \alpha}}
\]

I show that for the case of the non-zero wall absorption values, the structure of the matrix is sparse, it has a dominating diagonal part and small magnitude off-diagonal elements. These off-diagonal elements are responsible for the coupling between different modes.

For the case of a uniform impedance distribution over the walls, the truncation does not lead to observable error in the modal coordinates \( Q_n \). For this case, the modal coordinates can be directly expressed as

\[
Q_n = \frac{i k z_0 \int_{\Gamma} \Psi_n(x) \hat{v}_n(x, \omega) n(x) d\Gamma}{k^2 + i k \int_{\Gamma} \Psi_n^2(x) \hat{v}_n(x, \omega) d\Gamma - k^2}
\]

where, for the uniform impedance distribution, the integral in the denominator of the right-hand side of the equation can be expressed analytically.
Modeling of wall openings by absorbing boundary condition

As the described spectral finite element method can only handle shoe-box shaped domains with a closed boundary, it is not able to handle the case of wall openings. However, as wall openings can often be modeled with absorbing boundary condition, their influence on the re-radiated noise in the room can be approximated.

In this practically important case, the wall impedance distribution has to be approximated as the union of rectangular faces with different wall absorption coefficients. I show in the thesis that for this case, the structure of the damping matrix is more dense, and the truncation does not lead to acceptable accuracy of the method. I show that due to the wall openings, the effect of modal coupling is very significant, even for the zero absorption case $\alpha = 0$. It is shown that in typical cases when the total surface of the wall openings is about 10% of the total surface of the walls and floors, the effect of the wall openings on the internal pressure response is much more important than the absorbing properties of the room’s surfaces.

4.1 Application of the results

As a practical application, the described methodology has been used to determine the vibration specifications of the new Budapest underground line m4.

The construction works of the new line m4 have started in the end of 2006. At the moment when the present thesis is submitted, the tunnel of the new line is being drilled and simultaneously, the concrete tunnel wall is being constructed. In the following months, the structural planning of the railway track system will be finalized.

The tunnel of the line m4 is a deep-bored tunnel. Its length is 7300 m, and the total line contains 10 surface-cut stations. The tunnel depth varies between xxx m and xxx m, the deepest parts run under the Danube. Due to small depth of the tunnel, significant ground-borne noise and vibration is expected in existing nearby buildings. Therefore, the planning of the vibration isolation system is carried out with special attention to the noise and vibration problem.

The Laboratory of Acoustics at the Department of Telecommunications of the Budapest University of Technology and Economics took part in the design of the vibration isolation system, and the methodology described in the present thesis has been applied in the planning. The task of the Laboratory was to define the vibration specifications at the railway tunnel. This specification means that the maximal allowed tunnel vibration velocity has been determined in such a way that the ground-borne vibration and noise in nearby buildings do not exceed the limits given by the Hungarian standards.

The definition of the tunnel vibration specification has been carried out in the following steps:

- First, a measurement campaign has been organised. The aim of the measurements was threefold.
1. The material and geometrical properties of the soil has been measured in three locations in Budapest by means of a Spectral Analysis of Surface Waves method. These parameters are essential input of the numerical model.

2. Transfer functions have been measured between the tunnel of the existing underground lines m2 and m3 and nearby buildings. These measurements have been used for validation purposes.

3. The typical vibration velocity of the tunnel’s wall has been recorded in the tunnel of the lines m2 and m3 during the passage of metro trains.

- In a second step, the coupled numerical model has been validated, using the transfer functions measured at the lines m2 and m3. During the validation, the coupled numerical model has been used to compute the vibration transfer between the tunnel’s wall and the surrounding buildings, and these results have been compared to the measured transfer functions.

- In a last step, the validated numerical model has been used to compute the transfer between the tunnel of the line m4 and nearby buildings. These transfer functions were used to maximize the vibration velocity of the tunnel wall.

Publications

International journal papers


IJ3 P. Fiala, S. Gupta, G. Degrande, and F. Augusztinovich. A comparative study of different measures to mitigate ground borne noise and vibration from underground trains. *Journal of Sound and Vibration*. Submitted for publication

National journal papers


International conference papers


