Nonlinear wave propagation and ultrashort pulse compression in step-index and microstructured fibers

Ph.D. thesis

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Foreword

I had been working for the Japanese Furukawa Electric Co. Ltd. for one year when I started to work on my PhD in 2001. My company allowed me to join to the Atomic Physics Department of BUTE and prepare a PhD degree besides my regular subjects at the company. At the beginning, Prof. Péter Richter and Dr. László Jakab gave me a helping hand to start to work in their department. I would like to thank them for providing me the invaluable opportunity to prepare for my PhD degree and to get involved in scientific research projects which could improve my scientific view, understanding and knowledge.

I would also like to thank the management of Furukawa Electric Institute of Technology Ltd. (especially Dr. Gyula Besztercey) for letting me continue a postgraduate education, supporting me in my work and allowing to continue the research on similar topics at the company after finishing my PhD scholarship, that I had done in the University.

At the beginning of my scholarship, I was interested in the numerical solutions of the Nonlinear Schrödinger Equation (NLSE) as a mathematical interpretation of the nonlinear wave propagation in single mode optical fibers (SMF). This interest had determined the direction of my initial research on optical fibers in the University. Here, our main focus was to look for possible applications of the numerical results in such parameter regions which are difficult to achieve experimentally.

I was very glad to have a good contact with Professor István Frigyes during this work and later on. I thank him for his ideas, his critical remarks on the results and his time for our long discussions.

In 2004, I met Dr. Róbert Szipőcs in the Research Institute for Solid State Physics and Optics, Hungarian Academy of Sciences. He and his group had been working on optical fiber related experiments in their laboratory for three or four years by then. I was happy to work with such talented young scientists as Júlia Fekete, Ákos Bányász and Péter Antal who could prepare experiments to my simulations and the discussions with them led to collaborate papers. I thank Júlia
Fekete and Péter Antal their useful comments in connection with my dissertation.

This collaboration developed fruitfully until 2005 when we formed a consortium including Furukawa, Dr. Szipőcs’s company (R&D Ultrafast Lasers Ltd.) and six Hungarian research institutes. This consortium was given the name FEMTOBIO and focuses mainly on the biomedical applications of ultrashort laser pulses. My field of research in this group is developing ultra-short and high intensity fiber laser and amplifier as a cost effective seed for recent and future applications. This activity is strongly related to my previous works.

I would like to thank Dr. Róbert Szipőcs for the opportunity of working in his group and in his laboratory. I would also like to thank him for the many discussions as well about science as well as about general subjects from which I could learn so much.

Finally, I thank my family for their support, motivation and love.

Zoltán Várallyay
Budapest, March, 2007
## List of acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>1PA</td>
<td>One-Photon Absorption</td>
</tr>
<tr>
<td>2D</td>
<td>Two Dimensional</td>
</tr>
<tr>
<td>2PFM</td>
<td>Two-Photon Fluorescent Microscopy</td>
</tr>
<tr>
<td>3D</td>
<td>Three Dimensional</td>
</tr>
<tr>
<td>AO</td>
<td>Acousto-Optic</td>
</tr>
<tr>
<td>AOM</td>
<td>Acousto-Optic Modulator</td>
</tr>
<tr>
<td>CN</td>
<td>Crank-Nicolson (method)</td>
</tr>
<tr>
<td>CPA</td>
<td>Chirped Pulse Amplifier</td>
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<tr>
<td>CW</td>
<td>Continuous Wave</td>
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<tr>
<td>DCF</td>
<td>Dispersion Compensating Fiber</td>
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<tr>
<td>DFF</td>
<td>Dispersion Flattened Fiber</td>
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<tr>
<td>DIM</td>
<td>Direct Intensity Modulation</td>
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<tr>
<td>DSF</td>
<td>Dispersion Shifted Fiber</td>
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<tr>
<td>EDFA</td>
<td>Erbium Doped Fiber Amplifier</td>
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<tr>
<td>FI</td>
<td>Faraday Isolator</td>
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<tr>
<td>FWHM</td>
<td>Full Width at Half Maximum</td>
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<tr>
<td>FWM</td>
<td>Four-Wave Mixing</td>
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<tr>
<td>GDD</td>
<td>Group-Delay Dispersion</td>
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<tr>
<td>GTI</td>
<td>Gires-Tournois Interferometer</td>
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<tr>
<td>GVD</td>
<td>Group Velocity Dispersion</td>
</tr>
<tr>
<td>HC</td>
<td>Hollow Core</td>
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<tr>
<td>HNL</td>
<td>Highly-Nonlinear</td>
</tr>
<tr>
<td>IM</td>
<td>Intensity Modulation</td>
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<tr>
<td>KdV</td>
<td>Korteweg de Vreis (equation)</td>
</tr>
<tr>
<td>LMA</td>
<td>Large-Mode Area</td>
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<tr>
<td>MCVD</td>
<td>Modified Chemical Vapor Deposition</td>
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<td>MMF</td>
<td>Multi-Mode Fiber</td>
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<tr>
<td>MOF</td>
<td>Microstructured Optical Fiber</td>
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<tr>
<td>Abbreviation</td>
<td>Definition</td>
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<tr>
<td>MZ</td>
<td>Mach-Zehnder (interferometer)</td>
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<tr>
<td>NA</td>
<td>Numerical Aperture</td>
</tr>
<tr>
<td>NLS</td>
<td>Nonlinear Schrödinger (equation)</td>
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<tr>
<td>PBG</td>
<td>Photonic Bandgap</td>
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<tr>
<td>PC</td>
<td>Photonic Crystal</td>
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<tr>
<td>PCF</td>
<td>Photonic Crystal Fiber</td>
</tr>
<tr>
<td>PCFs</td>
<td>Photonic Crystal Fibers</td>
</tr>
<tr>
<td>PML</td>
<td>Perfectly Matched Layer</td>
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<tr>
<td>QW</td>
<td>Quarterwave</td>
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<tr>
<td>ROF</td>
<td>Radio Over Fiber</td>
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<tr>
<td>SBS</td>
<td>Stimulated Brillouin Scattering</td>
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<tr>
<td>SM</td>
<td>Single-Mode</td>
</tr>
<tr>
<td>SMF</td>
<td>Single-Mode Fiber</td>
</tr>
<tr>
<td>SOA</td>
<td>Semiconductor Optical Amplifier</td>
</tr>
<tr>
<td>SPIDER</td>
<td>Spectral-Phase Interferometry for Direct Electric-field Reconstruction</td>
</tr>
<tr>
<td>SPM</td>
<td>Self-Phase Modulation</td>
</tr>
<tr>
<td>SRS</td>
<td>Stimulated Raman Scattering</td>
</tr>
<tr>
<td>TOD</td>
<td>Third Order Dispersion</td>
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Chapter 1

General introduction

1.1 Brief history

In the middle of the 1800s, some experiments demonstrated that light could be conducted through a curved stream of water or curved glass rod by total internal reflection (1841 Daniel Colladon, 1842 Jacques Babinet, 1854 John Tyndall). The first all-glass optical fiber which had a higher index core surrounded by the lower index cladding was devised only almost a hundred years later. Although uncladded glass fibers were fabricated in the late 1920s (J. L. Baird, British Patent 285,738 (1928)) the realization that optical fibers benefit from a dielectric cladding was discovered in the 1950s when van Heel and Hopkins and Kapany published independently in the same issue of Nature [1, 2].

The modern ages of optical fibers start from the 1960s with the appearance of the first lasers. These fibers were extremely lossy but new suggestion on the geometry with single mode operation [3] which was obtained by theoretical calculations based on the Maxwell-equations, and the development of a new manufacturing process [4] (this was the so called MCVD process) led to the achievement of the theoretical minimum of loss value (0.2 dB/m at 1.5 µm) still in 1979 [5].

The investigation of nonlinear phenomena in optical fibers have been continuously gained by the decreasing loss. Loss reduction in fibers made possible the observation of such nonlinear processes which required longer propagation path length at the available power levels in the 1970s. Most of the investigations related to nonlinear phenomena in optical fiber were executed in the Bell Laboratories. Stimulated Raman Scattering (SRS) and Brillouin scattering (SBS) were studied first [6, 7]. Optical Kerr-effect [8], parametric four-wave mixing (FWM) [9] and
self-phase modulation (SPM) [10] were observed later. The theoretical prediction of optical solitons as an interplay of fiber dispersion and fiber nonlinearity was done as early as 1973 [11] and the soliton propagation was demonstrated seven years later in a single mode optical fiber [12]. The first soliton fiber laser operating around the 1.55 µm wavelength region was demonstrated in 1984 [13].

In the 1980s, the pulse compression procedure was advanced by the exploitation of fiber nonlinearity. This possibility led to the generation and control of ultrashort optical cycles at the beginning of 1980s [14] and pulses as short as 8 fs were generated by 1985 [15] when amplified pulses with 40 fs initial duration were injected into a single mode fiber utilizing the SPM effect for spectral broadening. In 1987, 6 fs pulses were generated in single mode fiber compensating the cubic phase distortions by prisms and diffraction gratings [16]. Sub-5-fs pulses, however, were obtained only at the end of the 1990s, applying 2 mm long SMF in a cavity-dumped laser oscillator [17]. The generation of ultrashort pulses shorter than 5 fs was also demonstrated in gas filled hollow core fiber (or capillary) as a nonlinear medium [18].

The common feature of the previous techniques is that they require laser pulses at energy levels well above 10 nJ which is difficult to obtain directly from ultrashort pulse laser oscillators without expensive amplification stages. In pulse compression, the breakthrough was the appearance of small core area photonic crystal fibers (PCF) [19] in which SPM can be much larger than in conventional fibers, therefore sub-nJ pulses could be compressed.

PCF is a new class of fibers in that the cladding is formed of a periodic array of holes like a crystal lattice in which the core is formed by the absence of the centermost air hole and/or also the first or more surrounding rings of holes. Holes form the cladding and run along the entire fiber (See figures and further explanation in Chapter 5).

The idea to prepare PCFs as their name show, goes back to the birth of photonic crystals (PC). The ability to tailor structures on the micro and nano scale in the late 1980s provided the opportunity to investigate the relation between the structure of matter and light. Within this framework, the photonic crystal emerged and became an extensively studied scientific area since 1987 when Eli Yablonovitch advised the idea of “photonic bandgap structures” [20]-[23]. 2D and 3D photonic crystal structures are periodic artificial, dielectric structures in which light behaves the same way as electron waves in natural crystals. Under suitable circumstances, the photonic crystal may open up a frequency band in which the propagation of electromagnetic wave is forbidden. This frequency band is generally called photonic bandgap (PBG) as a nomenclature borrowed from solid-state physics.
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PCFs appeared in the middle of the 1990s [19] and the special class of hollow-core PBG fibers which have a lower index core and where the light is guided by the photonic bandgap effect was demonstrated in 1998 [21, 25]. These type of fibers have many novel properties.

PCFs can have a large waveguide contribution to the material dispersion, therefore anomalous dispersion can be achieved below the 1.27 µm region [26, 27]. This allows the dispersion management of the fibers even in the visible region with a proper design [28]. This property of the fiber was used recently in Ytterbium fiber laser oscillators for dispersion compensation [29], because other fibers have normal dispersion at the radiation wavelength of Ytterbium but PCF can have an anomalous one.

PCFs revolutionized the nonlinear optics since the end of 1990s. These types of fibers opened up new areas in physics such us ultra-broad supercontinuum generation in silica microstructured fiber even in the visible region [28]. The process of continuum and super-continuum generation had been known for many years but initial experiments used very high power lasers (> µJ) with ultra short pulses (< 1 ps), focused into glass, sapphire or even water. The breakthrough provided by PCF in 2000 was the design of the fiber that allowed the use of much lower powers to produce the continuum effect and the zero dispersion wavelength of fiber might be close to the pump wavelength of Ti:Sapphire. The continuum is also particularly broad, often spanning over two optical octaves. These novel light sources are cheap and effective sources for spectroscopy, frequency metrology and optical coherent tomography.

Ultrashort pulse generation via nonlinear compression is also reconsidered with much lower optical pulse energies (< 1 nJ). Tenfold pulse compressions were demonstrated in a few experiments [30, 31] at nJ or sub-nJ optical pulse energies, which resulted in typical compressed pulse durations of 20 to 35 fs starting from 100-150 fs. The length of the used microstructured fiber was only a few centimeter. The possibility of compressing supercontinuum generated in a 5 mm long microstructured fiber using a rather complicate and expensive adaptive compression technique based on spectral-phase interferometry for direct electric-field reconstruction (SPIDER) was also reported [32]. The obtained pulse width was 5.5 fs starting from 15 fs transform limit.

A novel and effective pulse compression method is recently based on soliton-effect compression in a very new type of microstructured fibers which have sub-micron core diameter. This waveguide is called nano-wire [33-37]. These waveguides provide suitable conditions for broadband soliton-effect compression of ultrashort pulses. In a recent experiment, 6.8 fs, few-cycle duration was demon-
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strated starting from 70 fs \[38, 39\] and computations show the reliable single-cycle compression of sub-nJ pulses (< 3 fs). This compression technique in photonic nanowires provides a simple method for the self-compression of sub-nJ pulses to few-cycle durations without any additional optical elements.

Higher intensity pulse compression (> 5 – 10 nJ) however, can not be carried out by silica core nanowires or small-core area microstructured fibers because the photon-density per volume unit would be such a high value that could damage the fiber itself (well above 10-20 kW peak power of the pulse). A possible alternative in this energy region are the large-mode area (LMA) photonic crystal fiber \[40\] and the above mentioned hollow core photonic bandgap fibers.

LMA fibers with step-index profiles are usually multimode, cares must be taken of the suppression of higher-order mode excitation to obtain near-diffraction-limit outputs. Novel PCF technology however yields the possibility of carrying only the fundamental mode in an LMA fiber \[40\]-\[42\]. Using LMA PCF higher than 1000 \(\mu m^2\) effective core area was achieved recently without losing the single mode operation in a fiber amplifier \[43\]. This fiber reduces the nonlinearity more than 100 times compared to a single mode fiber which has an effective core area of approximately 80 \(\mu m^2\) at 1550 nm. The dispersion of LMA fibers, however, can not be tailored using structural modifications. LMA fibers have about the same magnitude of dispersion as material dispersion does. LMA fibers are therefore suitable for chirped pulse amplifiers (CPA) \[44\]-\[46\], but not for delivering ultrashort pulses (below 1.3 \(\mu m\)) where material dispersion changes rapidly. CPA technique was first proposed in doped step-index fibers in order to avoid the harmful effect of nonlinearity and fiber damage \[47\].

PBG fibers enable light guidance in a low refractive index material such as air, vacuum or gas. PBG fibers in which a high percentage of the light (> 95%) can be guided in a hollow core, offer a significantly reduced nonlinearity with respect to silica core fibers \[48\]. In PBG fibers, achieving lower losses than conventional doped solid core fibers are also possible \[49\] and light transmission at wavelengths where the material absorption would otherwise be prohibitive \[50, 51\]. Photonic bandgap fibers have a typical third order function-like dispersion property \[52, 53\] which increases continuously from the normal dispersion region to the anomalous region with an inflection point at the middle of the bandgap. This behavior is independent of the constituting materials, this is a basic feature of the bandgap structure \[53\]. This dispersion property of PBG fiber, however, is somewhat affected by the waveguide contribution \[54\] and also by harmful effects such as mode anti-crossing \[55, 56\]. The problem of avoiding cladding or surface modes (mode anti-crossing effect) and tailoring the dispersion properly by the waveguide contri-
bution or by the introduction of resonant structures are the subjects of recent and future researches.

We note here that designing photonic crystal structures by using finite element numerical method was already done by Hungarian scientist in the Central Research Physics Institute, Budapest \cite{57}.

A detailed overview on nonlinear fiber optics and their applications covering almost all the physical effects and their theories are given in Ref. \cite{58} and \cite{59}. Microstructured fibers are discussed in Ref. \cite{60} and \cite{61} extensively.

### 1.2 Scope of the dissertation

The matter of the dissertation can be divided to three topics which are connected to each other by the investigation of nonlinear wave propagation in single-mode or microstructured fibers. Furthermore, all chapters are dealing with finding suitable pulse or fiber parameters for the realization of an optical system.

In the first chapter, we investigate the microwave modulated light transmission through single mode fiber with the inclusion of nonlinearity. This chapter is using the nonlinearity as an advantageous effect compensating dispersion caused modulation suppression and investigate the expected soliton propagation of intensity modulated signals.

We continue to use our numerical tool to design such pulse delivery system applied in a two-photon fluorescent microscope (2PFM) which is based on the chirped pulse transmission through single mode, step-index fiber and recompression. Avoiding the harmful effects of nonlinearity is the aim in this system. The simulation and corresponding measurements can predict the usable pulse energy, applied prechirp and compression chirp allowed to deliver through a single mode fiber in order to achieve the two-photon excitation after.

The third part of the dissertation deals with pulse compression using microstructured fibers. Prechirp, in this chapter, has also a significant role to avoid dispersion caused temporal distortions and fine tune the broadening of the spectrum by changing the peak power of the pulse. Finding the optimum input and output chirps is the aim here to obtain sub-6 fs compressed pulses. The quality and duration of the compressed pulses are investigated also in dispersion modified microstructured fibers.
Chapter 2

Theory of optical fibers

2.1 Optical waveguides and fibers

Optical waveguides constitute a general structure in that typically a higher refractive index region is surrounded by a lower refractive index dielectric material. This arrangement assures the light propagation in the higher refractive index part of the waveguide by total internal reflection. The waveguide mechanism works until the angle of incidence at the boundary of the two dielectric materials is higher than the critical angle of the total internal reflection. Let the higher index material have a refractive index of $n_h$ and the surrounding, low index one $n_l$. The critical angle incidence for internal reflection can be given by

$$\Phi_c = \sin\left(\frac{n_l}{n_h}\right). \quad (2.1)$$

Step-index fibers are waveguides with core and cladding dielectric components as higher and lower refractive index parts, respectively. They show a cylindrical symmetry and the core and cladding regions are placed concentrically (See Figure 2.1).

The numerical aperture of the fiber (the maximum space angle with that the light can be injected into a fiber) can be derived with the assumption of $\Phi$ is the critical angle of the total internal reflection ($\Phi_c$). Namely, $\Phi$ can be larger than $\Phi_c$ (See explanation in Fig. 2.1). In this case $\Theta_2$ can be obtained by subtracting $\Phi_c$ from $\pi/2$ (neglecting the small bending of the fiber) and using the Snellius-Descartes law at the fiber input for obtaining $\Theta_1$ (See Fig. 2.1). The numerical aperture can be expressed by the refractive index of the core and cladding using
Figure 2.1: Structure of optical fibers with a propagating mode represented as a ray with incident angle $\Theta_1$, refracted angle $\Theta_2$ and total internal reflection with an angle of $\Phi$ at the boundary of core and cladding. The refractive index of the core and cladding are $n_h$ and $n_l$, respectively.

some trigonometric identities during the derivation:

$$NA = \sin(\Theta_1) = \frac{1}{n_{\text{env}}} \sqrt{n_h^2 - n_l^2}$$  \hspace{1cm} (2.2)

where $n_{\text{env}}$ is the refractive index of the environment surrounding the fiber which can be approximated by 1 in case of air and atmospheric gases. In Eq. (2.2), the refractive index of core and cladding can be wavelength dependent, therefore the numerical aperture of the fiber varies as a function of wavelength.

Two parameters that can characterize an optical fiber are the relative core-cladding index difference

$$\Delta = \frac{n_h - n_l}{n_h}$$  \hspace{1cm} (2.3)

and the so-called $V$ parameter

$$V = k\rho \sqrt{n_h^2 - n_l^2}$$  \hspace{1cm} (2.4)

where $k = 2\pi/\lambda$ is the free-space wave number, $\rho$ is the core radius and $\lambda$ is the wavelength. About $V$-parameter in connection with microstructured fibers, one can find information in Ref. [63].

The eigenvalue equation for modes can be used to determine the values of $V$ at which different modes reach cut-off (See, for instance, [64]). It can be seen, that the $V$ parameter depends on the fiber geometry and the core-cladding index difference. The $V$ parameter tells us how many modes are supported by the optical
fiber, therefore the $V$ parameter is a critical fiber design parameter. A step-index fiber supports only one mode if the $V$ parameter is less than 2.405.

An approximate expression can be used to determine the number of modes in a step-index fiber by the following equation

$$M \approx \left[ \frac{V}{\pi} \right]^2 = \left[ \frac{NA}{\lambda/(2\rho)} \right]^2$$

where we denoted the number of modes by $M$ and we used (2.2) and (2.4) to express $M$ by the numerical aperture too.

2.1.1 Fiber types

Standard fibers for communication purposes are step-index or graded-index fibers manufactured nowadays. Graded-index fibers have a core with radially decreasing refractive index from the center to the core boundary (See Fig. 2.2).

These fibers are usually referred to transmission fibers. To increase the transmission capacity and the wavelength range where these fibers are able to be used, dispersion compensating and active fibers were introduced.

Considering the geometrical properties and the number of guided modes the fibers can be categorized further as multi-mode and single mode fibers.

Multi-mode fibers (MMF) are inexpensive solutions for building LAN in an office, for example. The name MMF comes from the fact that the light travels down the fiber in multiple paths. Graded-index fibers are usually used for LAN/WAN equipments because the light path in this case is sinuous and regular while in the case of step-index MMF the light path is highly angular and irregular.

The graded index profile is usually given by the following form

$$n(r) = \begin{cases} n_1 \sqrt{1 - 2\Delta \left( \frac{r}{\rho} \right)^m}, & r \leq \rho \\ n_2, & r > \rho \end{cases}$$

where $n_1$ is the nominal refractive index $n_1 = n(r = 0)$, $n_2$ is the refractive index of the homogeneous cladding, $\rho$ is the radius of the core, $\Delta = (n_1^2 - n_2^2)/(2n_1^2)$ and $m$ is a parameter defining the shape of the profile.

From the end of 1990s, highly nonlinear (HNL) fibers were developed for supercontinuum generation. These fibers are SM fibers with a relatively small core area and high NA in order to yield high nonlinearity (see the relation between nonlinearity and core area in subsection 2.2.3).
An other category of fibers is microstructured optical fibers (MOF) which are usually referred to photonic crystal fibers (PCF). These fibers can be ranked in two main classes: high-index core fibers or index guiding fibers and photonic bandgap fibers (PBG) or bandgap guiding fibers. High index core fibers can be large mode area fibers and highly nonlinear fibers. PBG fibers can be Bragg fibers and hollow core (HC) or air guiding fibers.
2.1.2 Attenuation in the fiber

The light propagating through a medium loses some percentage of its power if the dielectric material is not perfectly transparent. Phenomenologically, this phenomenon can be demonstrated by a complex susceptibility,

\[ \chi = \chi' + i\chi'' \]  

(2.7)
corresponding to the complex permittivity \( \epsilon = \epsilon_0(1+\chi) \) \[62\]. The wavenumber will be complex valued \( k = \beta - i\alpha/2 \) where the imaginary part of the wavenumber will be responsible for the attenuation of light (See subsection 2.2.3 for the relations between \( \alpha \) and \( \chi \)).

The transmitted power \( P_{\text{out}} \) of a light beam with an initial power of \( P_{\text{in}} \) can be given by the following expression after the propagation length \( L \)

\[ P_{\text{out}} = P_{\text{in}} \exp(-\alpha L) \]  

(2.8)
where the attenuation constant \( \alpha \) is a measure of total fiber losses from all sources. \( \alpha \) in Eq. (2.8) has a unit of 1/m in SI. It is customary however to express \( \alpha \) in units of dB/km. The conversion between a ratio \( R \) in SI and in decibel can be easily done by the general definition

\[ R(\text{in dB}) = 10 \log_{10} R(\text{in SI}) \]  

(2.9)
The most common use of decibel scale occurs for power ratios as it is the case in Eq. (2.8)

\[ \alpha_{[\text{dB/m}]} = -\frac{10}{L} \ln \left[ \frac{P_{\text{out}}}{P_{\text{in}}} \right] = -\frac{10}{L} \ln \left[ \frac{P_{\text{out}}}{P_{\text{in}}} \right] \ln 10 = 10 \ln 10 \alpha_{[1/m]} \]  

(2.10)
where \( \ln \) stands for the natural logarithm and we used Eq. (2.8) to substitute the ratio of output and input power with \( \alpha_{[1/m]}L \). Subscripts next to the \( \alpha \) parameters are intended to show the unit of the parameter. In order to get the loss in dB/km we should simply multiply the above equation by thousand

\[ \alpha_{[\text{dB/km}]} = \frac{10^4}{\ln 10} \alpha_{[1/m]} \]  

(2.11)
This expression is often used in the simulations for obtaining the loss in SI from the loss data provided by the fiber manufacturers.
The loss in a fiber is also wavelength dependent. Different frequency components of the propagating light are attenuated with different magnitudes. The factors which contribute to the loss spectrum are the Rayleigh scattering, water (OH−) absorption and metal-oxide absorption peaks. Silica glass, for example, has electronic resonances in the UV and vibrational resonances in the FIR region. Therefore, this type of glasses can transmit light in the 0.5 – 2.2 µm region.

Rayleigh scattering is a fundamental loss mechanism arising from the density fluctuations frozen into the fused silica during manufacturing. Resulting local fluctuations in the refractive index scatter light in all directions. The Rayleigh scattering loss varies with \( \lambda^{-4} \) therefore its effect is dominant at short wavelengths.

Another loss factor is the bending loss which may scatter light at the core-cladding interface. In communication systems the splicing loss and connector losses may also contribute to the attenuation of the transmitted light.

### 2.1.3 Dispersion

Propagating light interacts with the bound electrons of a dielectric material whose response, in general, depends on the optical frequency. This property is referred to as chromatic dispersion and manifests through the frequency dependence of the susceptibility \( \chi(f) \), refractive index \( n(f) \) and speed of light \( c = c_0/n(f) \) where \( c_0 \) is the speed of light in vacuum. Relations between the real part of the wavenumber (\( \beta \): propagation constant), susceptibility and dispersion are described in subsection 2.2.3.

There are characteristic resonances of the medium at which the medium absorbs the electro-magnetic radiation through the oscillations of bound electrons. Far from the medium resonances the refractive index of the glass can be approximated by the Sellmeier equation (See, for example, [64])

\[
n(\lambda) = \left[ 1 + \sum_{j=1}^{m} \frac{B_j \lambda^2}{\lambda^2 - C_j} \right]^{1/2} \tag{2.12}
\]

where \( B_j \) and \( C_j \) are fitting parameters with \( j = 1, 2, \ldots, m \) where \( m \) can be considered the number of resonances that contribute to the frequency range of interest and this way \( C_j \) is the \( j \)th resonance wavelength and \( B_j \) is the strength of the \( j \)th resonance.

In the case of silica glass the Sellmeier parameters have the following values if \( m = 3 \) and \( \lambda \) is calculated in \( \mu \text{m} \) in Eq. (2.12): \( B_1 = 6.961663 \times 10^{-1}, B_2 = \ldots \)
Chromatic dispersion

Because of the dispersion, propagating pulses become broader as a function of fiber length. This effect is the result of the above mentioned frequency dependence of the speed of light in the guiding medium. We may note that shorter pulses are affected more strongly by dispersion than longer ones. Longer pulses have a narrower spectrum and because of the Fourier-transform relation

$$\Delta \nu \Delta \tau = \text{const} \quad (2.13)$$

where $\Delta \nu$ is the spectral and $\Delta \tau$ is the transform limited temporal width.

The dispersion coefficient can be derived from the propagation constant for that we expand $\beta(\omega)$ in a Taylor-series around the center frequency $\omega_0$

$$\beta(\omega) = n(\omega) \frac{\omega}{c_0} = \beta(\omega_0) + \left( \frac{d\beta}{d\omega} \right)_{\omega=\omega_0} (\omega - \omega_0) + \frac{1}{2} \left( \frac{d^2\beta}{d\omega^2} \right)_{\omega=\omega_0} (\omega - \omega_0)^2 + \cdots \quad (2.14)$$

The derivatives of $\beta(\omega)$ are related to the refractive index $n(\omega)$ and its derivatives through the relations

$$\frac{d\beta}{d\omega} \equiv \beta_1 = \frac{1}{v_g} = \frac{n_g}{c_0} = \frac{1}{c_0} \left( n + \omega \frac{dn}{d\omega} \right) \quad (2.15)$$

$$\frac{d^2\beta}{d\omega^2} \equiv \beta_2 = \frac{1}{c_0} \left( 2 \frac{dn}{d\omega} + \omega \frac{d^2n}{d\omega^2} \right) \quad (2.16)$$

where $n_g$ is the group index and $v_g$ is the group velocity. The envelope of an optical pulse moves at the group velocity ($v_g = 1/\beta_1 = c_0/n_g$), while the parameter $\beta_2$ represents dispersion of the group velocity and is responsible for pulse broadening. The dispersion parameter or dispersion coefficient which is often used to describe dispersion properties is given by

$$D(\lambda) = \frac{d\beta_1}{d\lambda}. \quad (2.17)$$

We can say that $D$ gives the time difference between two frequency components with unit wavelength difference after unit propagation length.
The dispersion coefficient can be expressed by the group velocity dispersion, $\beta_2$, and the refractive index as well using Eq. (2.15), (2.16) and (2.17)

$$D(\lambda) = -\frac{2\pi c_0}{\lambda^2} \beta_2 = -\frac{\lambda}{c_0} \frac{d^2 n}{d\lambda^2}$$  \hspace{1cm} (2.18)

The expression after the second equal sign is often used to determine the dispersion of a waveguide whose refractive index is obtained from the solution of the Helmholtz-eigenvalue equation. This type of analysis is important when the waveguide dispersion contributes significantly to the total group velocity dispersion.

The dispersion parameter as a function of wavelength is usually described by the coefficients of a Taylor series

$$D(\lambda) \approx D_0 + S(\lambda - \lambda_0) + \frac{T}{2} (\lambda - \lambda_0)^2 + \frac{F}{6} (\lambda - \lambda_0)^3$$  \hspace{1cm} (2.19)

where $D_0, S, T$ and $F$ are the dispersion (at $\lambda_0$), dispersion slope, third order dispersion and fourth order dispersion, respectively. The $\lambda_0$ parameter is the reference wavelength around which $D(\lambda)$ was expanded. Fiber manufacturers usually provide $\lambda_{\text{ref}} = \lambda_0$ and $D, S$ and/or higher order terms for describing the dispersion properties of their fibers.

During the numerical solution of the wave propagation (See Section 2.3) the coefficients of the Taylor series for the mode propagation constants ($\beta_1, \beta_2, \ldots$) are used to calculate the effects of dispersion. They can be obtained from the dispersion coefficients as follows

$$\beta_2 = -D_0 \frac{\lambda_0^2}{2\pi c}$$  \hspace{1cm} (2.20)

$$\beta_3 = \left(\frac{\lambda_0}{2\pi c}\right)^2 (\lambda_0^2 S + 2\lambda_0 D_0)$$  \hspace{1cm} (2.21)

$$\beta_4 = -\left(\frac{\lambda_0}{2\pi c}\right)^3 (\lambda_0^3 T + 6\lambda_0^2 S + 6\lambda_0 D_0)$$  \hspace{1cm} (2.22)

$$\beta_5 = \left(\frac{\lambda_0}{2\pi c}\right)^4 (\lambda_0^4 F + 12\lambda_0^3 T + 36\lambda_0^2 S + 24\lambda_0 D_0)$$  \hspace{1cm} (2.23)

The total group velocity dispersion of a waveguide, as briefly mentioned above, consists of two parts:

1. **material dispersion**
inter-modal dispersion may belong to dispersion effects in general. Intermodal dispersion occurs in MMF because the different modes are associated with different values of $\beta$ and hence different velocities. Polarization mode dispersion is also the same phenomenon as inter-modal dispersion but the relevant modes are here originally degenerate.

The contribution of waveguide dispersion to the total dispersion depends on fiber design parameters such as core radius $\rho$ and core-cladding index difference $\Delta$. This feature makes it possible to design such fibers whose zero dispersion wavelength are shifted towards the longer wavelengths. These fibers are called dispersion-shifted fibers (DSF). Dispersion compensating fibers (DCF) are the fibers in which the $\beta_2$ value is positive ($D$ negative) in the wavelength region below 1.6 $\mu$m. DCFs usually have more cladding layers as transmission fibers (See Fig. 2.2). This is the same for dispersion-flattened fibers (DFF) which provide a flat dispersion profile for a few tens or hundreds of nano-meters of wavelengths.

In case of MFs or PCFs, the structure indicates that the waveguide contribution to the dispersion can be tuned by the hole size $d$ and the pitch (hole-to-hole spacing) $\Lambda$. These types of fibers may have such a large dispersion contribution from the guiding mechanism that anomalous dispersion can be achieved even in the visible region [28].

Group-delay dispersion

Using the terminology of people working with femtosecond lasers we respectively define the followings: phase shift after the propagation length $L$

$$\phi(\omega) = n(\omega) \frac{\omega}{c} L$$  \hspace{1cm} (2.24)

group-delay which is the frequency dependence of phase-shift

$$\tau(\omega) = \frac{d\phi}{d\omega} = \frac{1}{c} \left( n(\omega) + \omega \frac{dn(\omega)}{d\omega} \right) L$$  \hspace{1cm} (2.25)

Group delay dispersion (GDD) is the frequency dependency of the group delay, or quantitatively the corresponding derivative with respect to angular frequency

$$\phi_2(\omega) = \frac{d\tau(\omega)}{d\omega} = \frac{d^2\phi}{d\omega^2}.$$  \hspace{1cm} (2.26)
The third order dispersion (TOD) is obviously the third derivative of phase with respect to \( \omega \).

GDD is usually specified in fs\(^2\) or ps\(^2\). Positive (negative) values correspond to normal (anomalous) chromatic dispersion. (GDD) always refers to some optical elements or to some given length of a medium. If one specifies the GDD per unit length (in units of s\(^2\)/m or ps\(^2\)/m, etc.), it gives the above mentioned GVD (\( \beta_1 \), see Eq. (2.15)).

We can see consequently that

\[
\phi(\omega) = \beta(\omega)L.
\]  

(2.27)

In order to derive a relation between \( D(\lambda) \) and GDD (\( \phi_2 \)) we can write \( D(\lambda) \) using Eq. (2.27) in the form

\[
D(\lambda) = \frac{d\tau}{d\lambda}L = \frac{d\tau}{d\omega} \frac{d\omega}{d\lambda}L = -\frac{\omega^2}{2\pi c_0} \phi_2(\omega)L
\]  

(2.28)

where \( d\omega/d\lambda = -2\pi c_0/\lambda^2 \).

2.1.4 Nonlinearity

The origin of nonlinear response is related to anharmonic motion of bound electrons under the influence of an applied field. Mathematically, this phenomena can be expressed by the nonlinear dependence of induced polarization vector (\( P \)) from the electric field (\( E \)) (see the relations in Section 2.2.3). Most of the nonlinear effects in optical fibers are originate from nonlinear refraction which is a phenomenon referring to the intensity dependence of refractive index

\[
n(\omega) = n_0(\omega) + n_2|E|^2
\]  

(2.29)

where \( n_0(\omega) \) is the linear part and \( n_2 \) is the nonlinear-index relates to the third order susceptibility (see Section 2.2.3). The intensity dependents of the refractive index leads to SPM, for example. SPM refers to self-induced phase shift experienced by an optical field during its propagation in optical fibers. Its magnitude can be given by

\[
\phi(\omega) = n(\omega)k_0L = (n_0 + n_2|E|^2)k_0L
\]  

(2.30)

where \( k_0 = 2\pi/\lambda \), L is the fiber length and \( \phi_{NL} = n_2k_0L|E|^2 \) is the nonlinear phase-shift.
An other class of nonlinear effects in optical fibers are the inelastic scatterings. In this case, the optical field transfers energy to the nonlinear medium. Such effects are the stimulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS). The main difference between the two is that optical phonons participate in SRS while acoustic phonons participate in SBS. Both scatterings can be explained the same way. A photon of the incident field is annihilated to create a photon at lower frequency and a phonon with the right energy (and momentum) conserving energy and momentum. Further explanation of the nonlinear effects are in Section 2.2.3.

2.2 Nonlinear propagation in optical fiber

The nonlinear Schrödinger equation is derived in the followings starting from the Maxwell’s equation. The used approximations are discussed in detail and higher order approximations and additional effects are described too. We note that \( c \) is used instead of \( c_0 \) in further sections because the necessary physical constant we need is the speed of light in vacuum.

2.2.1 Maxwell’s equations and Wave equation

The complete equation system that can describe all electromagnetic phenomena are the Maxwell’s equations whose differential and integral forms are presented here

\[
\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \quad \oint \mathbf{H} \, d\mathbf{r} = \int_{S} \left( \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \, d\mathbf{S}, \quad (2.31a)
\]

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \oint \mathbf{E} \, d\mathbf{r} = -\frac{\partial}{\partial t} \int_{S} \mathbf{B} \, d\mathbf{S}, \quad (2.31b)
\]

\[
\nabla \mathbf{B} = 0, \quad \oint \mathbf{B} \, d\mathbf{S} = 0, \quad (2.31c)
\]

\[
\nabla \mathbf{D} = \rho, \quad \oint \mathbf{D} \, d\mathbf{S} = \int_{V} \rho \, dV \quad (2.31d)
\]

where \( \mathbf{H} \) is the magnetic field vector, \( \mathbf{E} \) is the electric field vector, \( \mathbf{B} \) and \( \mathbf{D} \) are the magnetic and electric flux densities, respectively. The current density vector
is $J$ and the charge density is $\rho$. The notation $(c)$ under the sign of the integral means that the integral is carried out for a closed curve, $S$ is for surface and $V$ is for volume.

The corresponding constitutive relations are given by

\begin{align}
D &= \varepsilon_0 E + P, \\
B &= \mu_0 H + M, \\
J &= \sigma E,
\end{align}

where $\varepsilon_0 = 8.85 \times 10^{-12}$ As/Vm is the vacuum permittivity, $\mu_0 = 1.2566 \times 10^{-6}$ Vs/Am is the vacuum permeability, $\sigma$ is the conductivity, its unit of measure is A/Vm, and $P$ and $M$ are the induced electric and magnetic polarizations.

In optical fibers the following quantities are zeros: $J$, $\rho$ (no free charges) and $M$ (nonmagnetic medium). Therefore if we take the curl of Eq. (2.31b) and using (2.31a), (2.32a) and (2.32b), yields Eq. (2.33) where we also used the relation $\mu_0 \varepsilon_0 = 1/c^2$

$$
\nabla \times \nabla \times E + \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = -\mu_0 \frac{\partial^2 P}{\partial t^2}.
$$

There is a well-known vector-analytical relation for $\nabla \times \nabla \times E$ that we can apply in Eq. (2.33):

$$
\nabla \times \nabla \times E = \nabla (\nabla E) - \nabla^2 E = -\nabla^2 E,
$$

because the fiber can be considered isotropic and $\rho = 0$, therefore $\nabla E$ vanishes ($\nabla D = \varepsilon_0 \nabla E = 0$). With this substitution Eq. (2.33) becomes:

$$
\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P}{\partial t^2}.
$$

2.2.2 Induced polarization vector and susceptibility tensor

In order to solve Eq. (2.35) one has to determine the relation between the induced polarization vector $P$ and electric field vector $E$. In general, a quantum mechanical approach is needed but if the applied optical frequency is far from the medium resonances which means the wavelength of the field is between 0.5 and 2.2 $\mu$m ($f_c = 140 - 600$ THz) then the electric-dipole approximation is valid. Assuming that the medium response is local, the induced polarization vector can be written
as

\[
\mathbf{P}(r, t) = \varepsilon_0 \int_{-\infty}^{\infty} \chi^{(1)}(t - \tau) \mathbf{E}(r, \tau) d\tau + \\
\varepsilon_0 \int \int_{-\infty}^{\infty} \chi^{(2)}(t - \tau, t - \theta) \mathbf{E}(r, \tau) \mathbf{E}(r, \theta) d\tau d\theta + \\
\varepsilon_0 \int \int \int_{-\infty}^{\infty} \chi^{(3)}(t - \tau, t - \theta, t - \eta) \mathbf{E}(r, \tau) \mathbf{E}(r, \theta) \mathbf{E}(r, \eta) d\tau d\theta d\eta + \ldots
\]

(2.36)

If the medium response is instantaneous compared to the pulse duration \((\tau/T_0 \ll 1\) where \(T_0\) is the pulse width and \(\tau\) is the nonlinear response time of the medium) then Eq. (2.36) may be approximated by

\[
\mathbf{P}(r, t) \approx \varepsilon_0 \left[ \chi^{(1)} \mathbf{E}(r, t) + \chi^{(2)} \mathbf{E}(r, t) \mathbf{E}(r, t) + \chi^{(3)} \mathbf{E}(r, t) \mathbf{E}(r, t) \mathbf{E}(r, t) \right]
\]

(2.37)

where \(\chi^{(j)}\) is the \(j\)th order susceptibility, a tensor of rank \(j + 1\).

- \(\chi^{(1)}\) – is the linear susceptibility. Its effects are included in the linear refractive index \(n_0\) and the attenuation coefficient \(\alpha\).

- \(\chi^{(2)}\) – is the second order susceptibility. The second order susceptibility is responsible for the second-harmonic generation and sum-frequency generation. It is nonzero only for media that has a lack of inversion symmetry at molecular level. \(\text{SiO}_2\) is a symmetric molecule, therefore \(\chi^{(2)}\) vanishes for silica glasses.

- \(\chi^{(3)}\) – is the third order susceptibility. It is responsible for the third-harmonic generation, four-wave mixing and nonlinear refraction.

The following notation will be used:

\[
\mathbf{P}_L = \varepsilon_0 \chi^{(1)} \mathbf{E},
\]

(2.38)

\[
\mathbf{P}_{NL} = \varepsilon_0 \chi^{(3)} \mathbf{E}\mathbf{E}\mathbf{E}
\]

(2.39)

where

\[
\mathbf{P} = \mathbf{P}_L + \mathbf{P}_{NL}.
\]

(2.40)

and \(P_L\) denotes the linear part while \(P_{NL}\) the nonlinear part of the induced polarization vector.
2.2.3 Deriving the nonlinear Schrödinger equation

We can evaluate now a basic propagation equation from Eq. (2.35) using the (2.38) and (2.39) relations between $E$ and $P$. Here, we can make some simplifying assumptions:

- $P_{NL}$ is treated as a small perturbation compared to $P_L$ (nonlinear effects are weak in silica fibers).
- The optical field is assumed to maintain its polarization along the fiber length.
- The optical field assumed to be quasi-monochromatic ($\Delta \omega/\omega_0 \ll 1$ where $\omega_0$ is the center frequency and $\Delta \omega$ is the spectral width).

According to the slowly-varying-envelope approximation it is useful to separate the rapidly varying part of the electric field by writing it in the form of

$$E(r, t) = \frac{1}{2} \hat{x}[E(r, t) \exp(-i\omega_0 t) + E^*(r, t) \exp(i\omega_0 t)], \quad (2.41)$$

where $\hat{x}$ is the polarization unit vector of the light assumed to be linearly polarized along the $x$ axis, $E(r, t)$ is a slowly-varying function of time (relative to the optical period) and $E^*$ means the complex conjugate of $E$.

Eq. (2.41) is substituted into Eq. (2.39) and (2.38) and a similar form is used in the polarization vector as in Eq. (2.41):

$$P_L(r, t) = \frac{1}{2} \hat{x}[P_L(r, t) \exp(-i\omega_0 t) + P_L^*(r, t) \exp(i\omega_0 t)], \quad (2.42)$$

$$P_{NL}(r, t) = \frac{1}{2} \hat{x}[P_{NL}(r, t) \exp(-i\omega_0 t) + P_{NL}^*(r, t) \exp(i\omega_0 t)]. \quad (2.43)$$

A Fourier-transformation is applied on Eq. (2.35) and Eq. (2.42) and (2.43) in that where we express $P_L(r, t)$ and $P_{NL}(r, t)$ with their relation to $E(r, t)$. The obtained wave equation will have a form of

$$\nabla^2 \tilde{E} + \varepsilon(\omega)k_0^2 \tilde{E} = 0 \quad (2.44)$$

where $\tilde{E}$ denotes the Fourier-transform of $E(r, t)$, $k_0 = \omega_0/c$ and

$$\varepsilon_0(\omega) = 1 + \chi^{(1)}_{xx} + \frac{3}{4} \chi^{(3)}_{xxxx} |E(r, t)|^2. \quad (2.45)$$
CHAPTER 2. THEORY OF OPTICAL FIBERS

Eq. (2.44) is known as Helmholtz equation and can be solved by using the method of separation of variables

\[ \tilde{E}(r, \omega - \omega_0) = F(x, y)\tilde{E}(z, \omega - \omega_0)e^{i\beta_0 z} \] (2.46)

where \( \tilde{E}(z, \omega - \omega_0) \) is a slowly varying function of \( z \) and \( F(x, y) \) is a function which corresponds to the transverse electric modes in the \((x, y)\) plane if the \( z \)-axis is identical to the propagation direction. We note here, that both side of Eq. (2.46) contain the \( E \) function but at the left hand side it depends on all spatial coordinates while at the right hand side \( E \) is only \( z \)-dependent. In the following, only \( E(z, t) \) will be used in the derivation process therefore the argument of the function will not be noted in all cases.

Writing back Eq. (2.46) into Eq. (2.44) we obtain

\[ \frac{1}{F} \left( \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} \right) + \varepsilon(\omega)k_0^2 = \frac{1}{\tilde{E}e^{i\beta_0 z}} \frac{\partial^2}{\partial z^2} (\tilde{E}e^{i\beta_0 z}). \] (2.47)

The two sides of the equation depend on different variables. Therefore the right hand side and the left hand side must be equal with the same constant. Thus we obtain the following differential equations:

\[ \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \left[ \varepsilon(\omega)k_0^2 - \beta^2 \right] F = 0, \] (2.48)

\[ \frac{\partial^2 \tilde{E}}{\partial z^2} + 2i\beta_0 \frac{\partial \tilde{E}}{\partial z} + \left[ \beta^2 - \beta_0^2 \right] \tilde{E} = 0 \] (2.49)

where \( \beta \) is the wavenumber and it is determined by solving the eigenvalue equation (2.48). In Eq. (2.49), the second derivative can be neglected because \( \tilde{E}(z, \omega) \) is a slowly varying function of \( z \).

The eigenvalue \( \beta \) can be written in the form of

\[ \beta(\omega) = \beta(\omega) + \Delta\beta, \] (2.50)

where \( \Delta\beta \) is a perturbation term and \( \beta(\omega) \) is the frequency dependent mode-propagation constant. Thus, from Eq. (2.49) we obtain

\[ \frac{\partial \tilde{E}}{\partial z} - i \frac{1}{2} \left\{ \left[ \beta(\omega)^2 + 2\beta(\omega)\Delta\beta \right] \frac{1}{\beta_0} - \beta_0 \right\} \tilde{E} = 0. \] (2.51)

It is useful to expand \( \beta(\omega) \) in a Taylor-series around the carrier frequency \( \omega_0 \) as it is given in Eq. (2.14) and we use the same notation as in Subsection 2.1.3 for
the derivatives \((\beta_1, \beta_2, \text{etc.})\). Writing back the Taylor expanded form of \(\beta(\omega)\) to Eq. (2.51) and neglecting the terms that are higher than second order such as \(\beta_1 \Delta \beta\) and \(\beta_2 \Delta \beta\). Thus we may obtain the following equation in the Fourier space from (2.51):

\[
\frac{\partial \tilde{E}}{\partial z} - i \beta_1 (\omega - \omega_0) \tilde{E} - \frac{i}{2} \beta_2 (\omega - \omega_0)^2 \tilde{E} - i \beta_0 \Delta \beta \tilde{E} = 0. \tag{2.52}
\]

Now, performing the inverse Fourier-transformation on Eq. (2.52) and taking into consideration the following equations:

\[
\mathcal{F}^{-1}\{(\omega - \omega_0) \tilde{E}(z, \omega - \omega_0)\} = i \frac{\partial E(z, t)}{\partial t}, \tag{2.53}
\]

and

\[
\mathcal{F}^{-1}\{(\omega - \omega_0)^2 \tilde{E}(z, \omega - \omega_0)\} = - \frac{\partial^2 E(z, t)}{\partial t^2}. \tag{2.54}
\]

\(\mathcal{F}^{-1}\) means the operation of inverse Fourier-transformation. We obtain the next equation

\[
\frac{\partial E}{\partial z} + \beta_1 \frac{\partial E}{\partial t} + \frac{i}{2} \beta_2 \frac{\partial^2 E}{\partial t^2} - i \Delta \beta E = 0. \tag{2.55}
\]

The term with \(\Delta \beta\) includes the effect of fiber loss and nonlinearity. It can be evaluated from Eq. (2.48) using a first-order perturbation theory (detailed derivation and references are given in Ref. [58]):

\[
\Delta \beta = -\frac{\alpha}{2} + i \gamma |E|^2, \tag{2.56}
\]

where \(\gamma\) is the nonlinear coefficient defined by

\[
\gamma = \frac{n_2 \omega_0}{c A_{\text{eff}}}. \tag{2.57}
\]

\(A_{\text{eff}}\) is the effective core area in (2.57) which is inversely proportional to the nonlinearity. \(n_2\) is the so-called nonlinear refractive index which perturbs the linear index at higher intensities \(n = n_0 + n_2 I\).

The effective core area is given in the form of

\[
A_{\text{eff}} = \frac{\int \int_{-\infty}^{\infty} |F(x, y)|^2 dx dy}{\int \int_{-\infty}^{\infty} |F(x, y)|^4 dx dy}. \tag{2.58}
\]
where \( F(x, y) \) is the transverse mode field distribution that can be obtained from the eigenvalue equation (2.48). Substituting Eq. (2.56) into Eq. (2.55) and making a variable transformation with

\[
T = t - \frac{z}{v_g} = t - \beta_1 z, \tag{2.59}
\]

one can obtain the NLS equation (2.60). The variable transformation yields the frame moving with the group velocity of the pulse envelope. This is the reduced time useful for describing pulse propagation in a coordinate system fixed to the pulse.

The obtained differential equation describes the light propagation in a lossy, dispersive and nonlinear fiber can be written as [58]

\[
\frac{\partial E(z, T)}{\partial z} = -\frac{\alpha}{2} E - i\beta_2 \frac{\partial^2 E}{\partial T^2} + i\gamma |E|^2 E \tag{2.60}
\]

which is often referred as NLS equation in the case of \( \alpha = 0 \). Attenuation is described by the first term at the right-hand side in Eq. (2.60), GVD corresponds to the second term and nonlinearity, or SPM is the third term with the intensity dependence.

In the followings, we describe more general forms of the NLS equation using higher order approximations and including an inelastic stimulated scattering effect.

### 2.2.4 Higher order dispersion and nonlinearity

Equation (2.60) does not include inelastic scattering such as Raman or Brillouin scattering which becomes important above a threshold of pulse peak intensities and below a certain time duration of the pulses. Raman effect is usually important below the 1 ps time scale if the Raman threshold is reached which can be approximated as follows

\[
P_{cr}^0 \approx 16 \frac{A_{eff}}{L_{eff} g_R} \tag{2.61}
\]

where \( L_{eff} = (1 - \exp(-\alpha L))/\alpha \) is the effective fiber length with the pulse attenuation \( \alpha \) and fiber length \( L \). \( A_{eff} \) is the effective core area in Eq. (2.61) and \( g_R \) is the Raman gain curve as a function of frequency shift. The maximal value of \( g_R \) is about \( 10^{-13} \) m/W for fused silica which is approximately 13.5 THz shift from the reference frequency.

Below 1 ps the spectral width can be broad enough that Raman gain transfers energy from the low-frequency components to the higher frequency components.
This results in the self-frequency shift of the pulse whose physical origin comes
from the delayed nature of Raman response.

In this approximation the nonlinear response of the medium is comparable with
the pulse width. Thus Eq. (2.36) should be used in the derivation of generalized
nonlinear Schrödinger equation (GNLSE). Assuming the following functional form
of the nonlinear susceptibility

\[ \chi^{(3)}(t-\tau, t-\theta, t-\eta) = \chi^{(3)} R(t-\tau) \delta(t-\theta) \delta(t-\eta) \]  

(2.62)

where \( R(t) \) is the nonlinear response function normalized the same way as the
delta function \( \int_{-\infty}^{\infty} R(t) dt = 1 \).

Higher order dispersion terms can be easily added including higher order Taylor
coefficients from the expansion of \( \beta(\omega) \) during the derivation process of (2.60) at
the step Eq. (2.51).

Substituting Eq. (2.62) into Eq. (2.36) and performing a similar derivation
process to the case of Eq. (2.60) this yields [65]

\[ \frac{\partial E(z,t)}{\partial z} = -\frac{\alpha}{2} E - \left[ \sum_{m=1}^{M} \frac{i^{m-1}}{m!} \beta_m \frac{\partial^m}{\partial t^m} \right] E \]

\[ + i\gamma \left( 1 + \frac{i}{\omega_0} \frac{\partial}{\partial t} \right) \left( E(z,t) \int_{-\infty}^{\infty} R(t') |E(z,t-t')|^2 dt' \right). \]  

(2.63)

The response function \( R(t) \) includes the electronic (instantaneous) and vibrational
(delayed) Raman response

\[ R(t) = (1 - f_R) \delta(t) + f_R h_R(t) \]  

(2.64)

where \( f_R \) is the fractional contribution of the delayed Raman response to the
nonlinear polarization and \( h_R(t) \) is the Raman response function.

Eq. (2.63) can be simplified with the assumption \( \Delta \tau \gg 10 \) fs to the following
expression

\[ \frac{\partial E(z,T)}{\partial z} = -\frac{\alpha}{2} E \left[ \text{Loss} \right] - \left[ \sum_{m=2}^{5} \frac{i^{m-1}}{m!} \beta_m \frac{\partial^m}{\partial t^m} \right] E \]

\[ + i\gamma \left[ \text{SPM} \frac{|E|^2 E}{\partial T} + \frac{i}{\omega_0} \frac{\partial}{\partial T} (|E|^2 E) - T_R E \frac{\partial |E|^2}{\partial T} \right]. \]  

(2.65)
where

\[ T_R \approx f_R \int_0^\infty t h_R(t) dt \] (2.66)

where \( h_R \) is the Raman response function can be given by an approximate formula which has a Lorentz shape in the Fourier space

\[ h_R = \frac{\tau_1^2 + \tau_2^2}{\tau_1 \tau_2^2} \sin \left( \frac{t}{\tau_1} \right) \exp \left( -\frac{t}{\tau_2} \right) \] (2.67)

where \( \tau_1 \) and \( \tau_2 \) are adjusting parameters with typical values in silica 12.2 fs and 32 fs, respectively.

Using this form of the Raman response function, the integration of Eq. (2.66) can be performed analytically:

\[ T_R \approx f_R \frac{2 \tau_1^2 \tau_2}{\tau_1^2 + \tau_2^2}. \] (2.68)

This can be used to approximate Raman scattering effect in the last term of (2.65).

2.3 Numerical algorithm

2.3.1 Split-step Fourier method

The Split-Step Fourier (SSF) Method applies the linear propagation (diffraction) operator and index nonhomogeneity in separate steps (see Fig. 2.3). The linear propagation operator (\( \hat{L} \)) is applied in the Fourier space and simply represents the \( k \)-sphere appropriate to the polarization, direction of propagation and material symmetry. The index nonhomogeneity is a result of a waveguiding structure or third-order nonlinearity.

The SSF method is commonly used to integrate several types of nonlinear partial differential equations. In simulating NLS systems, SSF is predominantly used, rather than finite difference method (FDM), as SSF is often more efficient \[66, 67\].

Considering one of the simplest NLS type system, the equation contains the terms of attenuation, dispersion and nonlinearity (See Eq. (2.60)).

In order to solve Eq. (2.60) by the SSF method, we write the differential equation in the following functional form

\[ \frac{\partial A(z,t)}{\partial z} = [\hat{L} + \hat{N}]A(z,t), \] (2.69)
CHAPTER 2. THEORY OF OPTICAL FIBERS

Nonlinear Segments
Dispersive Segments
Input Pulse Output Pulse
\( \Delta z \)

Figure 2.3: Schematic illustration of the calculation. The material is represented by a sequence of thin segments

where \( \mathbf{\hat{L}} \) and \( \mathbf{\hat{N}} \) are the linear and nonlinear parts of (2.60), respectively, where

\[
\mathbf{\hat{L}} = -\frac{\alpha}{2} - \frac{i}{2} \beta_2 \frac{\partial^2}{\partial T^2}, \tag{2.70}
\]

\[
\mathbf{\hat{N}} = i\gamma |E|^2. \tag{2.71}
\]

Integrating (2.69) along \( z \) using a small space interval \( \Delta z \), the solution can be written in the form of

\[
E_m(z + \Delta z, t) = \exp\left[\Delta z (\mathbf{\hat{L}} + \mathbf{\hat{N}})\right] E(z, T), \tag{2.72}
\]

where the effects of the linear operator (2.70) can be easily implemented because the time derivatives become multiplications in the Fourier space by: \( (i\omega)^n \) where \( n \) is the order of the derivative

\[
\exp[\Delta z \mathbf{\hat{L}}] E(z, T) = \left\{ \mathcal{F}^{-1} \exp[\Delta z \mathbf{\hat{L}}(i\omega)] \mathcal{F} \right\} E(z, T), \tag{2.73}
\]

where \( \mathcal{F} \) denotes the Fourier transformation, \( \mathcal{F}^{-1} \) the inverse Fourier transformation and \( \mathbf{\hat{L}}(i\omega) \) is the Fourier transform of \( \mathbf{\hat{L}} \) which is obtained from Eq. (2.70).
First Order SSF

The essence of the first order SSF method is that the exponential operator acting on \( E(z, T) \) in (2.72) is divided into two parts

\[
E_m(z + \Delta z, t) \approx \exp \left[ \Delta z \hat{L} \right] \exp \left[ \Delta z \hat{N} \right] E(z, T).
\]  

(2.74)

The computation of the propagation of the slowly varying envelope is realized in four steps within a space interval \( \Delta z \):

- **Step 1.** Nonlinear step: compute \( E_1 = e^{\Delta z \hat{N}}E(z, T) \) (by finite differences)
- **Step 2.** Forward FT: Perform the forward FFT on \( E_1 \): \( E_2 = \mathcal{F}E_1 \)
- **Step 3.** Linear step: compute \( E_3 = e^{\Delta z \hat{L}}E_2 \)
- **Step 4.** Backward FT: Perform the backward FFT on \( E_3 \): \( E(z + \Delta z, t) = \mathcal{F}^{-1}E_3 \)

It is to be noted that in order to work the algorithm properly, the sampling points in the temporal and frequency space must comply the following relation coming from digital signal processing

\[
\delta f = \frac{1}{M\delta t}
\]

(2.75)

where \( \delta f \) is the frequency resolution in the Fourier space, \( \delta t \) is the time resolution in the temporal space and \( M \) is the number of sampling points.

The Symmetrized (Second Order) SSF

The main difference between the first order Split-Step and the Symmetrized SSF method is that the effect of nonlinearity is included in the middle of the segment. Schematic illustration of this method can be seen in Fig. 2.4. In this procedure the Eq. (2.72) is replaced by

\[
E(z + \Delta z, T) \simeq \exp \left[ \frac{\Delta z}{2} \hat{L} \right] \exp \left[ \int_z^{z+\Delta z} \hat{N}(z')dz' \right] \exp \left[ \frac{\Delta z}{2} \hat{L} \right] E(z, T)
\]

(2.76)

where \( \hat{L} \) and \( \hat{N} \) are the linear and nonlinear operators. The integral in (2.76) can be approximated by the trapezoidal rule

\[
\int_z^{z+\Delta z} \hat{N}(z')dz' \approx \frac{\Delta z}{2} \left[ \hat{N}(z) + \hat{N}(z + \Delta z) \right].
\]

(2.77)

This method can be realized in seven steps within a spatial step \( \Delta z \):
Figure 2.4: Schematic illustration of Symmetrized SSF method. The nonlinearity acts in the center of the segment.

- **Step 1.** FFT: $E_1 = \mathcal{F}E(z, t)$
- **Step 2.** Half linear step: $E_2 = e^{\Delta z \hat{L}} E_1$
- **Step 3.** IFFT : $E_3 = \mathcal{F}^{-1} E_2$
- **Step 4.** Nonlinear step: $E_4 = e^{\Delta z \hat{N}} E_3$
- **Step 5.** FFT: $E_5 = \mathcal{F} E_4$
- **Step 6.** Other half linear step: $E_6 = e^{\Delta z \hat{L}} E_5$
- **Step 7.** IFFT : $E_7 = \mathcal{F}^{-1} E_6$

The symmetrized SSF method gives the possibility to calculate the nonlinear operator with the help of an iteration process accurately. Using (2.76), the envelope at the beginning of the segment and at the end of the segment can be used to iterate (2.77) such a way that we place $N(z)$ instead of $N(z + \Delta z)$ in the first iteration step. Usually two iterations are enough in each split-step segment to reach high accuracy.
2.3.2 Finite difference method

Using the Cranck-Nicolson (CN) method ([68]), the \((\partial^2 E/\partial T^2)_{m+1/2}\) second partial derivative is expressed as an average of the points \((m, n)\) and \((m + 1, n)\), where \(m\) is the index of the spatial mesh points and \(n\) is for the time discretization. The discretized form of the NLS equation (2.60) applying the CN scheme can be given by

\[
\frac{E_{n}^{m+1} - E_{n}^{m}}{h} + \frac{i \beta_{2}}{2} \left[ \frac{E_{n+1}^{m} - 2E_{n}^{m} + E_{n-1}^{m}}{\tau^2} + \frac{E_{n+1}^{m+1} - 2E_{n}^{m+1} + E_{n-1}^{m+1}}{\tau^2} \right] - i \gamma |E_{n}^{m}|^2 E_{n}^{m} = 0 \tag{2.78}
\]

where \(m\) is the spatial and \(n\) is the temporal discretization (grid points) in the algorithm. \(h\) is the step size in space while \(\tau\) is the step size for (retarded) time.

By sorting the terms in Eq. (2.78), the following form is obtained

\[
E_{n}^{m+1} + \frac{i \beta_{2} h}{2 \tau^2} (E_{n+1}^{m+1} - 2E_{n}^{m+1} + E_{n-1}^{m+1}) =
E_{n}^{m} - \frac{i \beta_{2} h}{2 \tau^2} (E_{n+1}^{m} - 2E_{n}^{m} + E_{n-1}^{m}) + i \gamma h |E_{n}^{m}|^2 E_{n}^{m}. \tag{2.79}
\]

For the sake of simplicity, let now \(\sigma, \zeta \) and \(\eta\) be the following

\[
\sigma = \frac{\beta_{2} h}{2 \tau^2}, \quad \zeta = \gamma h, \quad \text{and} \quad \eta = 1 + i2\sigma + i\zeta |E_{n}^{m}|^2. \tag{2.80}
\]

with that Eq. (2.79) can be written in the form of

\[
(1 - i2\sigma) E_{n}^{m+1} + i\sigma (E_{n+1}^{m+1} + E_{n-1}^{m+1}) = \eta E_{n}^{m} - i\sigma (E_{n+1}^{m} + E_{n-1}^{m}). \tag{2.81}
\]

Furthermore, let be

\[
a = 1 - i2\sigma, \quad \text{and} \quad b = i\sigma. \tag{2.82}
\]

This allow us to reshape Eq. (2.78) to the following matrix equation

\[
\begin{pmatrix}
a & b \\
b & a & b \\
& b & \ddots & \ddots \\
& & b & \cdots & \cdots \\
& & & \ddots & a \\
& & & & \cdots & a \end{pmatrix}
\begin{pmatrix} E_{1}^{m+1} \\ E_{2}^{m+1} \\ \vdots \\ E_{N}^{m+1} \end{pmatrix}
= \begin{pmatrix} \eta_{1} & b \\ b & \eta_{2} & b \\ & b & \ddots & \ddots \\ & & b & \cdots & \cdots \\ & & & \cdots & \eta_{N} \end{pmatrix}
\begin{pmatrix} E_{1}^{m} \\ E_{2}^{m} \\ \vdots \\ E_{N}^{m} \end{pmatrix}. \tag{2.83}
\]

This algebraic equation system can simply be written in the form of

\[
A E = B, \tag{2.84}
\]

where \(B\) is the whole right-hand side of the Eq. (2.83) while \(A\) is the coefficient matrix of the envelope function indicating the solution for \(E_{n}^{m+1}\).
Improved calculations of nonlinear phase-shift in FDM

The above drafted FDM is not able to solve for the nonlinear phase shift accurately without the introduction of an iteration process for finding \( E^{m+1} \) in every step. This iteration slows down the method significantly.

For the more accurate treatment of the nonlinear part of the equation, some FD schemes were proposed to make the solution more reliable [69]-[72]. One of the most often used discretization of Eq. (2.60) if \( \alpha = 0 \) is given by

\[
\frac{E^{m+1}_n - E^m_n}{h} + \frac{i\beta_2}{2} \left[ \frac{E^{m+1}_{n+1} - 2E^m_n + E^m_{n-1}}{\tau^2} + \frac{E^{m+1}_{n+1} - 2E^{m+1}_{n} + E^{m+1}_{n-1}}{\tau^2} \right] - \frac{i\gamma}{2} \left| E^{m+1}_n \right|^2 + \frac{i\gamma}{2} \left| E^m_n \right|^2 \left( E^{m+1}_n + E^m_n \right) = 0,
\]

where we used the same notation as in the previous subsection.

One has to solve the above difference equation iteratively in every step because the equation is implicit. In order to avoid the iterative process, we may consider an extrapolation formula to approximate the nonlinear term and obtain a linearized CN scheme.

\[
\frac{E^{m+1}_n - E^m_n}{h} + \frac{i\beta_2}{2} \left[ \frac{E^{m+1}_{n+1} - 2E^m_n + E^m_{n-1}}{\tau^2} + \frac{E^{m+1}_{n+1} - 2E^{m+1}_{n} + E^{m+1}_{n-1}}{\tau^2} \right] + \frac{3}{2} i\gamma \left( \left| E^m_n \right|^2 E^m_n - \left| E^{m-1}_n \right|^2 E^{m-1}_n \right) = 0.
\]

This is a semi-implicit method, because only a linear tridiagonal system of equations needs to be solved at each spatial point.

In Ref. [71], a new linearized CN scheme was published which was used to approximate only the nonlinear coefficient and not the entire nonlinear term. The scheme can be written in the following form

\[
\frac{E^{m+1}_n - E^m_n}{h} + \frac{i\beta_2}{2} \left[ \frac{E^{m+1}_{n+1} - 2E^m_n + E^m_{n-1}}{\tau^2} + \frac{E^{m+1}_{n+1} - 2E^{m+1}_{n} + E^{m+1}_{n-1}}{\tau^2} \right] + \frac{i\gamma}{2} \left( \frac{3}{2} \left( \left| E^m_n \right|^2 \right) - \frac{1}{2} \left( \left| E^{m-1}_n \right|^2 \right) \right) \left( E^{m+1}_n + E^m_n \right) = 0.
\]

which yield an accurate solution of this system and can be easily adopted to more general forms of the NLS equation.
Chapter 3

Soliton propagation of microwave modulated signals

When I started to deal with the simulation of light propagation in optical fibers, I primarily focused on looking for such parameter sets which resulted in the breathing and recurrence phenomena of soliton propagation in the temporal space. A possibility on calculating the microwave modulated signal propagation in SMF gave me the idea to do this with the modulated signals as well. Although the sine or cosine modulated signals behave differently as pulses under the effect of dispersion and SPM still I managed to find such modulation frequency and signal intensity where soliton propagation could be demonstrated numerically.

Finding soliton propagation of microwave modulated signal in SMF is a result of a long investigation period in connection with the interplay of dispersion and SPM. In the followings, we introduce briefly the system relates to the models. We describe the theory for the simulation and the physics behind after. Then the performed experiment and the corresponding calculations are discussed. Simulations for finding the position of dispersion caused modulation suppression along the fiber is described after. Lossless propagation is investigated in the next subsection in order to have enough power after long propagation lengths to maintain nonlinear effect. This leads to soliton propagation of IM signals which may have practical significance by compensating the loss.
3.1 Introduction

Radio over fiber (ROF) systems have attracted attention for distributing microwave signals through SMFs at the end of the 1990s \cite{73}. The aim was to solve the communication cost effectively between a central station and several base stations connected with optical fibers where microwave subcarriers are transmitted. At the base stations, photodetectors recover the microwave optical signals and radiate them to any wireless device. ROF has the advantage of centralizing the expensive electronics in the central station.

Basically, microwave signals can be transmitted three different ways over optical fibers with intensity modulation (IM) which type of modulations are applied in many radio over fiber systems \cite{74, 75}. First, it can be done by the direct intensity modulation (DIM) where an electrical parameter of the light source is modulated by the information-bearing radio frequency (RF) signal. Another method applies an unmodulated light source and an external light intensity modulator. Third, microwaves and millimeter waves can be optically generated via remote heterodyning.

Microwave optical transmissions operating in the telecommunication window are limited by the chromatic dispersion of SMF \cite{76}-\cite{78}. Chromatic dispersion can cause modulation suppression, linear distortion or even complete canceling of intensity modulation by the RF signal. The suppression of the IM signal may occur by the different group-velocity of the sidebands. The unequal phase between the sidebands leads to a modulation function during the detection which becomes zero if the phase difference is equal with $\pi$ \cite{76}.

Several techniques have been proposed to overcome this effect such as optical single sideband modulation \cite{79, 80}, dispersion compensation using chirped fiber gratings \cite{81}, variable chirp in external modulators \cite{82, 83}, SPM effect originated from the fiber \cite{84, 85}, midway optical phase conjugation \cite{86}, four-wave mixing in dispersion-shifted fibers \cite{87, 88} and interplay between the intensity dependent phase modulation in semiconductor optical amplifiers (SOAs) \cite{89}.

Optical fiber and radio over fiber systems usually operate around the 1550 nm wavelength range where they benefit from low propagation loss. The effects of dispersion and non-linearity however can not be neglected. I shall call the locations where intensity modulation disappears “notches” throughout this dissertation because of their appearance on the transfer function of the fiber over a relatively wide range of modulation frequencies (See Fig. 3.1). Notch is the location on the transfer function where dispersion caused modulation suppression occurs. The transfer function is defined as the quotient of the modulated input and the detected output.
Figure 3.1: Calculated transfer function of the fiber as a function of modulation frequency between 1 GHz and 30 GHz. The fiber length was 30 km, modulation depth 0.25, average power 1 mW and the center wavelength of the carrier is 1550 nm. First notch appears at 11.3 GHz intensity modulation.

signal powers as a function of frequency.

Above a given density of photon flux, SPM may become significant [84]. At relatively high average input intensities (> 5 – 10 mW) SPM modifies the transfer function of the fiber [85] and this may result the complete cancellation of dispersion caused suppression at elevated powers [87, 88] and periodic oscillations of the transfer function [90].

As the signal intensity increases the notches caused by chromatic dispersion shift to higher modulation frequencies. In accordance with one of our study [90], calculations show that the effect of dispersion can be compensated by SPM totally, at least in the case of lossless propagation. A critical input power as a function of carrier frequency can also be determined which separates the dispersion dominated and non-linearity dominated regions.

I present in the followings the joint effects of dispersion and nonlinear refraction in a region of intensity where non-linearity related distortions become important. I report our prepared measurements in the Broadband Communication Department of BUTE and corresponding simulations. I give the exact locations of notches predicted by computer simulation using various fiber parameters.

At the end of this chapter, I also report the soliton propagation of a 10 GHz modulated optical signal which has not reported in conjunction with microwave
modulated light waves in the literature previously. Although microwave modulated light propagation has a substantially different nature as pulse propagation but soliton propagation may happen because of SPM and FWM.

Transmission of unmodulated RF carriers are discussed here but the results lead to conclusions applicable to the modulated RF signals as well [88].

3.2 Theory

The propagation of light in SMF in the picosecond regions and larger time-scales is approximated by the NLS equation (see Eq. (2.60)). If the intensity \(|E(z,t)|^2\) of the investigated signal is large enough the SPM term in Eq. (2.60) may have a significant role in signal transmission. Especially in pulse propagation [58]. In agreement with previous studies [84]-[88], we found this is valid in the case of RF modulated signal propagation as well. In case of nonlinear pulse propagation in anomalous dispersive media, the broadening factor on a given length of fiber is less than if dispersion alone is considered. In the case of RF modulated signal propagation, the nonlinear term appears to decrease the effect of \(\beta_2\) virtually, resulting in an increase of the frequency at which the RF notch appears [84]-[88].

A qualitative explanation of the sharp dip (notch) in the transfer function can be given by the different propagation speeds of the two first-order modulation sidebands. Thus they arrive to the photo-detector, placed at the output of the fiber, with unequal phases. Consequently, the photo-detector produces two interfering signals resulting in a modulation transfer function that is less than unity,

\[
K(L, \omega_m) = \cos \left( \frac{\beta_2 L}{2} \frac{\omega_m^2}{2} \right)
\]

indicating that modulation is suppressed. In (3.1), \(\omega_m\) is the modulating RF angular frequency and \(L\) is the propagation length. We can see that modulation is completely suppressed if the argument of the cosine is an odd multiple of \(\pi/2\).

In the case of anomalous dispersion the first notch appears when the argument is \(-\pi/2\):

\[
L_n^l = \Theta(\beta_2) \frac{\pi}{\beta_2 \omega_m^2} \quad \text{where} \quad \Theta(\beta_2) = \begin{cases} 
1 & \text{if } \beta_2 \geq 0 \\
-1 & \text{if } \beta_2 < 0
\end{cases}
\]

(3.2)

and subscript \(n\) refers to notch and superscript \(l\) to the linear case. Expression (3.2) is valid until the fiber can be considered as linear. If SPM has a significant contribution to the signal evolution, notches will appear at locations different from those that could be calculated from (3.2).
3.3 Measurement

Fig. 3.2 illustrates our experimental setup. The frequency of the RF signal is varied between 50 MHz and 20 GHz in 800 steps. These frequencies modulate a 1 mW, 1550.8 nm continuous-wave (CW) laser using a Mach-Zehnder (MZ) modulator with 25% modulation depth. This type of MZ modulator (HP 83422A) has a relatively large 6.5 dB attenuation reducing its optical output power to 0.22 mW.

The RF signal propagates through 30 km of SMF and at the fiber output a network analyzer registers the field parameters. An optical spectrum analyzer monitors the frequency and optical power during the experiments. We used attenuators and an erbium-doped fiber amplifier (EDFA) before the fiber input to measure the transfer function of the fiber at various average input intensities. The gain of the EDFA is 20.6 dB raising the 0.22 mW input to 25.5 mW when no attenuation is used before the fiber input. An attenuator was applied to reduce the optical intensity by about 4 dB before injecting the light into the fiber, and some additional attenuator was used before the detector to protect it from damagingly high intensities.

We used a standard single-mode optical fiber with 0.22 dB/km attenuation and 16.8 ps/(nm⋅km) chromatic dispersion at 1550 nm. The effective core area of the fiber, necessary for calculating the nonlinear coefficient \( n_2 \), was 75 \( \mu \)m\(^2\).

During the measurements, we have not observed harmful effects caused by counter propagating signals indicated by inelastic scatterings therefore we have
not applied any optical isolator.

### 3.4 Simulation

A second order (symmetric) SSF method (Section 2.3.1) with a sinusoidally modulated input field is used to solve (2.60). The modulated output of the MZ modulator is

\[ E_{\text{out}}(z = 0, T) = E_{\text{laser}}(T)\sqrt{d(T)} \exp[j\Delta \phi(T)] \]  

(3.3)

where

\[ d(T) = 1 + m(\sin(\omega_m T)) \]  

(3.4)

is the power transfer function, \( m \) is the modulation depth, \( \omega_m \) is the modulation angular frequency and \( \Delta \phi(T) \) is the phase difference between the two branches of the modulator which can be set to zero if an ideal amplitude modulator is to be simulated. The input field in the calculated time window provided by the CW laser, like in the measurements, is \( E_{\text{laser}}(T) \), the square root of the laser intensity. The phase difference in (3.3) can be given by

\[ \Delta \phi(T) = C_1 - C_2 \left[ \sin(\omega_m T) - \frac{1}{2} \right] \]  

(3.5)

where \( C_1 \) and \( C_2 \) are constants representative of the modulator, chosen to be 0.06 and 0.073, respectively, to fit the simulated signal to the experimental values. The used \( T \), in the above expressions, is the retarded time same as used in the NLS equation (see Eq. (2.59) in section 2.2.3).

The fiber parameters used in the modeling were the same as those in the measurements (loss: 0.22 dB/km, chromatic dispersion: 16.8 ps/(nm km), effective core area: 75 \( \mu \)m\(^2\)). Silica based fibers usually have a nonlinear refractive index as large as \( 2.6 \times 10^{-20} \) m\(^2\)/W and third order chromatic dispersion in the range of \( 0.05 - 0.1 \times 10^3 \) ps/(nm\(^2\)km) in the vicinity of 1550 nm [58]. All the simulations were prepared using the above parameters. Results were obtained by varying the input intensity of the CW laser and the modulation frequency. We have also made calculations varying the optical power level and the fiber length using a constant 10 GHz RF modulation. In order to detect the RF component of the signal alone, a narrow band (100 MHz) Gaussian filter centered at 10 GHz was applied.
3.5 Results and discussion

3.5.1 Measurements and calculations

Fig. 3.3 shows the measured and calculated output RF power normalized to the modulated input power as a function of the modulation frequency for three different optical input powers. One can see that as the optical power is increased the notches appear at higher modulation frequencies. We note that this trend is valid in case of as well (see Fig. 3.4 below). The RF power corresponding to the 25.5 mW signal has a positive slope between 50 MHz and 7 GHz and remains above the input RF average intensity up to 8 GHz. As further explained below, this occurs because the Kerr non-linearity converts the optical power (central peak in the spectrum) to RF power (sidebands). We note that according to our simulations if a different type of modulator is used, e.g., an ideal amplitude modulator or a MZ modulator with smaller extinction ratio, the rise in RF power will occur at a much smaller RF signal input intensity, at about 10 mW.

The measured fiber responses beyond 30 km propagation length are in excellent agreement with calculated results. These correspondences make it possible to ex-
tend our investigations of parameter dependencies into regions, which are difficult to be measured. In what follows we make some numerical investigations into these regions.

### 3.5.2 Notch positions

From an engineering point of view, it might be important to know the notch location and their dependence on the optical input power. These simulations can be seen in Fig. 3.4, where the average intensity of the RF signal is plotted as a function of fiber length, retaining the parameters used previously. As one can see, when the input power is increased notches appear at longer distances. Here we also plotted the fiber response to an unusually high intensity RF signal, showing a behavior unexpected from that displayed at smaller RF intensities, namely, that modulation suppression occurs much earlier than before.

The dependency of notch position on input power can not be expressed by basic functions. In order to show these locations as caused by simultaneous chromatic dispersion and SPM, we performed computations with different nonlinear refractive indices and different dispersion parameters as well. These are shown in Figs. 3.5 (a) and (b). Fig. 3.5 (a) shows the notch location as a function of input intensity, using the nonlinear refractive index as a parameter at a fixed dispersion
Figure 3.5: Notch position as a function of average input intensity for different non-linear refractive indices (a) and different chromatic dispersion values (b) of SMF. Dispersion was fixed at 16.8 ps/(nm·km) in case of (a) and nonlinear refractive index at $2.6 \times 10^{-20}$ m$^2$/W in case of (b).

Notice that both in Figs. 3.5 (a) and (b) at power levels below a certain threshold the notch position shifts to longer fiber lengths as the input power increases. Above a certain input power however, the notch positions drop abruptly, i.e., they reappear at a much reduced fiber length. As expected, keeping the chromatic dispersion fixed, when non-linearity is increased (larger nonlinear refractive index) dispersion caused suppression occurs at larger distances. However, at a given power level, there is a change in this behavior and higher non-linearity causes a drop in notch position at a smaller input power level. A similar relationship holds when the magnitude of chromatic dispersion is increased while holding the nonlinear refractive index fixed. In this case, when the chromatic dispersion is decreased notches appear at longer path lengths, but drop at smaller input powers.

Considering the shape of the plotted functions in Figs. 3.5 (a) and (b) three different regions can be listed, namely:

- exponentially increasing region,
- region of downturn,
• region of abrupt drop.

At a given modulation frequency \((\omega_m)\), using the above fiber and signal parameters, these regions can be separated as follows. The exponentially increasing region reaches until the effect of non-linearity, defined by \(\gamma P_0\) where \(P_0\) is the optical input power, equalizes the dispersive one \((\beta_2 \omega_m^2)\), i.e., until \[\gamma P_0 \beta_2 \omega_m^2 \leq 1 \tag{3.6}\]

where the numerator is practically the coefficient of the envelope function in the last term of Eq. (2.60) with \(P_0 = |a(z = 0, t)|^2\), and the denominator is related to the GVD term, the second in (2.60).

Further behaviors can be assigned also to the loss because in loss compensated case notches do not appear after a critical intensity at all (see next subsection).

The second region, where the average RF signal intensity as a function of input intensity turns downward, non-linearity exceeds the magnitude of the chromatic dispersion at the fiber input. Notch position however does not appear at longer length but at shorter one. This is because optical power is attenuated progressively during the propagation and SPM related changes in spectrum can not happen so effectively.

In practice, this region is not significant because this behavior occurs only at very high intensities. However, if the fiber is dispersion compensated or dispersion shifted, this region might become important at reduced chromatic dispersion values.

Abrupt drop happens when the non-linearity and GVD related term exhibit the following relation \[\gamma P_0 \beta_2 \omega_m^2 \approx \frac{3}{4\pi} \approx 2.3 \tag{3.7}\]

Above this region we call the propagation of microwave modulated signal irregular.

This irregular behavior caused by the intense non-linearity can be observed in Fig. 3.4 at the plotted average intensity of the 160 mW signal. Fig. 3.5 shows however that the abrupt drops still happen regularly but further behaviors (average intensity as a function of propagating length, position of the second, third notch, etc.) are impossible to predict analytically.

We must note that the evaluated expressions slightly depends on the used modulator as well. The place of dispersion caused suppression of RF signal belong to a system outlined in the previous sections.
3.5.3 Lossless propagation

The attenuation of a signal in optical telecommunication systems and also in fiber-radio systems can be compensated along the transmission medium. A possible method to amplify the signal without the use of doped amplifier media is called Raman amplification. Through a phenomenon called induced Raman scattering the fiber is capable to amplify the signal using an approximately 13.5 THz blue-shifted high power pump. If the loss can be compensated this way the problem is traced back to the following periodic boundary value problem of solving the NLS equation

\[
\frac{\partial u}{\partial x} = -\zeta^2 \frac{\partial^2 u}{\partial t^2} + |u|^2 u
\]  

with a boundary condition of

\[
u(0, t) = a \sqrt{1 + m \sin(bt)}
\]  

where \(\zeta, a, m\) and \(b\) are positive real constants and \(u \equiv E\) now. This problem is somewhat similar to the one proposed and solved by Zabusky and Kruskal in connection with the KdV equation [92]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \zeta^2 \frac{\partial^3 u}{\partial x^3} = 0
\]

where

\[
u(0, x) = \cos(\pi x) \quad 0 \leq x \leq 2
\]

and \(u, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}\) are periodic on \([0, 2]\) for all \(t\); they choose \(\zeta = 0.022\). This numerical solution resulted a soliton solution with the recurrence of initial states.

I prepared numerical simulations neglecting loss from the previous calculations. I used various input powers using the original fiber parameters: 16.8 ps/(nmkm) chromatic dispersion and \(2.6 \times 10^{-20} \text{ m}^2/\text{W}\) nonlinear refractive index.

As illustrated in Fig. 3.6 one can see that above a critical input power the effect of dispersion is virtually completely compensated by non-linearity. As a consequence, there are no notches at all in this domain. At 10 mW input power the effect of dispersion in the form of modulation suppression is still apparent, but at 20 mW input the signal does not show this behavior. Between these two values therefore, a critical intensity exists where dispersion and Kerr non-linearity cancel each other [90].

Above this critical intensity, SPM exceed dispersion and as it can be seen in Fig. 3.7 the average intensity of the RF signal becomes higher at certain places
than the input. We plotted in Fig. 3.7 a 400 km long propagation with 20 km steps. The input signal was a 10 GHz, 20 mW sine modulated signal. Soliton-like recurrence can be observed in the changes of signal shape within this length. The periodicity of these oscillations can be observed in Fig. 3.6 too, where in case of a 60 mW input, the average RF signal oscillates several times along a 200 km path, although the amplitude of these oscillations are decreasing. Smaller intensity signals oscillate less than the ones with higher intensity within the same propagation length.

This kind of propagation can happen because a part of the optical power transfers to the RF signal due to SPM (see Fig. 3.8). Fig. 3.8 shows how the peaks of the spectrum is changing as a function of propagation path length. We plotted here only the central peak ($f_0$) and the two side bands ($f_0 \pm f_m$ where $f_m$ is the modulation frequency). Center peak has an input spectral intensity around 13 dBm. Side bands starts from smaller intensities and strong oscillations can be observed which coincide with the oscillations of the peak intensities of signal in the time domain (Fig. 3.7). Where central peak intensity is decreasing the two side bands are increasing in intensity and vice versa. This spectral intensity changes of side bands and carriers make possible the observation of soliton propagation [91].

![Figure 3.6: Relative intensity of a 10 GHz sine modulated RF signal as a function of distance in a lossless fiber. As the input intensity increases the plots show the dominance of SPM over GVD.](image-url)
Figure 3.7: The propagation of 10 GHz sine modulated signal with 25% modulation depth and 20 mW input power. RF signal peak intensities become higher than input ones after 100-140 km propagation in accordance with the increase of the average RF intensity in Fig. 3.6.

Figure 3.8: Oscillatory behavior of the peak intensities in the spectrum during the propagation. $f_0$ stands for the frequency of the perfect CW light source and $f_m$ is the modulation frequency. The small inset is intended to show the input spectral shape and the corresponding notation of peaks.
Chapter 4

Chirped pulse fiber delivery system for femtosecond pulses

This work is a theoretical investigation on a chirped pulse delivery system which is a part of a modern, real time, 3D, two-photon fluorescence microscope (2PFM) system now. This system is used for neurology related research in the Pharmacology Department of the Medical Research Institute of the Hungarian Academy of Sciences.

We met here the harmful effect of nonlinearity which must have been avoided because of pulse distortions which made the two photon imaging system useless. The aim of my calculations were to obtain and specify a pulse delivery system based on SMF which can transmit approximately 0.5 nJ ultra short laser pulses with less than 50 fs FWHM time duration. Transmitting pulses directly with these parameters is impossible using SMF. Pulse stretching, however, can lower the peak intensity sufficiently to maintain nearly the same shape till the output end of the fiber. At this point the compression has to be done and the system is ready for two-photon excitation.

The questions are: how large GDD should be applied to stay below 50 fs after recompression? How much pulse energy can be launched into the fiber for distortion free delivery using fixed prechirp and compression before and after the fiber?

During answering these questions we realized that spectral narrowing of the pulse can occur under certain circumstances.

In the followings, we describe briefly the phenomenon of two-photon excitation. The experiment is discussed in the second part of this chapter and the chirped pulse delivery system is introduced. The calculations are presented in the fourth section.
Figure 4.1: Jablonski diagram for two photon absorption. $h\nu_A$ is the photon for one photon excitation, $h\nu_{A1}$ and $h\nu_{A2}$ the photon energies for two-photon excitation and $h\nu_E$ is the emission. Usually $\nu_{A1} + \nu_{A2} \neq \nu_F$ because of small, additional molecular transitions.

4.1 Theory of two-photon excitation and multi-photon microscopy

Two-photon excitation is a physical process which require that the low energy molecules or fluorophores in a quantum event absorb two photons quasi simultaneously. During the absorption process, an electron of the molecule is transferred to an excited-state molecular orbital resulting in the emission of a fluorescence photon (See Fig. 4.1) [93]. The emitted photon has almost twice as large energy than the absorbed lower energy photons.

Multi-photon absorption lies on the same theory as two-photon excitation. Depending on the number of the absorbed photons the emitted photon will have so many times larger frequency. We note here, that the human cells contain such natural fluorophores in the mitochondrial system which are able to absorb three-photons simultaneously resulting in a nearly three times higher frequency photon.

The probability of the simultaneous two- or more-photon absorption is extremely low. Therefore, the application of high intensity, focused laser pulses are required. Typical pulse durations coming from a Ti:Sapphire oscillator should be well below 1 ps and their pulse energy above 0.5 nJ in order to obtain high photon flux in the focal point (0.1-10 MW/cm$^2$) resulting in significant amount of emitted light. This is because the virtual absorption of a photon of non-resonant energy lasts only for a very short period of time ($10^{-15} - 10^{-18}$ seconds). During this
time a second photon must be absorbed to reach an excited state. The number of absorbed photons in a sample excited by a laser beam focused by microscope objectives is described by the following equation \[94\]

\[
N_A = \frac{P_0^2 \sigma_{2P}}{\Delta \tau R_{pr}} \left( \frac{NA^2}{2hc\lambda} \right)^2
\]  

(4.1)

where \(\sigma_{2P}\) is the two-photon absorption cross-section, \(P_0\) is the average power, \(NA\) is the numerical aperture of the objective, \(\Delta \tau\) is the pulse width, \(R_{pr}\) is the repetition rate, \(c\) is the speed of light in vacuum, and \(\lambda\) is the excitation wavelength.

The most important differences compared to one-photon absorption (1PA) are the quadratic dependence on the average power (linear in 1PA) and the power-of-four dependence on the NA of the microscope objective.

The mentioned small probability ensures also the light emission from the focal point only. This is because the lower intensity light at other parts of the investigated sample can not produce two- or more-photon excitations. Therefore, very sensitive technique can be established on the above phenomenon in order to follow, for example, ion concentrations in neurons \[95\]. This method is called 2PFM which combines the two photon absorption phenomenon with a laser scanning technique.

The system realized in our work \[96\] for 2PFM is based on the idea to deliver the laser beam to the sample through optical fibers in order to be able to investigate living animals \[97\]. The problems must be solved in this case, are the pulse spreading, spectral broadening and scanning of the sample by moving the end of the fiber precisely. This relatively slow, scanning technique however can be changed by addressing many fibers by acousto-optic deflectors in order to obtain information within a micro-second useful for investigating neural processes \[98\]-\[100\].

### 4.2 Experimental setup

The used experimental setup is shown in Fig. 4.2. The intense laser pulses are provided by a mode-locked Ti:sapphire laser oscillator (FemtoRose 20 MDC \[101\]) with a central wavelength of 795 nm and a FWHM bandwidth of \(\sim 20\) nm.

A Faraday isolator (FI) was placed after the Proctor and Wise four-prism sequence in order to avoid any disturbance of the laser operation by backreflections. This FI reduced the negative dispersion by 2700 fs\(^2\).

Switching between the optical fibers is carried out by computer controlled acousto-optic modulator AOM \[103\]. The AO switches introduce an additional
positive GDD of $\sim 1500 \text{ fs}^2$. Angular dispersion of the AO switches is compensated by antireflection coated prisms ($P_x$, $P_y$) made of SF11 glass.

The AO switches are then imaged onto the fiber coupling lenses ($L_i$, $i = 1, \ldots, n$) by a large diameter doublet lens $L_1$ free of spherical and chromatic aberration.

The output ends of the optical fibers are imaged onto the sample by a 10:1 telescope imaging system. The imaging system consists of a collimating objective (OBJ$_1$, 4$\times$, NA$\sim$ 0.13), the high-dispersion ZnSe prism, and a focusing objective (OBJ$_2$, 40$\times$, NA$\sim$ 0.8). The spectral filtering is performed by a dichroic mirror (D). Fluorescent photons are detected by photomultiplier tubes (PMT$_1$, PMT$_2$) combined with spectral filters.

The lower part of Fig. 4.2 is a conventional scanning two-photon microscope.
used for the selection and alignment of the points to measure.

4.3 Pulse delivery system

Standard SM fiber (Nufern, 780HP [104]) was used to deliver pulses to the selected points of the investigated sample. This pulse delivery system ensures the Gaussian beam profile of the femtosecond pulses. SM fiber, however, causes temporal and spectral distortions on ultra-short and intense laser pulses which problem may pose the usage of different type of fibers as pulse delivery system such as single mode LMA MS fibers or HC fibers [105]. We must note here that both type of fibers have disadvantages such us increased mode size (LMA) and the water adsorption related losses (in the case of HC).

In order to avoid the nonlinear distortions, the successfully applied chirped pulse amplification (CPA) technique on broadband and ultra-short laser pulses [47] can be applied here as well. This procedure consist of the following steps:

1. stretching the initially short pulses by the addition of relatively large GDD. This reduces the peak intensity and the nonlinear distortion in the fiber media as well.

2. propagation of the stretched pulse in the fiber

3. compression of the output pulse introducing nearly the same GDD with the opposite sign as at the input end of the fiber.

Since the free space between the two objectives (OBJ1 and OBJ2, See Fig. 4.2) is limited, high-space demanding negative GDD setup was used before the fiber. The negative chirp (-14000 fs²) was introduced by a Proctor and Wise four-prism sequence [102]. The positive chirp after the fiber output was realized by a highly dispersive zinc-selenide (ZnSe) unit built into the imaging system, which is a right-angle prism and both additional optical elements possessing positive material dispersion.

4.4 Spectral and temporal characteristics of the delivered pulses

I prepared simulations on pulse propagation through the optical fibers in order to control the prechirp and optimize pulse parameters for the above described system.
To calculate the optimum arrangement and the corresponding distance values of the Proctor and Wise four-prism sequence of SF10 prisms, the dispersion of the optical components were measured.

Stability of pulse duration at different pulse energies were investigated by the SSF method. Fig. 4.3 (a) shows the effect of recompression on the initially stretched pulse by the high positive dispersion originating from the imaging system. The plotted pulse shapes corresponds to a pulse with 10 mW average power. Parameters used in the simulations are gathered in Table 4.1.

Pulses stretched to several hundred femtosecond time duration could be compressed to 40 fs approximately. Fig. 4.3 (b) shows that the pulse duration remains 40 fs if the coupled average power is 10, 20 or 40 mW.

However, pulses with the same duration at 80 mW average power (pulse energy $\sim 1$ nJ) can be recompressed only to $\sim 100$ fs due to the simultaneous nonlinear and dispersive effects in the fiber. The pulse shape become strongly distorted

Table 4.1: Physical parameters of the system used for the simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fiber parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fiber length</td>
<td>130</td>
<td>mm</td>
</tr>
<tr>
<td>Dispersion (@800 nm)</td>
<td>-95</td>
<td>ps/nm·km</td>
</tr>
<tr>
<td>Dispersion slope</td>
<td>0.4</td>
<td>ps/nm²·km</td>
</tr>
<tr>
<td>Attenuation (@780 nm)</td>
<td>4</td>
<td>dB/km</td>
</tr>
<tr>
<td>Nonlinear index</td>
<td>$3.2 \times 10^{-20}$</td>
<td>m²/W</td>
</tr>
<tr>
<td>Effective core area</td>
<td>15</td>
<td>µm²</td>
</tr>
<tr>
<td>NA</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td><strong>Pulse parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average power</td>
<td>10, 20, 40, 80</td>
<td>mW</td>
</tr>
<tr>
<td>Repetition rate</td>
<td>76</td>
<td>MHz</td>
</tr>
<tr>
<td>Central wavelength</td>
<td>795</td>
<td>nm</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>20, 40</td>
<td>nm</td>
</tr>
<tr>
<td>FWHM (chirp free)</td>
<td>33</td>
<td>fs</td>
</tr>
<tr>
<td>Additional input chirp</td>
<td>-14100</td>
<td>fs²</td>
</tr>
<tr>
<td><strong>Dispersion compensating unit</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beam path in ZnSe</td>
<td>12</td>
<td>mm</td>
</tr>
<tr>
<td>GDD</td>
<td>10000</td>
<td>fs²</td>
</tr>
</tbody>
</table>
Figure 4.3: (a) Recompression of the output pulse with 10 mW average power after the fiber sample to 36 fs. Small inset zooms to the input and output pulses plotted with the same line types. (b) Pulse shapes corresponding to different intensities after recompression. 40 mW pulses still suffer acceptable distortions.

at this intensity level. I note that in [96], we did not included the third order dispersion contribution of the fiber into the calculations but this effect only results somewhat asymmetric shape of the pulse.

The background of the incompressibility of 80 mW signal is due to the spectral narrowing happens during the propagation. We observed numerically and experimentally that the strongly chirped pulses suffer spectral narrowing [99] if the guiding medium has an opposite chirp contribution to the pulse evaluation (See Fig. 4.4). The input pulse had \(-14000 \text{ fs}^2\) in our case and the SM fiber had positive frequency chirp contribution induced by SPM in the visible region. Silica glass has positive GDD (negative dispersion coefficient) below the 1.3 \(\mu\text{m}\) wavelength region.

Simulations showed also that 25 fs pulses can be generated at the sample even at 80 mW if we use 40 nm FWHM bandwidth input pulses instead of 20 nm bandwidth [100]. This is due to the much higher stretching effect of the same dispersion for shorter pulses, which causes smaller nonlinearity in the fiber.

The fiber delivery system was tested by measuring the temporal and spectral properties of the femtosecond pulses at different energy levels. In the experiment, we used an SMF with a length of 130 mm (See fiber parameters in Table 4.1). After collimating the light exiting the fiber, the negative dispersion was compensated by
the ZnSe prism (the length of the beam path in ZnSe was 12 mm, which provided GDD $\approx 10000\text{fs}^2$). The measured spectra at the input/output and a corresponding autocorrelation trace at the output are shown in Fig. 4.4 (a).

Spectral FWHM for the pulses with 10, 20, 40 and 80 mW average power were monotonically decreasing and the simulations provided their width as 18.7, 17.4, 14.7 and 10.8 nm, respectively. The simulated spectral narrowing is shown in Fig. 4.4 (b).

As a summary of this work we may claim that a distortion-free pulse delivery system was modeled and experimentally achieved by the chirped pulse concept usually used in chirped pulse amplifiers. Femtosecond pulses were delivered through a SMF without significant temporal and spectral distortions. Simulation determined the intensity range of pulses can be applied in the built system without significant impact on pulse quality. Simulation and experiments also showed that spectral narrowing can be achieved with highly prechirped pulses in a medium with opposite GDD contribution and SPM.
Chapter 5

Optimizing ultrashort pulse compression in microstructured fibers

Pulse compression in a nonlinear medium which causes spectral broadening is an area investigated for many years. The appearance of the novel PCF or MOF revolutionized this scientific area as well as many other fields. Small core area PCFs provided the possibility of using only a small piece of them (avoiding large distortions by higher order dispersion) because of their very small core area resulting in significant spectral broadening even in case of a short fiber length. This way the duration of the pulse may become much shorter than the initial one.

Our group has already had some publications on this field but we decided to conduct a deeper analysis of nonlinear PCF compression using newer fibers with red-shifted zero dispersion wavelength and involving computer optimization. The experience of my previous works intimate that pulse prechirp should have significance on pulse quality and duration after the compression. This is because prechirp can control the spectral and temporal distortions in the fiber with some limitations. Finding an optimization procedure is difficult in case of HNL fibers because more than one optimum is expected as a function of input and output chirps.

We show in the followings, how input and output chirp of the pulse should be selected to reach the shortest available pulse duration in a given experimental arrangement with certain quality factor. We also show the compression possibilities using dispersion flattened PCFs as a cost effective and efficient solution of producing sub-6 or even sub-5 fs pulses launching 20-30 fs pulses in the fiber.
CHAPTER 5. ULTRASHORT PULSE COMPRESSION IN MOFS

5.1 Introduction

Pulse compression experiments using PCFs dates back to the beginning of this decade after the construction of the first dispersion tailored small core area PCF for supercontinuum generation [28]. Because the small core area results in larger SPM during the same propagation length, it can be used for broadening the spectrum considerably even at low pulse energies (around 1 nJ). Using a few centimeter long fiber sample, appropriate spectral broadening can be achieved without too large temporal distortions caused by the chromatic dispersion [30].

The first PCF samples that can be seen in Fig. 5.1 were prepared in the University of Southampton, Southampton, UK [19] and in the Bell Laboratories, Lucent Technologies, New Jersey, US [28].

In my dissertation, I show that it is possible to obtain compressed sub-6 fs pulses.
CHAPTER 5. ULTRASHORT PULSE COMPRESSION IN MOFS

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pulses using nanojoule or sub-nanojoule seed pulses at around 800 nm by utilizing only a small-core area PCF and prism-pair/chirped mirror compressors. In the study Ref. [30], the compressed pulse duration was primarily limited by the maximum available wavelength difference between the laser central wavelength (750 nm) and the zero-dispersion wavelength (767 nm) of the PCF sample. Novel PCFs with red-shifted zero-dispersion wavelengths, however, can improve both the quality and the duration of the compressed pulses when we choose input and output chirp compensation parameters properly. It is worth pointing out that 1 nJ seed pulse energies with the required pulse durations can be obtained easily from low pump threshold, mode-locked Ti:sapphire laser oscillators pumped by only 1.2 W at 532 nm [107].

As a proof of my calculations, I describe our corresponding experiment with similar experimental conditions as the simulations. I also extend the simulations to the compression of sub-nJ pulses with initial time duration at around 1 ps. Later, I discuss the possibility of building cost efficient, compact sub-100 fs laser sources by utilizing nonlinear spectral broadening and dispersion compensation. As a seed pulse, we consider low cost, compact ultrashort pulse laser diodes with transform limited pulse duration of around 1 ps [108, 109]. Their typical pulse energy is well below 1 nJ, which does not result in considerable spectral broadening in standard SMFs. However, nonlinear spectral broadening (and possible amplification) in doped [110] small core area PCF-s allows the reduction of the pulse duration to the sub-100 fs regime. Previously, this time domain could only be achieved by more expensive solid-state lasers.

5.2 Theory

We calculate the pulse propagation through PCFs as a nonlinear Schrödinger type system (See Eq. (2.60) in Section 2.2.3). The input pulses used in our simulation exhibit a sech² temporal intensity envelope function that is typical for femtosecond solid state laser oscillators. We used dispersion data provided by the manufacturer (type “2.2 Nonlinear PCF” fiber, Crystal Fibre, Denmark [111]) in our calculations. Since the dispersion was not available for the required broad spectral range necessary for our calculations, it was approximated by a Taylor-expansion:

\[ D(\lambda) = D_0 + S(\lambda - \lambda_0) + \frac{T}{2}(\lambda - \lambda_0)^2 + \frac{F}{6}(\lambda - \lambda_0)^3. \]  

(5.1)

In Eq. (5.1), \( \lambda_0 \) is the central wavelength (reference wavelength) of the seed pulse, \( D_0 \) is the chromatic dispersion at the central wavelength, \( S \) is the dispersion-
slope, $T$ is the third-order dispersion and $F$ is the fourth-order dispersion having the corresponding values of $D_0 = -27.15 \cdot 10^{-6}$ s/m$^2$, $S = 0.51772 \cdot 10^3$ s/m$^3$, $T = -3.277854 \cdot 10^9$ s/m$^4$, $F = 1.642713 \cdot 10^{16}$ s/m$^5$, respectively.

We found that the compression level strongly depends on the initial chirp of the pulse injected into the fiber for possible shortest pulse durations corresponds to the given spectrum in the sub-100 fs regime. We must note here that in coherent optical systems, the shortest pulse duration corresponds to a given spectrum is called transform limited pulse width. In this case, the pulse and spectral shapes are regular and the expression for $\Delta \nu$ and $\Delta \tau$ given in Section 2.1.3 is acceptable. In the case of strongly modulated spectra, the width of the spectral shape can be defined quite elaborately. Furthermore, systems with very broad spectral regions are difficult to consider as a coherent system (coherent length is inversely proportional with the spectral width). Because of the above reasons we will not use the expression “transform limited” for such pulses which have broad and irregular spectral shape. Instead of transform limited we apply the “possible shortest pulse duration” terminology throughout this dissertation.

Providing a small linear chirp to the input pulse, the pulse duration becomes slightly longer but it results in lower pulse shape distortion during propagation in the fiber (the distortion is caused by the strong third-order dispersion of the PCF). However, the required linear pre-chirp may result in less efficient spectral broadening during propagation. One may control the spectral broadening at a certain energy level in this way, and avoid frequency components that may harm the quality of the compressed pulse.

In our calculations, the following expression was used for describing the pre-chirp of the laser pulse seeding the PCF sample:

$$E_{\text{chirped}}(0, T) = \mathcal{F}^{-1} \left\{ \exp \left( i\phi_{2}^{\text{in}} \omega^2 + i\phi_{3}^{\text{in}} \omega^3 \right) \mathcal{F} \left\{ E(0, T) \right\} \right\}$$  \hspace{1cm} (5.2)

where $\mathcal{F}$ stands for Fourier transformation, $E(z, T)$ is the complex envelope function of the seed pulse with no chirp (where $z$ is the space coordinate in the propagation direction and $T$ is the retarded time), $\omega$ is the angular frequency and $\phi_2^{\text{in}}$ and $\phi_3^{\text{in}}$ are the second- and third-order pre-chirp parameters (GDD and TOD), respectively. The definition of GDD and TOD can be find in Section 2.1.3.

Compression after the fiber sample is described in the same way for the complex envelope function using output chirp parameters (\phi_2^{\text{out}}, \phi_3^{\text{out}}).
5.3 Experiment

Our Ti:sapphire laser oscillator (FemtoRose 20 MDC [101]) operated at 797 nm, and delivered 24 fs sech² pulses at a repetition rate of 76 MHz. A PCF piece with a length of 22 mm was the shortest that could be cut with our fiber cleaver, although the optimal length predicted by our simulation was shorter (see Sect. 5.4.1 below). Accordingly, the input pulse energy had to be further reduced in order to get spectral shapes similar to what can be obtained with a 6 mm long PCF used in our simulations.

The experimental setup is shown in Fig. 5.2. In order to provide the optimal pre-chirp parameters, a pre-compressor was built comprising of an SF10 prism pair and a pair of chirped mirrors. A FI was also installed into the pre-compressor to avoid feedback from the fiber [30]. The positive dispersion introduced by the FI (GDD of 2700 fs²) had to be compensated as well during pre-compression. We could set the pre-chirp between 100 fs² and 400 fs² in this way. The pre-compressor provided an input TOD of approximately -6000 fs³.

We obtained two-fold compression starting from 24 fs pulses with $\phi_{in}^{2} = 400$
fs$^2$ and $\phi_3^{in} = -6000$ fs$^3$ \[113\]. The measured and computed autocorrelation traces are shown in Fig. 5.3 (left) and the corresponding spectra are plotted in Fig. 5.3 (right). In the inset, the retrieved temporal pulse shape is shown with FWHM pulse duration of 12 fs.

We note that in order to fit our simulations to the measured experimental data, we had to increase the effective core area of the PCF in our model: it had to be doubled to the new value of around 10 $\mu$m$^2$. It might have been partially caused by improper orientation of the PCF sample (see Ref. \[30\]). An additional practical problem originated from the use of the Faraday-isolator that had to be placed in front of the PCF: its relatively high dispersion had to be compensated by an SF10 prism pair which limited the bandwidth and hence the transform limited pulse duration of the seed pulse at around 25 fs.

Because of the relatively good agreement between the experiment described and our corresponding simulations we continue our investigations with numerical modeling in the following section. The same fiber parameters and different input pulse durations and chirp parameters are used for our investigations in the simulations except the effective core area. We take this fiber parameter from an earlier study \[30\] and use in the following section. In Section 5.4.2 we turn back to the value in the experiment ($A_{\text{eff}} \approx 5 \mu$m). This is because in the next sec-
Figure 5.4: Peak intensity of 1 nJ optimally compressed pulses as a function of input GDD and TOD. The result corresponds to a small core area PCF with zero dispersion wavelength of 860 nm. The seeding laser pulse has a central wavelength of 760 nm and a transform limited FWHM pulse duration of 12 fs.

Figure 5.4: Peak intensity of 1 nJ optimally compressed pulses as a function of input GDD and TOD. The result corresponds to a small core area PCF with zero dispersion wavelength of 860 nm. The seeding laser pulse has a central wavelength of 760 nm and a transform limited FWHM pulse duration of 12 fs.

5.4 Modeling

5.4.1 Optimization

For the optimization of the input chirp parameters ($\phi_{2}^{in}, \phi_{3}^{in}$) and compression parameters ($\phi_{2}^{out}, \phi_{3}^{out}$) at a given fiber length, input pulse energy and pulse duration, we assumed that the shortest compressed pulse exhibits the highest peak power. This assumption was used in a brute-force optimization method to find the best second- and third-order input and output GDD and TOD parameters of a sech$^2$ input pulse through a given range of the chirp parameters [112, 113]. Fig. 5.4 shows the peak intensity of the compressed pulse as a function of input GDD and TOD after providing the best compression parameters for each calculated point.

In our calculations, the length, nonlinear refractive index and effective core area of the fiber were respectively chosen as 6 mm, $2.5 \cdot 10^{-20}$ m$^2$/W and 2.5 µm$^2$. 
The seeding laser pulse has a central wavelength of 760 nm, pulse energy of 1 nJ that corresponds to 76 mW average output power at a repetition rate of 76 MHz. The FWHM pulse duration was chosen to be 12 fs. Such pulse durations can be obtained from a mirror-dispersion controlled Ti:sapphire oscillator, see Ref. [107]. We must note that the pulse parameters used in the presented optimization process corresponding to Fig. 5.4 and Fig. 5.5 are slightly different from our experimental conditions. The effective core area is the same as used in Ref. [30].

According to the optimization map (see Fig. 5.4), the shortest compressed pulses can be generated at around $-200$ fs$^2$ input GDD and $-200$ fs$^3$ input TOD. The best compression values that correspond to this peak are $\phi_{2}^{\text{out}} = -100$ fs$^2$ and $\phi_{3}^{\text{out}} = -350 \cdots -400$ fs$^3$. The corresponding computed temporal and spectral intensity distributions are shown in Fig. 5.5(a). The compressed pulse that belongs to the red spot around $\phi_{2}^{\text{in}} = 140$ fs$^2$ and $\phi_{3}^{\text{in}} = -1530$ fs$^3$ is shown in Fig. 5.5(b). In this latter case, the compressed pulse width is quite short (4.7 fs) but the quality of the pulse is worse than that of the compressed laser pulse shown in Fig. 5.5(a). The reason for this is that the spectrum displayed in Fig. 5.5(b) extends well above the zero dispersion wavelength (860 nm) into the anomalous dispersion region of our
PCF sample. This part of the spectrum cannot be compressed properly, which results in a pedestal and satellite pulses of the compressed pulse. The obvious asymmetric spectral broadening both in Fig. 5.5(a) and Fig. 5.5(b) is caused by the considerable third-order dispersion of our PCF around 800 nm.

5.4.2 Compression of longer pulses

In the following, we present some attracting features of pulse compression using small core area PCF-s with the same parameters described in Sec. 5.3. We performed a number of calculations using different input pulse widths and we searched for the shortest compressed pulse durations as described in the previous section. We started our investigation with 1 ps pulses that can be obtained from ultrafast laser diodes. Then we use shorter and shorter pulses which could be related to different types of fs pulse solid-state laser oscillators [114]. The compression results are summarized in Table 5.1 and Figure 5.6.

Here, we define the Quality Factor (QF) of a compressed laser pulse as the ratio of the energy in the main peak and the total energy of the pulse taking on values between 0 and 1:

\[
QF = \frac{\int_a^b |A(z, T)|^2 dT}{\int_{-\infty}^{\infty} |A(z, T)|^2 dT}
\]

(5.3)

In Eq. (5.3), the numerator is computed as a temporal integral of the intensity function between time \(a\) and \(b\) corresponding to the two local intensity minima around the main peak. The Quality Factors calculated for the different compression and pulse parameters are listed in Table 5.1.

We found that 1 ps, 1 nJ pulses can be compressed to 50 fs resulting in a 20-fold compression ratio (see Table 5.1). We note that a similar compression was experimentally demonstrated at around 1550 nm using soliton compression in a dispersion flattened fiber [116]: a 965 m long polarization maintaining SMF was used and 54 fs pulses were obtained after the propagation along the slow axis with 71 mW input power at a repetition rate of 10 GHz.

In the case of our PCF sample, the compression ratio and the quality factor are strongly limited by third-order dispersion in the fiber. Although the PCF that we used in our simulations has much higher third-order dispersion at around 800 nm than the dispersion flattened fiber at 1550 nm, our simulation shows that it is still possible to obtain 50 fs compressed pulses.

We note that no pre-chirp was needed for the pulses with transform limited time durations between 100 fs and 1 ps, as it had no measurable effect on the
Figure 5.6: Compressed pulse width and applied GDD for the best compression as a function of initial pulse duration. We plotted also the temporal and spectral shape of some results. Strong asymmetric spectral broadening is also presented due to the higher order dispersions in the relatively wide spectral ranges.
Table 5.1: Pulse compression in PCF with zero dispersion wavelength of 860 nm computed for 1 nJ seed pulses of different transform limited pulse durations with a central wavelength of 797 nm and for different fiber lengths. The optimal pre-chirp, compression parameters and the QFs are listed along with the shortest compressed pulse durations.

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CHAPTER 5. ULTRASHORT PULSE COMPRESSION IN MOFS

spectral broadening process in the fiber. For instance, we tried to optimize pre-chirp for 250 fs seed pulses, but only extremely large negative TOD resulted in slight reduction of the final compressed pulse duration. Short seed pulses with time durations below 100 fs, however, suffer irreversible distortions during propagation due to higher-order dispersion. Therefore we have to use some linear pre-chirp to stretch the seed pulse slightly. We note that for shorter seed pulses we need higher optimal relative temporal stretching \( \tau_{\text{stretched}} / \tau_0 \). During optimization we also aimed for minimum necessary pre-chirp.

One can observe in Table 5.1 that high compression ratios can be obtained for considerably different linear and higher order pre-chirp parameters (see, for instance, the different parameters for 100 fs seed pulses resulting in compressed pulse durations in the 13.8-15.3 fs range). These results are similar to the calculations presented in Fig. 5.6. Having a 100 fs seed pulse the shortest compressed pulse is 13.8 fs long, which is slightly longer than what was presented in Ref. [30] (≈ 13 fs). Shorter pulses can be generated only at the expense of quality (like in Fig. 5.5 (a) and (b)).

5.4.3 Compression with dispersion flattened PCFs

Our PCF sample had large higher order dispersion contribution to the pulse evaluation which limited the usable length of the fiber because of the irreversible pulse shape distortions. This also limits spectral broadening obtained by SPM on a shorter fiber length.

This harmful effect can be avoided using such PCFs in which the waveguide dispersion is suited by careful structural modifications [117]. This can be done by changing the size and distance of the holes in the cladding region [26], [117]-[124]. The physics behind it is that the total dispersion of a waveguide comes partially from the material dispersion and partially from the waveguide contribution to the dispersion. The waveguide contribution can be large enough if the propagating mode is confined in a relatively small core \( r < 2 - 4 \times \lambda \). This can result in flat total dispersion curve of the waveguide and improve the quality of pulse compression significantly. The first demonstrated ultra-flattened and near zero dispersion PCF is shown in Fig. 5.7. This fiber shows dispersion of \( 0 \pm 1.2 \) ps/(nm · km) over 1 \( \mu \)m-1.6 \( \mu \)m wavelength [125].

We present calculations here using the same parameter set done in Sec. 5.3. An unchirped \( \phi_2 = 0 \) fs\(^2\), \( \phi_3 = 0 \) fs\(^3\)) 24 fs input pulse with 1 nJ is launched in a 6 mm length PCF whose dispersion is characterized by Eq. (5.1). The nonlinear refractive index and effective core area are \( 2.5 \cdot 10^{-20} \) m\(^2\)/W and 5 \( \mu \)m\(^2\), respectively. We
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Figure 5.7: The first dispersion flattened PCF with 11 periods of air holes around the core. The average hole diameter was 0.57 \( \mu \text{m} \) and the hole to hole spacing (pitch) was 2.47 \( \mu \text{m} \). Figure was obtained from Ref. [125].

Figure 5.8: Compressed shortest pulse duration and the quality factor of the pulse as a function of dispersion slope.

only changed the dispersion slope parameter \( (S) \) characterizing the fiber dispersion and searched for the optimum compression parameters \( (\phi_2^{\text{out}} \text{ and } \phi_3^{\text{out}}) \) after the propagation. Results are summarized in Fig. 5.8.
After a certain dispersion slope value, pulses can not be compressed in a good quality anymore. Although the main peak could be quite short, less than 6 fs actually, but 30-40% of the pulse energy is outside of the main peak. This is the case when pre-chirp is required in order to broaden the pulse not to suffer unwanted, irreversible distortions caused by higher order dispersion. This limits however the spectral broadening.

Summarizing our simulations, we can conclude that using a PCF with zero dispersion wavelength of 860 nm and dispersion properties described by Eq. (5.1), twentyfold compression can be obtained in case of 1 nJ input pulses with time duration of 1 ps. Additionally, starting from 12 fs transform limited seed pulses, which can be obtained from a relatively low cost fs pulse Ti:Sapphire laser, 6 fs compressed pulses can be generated. By the reduction of the higher order dispersion in a small core-area fiber, high quality (≥ 94%) 6-7 fs pulses can be generated starting from 24 fs transform limited pulse duration which means a fourfold compression in this region.
Summary

During the past few years, my research interest focused on nonlinear wave propagation in single mode step index and microstructured fibers. These investigations are strongly related to the designation of such step-index or microstructured fibers which change the dispersion property of the fiber through the waveguide contribution. This improvement may change the quality and the achievable duration of the compressed ultrashort pulses, for instance. I would like to summarize here the new scientific results in my research based on the above mentioned topics.

I investigated the nonlinear propagation of microwave modulated signal transmission over standard single mode fiber theoretically and experimentally. After the good agreement between simulation and experiment, I investigated signal propagation in relatively higher intensity regions. I discovered the soliton propagation of sinus modulated signals modeling that as a nonlinear Schrödinger type system in the GHz region. The propagation was investigated in a few hundred kilometer standard single mode fiber. I looked for such parameter sets which could provide the stable and distortion-free propagation of the 10 GHz intensity modulated CW laser source. The average intensity which had to be launched in the fiber was 20 mW. This signal could achieve slightly narrower signal formation after certain propagation path length (100-150 km) as an interplay between dispersion and self-phase modulation, and got back the original shape during the same fiber length. This recurrence phenomenon was observed only in case of soliton pulse propagation earlier. I demonstrated the soliton formation of microwave modulated signals as well [91], although the propagation properties of modulated signals are fundamentally differ from that of pulse propagation.

For lower intensity signals, I found approximate formulas for the determination of the position of dispersion suppression as a function of the quotient of group velocity dispersion and nonlinear coefficient [90, 91]. I also advised the experimental demonstration of the soliton propagation of microwave modulated signals in SMF utilizing the loss compensation possibilities of Raman amplification [91].
In order to avoid the harmful effect of nonlinearity in an ultra short pulse transmission system, based on SMF, we used chirped pulse delivery system in which the initial chirp stretches the pulses preventing them from nonlinear distortions. I calculated and optimized the chirp parameters of the pulse in order to maintain its shape undistorted and below an FWHM of 50 fs after recompression. During the simulations, we observed the spectral narrowing effect of highly prechirped, ultra-short pulses [99, 100] which was demonstrated experimentally as well [98]-[100].

The ultrashort, chirped pulse delivery system is a part of a 2PFM which require good beam quality and high peak power to achieve multi-photon excitation on the targeted sample. I investigated numerically the intensity limit where ultrashort pulses (∼55 fs) coming from a Ti:Sapphire oscillator with ∼20 nm bandwidth could be transmitted without temporal and spectral distortions. I found that applying ∼−14000 fs$^2$ prechirp using a Procter and Wise four-prism sequence and applying a dispersion compensation after the fiber output with ∼10000 fs$^2$ positive GDD, the pulses with 10, 20, and 40 mW average power could be distortion-freely delivered. This setup provided almost the same FWHM for these pulse powers which is also a unique feature of the system [96]. The obtained FWHM was around 40 fs. Pulses with 80 mW average power, however, could not be recompressed below 100 fs pulse duration without changing prechirp. The strong spectral narrowing and the large temporal distortions caused this limit.

Ultrashort pulse compression in photonic crystal or microstructured fiber was also investigated theoretically and experimentally. In our experiment a two fold compression was achieved starting from 24 fs transform limited seed pulses from a Ti:Sapphire laser [113]. The simulations with similar physical parameters showed a good correlation to the experiments after the adjustment of the effective core size to the experimental results.

I observed that below a given pulse duration (∼100 fs) which slightly depends on the higher order dispersion contribution of the fiber, optimal pulse compression can only be achieved by adding chirp to the ultrashort pulse before the input end of the fiber [114, 115]. This chirp (GDD, TOD) has an opposite sign than that is contributed to the pulse evaluation in the fiber. This pre-chirp can prevent the pulse from irreversible temporal distortions during the nonlinear propagation. I advised a computation method which can optimize the pulse compression in microstructured fibers and gives an optimization map with pulse peak intensities as a function of input GDD and TOD after ideal compression at the output end. This optimization procedure was carried out for different pulse durations (1 ps - 12 fs) which can be associated with different types of lasers (mode-locked diode to solid state lasers) [112].
I analyzed the possibility of using different microstructured fibers with different dispersion properties by simulation and I showed which dispersion parameters of the fiber should be the most appropriate to obtain ideal pulse compression without adding any pre-chirp to the pulse [112]. The investigated dispersion curves can be achieved in MOFs with structural modifications.
Thesis points

1. I investigated theoretically and experimentally the microwave modulated signal transmission over standard single mode optical fibers. I calculated the soliton formation and stable soliton propagation of 10 GHz sinus modulated signal for 20 mW and higher average powers [90, 91]. I advised the Raman amplification for eliminating the attenuation of the carrier in the case of long propagation path lengths [91].

2. I discussed the appearance of dispersion caused modulation suppression along the fiber as a function of the nonlinearity and chromatic dispersion [91]. The obtained results were summarized in approximating formulas.

3. I demonstrated numerically that the negatively chirped ultra-short pulses suffer spectral narrowing during its transmission on a single mode fiber in the normal dispersion regime as an interplay of chromatic dispersion and self-phase modulation [99]-[100]. I calculated also the limit intensity for the delivery of the initially chirped pulses with 20 nm bandwidth can be recompressed to nearly 40 fs at the end of the transmission system without temporal distortions [96].

4. I investigated theoretically the nano-Joule and sub-nano-Joule ultrashort pulse compression in small core area microstructured fiber. I showed that both, pre-compensation (initial chirp on the pulse) and dispersion compensation are required in order to obtain high quality sub-6 fs compressed pulses starting from 12 fs pulse duration applying only prism pairs and chirped mirrors beside the highly-nonlinear fiber [113, 114, 115].

5. I analyzed pulses with picosecond and several times hundred femtosecond durations and showed that proper dispersion compensation may result in twentyfold pulse compression on this time scale [112]. I showed that using a microstructured fiber with lower third order dispersion contribution to the
pulse evaluation, one may get record sub-6 or even sub-5 fs pulses starting from 24 fs transform limited pulse duration [112].
List of publications


7. Z. Várallyay, J. Fekete, Á. Bányaśz, R. Szipőcs, “Sub-nanojoule pulse compression in small core area photonic crystal fibers below the zero dispersion
wavelength,” Trends in Optics and Photonics, 98, 571-576 (2005).


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