Non-Linear Prediction Model for the European Small Aircraft
Accessibility for 2020

by

Dániel ROHÁCS

In partial fulfilment of the
requirements for the degree of
Philosophy Doctor
in
aeronautical science

Supervisor: Tamás Gausz

Budapest, October 2007
Nyilatkozat

Alulírott Rohács Dániel kijelentem, hogy ezt a doktori értekezést magam készítettem és abban csak a megadott forrásokat használtam fel. Miden olyan részt, amelyet szó szerint, vagy azonos tartalomban, de átfogalmazva más forrásból átvettem, egyértelműen, a forrás megadásával megjelöltem.

Budapest, 2007 Június 29.

Aláírás
Table of contents

Abbreviations ...............................................................................................................5

1. Introduction .........................................................................................................7

2. Small aircraft activity and development: a state-of-the-art in 2006 .....................10

2.1. Worldwide small aircraft concepts ...........................................................10

2.2. Small aircraft enabling technologies: aircraft design and operational concepts ...14

2.3. Potential aircraft for small aircraft purposes in 2006 .................................19

2.4. Potential demand and prediction models ......................................................22

3. Analysis of the European small aircraft air traffic in 2006..................................27

3.1. Data source of the analysis.............................................................................27

3.2. Methodology of the analysis .........................................................................30

3.3. Results of the small aircraft air traffic analysis .............................................32

3.4. Lessons learned from the analysis: initial data for the prediction model.........36

4. Methodology and modelling ..............................................................................40

4.1. Analysis of the generally used approaches for small aircraft modelling ........40

4.2. Initial linear model.........................................................................................42

4.3. Innovation diffusion theory approach: S-curve application ............................47

4.4. Non-linear analogical approach......................................................................50

5. Experimentations / simulations ........................................................................54

5.1. The proposed approach ..................................................................................54

5.2. Model variables .............................................................................................58

5.3. Monte Carlo Simulation..................................................................................64

5.4. Sensitivity analysis and validation ..................................................................67

6. Results and conclusion......................................................................................71

6.1. The total operating costs of small aircraft and the travelling budget constraints in 2006..........................................................71

6.2. The probabilistic evolution of the total operating costs of small aircraft and the travelling budget constraints .........................................................75

6.3. The probabilistic evolution of small aircraft accessibility .........................79

6.4. Conclusion......................................................................................................82
7. Summary and theses.................................................................................................................. 85

7.1. Summary......................................................................................................................... 85

7.2. Future works and perspectives...................................................................................... 86

7.3. Contributions / theses ............................................................................................... 86

Appendix A: The technical characteristics of the selected small aircraft .................. 89

List of publications ............................................................................................................... 90

References ............................................................................................................................ 92
Acknowledgements

Firstly, I would like to thank my advisor, Dr. Vu Duong for his continuous guidance throughout this work and for giving me the opportunity that my research would be funded by the EUROCONTROL Experimental Centre.

My sincere thanks go to my supervisor, Dr. Tamas Gausz for his particular advises and discussions that helped me to progress. I also would like to express my appreciations to the Department of Aircraft and Ships at the Budapest University of Technology and Economics, to make my PhD studies feasible.

A special thanks goes to my tutors, Marc Brochard and Marc Dalichampt at the EUROCONTROL Experimental Centre (EEC) for their help, support, interest and valuable hints.

Within the EEC, it was a pleasure to make a part of the amazing team of INO, which enabled me to work in an international atmosphere and to meet different innovative investigations. Many thanks to all PhD students and staff who have provided a pleasant environment during my stay at INO.

I express my gratitude to Alexandre d’Aspremont to support my research with his knowledge and for my three month internship at Princeton University in 2006.

Finally, I thank my family; Jozsef, Ludmilla and Viktor to support me and always being there in good and bad times. I also send my deepest gratitude to my girlfriend Maria whose patient love enabled me overcome the difficult moments and to complete this work.
Abbreviations

ADS-B: Automatic Dependent Surveillance - Broadcast
AGATE: General Aviation Transportation Experiments
AIP: Aeronautical Information Publications
ATFM: Air Traffic Flow Management
ATC: Air Traffic Control
ATM: Air Traffic Management
AWIN: Aviation Weather Information
BADA: Base of Aircraft Data
BRS: Ballistic Recovery System
CAPS: Cirrus Aircraft Parachute System
CNS: Communication Navigation Surveillance
CFIT: Controlled Flight Into Terrain
CFMU: Central Flow Management Unit
COSAAC: Common Simulator to Assess ATFM Concepts
ESTOL: Extreme Short Take-Off and Landing
FAA: Federal Aviation Administration
FAR: Federal Aviation Requirements
FL: Flight Level (nominal altitude in feet divided by 100)
FP: Flight Plan
FSW: Friction Stir Welding
GA: General Aviation
GAP: General Aviation Propulsion
GDP: Gross Domestic Product
GG: Gas Generator
GPS: Global Positioning System
HVO: High Volume Operations
ICAO: International Civil Aviation Organization
IFR: Instrumental Flight Rules
JAA: Joint Aviation Authorities
JAR: Joint Aviation Requirements
MEMS: Micro-Electro-Mechanical Systems
MGGF: Multi-Gas Generator Fan
MTOW: Maximum Take-Off Weight
PATS: Personal Air Transportation System
PAV: Personal Air Vehicles
PDF: Probability Density Function
PETA: Pulsed Ejector Thrust Augmenter
PLJ: Personal Light Jet
PPL: Private Pilot License
QA: Quality Assurance
QC: Quality Control
RFL: Requested Flight Level
SA: Small Aircraft
SATS: Small Aircraft Transportation System
SID: Standard Instrumental Departure
STAR: Standard Terminal Arrival Routes
STOL: Short Take-Off and Landing
SVS: Synthetic Vision System
TB: Travelling Budget
TBO: Time Between Overhaul
TCAS: Traffic Collision Avoidance System
TMB: Travelling Money Budget
TOC: Total Operating Cost
VFR: Visual Flight Rules
VOR: VHF Omni-directional Radio Range
VQSTOL: Very Quite and Short Take-Off and Landing
VTOL: Vertical Take-Off and Landing
UAV: Unmanned Aerial Vehicle
1. Introduction

Mobility studies showed that between 1966 and 2006, each percent of Gross Domestic Product (GDP) growth resulted from one to two percent of motorized passenger traffic increase [1, 2] (see Figure 1.). Among other transportation systems in 2006, the further GDP development is projected to double the air traffic volume by 2020 [3, 4]. Even so, the air transportation is already reaching its limits of capacity (in numerous domains such as safety, security, environmental and airport considerations), thus it might not be able to cope with the predicted growth. Potential solutions to this problem include the demand capacity balancing that could ensure the expansion and improvement of the air transportation system by introducing new air vehicles or Small Aircraft (SA) [5, 6] that would use underutilized infrastructures such as small or regional airports. This is also reasonable, knowing that (i) the alternative transportation modes for example the high speed trains operate only along established high population corridors [7, 8], and (ii) the new EU member states lack of transportation infrastructures [8, 9].

From our definition this class of small aircraft carries up to 6 or 8 passengers and applies all single or multi-piston, turboprop and jet propulsion systems, since the available related works in 2007 cannot clarify which of them is more appropriate for the European market. In 2007, there are approximately 300 000 [10, 11, 12] private pilots in Europe that use more then 60 000 SA [10, 12].

On the other hand – similarly to the philosophy of the Small Aircraft Transportation System (SATS) [13], the Personal Air Transportation System (PATS) [2, 9], the UK JETPOD [14] and other relevant concepts – the emerging technologies might even allow such air transportation system to become personal-based [6] that could maximize the satisfaction of market requirements with on-demand, point-to-point and more flexible operation [6]. Using simplified cockpit environment [5, 15, 16] with highway-in-the-sky [17, 18, 19], synthetic vision technologies [15], automatic take-off / landing vehicle control [2, 7, 9] and other instruments [5, 20, 21] small aircraft could also be designed to be accessible to common / ordinary people that enable even a pilot with limited experience to fly, similarly to the difficulty of driving an automobile [2, 6, 7, 9].

As the initial cost of these small aircraft might be relatively high, the NASA predicts [7] that first, a regional operation might develop – especially the professionally piloted on-demand air taxi services – as the extended use of SA might decrease the total operating cost on a seat-km basis. Afterward, the cost reduction technologies (for example the
advanced aircraft design [22] or the novel propulsion systems [23, 24, 25]) could lead to appeal even the price sensitive travellers [7] and the total SA users might further growth by 25% [7]. At this stage, the development might lead to intra-urban [7] and personal-based [2, 7, 9] small aircraft operations.

In short, the SA transportation could improve the quality of life while ensuring wide mobility, and the accessibility or affordability of these aircraft might change [26, 27, 28, 29]. For that reason, the European small aircraft activity has to be analyzed for example to understand, what the impact of SA on the Air Traffic Control / Air Traffic Management (ATC / ATM) is. Since in 2007 the European small aircraft traffic is limited and therefore the available statistical records are inadequate, such analysis and the background for further investigations call for the establishment of a European SA prediction model. Therefore, the research problem of this thesis is to access and model the growth of the European small aircraft.

This problem first requires to gather and analyze the available data on the past development phases and to name the potential future directions that small aircraft might take. If these records are limited or not accessible, an initial investigation could be a European small aircraft air traffic analysis (in 2007) to restrict the area that should be focused, using arguments on numerous characteristics such as the applied propulsion system, the flight distance, or the flight altitude. Further tasks include the analysis of the potential model structures (such as linear, log-linear, logit) to evaluate which methods are in use and how they can be applied to the European small aircraft purposes. The last task would be the development of the demand or prediction model. In this investigation, the proposed solution covers an analogical approach using an S-curve from the innovation diffusion theory, since this provides a non-linear and country specific model that is capable to address the uncertainties in the small aircraft development, and to reflect the socioeconomic data or the transportation characteristics of the European context. Over other techniques this model is advantageous in numerous attributes that are further explained in the following chapters. As a final task, a Monte Carlo Simulation would be used, since this provides the results with probability distributions that could support further decision making and risk analysis. As the output might be a source to uncertainties, the model relevance would be demonstrated via sensitivity analysis and a historical data validation technique.

This PhD dissertation is divided into seven chapters.

The first, the introduction gives the problem definition, actuality, the tasks to be solved and the proposed methodology.

Chapter 2 provides a literature review of the related works. First, the worldwide small aircraft concepts are discussed to understand the vehicle class being analyzed, including the regional, extra- and intra-urban operation possibilities. Then the state-of-the-art addresses the technological challenges to demonstrate that using the short-, mid-, and long-term technologies, small aircraft might meet the above mentioned modifications in the cockpit environment, the total operating cost and the required pilot skills. This chapter also reviews the potential small aircraft in 2006 to support the model development with initial aircraft characteristics, and finally introduces the potential demand and prediction methods.

As the relevant European investigations are limited and therefore the available statistical data is insufficient to support a prediction model, the Chapter 3 describes an initial small aircraft air traffic analysis, which aims to restrict the area that should be
focused by giving arguments on numerous attributes including the most frequently used propulsion system or the flight distance.

Chapter 4 presents the methodology and modelling including the analysis of the generally used prediction / demand models, the development of the initial linear model and some more advanced methods based on the combination of the innovation diffusion theory with an analogical approach.

Chapter 5 discusses the proposed approach and introduces the model variables. It also addresses the problem of uncertainties with a Monte Carlo Simulation to support further decision making and risk analysis with probability distributions. This chapter demonstrates the relevance of the model via sensitivity analysis and a historical data validation based on the EUROCONTROL Central Flow Management Unit (CFMU) data [30].

Chapter 6 provides the results of the simulation, including the unit total operating cost and accessibility values along the whole simulation horizon.

Finally, the summary, the theses and the recommendations for the future works are presented in the Chapter 7.
2. Small aircraft activity and development: a state-of-the-art in 2006

2.1. Worldwide small aircraft concepts

To provide a framework for establishing the importance of our study, this chapter aims to locate and summarize the related activities by understanding the vehicle class being investigated and the development of the worldwide small aircraft concepts.

Generally, the purpose of these is to provide an on-demand air transportation with a novel small aircraft which reaches the same operational and safety level than the personal cars, and has a Total Operating Cost (TOC) of a mid range automobile [6].

One of the concepts is the federally founded NASA initiative, the Small Aircraft Transportation System (SATS) [13, 31] research program that seeks to harness unused capacity in the airspace system through the boost of small aircraft utilization and the increased service to areas that are not in focus for the commercial scheduled air carriers. This program also aims to prove that the application of the emerging technologies might enable the vision of small aircraft, and that a safer, more reliable and affordable air transportation system could born [13]. Additionally, since initial studies shown that the air traffic might require resources at or near major airports, this program also aims to integrate small aircraft flights in the airspace through developing, evaluating and demonstrating the technical and operational feasibility of cockpit developments and some advanced operational concepts, such as the High Volume Operations (see Chapter 2.2.) in non-radar airspaces at non-towered airports (see Figure 2.).

However the SATS Program had a live demonstration in 2005 that showed the operating capabilities of the concept (see chapter 2.2.), its limitation in 2006 is that the aircraft has a limited appeal to price sensitive travellers, as the ownership and the total operating cost decrease is not yet in focus.

According to Moore [7, 32], other NASA initiative is the Tailfan a “highly affordable next generation GA concept” (see Figure 3.). As the structural design of small aircraft requires different methods and materials than the larger air vehicles [32], this concept aims to apply “more appropriate” materials and structures to enhance the aircraft affordability and to achieve a noise reduction [7]. For the non-structural elements, the concept already passed an FAA endurance test that demonstrated the feasibility of using the automobile manufacturing process to obtain lower costs and more reliable products [33,
Therefore, instead of the FAA certification standard of Quality Control (QC), rather the Quality Assurance (QA) is applied at the propulsion system of the aircraft, which is an automotive V-8 Corvette LS-1 engine. However the drawback is a weight increase compared to traditional aircraft engines, the approach enables a total propulsion system cost reduction of over 60 %, while maintaining a reasonable Time Between Overhauls (TBO) [7]. Additionally, the balanced V-8 engine also permits the elimination of noise due to vibration and the propellers. Further objectives are to produce 2000 units/year with the initial price of 55590 EUR [7], and to represent the first step in the small aircraft market.

Besides the regional mission concepts, numerous investigations focus on the Short/Vertical Take-Off and Landing (S/VTOL) capabilities to enable small aircraft operations in the vicinity of urban areas. One out of them is the UK Jetpod, developed by Avcen Ltd in association with Company Jet Central Ltd [14] (see Figure 4.). This European pre-design study for advanced small aircraft aims to offer a flying taxi service, as an alternative to the ground transportation [35]. It also provides several models: (i) a low-cost world-class city air taxi, (ii) an easy and safe to fly personal twinjet aircraft, (iii) a civil air ambulance variant, and finally (iv) a civil or military Unmanned Aerial Vehicle (UAV). The basic of the propulsion system is the same for all models; two over-wing jet engines with partially diverted thrust that enable Very Quiet and Short Take-Off and Landing (VQSTOL) operations. The choice of the jet engines is rationalized with the technological advancements in the engine design, fuel efficiency and advanced materials that together enabled to decrease the operating costs. In 2005, the proposed propulsion technology passed a computational fluid dynamics test, and the Jetpod is ready to be brought off the drawing board through a proof of concept flight-testing phase. According to Avcen [14], the first military and civil passenger versions could be ready in 2009 and 2010 respectively. However, efforts are done to decrease the price of the Jetpod, its limitation in terms of small aircraft remains the retail price of 741200 EUR.

Other S/VTOL aircraft is the Tilt-Nacelle concept [7], developed under the partnership of Mdot Aerospace, Shapery Gyronautics, Georgia Tech Research Institute and NASA Langley (see Figure 5.). The aim of this is to address the general penalty of the V/STOL aircraft, the relatively large propulsion systems to satisfy the single engine failure requirements. Seeing that the Tilt-Nacelle applies a Multi-Gas Generator Fan (MGGF) to turn a tip-driven fan system [36], the five gas generator (GG) arrangement results in a 20% thrust lost if one GG fails. As already demonstrated on a sub-scale test, there are numerous other advantages of the Tilt-Nacelle including the application of lower pressure ratio and lower peak temperature turbines that (relative to the cruise turbine) permits less expensive turbo-machinery. Additionally, with the exhaust products exhausting around the periphery after the fan-turbine expansion, there is a potential to drive a circulation control trailing edge for external flow expansion [7, 37].
With respect to the distributed propulsion systems, the Pulsed Ejector Thrust Augmenter (PETA) concept [38] (see Figure 6) is developed in the cooperation of Boeing and NASA Langley. This, relative to the Tilt-Nacelle program, offers a true distributed propulsion system that – after NASA – shows “extreme redundancy and robustness” [39]. Several hundreds of small-pulsed engines are applied, which reduces the engine out penalty to a negligible level. However, demonstrations have not yet been performed, since the engines found to not fulfil any noise restrictions. Seeing that small aircraft with S/VTOL capability might be operated in the vicinity of major cities or in intra-urban areas, the noise is a critical constraint for all S/VTOL aircraft. Moreover, the technologies to limit the noise emissions are powerless in open and uncontrollable free-air environments such as the PETA or the Tilt-Nacelle [7]. Besides, the general drawback of these aircraft with S/VTOL capabilities in 2006, is that – compared to traditional propulsion systems – they require additional power, which limits the potentialities in a total operating cost decrease. As a result, it is more probable that the S/VTOL concepts will be utilized as air taxis, in order to amortize the higher vehicle acquisition costs.

On the other hand, several feasibility investigations aim to adapt small aircraft – or also called personal aircraft – to purely intra-urban requirements. One ambitious concept is the SkyCar, initiated by Moller Industries [40]. As the Figure 7 indicates, this automobile-size vehicle looks like a cross between a sports car and a small jet with a maximum capacity of 4 passengers [41]. The SkyCar is designed with four pairs of rotary engines using deduced fans that enable VTOL capabilities. The advantage of this propulsion system is that the failure of a single part is not flight-critical, unlike the rotor of a helicopter or the single engine of a small aircraft. Its proposed features also include the option of electronic pilot [41] that will follow electronic flight rules and therefore enable a personal operation with no or limited pilot skills. Besides, the fuel consumption is projected to be between 10 and 13 litres of gasoline per 100 km [40], with the potential of alternative fuel cells such as the methanol, the diesel, or the hydrogen. Once in mass production, the SkyCar is expected to reach the same level of cost as a mid-range BMW, (around 37060 EUR [42]) and offer a total operating costs between 0.06 and 0.15 EUR per seat km. According to Moller [43], these benefits over other aircraft might enable to 90 percent of the population to use SkyCar till 2026. As in 2006, while the concept already demonstrated a flight test and the FAA certification stands at December 31, 2008, experts from NASA remained critical vis-à-vis the design’s feasibility [44], especially to achieve the cost predicted by Moller. From a small aircraft point of view, other limitation is that the SkyCar demonstrated a noise level of 85 decibels at 50 feet, while according to NASA [7] the acceptable level to communities is more on the order of 55 dBA.
Another personal aircraft concept is the X-Hawk (see Figure 8.), developed by Urban Aeronautics [45]. This vehicle operates similarly to the Skycar or the Tailfan, therefore it employs unexposed rotors, which otherwise might be considered dangerous or impossible to use in urban environments. It is promoted for numerous applications to improve the mobility and utility of small aircraft that include rescue, utility and air taxi service. Compared to a helicopter, its main advantage is the quieter operation and the lower fuel consumption. However, its bottleneck in terms of small aircraft remains the price, 2.2 million EUR, expected around 2010.

Fig. 8. The X-Hawk concept to improve the mobility and utility of small aircraft [45].

Besides the Skycar and the X-hawk, this state-of-the-art also revealed that a limited amount of the personal aircraft concepts address the dual-mode aerial/roadable operations [46]. In other words, this is the capability to fly as an aircraft, fold the wings (manually or automatically) and drive on the road as a car. Before 2006, this approach was already in focus [7], but these investigations resulted in design compromises and penalties that made the vehicle both a poorly performing car and an aircraft (e.g. the Italian Aerauto in the early 1950s [47]). On the other hand, in 2006 the FSC-1 (see Figure 9.) concept of the LaBiche Aerospace [48] demonstrates the feasibility of a more reasonable flying car. The FSC-1 has already flown a 1/10th scale model and started the construction of a prototype for road and air testing (expected in 2007). However the first version will require a pilot and a driver’s license to operate, upon the approval of the FAA, the development is underway for a hands free flight system to eliminate the pilot license requirement [49]. While the advantage of the concept is the automatic transformation at the touch of a simple button, its bottleneck is the unknown impact of the transformation mechanism on the operating costs.

Fig. 9. The FSC-1 dual mode concept.

Other dual mode concept is the Transition (see Figure 10.) from the private company of Terrafugia based in Cambridge (Massachusetts) [50]. Being an aerial/roadable vehicle, its mission is “the expansion of personal mobility through the practical integration of land and air travel”. According to the company, this is a “real personal air vehicle”, as it will carry only two passengers and will enable to fly and drive back home using a single tank of premium unleaded gas for both operations. While an operational prototype is expected in 2008, the purchase price is already known, 109697 EUR [49]. Even so, for price sensitive travellers the concept might result in inaccessible total operating costs, so only a limited amount of the population might experiment the advantage of the concept.

Fig. 10. The Transition dual mode concept.

According to Moore [7], NASA also launched its initiative in the dual-mode operations. They developed two different vehicles, the Dual-mode (see Figure 11.) and the SpiralDuck (see Figure 12.). The first applies a combination of a single telescopic wing
and canard panel that could be folded to transform the aircraft to a car. However, this four passenger aircraft included a weight growth of 500 kg relative to an equivalent small aircraft, which makes it neither practical, nor affordable, especially when compared with alternative travel choices. On the other hand, the SpiralDuck applied a different aerodynamic form, which ensured the transformation with smaller failsafe wings, folding downwards. Inspired by the Aerodyne [51], the SpiralDuck also takes the benefits of the highly integrated propulsion-aerodynamic coupling that enables an approach up to 40 degrees and Extreme Short Take-Off and Landing (ESTOL) capabilities.

Nevertheless, to not overpass a reasonable induced drag on the roads, the vehicle’s overall gross weight calls for a limitation, which results in a decreased range and carrying capability.

While the majority of the concepts mentioned here demonstrates that an on-demand small aircraft transportation is feasible in both regional and intra-urban areas, their general limitation is that in 2006 the total operating costs – including the ownership costs – offer a limited appeal to price sensitive travellers. However, the technological developments in the context of the propulsion systems and advanced materials might enable to solve this bottleneck, and finally make the aircraft more affordable. Till then, to achieve the first steps in the development of the small aircraft transportation, these and other vehicles might be operated in air taxi services or in fractional ownerships [7].

Finally, knowing that Europe consist of different countries with numerous socioeconomic characteristics [52], other major drawback of these investigations (expect the Jetpod) is the focus on the American market. This might even influence the initial requirements (e.g. the required propulsion system), since at the same level of GDP per capita, the Europeans travel three times less than the North Americans [53].

2.2. Small aircraft enabling technologies: aircraft design and operational concepts

While the previous chapter demonstrated the feasibility of small aircraft in general, this part of the state-of-the-art aims to address the technology challenges in aircraft design and operational concepts that together might decrease the influence of the bottlenecks in 2006, for example the required pilot skills, the total operating cost, or the environmental impacts.

The technological examples mentioned below are divided into three categories [5, 7]:

- a short-term to modernize the small aircraft in 2006 with automation (e.g. automatic take-off/landing, autonomous emergency procedures), cockpit developments (e.g. weather information, synthetic vision) and the application of the first operation concepts (e.g. High Volume Operations),
- a mid-term set of technologies to expend the accessibility and utility of small aircraft via cost reduction technologies including novel aerodynamic form (to decrease the drag) and enhanced propulsion systems (with decreased weight and fuel consumption),

- a long-term set of technologies that could further enhance the affordability of small aircraft with unconventional concepts, alternative energies and lightweight structures such as the Micro-Electro-Mechanical System (MEMS), or the nanotechnologies.

With respect to the short-term technologies, the objective of the Advanced General Aviation Transportation Experiments (AGATE) is to ease the use of small aircraft and reduce the pilot skills’ requirements to the level of personal automobiles [7]. As in 2006, the work package already achieved encouraging results in automatic flight procedures and in the ability of completely inexperienced pilots to fly patterns.

Similarly to AGATE, the Personal Air Transportation System (PATS) addresses a novel small aircraft that could be operated by pilots with limited skills [6, 54]. However, this project rather focuses on the limitation of the unwanted effects and motions, since the control of a traditional small aircraft in 2006 is a more complex issue than it is for the case of personal cars. For example, while the acceleration is relatively a simple task for the automobiles, in aviation this requires more skills, as both the gas control and the elevator is required to maintain a constant angle of attack and avoid the changes in cross-effected motions [54]. To deal with such and other parasite effects, the PATS discovered the potentialities in automation, using a feed-forward model for both symmetric [2, 54] and asymmetric [54] motions. Although in 2004 this project found the evidence for the model feasibility with a computational test for the Cessna 172, the concept has not yet been demonstrated in real flight tests.

With respect to the cockpit development of small aircraft, a major progress is achieved with the Synthetic Vision Systems (SVS). Generally, the goal of these programs is to enhance the pilots’ situational awareness through a three dimensional perspective presentation of the outside world, regardless of any weather condition. The Figure 13. indicates the NASA SVS [15], which generates the artificial vision by the combination of advanced on-board sensors, digital terrain databases, geo-positioning records (with Global Positioning System satellite signals [20]), relevant traffic data from radars or Automatic Dependent Surveillance - Broadcast (ADS-B) [21] and digital processing technologies. Furthermore, to reduce accidents where weather is a contributing factor, the SVS is also supported by real-time aviation weather information services. An example for such an instrument is the Aviation Weather Information System (AWIN) [16] that offers 3-D description of the weather patterns such as wind-shears, thunderbolts, or storm cells. As demonstrated in 2003 [55], other safety outcome of the SVS is that it reduces the greatest contributing factor to fatal worldwide airline and general aviation accidents [56], the

---

**Fig. 13.** Synthetic Vision application to enhance situational awareness [15].
Controller Flight Into Terrain (CFIT). On the other hand, the application of the SVS or other advanced cockpit displays also includes artificial representation of the flight path by tunnel or pathway-in-the-sky representations [17, 18, 19]. Using these technologies, numerous simulation tests and flight trials [57, 58, 59] demonstrated that pilots gained increased situational awareness and reduced workloads. Besides these advantages, the Figure 14. indicates, that the cockpit developments also enable small aircrafts pilots to meet an environment that is closer to the level of personal cars, and which might require less skills [60]. In the same time, the drawback of the equipments and especially when TCAS (Traffic Collision Avoidance System) and ADS-B technologies are applied, is that they rise the purchase price of the aircraft [61, 62], which considering price sensitive travellers might even limit its application on SA.

Based on the cockpit developments and other technologies, the NASA already initiated an operational concept in the context of the SATS Program. The High Volume Operations (HVO) [63] search for ways to boost the capacity of small aircraft flights at airports with no control tower or radar coverage, even when the weather is below the minimums for visual flights. In 2005, the concept had a live demonstration with six different SA, simultaneously flying an innovative T-shaped instrument approach to a small airport in Danville (USA). This advanced procedure and the use of an on-board “pilot advisor” software showed that the arrival sequence of each plane could be reduced from 20 min (in 2006) to about 5 [63]. Additionally, pilots reported that the developed on-board datalink technologies enabled them to see and avoid each other. However, in 2006 it is not yet clear how the HVO could be approved with the regulatory environments, like FAA. Moreover the technology resulted in added operating costs, which probably limits the interest of price sensitive travellers.

In the context of the mid-term technologies, the cost reduction possibilities are addressed. Since in 2006 the most expensive subsystem of the small aircraft is the engine [7], the General Aviation Propulsion (GAP) program aims to develop a novel propulsion system with enhanced reliability, decreased weight and reduced operating cost. The program resulted in a diesel (Figure 15.) and a turbofan engine, called the FJX-2. The first combines a two-stroke operating cycle with innovative lightweight constructions that offers “competitive” weight with pistons in 2002. Combined with low-speed propellers, NASA declares “a very quite operation for both passengers and airport neighbours”, with a fuel consumption rate of about 25 % less than other propulsion systems with similar characteristics [23]. As for the turbofan, it applied the lessons learned from
research of automotive gas turbines, and used low-cost design techniques with advanced automated manufacturing methods. This gave a primary cost reduction by a factor of ten, and led the FJX-2 to be the first turbofan engine, which is cost competitive with pistons. After that the flight tests demonstrated the near-term commercial viability of the program for small aircraft, the GAP ended in 2002 [23].

Besides the propulsion systems, other potentiality to increase the small aircraft affordability with decreased fuel consumption lies in aerodynamic developments. Generally the scope of these includes airframe systems such as wing, fuselage, propulsion/airframe integration that results in aerodynamic optimization. However, in 2006, the innovative system oriented research revolutionizes the traditional approach to aerospace technology and creates methods to reduce the development and certification time for a new aircraft. Furthermore, instead of an aerodynamic minimization of a given airframe, it is rather the aircraft design that adapts to an optimum drag. As a result the concepts such as the Revolutionary Concepts (RevCom) [22] end in novel aerodynamic forms that brake with conventional aircraft designs. In the context of SA, the SATS’ vision of the aerodynamic efficiency (see Figure 16.) should enable to take the advantage of lower drag and therefore decreased fuel consumption.

This investigation discovered several concepts to show the possible boundary of the long-term technologies and to demonstrate their potential in further small aircraft affordability enhancement. One out of them deals with the propulsion system of the aircraft, by proposing alternative engines. According to the non-profit Foundation for Advancing Science and Technology Education (FASTec), the E-plane [24] is a two passenger concept that is powered by “advanced hydrogen fuel cells driving a highly efficient electric motor”. The fuel cell is relatively compact, measuring roughly 8 by 11 by 26 inches and weighting about 22 kg (see Figure 17.). As in 2004, three flight development stages are planed, including the basic propulsion system, the fuel cells and the electric power generated tests. In its final version, the E-Plane should fly solely on the power of a fuel cell and have about 800 km of range with emergency-assist from reserve lithium-ion batteries. However, the evidence of the technology is not yet demonstrated, and according to NASA [7] small aircraft still requires improvements in energy storage technology to reach competitive power to weight ratios.

Other investigations aim to enhance the propulsion system by creating oil-free engines. As a first milestone in 2000, The Free Turbine Engine Technology Project [25] already defined the operational envelope of an air foil bearing. The tests indicated that by eliminating the need for oil lubrication systems and rolling element bearings, and engine weight is reduced by 15 percent, while yielding power density improvements of 20 %, and
reducing engine maintenance cost. Although, in 2006 the project still requires additional research to achieve a full engine test and to demonstrate its applicability to small aircraft.

Besides the manufacturing and design processes, the enabling technologies are also present in the Micro-Electro-Mechanical Systems (MEMS) [54]. The 10 – 100 µm size micro devices integrate sensors, actuators, control and transducer elements on a same silicon substrate. While the application of the MEMS covers several domains, in aviation an example is the investigation of professor Ho [64]. In 1996 he demonstrated that MEMS is capable to control the motion of a SA (see Figure 18.), therefore the novel aircraft designs could pass over the use of traditional (aerodynamic) control surfaces. The flight tests also reported that MEMS is missing the fatigue and damage problems, as the specific dimensions are smaller then the characteristic dimension of a crystalline structure. Additionally, the technology is found to be inexpensive, due to the developed production expertise and the use of microelectronic hardware fabrication skills.

The long-term technology concepts could also take the benefits of the technology transfer [54]. For example in 2006, theoretical analyses show the possibility of arranging atoms and molecules. Technologically it is feasible to build products with almost every atom in the right place, and do it inexpensively (see Figure 19.). This procedure is often called nanotechnology, molecular nanotechnology or molecular manufacturing, which in general permits to make most products lighter, stronger, cheaper and more precise. Numerically this results in materials that are about 10 times stronger than a graphite fibre [65] and even fifty times lighter than steel of the same strength (e.g. a Cadillac that weights fifty kilograms). Even so, in 2006 the research and development for the first such systems are likely to rise its price, and limit its potential application in the context of small aircraft.

On the other hand, in 2005 even the most advanced small aircraft are also some of the noisiest GA (General Aviation) aircraft ever produced [66]. The flyover noise ratings are on the order of 70 dBA [66] even with engine sound suppression systems, while the acceptable level is more on the order of 55 dBA [7]. Although this problem should be addressed, especially once the technologies (such as S/VTOL or the dual mode concepts) might enable small aircraft to be operated in the vicinity or in intra-urban areas, the investigations with small aircraft noise restriction in mind are limited in 2006.

Altogether, the examples of the technological achievements mentioned above demonstrates that even in 2006 a small aircraft could born and that the potential pilots could meet limited requirements with an operational environment that is closer to the level of personal automobiles. Even so, this review also found that the mid/long-term concepts should focus on the operating cost diminution techniques to ensure an extended appeal to price sensitive passengers.
2.3. Potential aircraft for small aircraft purposes in 2006

To support the prediction model of this investigation with initial aircraft characteristics, this chapter aims to present a summary of the potential aircraft in 2006. Since the applied SA definition (see chapter 1.) enables the use of various propulsion systems, this review mentions a selected number of models for all single and multi engine pistons, turboprops and jets (see also the Appendix A for more detailed data on aircraft characteristics).

With respect to the pistons, an example is the SR 20, developed by the American Cirrus company [67]. This single engine piston aircraft is certified in 1998 by the FAA, and in 2004 by the European Aviation Safety Agency. One of its major advantages is the fully digital avionics cockpit environment that offers a primary flight-, and a multi-function display including several safety features, such as the Terrain Awareness Warning System (TAWS), or the SKYWATCH that alerts airborne traffic. Beside this, the aircraft is also equipped with a Ballistic Recovery System (BRS) called the Cirrus Aircraft Parachute System (CAPS) [68]. It employs a solid-fuel rocket to pull a 55-foot diameter parachute out of its housing (see Figure 20), which in case of emergency enables to lower the entire aircraft to the ground [69]. Altogether, these equipments revolutionized the safety of small aircraft and demonstrated the short-term feasibility of the technological concepts mentioned in the previous chapters. Probably due to these advantages, (in 2006) the SR20 is also operated by the South-Carolina based SATSAir [70] and the North-Dakota based Point2Point Airways on-demand air taxi services [71], with a unit total operating cost of 0.33 Euros per seat-km [70]. However, to appeal price sensitive travellers in 2006, this cost and the purchase price of 185296 EUR [67] remains the bottleneck of the Cirrus SR20.

Other single engine piston aircraft is the Lancair Columbia 400, certified in 1998 by the FAA [72]. According to Lancair, this is one of the “most technologically advanced high performance” four-seat aircraft. Its features include an advanced aerodynamic form and a composite construction that results in a smooth low drag external finish. Under the sponsorship of the NASA’s AGATE program, the 400 also provide an enhanced IFR (Instrumental Flight Rules) avionics platform called the Highway in the Sky, which incorporates a primary flight display presenting flight data in an integrated format, and a multifunction display for moving map, radar, up-linked weather and traffic avoidance systems. As the Figure 21. indicates, the Columbia 400 is the evidence for the fact that even in 2006 small aircraft pilots could meet a cockpit environment that is closer to the level of personal cars. Even so, as already mentioned in the chapter 2.2., these
instruments raise the purchase price of the aircraft. This, in case of the Columbia 400 in 2006 results in 360149 EUR [72], which is nearly the double of the Cirrus SR20.

The Adam Aircraft Industries [73] in the US believes that the light non-pressurized twins available in 2005 do not deliver the required performance of the operators that would like to meet a “high performance” piston aircraft with “reasonable” acquisition price. To satisfy this potential need, Adam Aircraft developed the turbocharged piston-powered A500 that was FAA certified in 2005. This eight-seat aircraft has been designed and developed with 21st century materials, and all composite structure that employs a push-pull configuration. The advantage of this arrangement over other conventional twin-engine installations is the centreline thrust that reduces the drag and maximizes the controllability of the aircraft if one engine fails. As a result, the A500 shows that the research in aerodynamic drag reduction with advanced concepts that brake with conventional aircraft designs is already started in 2006. Additionally, one could also observe that the fuselage of the A500 (showed in the Figure 22.) is similar to the SATS’s aircraft vision (illustrated in the Figure 16.), which also indicates the viability of the SA programs. Besides, other feature of the A500 is the avionics. It includes an autopilot system and a multifunction colour display that is an IFR certified GPS with communication, VOR (VHF Omni-directional Radio Range), glideslope, Jeppesen database (e.g. airport coordinates, approach procedures) and a colour moving map in a one package [73]. While the constructor has foreseen that the A500 might also be employed in on-demand per-seat air taxi services with a unit total operating cost that is lower than other aircraft having similar characteristics (0.2 EUR per seat-km) [73], in 2006 this review could not locate any of such companies. Additionally, one might also consider this small aircraft with its 8 seats to be too large to fulfil the requirements of a “personal” air transportation.

Other turbocharged aircraft is the DA42 Twin Star [74] from the European Diamond Aircraft, certified in 2003 by the European JAA and in 2004 by the US FAA. Similarly to the cockpit features of the SR20 and the Columbia 400, the four-seat Twin Star offers multifunction displays to represent the flight data, moving map, up-linked weather and traffic/obstacle avoidance. However, its real advantage over other models is the two Tieler Centurion 1.7 turbo diesel engines, which are designed to run either on Jet-A or diesel fuel and – according to Diamond – offer a fuel consumption that is lower than any other small aircraft with comparable characteristics. Since the Tieler Centurion engines are based on a Mercedes-Benz automotive design, the Twin Star (similarly to the NASA Tailfan concept) demonstrates the potential in the application of the manufacturing technologies of other industries. Its limitation however is that the aircraft costs 390844 EUR (in 2006) [74], which is more than the double of the SR20 and still 15 % more expensive relative to the Columbia 400 [74]. Even so, in 2006 the Point2Point Airways employs it among other aircraft (e.g. the SR20) for their air taxi service in North-Dakota [71].

On the other hand, the Diamond Aircraft is also the first European player to enter to the market of small aircraft models with jet propulsion systems, often called as Very Light Jets (VLJ) [75]. Generally these aircraft offers at least some of the following features: (i) advanced cockpit automation such as multifunction displays, (ii) automated engine and systems management, and (iii) an integrated autoflight, autopilot, flight-guidance systems.
As for the D-JET [76], it is a five-seat VLJ, or a single engine Personal Light Jet (PLJ) (see Figure 23.) that aimed at the owner-pilot end of the market and had been optimized for flights at lower altitudes (25000 ft). As in 2006, the aircraft is undertaking flight tests and seeks to meet the FAA certification by 2008. The official price tag was around 741200 EUR, but Diamond finally reported that the purchase value is 1 022 856 EUR [76]. While this is between 2.5 and 6 times more expensive than piston small aircraft with the same size, the advantage of the flight speeds still motivates the air taxi companies like Point2Point to order it [71].

While this is between 2.5 and 6 times more expensive than piston small aircraft with the same size, the advantage of the flight speeds still motivates the air taxi companies like Point2Point to order it [71].

Other single engine jet is the four-seat Sport-Jet [77] from the Colorado based Excel-Jet Limited, founded in 2002. Similarly to the D-JET, this aircraft targets the owner-flyer market with simplified design and cockpit features enabling single-person operations by pilots trained on piston-powered airplanes. For safety reasons, the maximum cruising altitude is limited to FL 25000, which still enables the aircraft to fly as fast as 375 ktas [77]. While the first flight was on May 2006, the bottleneck of the aircraft is that the further development is temporary suspended due to a crash on June 2006. According to the company, wake turbulence was the likely cause and the FAA certification is still planed by end 2007. Anyhow, relative to the Diamond D-Jet, the Sport-Jet might be more affordable for small aircraft operations, since Excel-Jet expects first productions to sell for 32 % cheaper [77].

With respect to twin engine VLJs, a six-seat aircraft is under production from Cessna, called the Mustang Citation [78]. It was first announced on October 2002, and flew on April 2005. In 2006 the Cessna Mustang is also the leader of its class to receive full type, and “fly into known-icing conditions” certifications from the FAA, the first VLJ to be delivered to a customer, and the first company to obtain the FAA Production Certificate for a VLJ. Unlike PLJs, the Mustang is not limited on its maximum cruising altitude, which is reported to be 41000 ft [78]. Besides the certificates, the Cessna is reluctant in calling the aircraft a VLJ and rather considers it as “an inexpensive business jet’. This is confirmed with the purchase price of 1.87 million EUR [78] that might better meet the legacy Cessna market and business owner-operators than the requirements of the potential price sensitive small aircraft users or air taxi operators. As in 2006, from the total of 250 [79] orders that the Cessna had for the Mustang, the European sales represent 30 % [79].

Following the Cessna Citation Mustang, in late 2006 the second aircraft that achieved the FAA VLJ certification is the Eclipse 500, developed by the Eclipse Aviation [80]. This aircraft is powered by two PW610F engines in tail-mounted nacelles, and can accommodate five passengers with one pilot. The Eclipse also took the advantage of the novel manufacturing technologies with the Friction Stir Welding (FSW) [81], in which the skin and the underlying aluminium structures are welded together rather than riveted, to reduce the number of rivets and therefore the aircraft standard empty weight. Besides materials, the general process of building the airframe was redesigned with techniques taken from the automotive industry that – according to Collins [81] – resulted in a “more robust cabin”, which can be pressurized to a “higher differential”. As in late 2006, the Eclipse is the most popular VLJ with approximately 2000 firm and 800 optional orders at the purchase price of 1.1 million EUR [80]. The largest initial costomer in 2007 is the
American DayJet company [82] with around 250 aircraft operating in an air taxi role [83]. Otherwise this aircraft had 50 firm and 50 optional orders from one of the first European start-up air taxi companies (the London based JetSet Air [84]) and an additional 140 planes [84] from other European buyers. These numbers confirm that already in 2007 the American aircraft models could be adapted, and that the European small aircraft transportation is viable.

Other VLJ is the HondaJet developed by the Honda Motor Company [85]. This aircraft had its maiden flight in December 2003, and in 2006 expect to obtain the FAA certification by 2009 or 2010. The HondaJet has six seats and uses an un-usual over the wing engine configuration to offer more passenger space within the fuselage and to reduce drag at higher flight speeds [86]. Honda claims that the combination of such aerodynamic form and the applied lightweight composite materials give the HondaJet a 30-35 % [85] higher fuel efficiency that other similar aircraft. Additionally, it is also the fastest VLJ that reaches the 420 ktas at flies up to 41000 ft [85]. However, with the purchase price around 2.6 million EUR [85], in 2006 the HondaJet is also the most expensive VLJ on the market.

Based on the A500 model and using 90 % of its suppliers, the Adam Aircraft also developed a twin engine VLJ called the A700 [87]. Relative to the piston model, this eight-seat aircraft offers higher performance and a maximum ceiling of 41000 ft [87]. As in 2006, two prototypes of the A700 exist, and the FAA certification is scheduled by 2007. Otherwise, similarly to the A500 model, Adam Aircraft has foreseen the potential of the A700 in air taxi operations with the estimated unit total operating costs given as 0.225 EUR/seat-km [87]. Unlike the A500, this model court the attention of two American air taxi companies – the PogoJet [88] and the Magnum Jet [89] – which together ordered 125 aircraft [89].

In this section, numerous potential small aircraft were discussed. It has been showed that the aviation industry focuses on that class of airplanes, as the number of concepts are more important than ever before 2006, especially in the Very Light Jet or Personal Light Jet segments. While these aircraft remained at purchase prices that might appeal only a limited number of the population, this review demonstrated that the developments enable the first stage of the small aircraft transportation with on-demand, per-seat, air taxi companies such as the SATSAir, the DayJet, or the Pont2Point Airways. In 2006 these American initiations are already adapted to the European market requirements with two start-up companies, the English JetSet Air and the Swiss JetBird [90], which is also the evidence for the actuality, and the importance of the European small aircraft exploration. However, most of the airplanes are from companies based in the US that knowing the geographical characteristics and the specific market requirements propose jet-powered engines. On the other hand, as the Diamond Aircraft demonstrates with its twin-diesel, the European small aircraft might call for different propulsion systems, which require further investigations (such as air traffic analysis) to finally name whether jet, turboprops or pistons are more likely to be used in Europe.

2.4. Potential demand and prediction models

While the previous chapters reviewed the potential aircraft and the technological challenges, the objective of this investigation still requires to develop the available industry forecasts and the demand / prediction methods related to SA.
In 2006, several small aircraft forecasts are available from different representatives of the aerospace industry, such as Embraer [91], FAA [92], Forecast International [93], Honeywell [94], NASA [95], Rolls-Royce [96], and the Teal Group [97]. Since SA is a novel air transportation that offers limited historical data, these investigations use different assumptions and methodologies to define the possible operations, which therefore cover all owner-pilot, fractional ownership, and air taxi operations. For that reason, the result of the above mentioned forecasts range from 2000 to 135000 units being manufactured till 2020.

**However, the drawback of these investigations is that the applied methodologies are not publicly available. Additionally, all focus on the American market and on the Very Light Jets, which might be powerless in considering the European socioeconomic characteristics (see chapter 2.1.), and regarding the fact that other propulsion systems might be applied (see chapter 3.3.).**

Due to these limitations, the background of our work is finally discovered with the generally used methods of the conventional air transportation. **While one might consider that general aviation (GA) is similar to small aircraft and relevant for this task, in fact it is ineffective for our purposes, since GA is partially out of focus and therefore poorly documented.** On the other hand, commercial aviation offers a wide range of investigations. In this domain, both the demand and prediction models are considered, since the available data might not permit to forecast the demand, but rather different dependent variables (such as the affordability of small aircraft) where other methods are in use (e.g. regressions, questioners).

With respect to the demand models, this class of forecasts aims to express the passenger demand \(D\) through a series of different factors \(X_1, X_2, \ldots, X_n\), called the independent variables:

\[
D = f(X_1, X_2, \ldots, X_n) \quad (2.1)
\]

This literature review found that the **key factors determining the air travel demand includes the economic growth (principally the GDP), the air fair (or price) and the demographical characteristics like the population density or the unemployment rate [8, 98, 99, 100, 101, 102].** Otherwise, some of the investigations distinguished different market characteristics (e.g. leisure/business [103] or short/long-haul flights [104, 105]), and even considered the alternative transportations systems such as the presence of low-cost carriers or the High Speed Trains [106, 107, 108].

Based on past years functional relationships between these influencing factors and the passenger demand, the reactivity of the dependent variable with respect to the variation of one factor \(X\) is expressed with the following equation [109]:

\[
\eta_n = \frac{\Delta D / D}{\Delta X_n / X_n} \quad (2.2)
\]

This is the demand elasticity \(\eta_n\), which therefore measures the responsiveness of the quantity demanded to a change in the factor \(X_n\), while keeping all other variables constant. Knowing the elasticities (of each \(X_n\)) and defining the future evolution of the independent variables, the projection of the passenger demand could be expressed with one of the following four functional forms of the equation 2.1. [110, 111]:

- the linear demand model,
- the log-linear demand model,
- the logit model, and
- the translog demand system.

The equation (2.3) represents the first, where $\beta_n$ are the respective parameters associated to the independent variables $X_n$ and $\alpha$ captures the value of $D$ when all $X_n$ are equal to zero. This class of models has been extensively used for demand and sales forecasting, since – compared to others – it is relatively easy to estimate and advantageous in interpreting the empirical results [99, 112, 113]. It is valuable that each elasticity of demand depends on the value of the variable [111], but the assumption of a linear effect might not reflect the realistic relationships in the context of the European small aircraft development.

$$D = \alpha + \beta_1 * X_1 + \beta_2 * X_2 + ... + \beta_n * X_n$$  \hspace{1cm} (2.3)

The log-linear (or double-logarithmic or Cobb-Douglas [114]) model specifies the logarithm of the traffic volume as a linear function of the logarithms of the potential independent variables (see equation 2.4) [115, 116]. In 2006 this literature review found that this is the most widely used functional form of the transportation demand models [109, 111, 113, 117, 118, 119] as the $\beta_n$ coefficients are the respective elasticities ($\eta_n$), and the log-linear function is capable of modelling non-linear effects. However, the main drawback of this model is that each elasticity is invariant across all data points, which is powerless in considering novel transportation means (such as small aircraft), where the responsiveness of the quantity demanded to a change in one factor $X$, might be non-linear.

$$\ln(D) = \alpha + \beta_1 * \ln(X_1) + \beta_2 * \ln(X_2) + ... + \beta_n * \ln(X_n)$$  \hspace{1cm} (2.4)

Other functional form for modelling the demand considers the market shares of alternative transport modes. This, called logit model [120, 121], extends the log-linear form to allow a mixture of categorical and common independent variables and to estimate one or more categorical dependent variables. Researchers have been used this class of models to investigate the sensitivity of the market shares of alternative transportation modes to a change in regulatory or managerial control variables such as the relative prices or the quality attributes [122, 123]. For instance in a study within the income influences the road transportation means, the logit models would calculate the ratio of the passenger car and public bus users. In the literature, the mathematical characterization of the logit models for modal split analysis is given with the following equation:

$$\ln \left( \frac{S_i}{S_m} \right) = \alpha_i + \sum_{k=1}^{K} \left( \beta_{ik} * \frac{X_{ik}}{X_{mk}} \right) + \sum_{n=1}^{N} \gamma_{ik} * X_n$$  \hspace{1cm} (2.5)

where

$S_i/S_m$ is the ratio of the demand $i$ to the base mode $m$,

$X_{ik}, X_{mk}$ are respectively the $k^{th}$ attribute of the mode $i$, and the base mode $m$,

$X_n$ is the $n^{th}$ common variable to all modes,

$\alpha_i, \beta_{ik}, \gamma_{ik}$ are the model parameters.
According to Oum [110], the major advantage of the logit model is that the two alternative case yield the logistic curve, which being an S-curve is “intuitively attractive and realistically describe” the mode switching behaviour of decision makers. On the other hand, the small aircraft prediction model might not focus on the analysis of the modal choice behaviour, which suggests the application of other functional forms.

With respect to the translog demand model [110, 111] it is derived from a “flexible” utility or production function that provides a quadratic approximation to the unknown true function. Because of its specifications, all variables \((X_n)\) are interacting with each other and with themselves, which – considering \(X_1\) and \(X_2\) – is characterized with the following functional representation:

\[
\ln(D) = \alpha_1 \ln(X_1) + \alpha_2 \ln(X_2) + \beta_{11} \ln(X_1^2) + \beta_{12} \ln(X_1X_2) + \beta_{22} \ln(X_2^2)
\]

where \(\alpha_1, \alpha_2, \beta_{11}, \beta_{12}, \beta_{22}\) are the parameters of the translog function.

While this method is widely applied to the cost functions of transport industries [110, 124, 125], its primary disadvantages for the small aircraft application include (i) the complexity in evaluating the coefficients, and (ii) the statistical concerns with over parameterization due to the presence of numerous interaction terms involving the explanatory cost factors. Finally, this review found that from the four generally used demand models, the log-linear might be the most suitable for SA application, due to its advantage in estimating the elasticity values. Additionally, as this technique is also the most widely used method in 2006, it provides numerous elasticity values that might be employed in the European SA context. However, this issue calls for further analysis to review whether the constant elasticities are reasonable to apply (see chapter 4.1.).

As for the prediction methods, first the regression and trend analysis is discovered [111, 126]. These use past data to establish the historical trends, and project future values of the dependent variables. While these are frequently used methods that cover both linear and non-linear functional forms, their major constraint with respect to the European SA is the need of using historical data. Additionally, according to NASA [7], these predictions are only of value as general trends, since performing extrapolations when new factors (e.g. simplified cockpit environment) might dramatically change the market might not provide reasonable results.

On the other hand, exponential smoothing is based on time series analysis of observations in which the most weight is given to the latest observation, while decreasing importance is attributed to earlier observations [111, 126]. Since this might better capture the most recent characteristics of small aircraft, the exponential smoothing might provide more appropriate results than regression, although the projections still require past records.

Another prediction method is the comparison, within the analyst contrasts the objective of the research with other relevant activities having similar characteristics [126]. One particular use of this technique is in projecting the SA development after the growth of early general aviation activity or even early jet age business jets / VLJ. However comparison might be applicable to the requirements of this investigation, it would be more than difficult – if not impossible – to bring into play the specific values of SA (e.g. pilot requirements).
On the other hand, survey techniques [111] might directly deal with the potential aircraft users, and therefore reflect the characteristics of the passengers (e.g. the importance of the total operating cost, the role of the cockpit requirements, the value of the average annual flight hours). Moreover several surveys could be done for different geographical areas across Europe, which could take into account the differences between socioeconomic characteristics and country specific attributes such as the presence of alternative transportation systems like the high speed trains. A typical example of such survey technique is a questionnaire given to general aviation users, business and other passengers that might afford a personal aircraft.

Finally, an alternative to the simple point predictions [111] – within the demand is projected with a single number “best estimate” – is an interval or range forecast. Unlike others, this technique addresses the uncertainty behind the evolution of the dependent variable by defining numerous alternative scenarios on the future development of the independent variables. The advantage of this is the capability to consider extraordinary events such as oil crisis or the shift in the regulation level related to noise restrictions, and to support further decision making and risk analysis. Since in the context of the European small aircraft, both the past and future developments are unclear or offering limited available data, this literature review found valuable to apply the scenarios. Additionally, to fulfil the specific requirements of a model, the prediction techniques might also be combined together, for example the scenarios with the comparison to estimate the potential small aircraft users with the ownership of personal cars. However this requires feasibility investigations and further analysis, which is given in the chapter 4.1.
3. Analysis of the European small aircraft air traffic in 2006

3.1. Data source of the analysis

The state-of-the-art revealed that the novel small aircraft might range from piston to jets (see chapter 2.2.). As the propulsion technology drives the flight performance, this also means a difference in the flight speed, maximum altitude, the rate of climb and other characteristics. Upon these, small aircraft flights might take place in various airspaces with different requirements (see Appendix A). For instance while low altitude operations might happen in uncontrolled areas after VFR, flights at higher altitudes (e.g. FL 250) might require control, thus more advanced pilots skills and cockpit instruments.

Therefore, to define the main characteristic and trends in small aircraft traffic, firstly the circumstances of the simulation origin (2006) should be analyzed, by taking into account those flights that are comparable to small aircraft. However, as the related investigations are not available for the European context, this task calls for developing the analysis by my own, and finally addresses the following issues [8, 11]:

- place SA flights in the airspace,
- restrict the area that should be focused on,
- provide the initial data for the demand / prediction models,
- give arguments on numerous flight characteristic (such as the propulsion technology and city pair preference, or the distribution of the flight altitudes, flight distances).

To explore the air traffic in Europe, this investigation selected to use the EUROCONTROL’s CFMU (Central Flow Management Unit) ATFM (Air Traffic Flow Management) records [30], as it contains complex databases on real flights for several years. Driven by the data availability, this analysis took a nine-month period in 2004 (to obtain a most relevant and adequate statistical data during a reasonable period of investigation: up to 6 months), which permits to

- receive an adequate statistical data (more than 900 000 flights),
- recognize the impact of seasons, and finally
- to address whether business or leisure operations are more often to happen.

To reach the data, the DANCE web interface [30] is applied due to its following advantages:

- gives a visibility on CFMU ATFM records,
- provides file format descriptions,
- proposes standard pre-generated and packaged files.

From a total of more than 70 fields to characterize the flights in DANCE, the followings are selected in order to fulfil the above mentioned objectives (see Figure 24.):

- the flight number,
- the identification number of the aircraft type (to select small aircraft)
the airport of departure,
- the airport of arrival (to analyze the city pairs),
- the Requested Flight Level (RFL): to identify the cruising altitudes,
- the date (to assess the number of daily flights and the potential effect of the seasons).

At the same time, DANCE does not enclose flight distance records [30]. To support its approximation, this investigation called for great circle calculations, as it considers the shortest distance between two points of the terrestrial surface (see chapter 3.2.). To obtain the GPS coordinates of both departure and arrival airports, the Aeronautical Information Publications (AIP) [127] are employed, since in air transportation, these provide the official data (see Figure 25.).

<table>
<thead>
<tr>
<th>ICAO ID</th>
<th>LAT°DEC</th>
<th>LAT°MIN</th>
<th>LAT°SEC</th>
<th>POS</th>
<th>LONG°DEC</th>
<th>LONG°MIN</th>
<th>LONG°SEC</th>
<th>POS</th>
<th>NAME</th>
</tr>
</thead>
<tbody>
<tr>
<td>EBAM</td>
<td>50</td>
<td>44</td>
<td>25</td>
<td>N</td>
<td>3</td>
<td>29</td>
<td>15</td>
<td>E</td>
<td>AMOUGIES</td>
</tr>
<tr>
<td>EBAW</td>
<td>51</td>
<td>11</td>
<td>25</td>
<td>N</td>
<td>4</td>
<td>27</td>
<td>42</td>
<td>E</td>
<td>ANTWERPEN DEURNE</td>
</tr>
<tr>
<td>EBBE</td>
<td>50</td>
<td>45</td>
<td>28</td>
<td>N</td>
<td>4</td>
<td>46</td>
<td>1</td>
<td>E</td>
<td>BEAUVECHAIN</td>
</tr>
<tr>
<td>EBBL</td>
<td>51</td>
<td>10</td>
<td>6</td>
<td>N</td>
<td>5</td>
<td>28</td>
<td>12</td>
<td>E</td>
<td>KLEINE BRÖSSEL</td>
</tr>
<tr>
<td>EBBR</td>
<td>50</td>
<td>54</td>
<td>8</td>
<td>N</td>
<td>4</td>
<td>29</td>
<td>10</td>
<td>E</td>
<td>BRUSSELS NATIONAL</td>
</tr>
<tr>
<td>EBBT</td>
<td>51</td>
<td>20</td>
<td>27</td>
<td>N</td>
<td>4</td>
<td>30</td>
<td>15</td>
<td>E</td>
<td>BRUSSELS-SCHAAT</td>
</tr>
<tr>
<td>EBBX</td>
<td>49</td>
<td>53</td>
<td>30</td>
<td>N</td>
<td>5</td>
<td>13</td>
<td>26</td>
<td>E</td>
<td>BERTIX</td>
</tr>
<tr>
<td>EBCI</td>
<td>50</td>
<td>27</td>
<td>39</td>
<td>N</td>
<td>4</td>
<td>27</td>
<td>15</td>
<td>E</td>
<td>CHARLOEI</td>
</tr>
<tr>
<td>EBCV</td>
<td>50</td>
<td>34</td>
<td>39</td>
<td>N</td>
<td>3</td>
<td>50</td>
<td>17</td>
<td>E</td>
<td>CHEUVES</td>
</tr>
<tr>
<td>EBFS</td>
<td>51</td>
<td>5</td>
<td>25</td>
<td>N</td>
<td>2</td>
<td>39</td>
<td>10</td>
<td>E</td>
<td>KOKSUDE</td>
</tr>
<tr>
<td>EBGB</td>
<td>50</td>
<td>14</td>
<td>36</td>
<td>N</td>
<td>4</td>
<td>38</td>
<td>45</td>
<td>E</td>
<td>FLORENNIES</td>
</tr>
<tr>
<td>EBGO</td>
<td>50</td>
<td>45</td>
<td>17</td>
<td>N</td>
<td>3</td>
<td>51</td>
<td>45</td>
<td>E</td>
<td>GERAARDI / OBERDOELAR</td>
</tr>
</tbody>
</table>

Fig. 24. Example of the DANCE database (note: flight numbers and dates are fictional due to prohibited data; aircraft types and airport codes follow the ICAO standards).

Fig. 25. Example of the database on airport GDP coordinates (note: “lat” stands for lateral and “long” for longitudinal).
With respect to the aircraft models, DANCE only provides their four letter ICAO designations [30]. Seeing that this code gives the abbreviations of the manufacturer and the model type without the information on any aircraft characteristics, it cannot support the selection of the appropriate SA flights. On the other hand, once associated with the MTOWs or at least the weight categories, one could base the investigation on the size of the aircraft. While these attributes are still accessible for the commercial aircraft, with respect to SA – and particularly the “unpopular” models – the available MTOW records are limited. Therefore, the selection of SA flights is rather based on the aircraft weight categories, upon the classification of the Federal Aviation Administration [128]. The major reason of using this source is its public availability, and the complex database that covers more than 3200 aircraft. Accordingly, it proposes the following weight classes:

- small aircraft of 12500 lbs (~5670 kg) or less,
- small plus – aircraft weighting between 12500 and 41000 lbs (~18598 kg),
- light aircraft of 15500 lbs (~7030 kg) or less,
- large aircraft from 41000 to 255000 lbs (~11567 kg),
- medium aircraft up to 300000 lbs (~136080 kg),
- heavy aircraft of 255000 lbs (~115668 kg) or more (all classes including overlapping regions according to FAA [128]).

As the name indicates, the first category (with its maximum of 5670 kg) should be applied in this study, since that covers the size of the aircraft, which is in focus. On the top of the weight classes, the FAA database also enables to associate the type of the propulsion technology and the number of the engines. Accordingly, all pistons, turboprops and jets are distinguished, which allows further investigations and to analyze the one that is the most frequently used. Taking all these advantages into account, the aircraft database took the form as presented in the Figure 26.

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Model</th>
<th>Type/Weight Class</th>
<th>ICAO code</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADAMS (2)</td>
<td>A-500, Carbon Aero</td>
<td>LJ/P/S</td>
<td>A500</td>
</tr>
<tr>
<td>AIRBUS</td>
<td>A-300-600</td>
<td>LJ/J/H</td>
<td>A300</td>
</tr>
<tr>
<td>BEECH</td>
<td>58 Baron</td>
<td>LJ/P/S</td>
<td>B58</td>
</tr>
<tr>
<td>BEECH</td>
<td>35 Bonanza</td>
<td>LJ/P/S</td>
<td>B35</td>
</tr>
<tr>
<td>BOEING</td>
<td>737-400</td>
<td>LJ/J/H</td>
<td>B734</td>
</tr>
<tr>
<td>BOEING</td>
<td>737-400</td>
<td>LJ/J/H</td>
<td>B764</td>
</tr>
<tr>
<td>CESSNA</td>
<td>536 Skymaster</td>
<td>LJ/P/S</td>
<td>C36</td>
</tr>
<tr>
<td>CESSNA</td>
<td>411 Conquest, Conquest 2</td>
<td>LJ/J/S</td>
<td>C41</td>
</tr>
<tr>
<td>CESSNA</td>
<td>650 Citation, Citation 1</td>
<td>LJ/J/S</td>
<td>C60</td>
</tr>
<tr>
<td>DORNIER</td>
<td>D-21 (F453)</td>
<td>LJ/J/S</td>
<td>D20</td>
</tr>
<tr>
<td>ECLIPSE</td>
<td>Eclipse 600</td>
<td>LJ/J/S</td>
<td>E60</td>
</tr>
<tr>
<td>EMERAIR</td>
<td>ERJ-145, ERJ-145 (R-29)</td>
<td>LJ/J/E</td>
<td>E145</td>
</tr>
<tr>
<td>EMERAIR</td>
<td>E170, E175, E170</td>
<td>LJ/J/M</td>
<td>E170</td>
</tr>
<tr>
<td>MITSUBISHI</td>
<td>MU-2, Marquise, Solitaire (LR-1)</td>
<td>LJ/J/S</td>
<td>MU2</td>
</tr>
<tr>
<td>PIPER</td>
<td>PA-28-140/150/161/251/261/180/181 Archer, Cadet,</td>
<td>LJ/P/S</td>
<td>P28A</td>
</tr>
<tr>
<td>TUPOLEV</td>
<td>Tu-154</td>
<td>LJ/J/M</td>
<td>T154</td>
</tr>
</tbody>
</table>

Fig. 26. Example of the FAA aircraft database (note: letters and numbers in the column “type/weight class” stand for land base aircraft; the number of the engines; the type of the engines with p: piston, t: turboprops and j: jet; and the weight class).
3.2. Methodology of the analysis

The available data source indicated that the air traffic analysis calls for a methodology that permits to run queries in an overall database, constructed from the framework of the following records:

- the air traffic data from DANCE (using the CFMU ATFM),
- the airport coordinates,
- the aircraft characteristics (e.g. weight category, propulsion system, number of engines).

To perform this task, the Microsoft Access software [129] is selected, due to its advantage in creating or modifying reports, and in dealing with data having thousands of rows.

As the Figure 27. indicates, this relational database management program connects the above listed records upon a predefined scheme of relationships. Based on the conditions surrounding the variables, the type of these connections varies between the followings:

- one to one: the joint field in both tables are equal (for one variable only one possible data might be attributed from the other table),
- one to many: for one data many information are attached in the other table,
- many to one: for all fields, only one record is found in the other table.
For example the arrow between the “FAA aircraft” and the “CFMU traffic” indicates a “one to many” relationship, which is reasonable, as the same aircraft might fly more than once and cause numerous fields in the traffic records. In addition, the rest of the connections are the same type, since during one day several flights might take place to numerous different airports.

On the other hand, the real power of using Access lies in its capability to combine the data from the multiple databases and to retrieve only those that meet certain specific conditions. In a first time, these (called) queries enabled to sort out all potential aircraft that offers the same weight class than SA, which gave about 2000 different records. After, to get the traffic data of these aircraft, a second query was applied. Finally, in order to not influence the analysis with overseas and training flights, only those records were selected, which had different, and European (arrival and departure) airport codes.

Besides, the following queries were made to recall the information that met the further conditions of the objectives:

- the number of SA flights per day,
- the SA city pair list,
- the most frequently used flight distance and RFL of SA flights,
- the most frequently used flight distance and RFL of traditional flights,
- the propulsion technology preference of SA,
- the most often employed SA.

As already mentioned in the previous chapter with respect to the flight distance, the problem was that DANCE does not enclose such records. As a result, this investigation employed rather the geographical distance between the airports of departure and arrival, based on their GPS coordinates [127]. Supposing that a spherical Earth model holds, the shortest distance between the airports is not a straight line – as in the Euclidean geometry –, but rather a geodesic or a great circle along the path of the terrestrial surface (see Figure 28.) [130]. The length of the arc related to this circle (whose centre is coincident with the one of the sphere) is the great-circle distance, and therefore the shortest path between the two geographical points. [131].

If $\phi_A, \lambda_A$; $\phi_B, \lambda_B$ are the latitude and longitude coordinates of the airports $A$, $B$, respectively, then the lateral and longitudinal differences are:

$$\Delta\phi = \phi_A - \phi_B$$

$$\Delta\lambda = \lambda_A - \lambda_B$$

where all coordinates are positives in Europe, since only the south $\phi$ and west $\lambda$ are treated as negative values.

Using the equations (3.1),(3.2) and supposing that $r$ is the average radius of the Earth (6372.795 km [130]), the great-circle distance $d$, might be estimated using the following formulas [130, 131, 132]:

$$d = r \times \arccos(\sin\phi_A \times \sin\phi_B + \cos\phi_A \times \cos\phi_B \times \cos\Delta\lambda)$$
\[ d = r \cdot \arcsin \left( \sqrt{\sin^2 \left( \frac{\Delta \phi}{2} \right) + \cos \phi_1 \cdot \cos \phi_2 \cdot \sin^2 \left( \frac{\Delta \lambda}{2} \right)} \right) \]  
\hspace{1cm} (3.4)

\[ d = r \cdot \arctan \left( \frac{(\cos \phi_2 \cdot \sin \Delta \lambda)^2 + (\cos \phi_1 \cdot \sin \phi_2 - \sin \phi_1 \cdot \cos \phi_2 \cdot \cos \phi_2 \cdot \cos \Delta \lambda)^2}{\sin \phi_1 \cdot \sin \phi_2 + \cos \phi_1 \cdot \cos \phi_2 \cdot \cos \Delta \lambda} \right) \]  
\hspace{1cm} (3.5)

While the equation (3.3) is constituted form the spherical law of cosines [130] and mathematically exact, according to Sinnott [132], it accounts for rounding errors at short distances due to the ill-conditioned inverse cosine.

With respect to the Heversine formula (3.4), it returns more accurate results, but suffers from miscalculations at antipodal points.

Therefore, this investigation selected the equation (3.5), since over the other formulas this is capable to estimate the variable \( d \) for all distances.

### 3.3. Results of the small aircraft air traffic analysis

Using the databases and the methodology mentioned in the chapter 3.2., this analysis accounts for a total of 47146 small aircraft flights in 2004, whose daily distribution is given in the Figure 29. [8, 133]. This clearly indicates two separate group of points, as the average number of flights are different during the weekdays and weekends, 924 and 1904 respectively. The reason for this dissimilarity might be caused by the majority of business flights (rather than leisure or pleasure), since generally these are the ones that take places during the weekdays. On the other hand, the average weekly flights are also estimated from
a trendline over the values of the investigation period. After tested on numerous different models, this investigation took the second order polynomial, as this found to be the most accurate. According to the Figure 29., this line (represented by a red colour) reflects the general effect of the seasons, as more flights appear during the summer period than it is for the winter. Otherwise, one could also observe a periodic shape for both of the upper and lower group of points. Perhaps this indicates the dependency on the meteorological conditions, since small aircraft might fly at relatively low altitudes, thus meet more turbulence. Naturally, a shift in small aircraft flexibility would require to decrease this effect, and to fulfil the possibility of flying at nearly any time.

To understand what could be behind these numbers, and how the small aircraft flights are distributed across the European airspace, this analysis represented the traffic of one typical day [11, 12]. To support this task, the COSAAC Software [134] is applied, since this is compatible with the CFMU ATFM records, and advantageous in graphical imaging. As the Figure 30. indicates, unlike the “traditional” flights (on the right), the distribution of the SA traffic (on the left) is non-homogenous, and one might be in problem to define the preferable routes or city pairs. One exception is the traffic between England

<table>
<thead>
<tr>
<th>Total frequency</th>
<th>City pair</th>
<th>Airport of departure</th>
<th>Airport of arrival</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>635</td>
<td>EGJS-EGUJ</td>
<td>GUERNSEY</td>
<td>JERSEY</td>
<td>39,9121306</td>
</tr>
<tr>
<td>325</td>
<td>EGSA-EGUJ</td>
<td>ALDERNEY/THE BLAYE</td>
<td>GUERNSEY</td>
<td>41,1771741</td>
</tr>
<tr>
<td>318</td>
<td>EGJS-EGUJ</td>
<td>SOUTHAMPTON</td>
<td>ALDERNEY/THE BLAYE</td>
<td>151,0328002</td>
</tr>
<tr>
<td>221</td>
<td>LEI-LEPA</td>
<td>IBIZA</td>
<td>PALMA DE MALLORCA</td>
<td>139,0027861</td>
</tr>
<tr>
<td>213</td>
<td>LEL-LEPA</td>
<td>BARCELONA</td>
<td>PALMA DE MALLORCA</td>
<td>201,5066003</td>
</tr>
<tr>
<td>203</td>
<td>LFRD-EGUJ</td>
<td>DINARD SAINT MALO</td>
<td>GUERNSEY</td>
<td>101,7437343</td>
</tr>
<tr>
<td>178</td>
<td>ESHA-ESSV</td>
<td>STOCKHOLM/KASKAVSTA</td>
<td>MJSA</td>
<td>151,0326929</td>
</tr>
<tr>
<td>143</td>
<td>EGJS-EGNIS</td>
<td>BLACKPOOL</td>
<td>ISLE OF MAN/RONALD'SW</td>
<td>110,0433077</td>
</tr>
<tr>
<td>141</td>
<td>EHSS-EHLE</td>
<td>GRONINGEN-EELDE</td>
<td>LELYSTAD</td>
<td>102,4067577</td>
</tr>
<tr>
<td>130</td>
<td>LSZH-EDN</td>
<td>ZURICH</td>
<td>FRIEDRICHSHAFEN</td>
<td>76,0616891</td>
</tr>
<tr>
<td>126</td>
<td>LFGB-LFPN</td>
<td>BEAUVAIS-TILLE</td>
<td>TOUSUSBUS LE NOBLE</td>
<td>79,3619365</td>
</tr>
<tr>
<td>122</td>
<td>LFRD-EGUJ</td>
<td>DINARD SAINT MALO</td>
<td>JERSEY</td>
<td>68,4899029</td>
</tr>
<tr>
<td>113</td>
<td>EETN-FEHK</td>
<td>TALLINN/JUEMISTE INT</td>
<td>HELSINKI-VANTAA</td>
<td>101,2543936</td>
</tr>
<tr>
<td>100</td>
<td>EGPF-EGPR</td>
<td>GLASGOW INTL</td>
<td>BARRA</td>
<td>224,9666352</td>
</tr>
<tr>
<td>90</td>
<td>EGSA-EGUJ</td>
<td>ALDERNEY/THE BLAYE</td>
<td>JERSEY</td>
<td>55,406746655</td>
</tr>
</tbody>
</table>

**Fig. 30.** The non-homogenous distribution of the daily total small aircraft traffic (on the left) relative to the total flights (small aircraft and traditional). Note: figures are represented with COSAAC Software [134].

**Fig. 31.** Top fifteen city pair of the European small aircraft traffic.
and some of its islands in the vicinity of the French coasts (e.g. Alderney, Jersey, Guernsey) where even scheduled flights are happening several times a day, which shifts them to the top of the most frequent city pair list (see Figure 31.).

As for the city pairs, while more than 12000 are found for the whole period, only 4.3 % of them occurred relatively frequently (at least once in a week) [135]. Additionally, some have equal arrival and departure airports, which is the evidence for the training or recreational flights. Therefore, this result shows that the number of scheduled small aircraft flights is limited, and that generally the most frequently used city pairs are driven by geographical conditions.

While the city pair list gives some insight to the flight distances, a complete analysis for all of the SA traffic is given in the Figure 32. [12]. Not so surprisingly – after the characteristics and geographical location of the cities – the most frequently appeared distance is 150 km, representing already 12 % of the total flights. Otherwise, the average value is 310 km, since 85 % of the results are shorter than 500 km, and only 3 % are at least 1000 km.

Although the most frequent flight distance associated to the SA flights is shorter than the one of the commercial traffic [11], in the complex airspace structure – where SA might take place – the evaluation of the cruising altitude density is crucial. This might help to assess how SA penetrates to the airspace and whether it impacts the traditional traffic. To perform such analyze, this investigation considered the cruising altitude, since according to the Flight Safety Foundation this represents the longest flight phase [136], and it is also accessible via the CFMU ATFM records. As the Figure 33. indicates, while 19 % of the traffic flies up to the FL (flight level) 50, the most of the flights (39 %) take place between the FL 50 and FL 100 [12]. Otherwise, the percentages of the aircraft at higher altitudes are decreasing gradually, resulting in 13 % with respect to the range of FL 100 - FL 150, and 11 % for the group between FL 150 and FL 200. Finally, with 18 %, just a limited amount of the traffic reaches at least the FL 200
[12], which is reasonable knowing that the optimal cruising altitude is also driven by the distribution of the flight distance.

On the other hand, this investigation also evaluates the propulsion technology preference to obtain more information on the flight characteristics of SA, and to assess whether it is in line with the flight distance and the cruising altitude distributions. The Figure 34. shows that with 30 %, single engine pistons are in majority and that all pistons together (single and multi engines) give 59 % of the total flights [11, 133]. While turboprops still reach 29 %, the share of the jet small aircraft is less important, approximately 12 % [11, 133]. These results are reasonable, as they reflect that generally – due to the lower fuel consumptions – pistons are selected for the flights shorter than 500 km. Otherwise, form 500 km turboprops or jets are more practical, as the cost of the added fuel consumption might be recovered with the increased performance that cuts the flight time.

This distribution of the propulsion technology also appears in the outcome of the most preferred small aircraft. The Figure 35. shows the first thirty out of them, since they cover 73 % of the total flights, and therefore considered to be representative. Accordingly, the piston propulsion remained in majority, while the number of turboprops and jets are
limited. On the other hand, the Figure 35. also expresses that the average capacity of the most frequently used small aircraft is about six passengers.

Seeing all the results, further investigations should consider that in 2004, the European small aircraft traffic flies to about 150 km, cruise around FL 100, and pistons are the most frequently used. Otherwise, the flights are non-homogenously distributed that results in more than 12000 city pairs for the whole investigation period.

3.4. Lessons learned from the analysis: initial data for the prediction model

While the air traffic analysis provided the main characteristics of the European small aircraft flights, to support a prediction model with initial data, numerous attributes remained unclear. First of all the fact whether business or leisure passengers are more often to fly. Knowing that the number of daily flights that took place during the weekdays was about twice more important over those happened at the weekends, the analysis first indicated a business market. On the other hand, the city pair list (see Figure 31.) showed that this result might be influenced by the geographical conditions (e.g. flights to and from islands), where flights happened rather due to the limited alternative transportation systems. Even so, the identification of the business or leisure flights is crucial, as they represent different requirements [137] that might change the focus of the prediction model. For instance business passengers are generally more sensitive to the flexibility of the transportation, which calls for a limited dependency on the meteorological conditions [8]. On the other hand, leisure travellers are rather price sensitive [8] that – due to the difference in the total operating costs – might even influence the applied propulsion system of small aircraft. Because of these reasons, the prediction model should finally consider both leisure and business flights. Moreover, to represent the different requirements of these passengers, all propulsion systems (such as piston, turboprop and jet) should be taken into consideration, which also enables to analyze whether the model is consistent.

Other uncertainty that remained after the chapter 3.3., is the potential impact of small aircraft flights on the traditional traffic [11, 12, 133, 135]. Even so, as this information might drive the SA characteristic – since upon the level of impact, the pilot responsibility, the level of on-board automation, the type of the cockpit instruments or other features might vary – it also influence the initial data of a prediction model (e.g. the cost of the aircraft). Therefore, this investigation calls for the analysis of the domain mentioned above, which due to the available data in the CFMU ATFM records first focuses on the distribution of the cruising phases (of both small aircraft and traditional flights), as according to the Flight Safety Foundation [136] these are the longest periods along a flight. The Figure 36. presents that the two type of traffics (of one day in 2004) are separated by 25000 feet between their most frequent altitudes, the FL 100 with respect to small aircraft flights, and the FL 350 to represent the traditional traffic [11, 135].
Otherwise, for the safety of air navigation, one might also find an optimal limit between the traditional and SA flights, where both would have a minimal impact on the other. By taking the minimum value of the cruising altitude distributions (see Figure 36.), this investigation found the FL 190 to be this limit, where 83% of traditional flights would take place above that altitude, and which would be influenced by 19% of the small aircraft traffic [11, 135]. Therefore, if that trend remains the same along the investigation period, the impact of small aircraft on the cruising phase of the traditional flights is limited, and no supplementary means might be required to manage the traffic. On the other hand, even if the SA altitude distribution remains the same, an increased number of flights could shift the previously mentioned 19% to higher values, and influence more the traditional flights. Additionally, the same happens if the small aircraft traffic characteristics would change, since for example the modification of the flight distance or the propulsion technology preference could raise the most frequently used cruising altitude.
Besides that I found the evidence for the fact that the two type of flights are separated along their cruising period, the traffic characteristics might be different at the airports vicinities, as the climb and descend phases of the traditional flights might be in the same area where most of the small aircraft flies. To analyze this problem, the COSAAC Software was applied [134], since that enabled a 3 dimensional graphical representation for the same CFMU ATFM records as considered previously. After selected one European sector at an airport vicinity, the outcome is presented in the Figure 37. [11, 135], where the signification of the yellow and red colours are respectively the outgoing and the incoming flights. Accordingly, the traffic is no more clearly separated as in the Figure 36., but rather represents a mixture of the small aircraft and traditional flights with a complex situation, where all climbing, cruising and descend traffics take place. As a result, the lower altitudes than the FL 100 meet a relatively high aircraft density, where the potential shift in the number of SA would further increase the complexity of managing the traffic [12, 135].

However to get to know how frequently this problem might occur on a European scale, one should also find the size of the (above mentioned) airport vicinities and represent them on a map. From a small aircraft point of view, these might be estimated by the areas within the traditionally traffic flies lower than the previously mentioned optimal limit, the FL 190. Then, the distance from the airport that is required for a commercial flight to reach this altitude might be approximated upon the aircraft performance, and the airports SIDs (Standard Instrumental Departure), or STARs (Standard Terminal Arrival Routes). With respect to the aircraft performance, this investigation applied the average values of the EUROCONTROL BADA [138] records, since that provides 295 different models. On the other hand, the departure and arrival procedures were based on numerous European AIPs [139] due to the official data that they offer. As a result, the average value is found to be 130 km [11, 135]. By tracing circles with this radius around those airports – where both small aircraft and traditional flights are present – I had the Figure 38. . Surprisingly the airport surroundings cover a relatively large geographical area, which
shows that some of the potential solutions to manage the traffic might not be reasonable. For example, while the deviation of small aircraft flights at the airport vicinities might enable the same service for the rest of the airspace users, this cannot be achieved without the revision of the FPs (Flight Plans). As this results in parasite effects (e.g. raised flight distance or modified departure/arrival airports) the Figure 38. also demonstrates that the role of the ATM might grow and novel methods could be required to deal with SA [6].

Finally, a major outcome of this analysis for the prediction model is that the characteristics of the Air Traffic Management might have an impact on the operating cost of SA (e.g. increased automation, enhanced cockpit instruments), which – for a reasonable prediction model – should be taken into consideration.
4. Methodology and modelling

4.1. Analysis of the generally used approaches for small aircraft modelling

After the review of the generally used demand and prediction models – in air transportation – (see chapter 2.4.) this chapter aims to analyze their potential application for the European small aircraft. The methods mentioned below differ in their depth of experience and applicability regarding SA. Therefore, to select a convenient approach, a deep analysis is required to see whether the following characteristics meet the model requirements of small aircraft:

- type of the model (such as linear, non-linear),
- allowed parameters in the model (like with or without past data, the level of uncertainty),
- potentiality in the end use of the results (such as the probability of certain results, applicability for extraordinary events like oil shocks, etc.)

First of all, those investigations were analyzed that focus on small aircraft (see also chapter 2.4. Potential demand and prediction models). Their major limitations are found to be the followings:

- the applied methodologies and assumptions are not publicly available, which therefore can not facilitate the analyzer to obtain a relevant knowledge on the potential model structures,
- the investigations are not dealing with the European market.

Additionally, the application of the existing (US) SA demand models to the European context is not promising, since Europe - instead of having a unique territory with more or less the same attributes - consists of different countries, with numerous socioeconomic characteristics and habits [8]. This European typicality also drives the alternative transportations means - such as the presence of High Speed Trains or the complexity of highway networks [8] - that makes reasonable to consider country specific small aircraft demand. Furthermore, due to specific geographical conditions in Europe relative to the US (for example dissimilar distances between the major cities [53]) even the optimal propulsion system might be different. This results again, in the inability to exploit the US models to the European small aircraft.

Therefore, other potential models – demand models – were analyzed, which rather applied in (conventional) air transportation (see chapter 2.4.) [111]. By doing so, this enabled to establish the background of the generally used mathematical models, and the list of numerous factors describing the commercial air travel demand.

In our investigation, the most often used technique was the log-linear model, due to its advantage in providing the elasticity values (that measures the responsiveness of the demand to one parameter change [140]) directly from the coefficients of each independent variable [111]. However the simple application of the European elasticity values (from the literature) is not manageable for small aircraft application, since in general there is no single elasticity value that is representative for the whole air travel demand [107]. There are several distinct domains and several different elasticities, caused by the wide range of flight distances, type of passengers, and economic characteristics. While, specific data is
especially desirable to capture the characteristics of small aircraft (such as the high value of time in business operation [107]) the drawback of the log-linear model is the need of using past data on the activity to predict. This is problematic in case of the European SA, since those parameters that drive its demand are limited (if not non-existent in the past), while the available information is insufficient to reflect the causal relationships between the dependent and the independent variables [111, 141]. In addition, these methods provide linear elasticities, which could not realistically characterize the non-linear attribute of SA’s market adoption [26, 142]. Therefore, the potential application or adaptation of the generally used demand model to SA requirements was rejected.

As for the potentialities of the prediction models, firstly the range predictions [143, 126] were analyzed. When considering these, their advantage lies in the capability to define extraordinary events with a positive or negative impact on the object to predict [8]. For example once small aircraft is operated in the vicinity of city centres, the increased noise emission might require the use of lower (noise) restrictions, which finally could postpone or even limit the further development of SA. Another example – a positive impact – might be in fuel cost decrease, due to a more strict regulation on the aircraft fuel consumption. This and other characteristics of the uncertain future development lead to specify predictions with different alternative forecast scenarios, within the evolution of the independent variables are addressed with different assumptions [126]. At that point, the relative importance of modelling over intuition becomes essential, since using too much of it might blur the professional judgment, while using none might be equally problematic [144]. However, the rationality of the applied assumptions is a sensitive part of the predictions based only on scenarios. Furthermore, while this method could provide some familiarity with the underlying mechanism, it is unable to evaluate which scenario is more (or less) probable to happen. In a prediction, such as the one of small aircraft, this drawback makes the technique incompetent in establishing the background for complex further investigations in decision making and risk analysis (for example the analysis of the impact of SA on the ATC/ATM).

Other potential method for small aircraft prediction is the regression and trend analysis [111]. From the past observation of the independent variables, the regression and trend analysis provide projections (on the dependent variables) with a point estimate. Therefore it assumes that the future evolution follows the past trend. This could limit the applicability of the approach, since the European small aircraft development might not reflect its past evolution, due to the latest technological achievements and developments in the domain of SA (see chapter 2.2.). Additionally, to compute any past trend, the available data in 2006 is insufficient, since the European SA activity is still limited (in 2006).

Even so, since this technique is widely used [126], to test its applicability and to show the evidence for its drawbacks, investigations in a multi-regression model were made. To develop the regression equation, different combinations of the independent variables have been examined, in order to obtain a more reliable equation that better represents the reality. Additionally, the signs (positive or negative) of the coefficients are predefined to be reasonable and logical. For example, the total operating cost should be negatively related to SA accessibility, otherwise the equation might not hold in the reality and should be rejected. Due to the limited small aircraft demand in the past, the dependent variable is represented by the data of 18 different SA similar aircraft sells (such as the Cessna 172, Piper Archer and others). Since the production period is aircraft specific and varies over the timescale of 1980 and 2006, finally an average yearly sells value is considered for each aircraft [145]. As for the independent variables, a complex and aircraft
specific data is available using the EUROCONTROL’s BADA documents that is a so-called Total Energy Model providing performance records [138]. With respect to aircraft list price, the 2006 values are used, or in case of an older production, it is estimated by an exponential trendline over the price of past years (see chapter 5.2.). Following these sources, a representative model – with the highest coefficient of determination (defining the model’s goodness of fit) [146], R-square (0.929), adjusted R-square (0.809) and cross validation value – was obtained by the following independent variables:

- the aircraft list price (on a per seat basis),
- the fuel consumption (on a per seat basis),
- the cruising speed,
- the aircraft performance,
- the mass,
- the production year.

Using the regression equation, and several scenarios to define the future evolution of the independent variables, numerous predictions are tested. The general observations and limitations are found to be the followings:

- regression could not allow to perform relevant European analysis, since country specific data on aircraft sells is limited, if not non-existent,
- sensitivity of the independent variables is insufficient, as only a limited change is observed along the whole investigation period,
- other variables without past data that also drive the small aircraft demand (e.g. the level of aircraft automation) could not be concerned without polluting the reliability and applicability of the method.

The analysis of the generally used prediction and demand models showed the limitation of these techniques. This review demonstrated that even the multi-regression model could not be fully employed, as it is incapable to take into account the independent variables with limited past data and an unclear (uncertain) future evolution. As a result, the model of the European small aircraft growth requires the development of a novel method, or the combination of two or more generally used technique.

### 4.2. Initial linear model

As discussed in chapter 4.1., the simple application or adaptation of the generally used models to the European small aircraft context is not promising, due to numerous limitations and drawbacks. Therefore, to obtain a valuable model, which replies to the specific requirements of the European SA, the results of the air traffic analysis (see chapter 3.4.) are employed. Its arguments on the most preferred propulsion technology, number of flights, cruising altitude and other characteristics clarified several attributes and variables [11, 12, 133, 135] that could be considered for the prediction model. While this enabled to name the key elements, their relationship, role and weight (in the influence on the small aircraft activity) remained unclear. Therefore at that time, an initial model was considered, with the following objectives:

- represent the possible relationships between the elements driving the SA demand,
- define the functionality of the key elements,
- obtain a prediction model, where the weight and role of the key elements could vary (since their influence might be different),
- provide a base for further investigations.

The proposed initial prediction model consists of four pars (see Figure 39.) [11, 135]. First of all, the market attributes, containing the factors that define the demand of air transportation [12, 133]. As preliminary data analysis shows [8, 147, 148] these factors could be the GDP, SA cost, various socioeconomic data [8] and the characteristics of the alternative (ground) transportation systems (e.g. high speed trains). Even so, as this prediction model focuses on the potential relationships between the small aircraft and traditional air traffic, the alternative ground transportation systems and therefore such mode switching behaviours are not analyzed. As a result, the input data – that is called here market attributes – is considered to consist of the GDP – as the main determining factor of the mobility – and two other variables; the technological development and the regulation. The technological development (see chapter 2.2.), because it might shift the SA accessibility; firstly in terms of costs via the total operating cost decrease with more reliable materials (e.g. MEMS [64], nanotechnologies [65]), and secondly in terms of aircraft manoeuvrability with advanced cockpit instruments (e.g. easy-to-follow 4D guidance [17], synthetic vision technologies [15]). As for the regulation (see chapter 4.1.), it was brought into play to capture the positive or negative influence of SA development on the environmental considerations (e.g. noise restrictions).

Following the economical theories [147, 148] and as the Figure 39. shows, the market attributes drive the small aircraft and the traditional traffic group. To characterize these parts of the model, this investigation considered two major elements; the number of aircraft and the cost, since from a small aircraft point of view both might influence the future characteristics of the European traffic. Otherwise, a supplementary element, called flexibility of market requirements is also a part of the SA group [12, 135], which – due to the lack of relevant data – aims to reflect the uncertainty in the flexibility of the market to the small aircraft cost variance, regardless of any GDP growth or technological development.

The final group of the model is called interaction on ATM [12, 135]. The objective of the presence of that part is to enable the analysis of the impact of small aircraft on ATM. Additionally, it also permits to consider, whether the characteristics of the ATM (for example the costs of automation) is in relationship with the SA demand. Therefore, this group is made up of some of the domains and potential problems in the Air Traffic Management

<table>
<thead>
<tr>
<th>Market attributes</th>
<th>SA traffic</th>
<th>Traditional traffic</th>
<th>Interaction on ATM</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>- number of a/c</td>
<td>- number of a/c</td>
<td>- avionics</td>
</tr>
<tr>
<td>tech. development</td>
<td>- market requirements</td>
<td>- costs</td>
<td>- ASM</td>
</tr>
<tr>
<td>regulation</td>
<td>(freedom, non-exp. pilot)</td>
<td></td>
<td>- separation responsibility</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(free flight, ASAS, etc.)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- automation</td>
</tr>
</tbody>
</table>

Fig. 39. The representation of the small aircraft adaptation stochastic process.
areas from a SA point of view, such as the airspace management (ASM), the level of cockpit instruments (avionics), the separation responsibility and the level of automation. The choice of these elements is based on the outcome of the small aircraft air traffic [8, 11] and future perspectives in ATM analysis [8]. For example, as initial investigations in air traffic analysis showed (see chapter 3.4.), the difficulty of small aircraft flights at airport surroundings might require an adapted ASM, which therefore is a part of the model. Following this logic, avionics defines the cockpit instruments and navigation tools (such as TCAS, GPS or ADS-B [20, 21]). Separation responsibility is represented by its importance, without taking into consideration whether pilots or controllers should deal with it. Similarly, automation means more its significance, without underlining that it might range from automation of controllers’ routine tasks to autonomous operation with advanced airborne system application (such as Airborne Separation Assurance Systems) and even free flight [8, 11].

Seeing the elements of the initial prediction model, numerous potential mathematical representations are feasible [11, 111, 135]. Generally, the evolution of dynamic systems (such as transportation systems) could be approximated with the following models given in continuous/discrete form:

\[
\begin{align*}
x(t_\text{i}) &= x_0, \\
x(t) &= f(x(t),u(t),p(\zeta)) + F(p(\zeta))n(t), \\
y(t) &= g(x(t),u(t),p(\zeta)) + G(p(\zeta))\eta.
\end{align*}
\]

where \(x, u, p, n, y, \eta\) are respectively the state, the control (input or regulatory), the parameter (system structural and operational characteristics), the state noise, the observation (output), and the measurement noise vectors. Furthermore, \(f\) and \(g\) are the system state and observation functions, while \(F\) and \(G\) are the system matrices with \(t\) time and \(\zeta\) random value.

For a small aircraft prediction model, \(u\) could be defined by the regulatory aspects (such as changes in safety requirements, or in taxation systems, application of novel technologies, etc.) The system characteristics \(p\) could depend on the evolution of unknown variables (e.g. the evolution of small aircraft, or socioeconomic characteristics) that is described by a random value, \(\zeta\) in its possible space, \(p_\Omega\). As for the state noise vector, following the general rule, it could be zero-mean, while the measurement noise vector might be a sequence of independent Gaussian random variables with zero mean and identity covariance.

On the other hand, the model (4.1) also represents the following stochastic (random) differential equation [111]:

\[
\dot{x} = f(x,t) + \sigma(x,t)\xi(t),
\]

This equation characterizes the diffusion process [111]. The first, the deterministic part (at the right side of the equation) describes the direction of the changes of the stochastic process passing through the \(x(t) = X\) at the moment \(t\), while the second part shows the scattering of the random process, with \(\xi(t)\) white noise. In the prediction and forecast methodology, several models are based on the equation (4.2), or on the diffusion theory. The model with S-curves is one out of them (see chapter 4.3.), which – relative to
the linear models – defines more realistically the non-linear aspect of the market adoption of a new product or service [142].

Furthermore, the equation (4.2) could also be applied to describe the system dynamics, by reformulating the model (4.1b) into the following form of controlled diffusion process:

\[ \dot{x} = \Phi(x,t) + b(t) + \sigma(x,t)\xi(t) , \]  

(4.3)

where \( \Phi \) is the deterministic vector describing the rate of change of the state vector \( x \) (as the product of the state and time increment functions); \( b \) is the vector of control effects; and finally \( \sigma \) is the transfer matrix describing impact of the noise disturbance on the state vector \( x \).

By replacing the state vector \( x \) by \( x = mx + \Delta x \), the equation (4.3) could be statistically linearized in the vicinity of \( x = m \), that results in:

\[ \dot{x} = \frac{d}{dt}(m + \Delta x) = F(m,\Delta x) + U(m,\Delta x)\Delta x + b(t) + \sigma(x,t)\xi(t) \]  

(4.4)

where \( U(m,t) \) is the sensitivity matrix, i.e. the matrix of partial derivatives of the (vector) function \( F(x,t) \) respectively to state vector \( x \), determined at \( x = m \).

However, this class of models could be applied for the prediction modelling, in the context of the European small aircraft, the exploitation of these techniques is limited, due to the lack of relevant data. In addition, according to the investigations in this domain [111], the prediction model in a generalized form is powerless, since the system should reflect the particular characteristics of the independent variables defining. These attributes might depend on the regulational aspects or the application level of the novel technologies, which gives rather a complex system with internal coupling and discrete (step) changes. Therefore, the initial prediction model of this investigation considers the superposition of the followings elements:

- an exponential (general) growth,
- the periodical changes (such as the requirements) and
- the discrete changes (like the characteristics of the general growth).

Consequently, the model (4.1b) might rather take the form of stochastic equations, such as the following:

\[ \dot{x} = f(x,u,t) \]  

(4.5)

where the vector \( x \) represents the dependent variables, \( u \) the input vector, and \( t \) the time, knowing that:
Since (4.5) is a non-linear differential equation, its linearization results in the equation (4.6) as:

\[
\dot{x}(t) = A^* \cdot x(t) + B^* \cdot u(t)
\]  

(4.6)

where

\[
A^* = \begin{bmatrix} a_{11} & \ldots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \ldots & a_{nn} \end{bmatrix}, \quad B^* = \begin{bmatrix} b_{t1} & \ldots & b_{tn} \\ \vdots & \ddots & \vdots \\ b_{nt} & \ldots & b_{nn} \end{bmatrix}
\]

and

\[
a_{ij} = \frac{\partial f(x,u,t)}{\partial x_i}, \quad b_{ke} = \frac{\partial f_k(x,u,t)}{\partial u_e}
\]

With \( T \) discretization time, the equation (4.6) could be discretized using \( t = k^*T \), where \( k \in N \) is the prediction horizon in years. Knowing that \( \Delta t \Rightarrow T \), this modifies the equation (4.6) to:

\[
\frac{x[k+1]-x[k]}{T} = A^* \cdot x[k] + B^* \cdot u[k]
\]  

(4.7)

With the rearrangement of (4.7), the initial prediction model results in the following form:

\[
x[k+1] = A \cdot x[k] + B \cdot u[k]
\]  

(4.8)

In the equation (4.8), the matrix \( A \) defines the internal dynamics of the system, and \( B \) stands for the outer (control) influence. Therefore, \( a_{ij} \) (where \( i \neq j \)) are coefficients that express the connections between \( a_{ij} \) and \( a_{ij} \) (e.g. the effect of automation on SA need).
The major advantage of the initial linear approach lies in the potentialities of using independent variables with a predefined role and weight in their influence on the small aircraft development. Using the matrix $A$, the model could assist to analyze the internal dynamics of the underlying mechanism, such as the effect of regulation or technological achievements on the small aircraft development. This enables further investigations, even with different aircraft characteristics, and the capability to find the right balance between initial conditions and potential solutions (e.g. the advantage of technological achievements against their impact on SA cost). Using the elements of the group ATM, the model also permits to analyze the characteristics of the impact of small aircraft on ATM.

Defining several scenarios (for the independent variables) and numerous potential matrix coefficients, the evidence for the model applicability was found [11, 12, 111, 135]. It reacted to the evolution of the independent variables, and responded to the shift of the regulation level. This last caused – assuming a negative impact – a delay in the small aircraft development that also influenced other variables. Due to the air traffic growth (small aircraft and traditional traffic), the variables of the group ATM were also influenced. For example the role of automation increased, which is in line with other predictions and especially with the special operation possibilities of SA [26] (with pilots having limited skills).

However, to provide a reasonable prediction, numerous drawbacks were discovered [111, 133, 135]. First of all, due to the limited data on the European small aircraft activity, the determination of the matrix coefficients was inadequate. Additionally, the assumed relationships between the independent variables might not always hold in the reality, and therefore the approach was rejected.

### 4.3. Innovation diffusion theory approach: S-curve application

For a valuable prediction model, the importance of a specific technique that replies to the non-linear characteristics of the European small aircraft development was already mentioned in the chapter 4.2.. As discussed, the equation (4.2) is the base of the innovation diffusion theory approach, the application of the S-curves. This approach is based on the idea of describing the demand – instead of a linear or log-linear form – by a more suitable model, some sort of an S-curve [142]. These curves are based on the innovation diffusion theory [149] that deals with the market penetration of the new products/systems into the

![Fig. 40. The market penetration (upper figure) and the number of individuals adopting the technology (lower figure) in function of the time [153].](image-url)
society. For the first time, the S-curve was introduced by Tarde [150]. According to its results, such approach might better represent the relationship between the technology adoption and time [151], since a new technology covers numerous market penetration and adoption phases with different characteristics (see Figure 40.). This is reasonable, because when a new technology is first introduced, it is often unproved, relatively expensive, and difficult to operate [142]. Standards are not yet established, and practices are to emerge. Therefore, only the innovators – a limited segment of the population – might experiment it at that time (see Figure 40.). On the other hand, once its value becomes clear, users and vendors rush to invest, the main market starts and the adoption curve heads upward. With further progress, the technology gets to the saturation level, where it could no longer be improved, such as the costs of surpassing the sound barrier in case of commercial air transportation. This point is the maturity [152], where the technology reached the limits of its underlying physical principles. Due to no potential for further development, at that stage only the laggards holding out and the S-curve becomes complete [142, 153].

The above mentioned is the engineering approach to the S-curve application for the technology development [154]. On the other hand, the same curve might also be employed to describe the society acceptance of the new products that depend on the communication channels, the individual decision making and the budget constraints (see Figure 40.). To describe such S-curves, numerous different mathematical models are offered [111, 155]. The basic extrapolatory model – using a logistic curve – was first proposed by Tanner [156], such as the followings:

\[ VO = \frac{S}{1 + a \cdot \exp(-b \cdot t)} \]  \hspace{1cm} (4.9)

This mathematical representation estimates the dependent variable (this time the vehicle ownership per capita: \(VO\)) at the simulation time \(t\), with a saturation level \(S\), and two model coefficients \(a\) and \(b\).

By introducing the casual variables \((v_1, v_2)\) and their coefficients \((c, d)\), the equation (4.9) could be modified to [157]:

\[ VO = \frac{S}{1 + a \cdot \exp\left(-b \cdot t + c \cdot \ln(v_1) + d \cdot \ln(v_2)\right)} \]  \hspace{1cm} (4.10)

Later, the model was extended with a power growth function [158], to permit a slower and more realistic approach to the saturation level. Therefore, by revising the equation (4.10), the extended formula is given by:

\[ VO = \frac{S}{1 + \left(a + b \cdot t + c \cdot \ln(v_1) + d \cdot \ln(v_2)\right)^n} \]  \hspace{1cm} (4.11)

where \(n\) is the additional coefficient, which once approaches the infinity approximates the equation (4.10).

The Gompertz curve, another widely used mathematical representation of the S-curve models, is characterized with the following equation (4.12):

\[ VO = S \cdot \exp[-a \cdot \exp(-b \cdot v)] \]  \hspace{1cm} (4.12)
where \( v \) is the major independent variable, otherwise all variables are the same as in the equation (4.9).

Between the models mentioned and others not cited here, the most often used technique is the Gompertz function [111] due to its flexibility, especially at the beginning and the final part of the curve. However, to clarify the potential application of this class of model for the prediction of transportation systems, the implied elasticity value needs to be analyzed. Using the relevant mathematical formula (see chapter 2.4.), the long-run elasticity is calculated as:

\[
\eta^{LR}_v = a * b * \exp(-b * v)
\]  

(4.13)

Due to the nature of this functional form [151] – unlike the elasticity values of the linear or log-linear demand models (see chapter 2.4.) – the \( \eta^{LR}_v \) is not constant, rather increases rapidly as the independent variable \( v \) raises through the early development stage, before declining gradually to zero as saturation is reached (see Figure 41.).

Therefore, using the innovation diffusion theory approach and especially the Gompertz function, one could more realistically describe the market penetration and technology adoption. In addition, investigations show [151, 156, 159, 160, 161] that on the long-run technology could also be understood as transportation systems, and therefore the S-curve models could be applied to approximate the development of these, such as the prediction of the total car ownership. Seeing all the characteristics mentioned above, the approach based on S-curves could bring into play for example the following advantages in prediction modelling:

- non-linear approach that – relative to the log-linear models – could realistically represent the attribute of small aircraft adoption,
- relevant technique to analyze how SA could penetrate to the market [142],
- model structure that does not suffer from the problem of gathering detailed socioeconomic and transportation characteristic data.
- offers non-linear elasticity to represent the responsiveness of the quantity demanded to a change of one independent variable (see Figure 41),
- includes a saturation level where the demand reaches its maximum value regardless of the independent variable(s) [151, 155] (unlike predictions based for example on regressions)
- enables to apply a partial adjustment mechanism (to account for lags in the adjustment).
While S-curve applications could offer several benefits when considering a prediction model, in aviation the use of this approach is limited [161]. It mainly appears in general descriptions of the aircraft development [161], without quantitative results or investigations in model development. However to apply it to small aircraft, this investigation revealed two possibilities. In the first, only a limited number of variables are required: the maximum expected market saturation, and the time to get to the maturity level. Once these are known, and assuming that the simulation starts at the current market penetration level, an S-curve could be fitted between the two points. Then, using this curve, the small aircraft demand, or the market penetration level could be worked out at any time of the simulation horizon.

The second technique calls for the idea of the Travelling Money Budget (TMB), first observed by Zahavi [162], as a fix proportion of income devoted for travelling. Investigations show [162, 163] that it follows a predictable pattern within each society, and it remains unchanged, even when unpredictable events are occurred. For example in response to oil shocks, travellers reduced other costs (e.g. the list price of the car), rather then decreasing the TMB. Since its main determining factor is more the income, knowing the unit total operating cost of small aircraft and the GDP (that drives the income), one could evaluate when the two curves intersect (see Figure 42.). At that point, SA would meet the budgetary constraints of the population that is associated to travelling (or the TMB), and becomes available [27, 28]. Using this data, and the population ratio that travels, this approach provides the initial point to fit an S-curve. In other words, by being a function of SA cost, the demand model would reflect the price sensitiveness of the population, which is also in line with basic economic theories [163, 164].

Nevertheless, the limitation of the innovation diffusion theory approach is the lack of knowledge in small aircraft market characteristics. This pollutes the two proposed techniques, since in the first both of the variables, while in the second the budget constraints (or the TMB) are unknown elements.

4.4. Non-linear analogical approach

The drawback of the innovation diffusion theory approach – the presence of unknown budget constraints – calls for an analogical approach, in order to approximate the uncertainties with known variables of a similar activity. Therefore, in case of the European small aircraft prediction model, the budget constraints could be estimated from the unit total operating costs of personal cars. This is promising, since (as mentioned in the chapter 2.1.) SA might become similar to personal cars, and it finally enables to gather a detailed and widely available transportation data. Additionally, when considering country specific records, this would bring into play specific socioeconomic characteristics and other preferences of each country [165]. For example, investigations show that the market share of the same class of cars (e.g. the high-class) in different European countries
ranges from 31 to 6 percent (see Figure 43.). However generally this is driven by the GDP, in some cases the reason is rather the attitude of the country, and the preference of a given market segment. For example in Denmark relative to the United Kingdom, this results in 7 % lower market share for the high-class cars, while the GDP is 36 % higher [165].

Consequently, the database that captures this characteristic – independent from the economic situation of the countries – might offer a more realistic description of the European market.

Following the economic theories [166] and the practices in transportation domains [167], the total operating cost is the sum of all indirect (see chapter 5.2.) and direct (see chapter 5.2.) costs such as follows:

$$ TOC_t = u_1 + u_2 + u_3 + \ldots + u_n, $$

where $TOC_t$ is the unit total operating cost, and $u_n$ are the independent variables at the simulation time $t$.

In the same time, to estimate the overall car market, the model determinates the total car ownership and population ratio. After experimenting with a number of different models for this task, finally the one proposed by Dargay is selected [151] that provides a non-linear prediction based on a Gompertz function given as:

$$ V_t = \gamma \cdot \exp[\alpha \cdot \exp(\beta \cdot GDP_t)], $$

Similarly to the equation (4.12), this formula gives the $V_t$ vehicle/population ratio (at the simulation time $t$), in function of the GDP per capita $GDP_t$, the saturation level $\gamma$, and two Gompertz curve coefficients, $\alpha$ and $\beta$. 

![Figure 43. The market share of the different class of personal cars varies in the European countries [165].](image)
The primary justification of using this work is that it provides a family of S-curves (see Figure 44.) and a long-term country specific prediction for most of the European countries. On the other hand, it also enables to bring into consideration the advantage of the Gompertz function, such as the non-linear description of the market adoption (see chapter 4.3.).

Knowing the total car ownership / population (equation 4.15) and the market share [165] of the classes of cars having a similar unit TOC to small aircraft, the accessibility – or the population ratio whom SA is affordable in terms of costs – is given with the following equation:

\[ SA_t = M_t \gamma \beta e^{\alpha t} e^{\beta GDP}, \]  

(4.16)

where \( M_t \) stands for the car market share, \( SA_t \) is the accessibility of SA at the simulation time \( t \), and the other coefficients are the same as in the equation (4.15.).

Therefore, the accessibility is mainly determined by two elements, the unit total operating cost evolution of SA and the budget constraint associated to travelling. Since the first is a factor of its components, the SA growth might also be driven by the influencing elements on the total operating costs, such as the technological achievements (see chapter 2.2.), or the different aircraft operational aspects (see chapter 5.3.). As for the impact of the budget constraint on the small aircraft accessibility, following the equation (4.16), it is defined by the factors listed below:

- the GDP,
- the population density,
- the country specific $\beta$ parameter of the Gompertz function (which reflects the income level where the income elasticity reaches its maximum value),
- the maximum income elasticity (coefficient $\alpha$ of the Gompertz function),
- the market saturation level (parameter $\gamma$ of the Gompertz function).

As a result, the approach analyzed here, finally accounts for numerous
- transportation characteristic,
- country specific and
- socioeconomic records.

Since this is unique in the range of the models analyzed, the further application of the analogical approach to the European small aircraft prediction modelling is promising.
5. Experimentations / simulations

5.1. The proposed approach

For a valuable prediction model, the importance of a specific technique (that replies to the characteristics of the European small aircraft) was already discussed in the lessons learned from the air traffic analysis (see chapter 3.4.). However, as previous investigations show (see chapter 4.1), the simple application or adaptation of the generally used methods to predict the air transportation activity is not promising due to their limitation and the lack of statistical data on the European small aircraft development. Therefore, to reply to the specific requirements and to cope with uncertainties, the proposed approach is based on the S-curve form the innovation diffusion theory to project the personal car ownership development as an analogical approach. This provides a non-linear and country specific model that brings into play the different socioeconomic data and transportation characteristics of the European context.

In the proposed model, the prediction is based on the unit TOC evolution of small aircraft, relative to the travelling budget constraints of the population, estimated from the unit TOC of personal cars [26, 28]. However, since the approximation of the automobile that has a similar unit TOC to SA is complex, relative to the chapter 4.4., a more advanced approach lies in the idea of associating the budget constraints to different social classes, represented by different car market segments. This – due to the dissimilarities in the price and income sensitiveness of the social classes [26] – results in a more realistic and logical approach. Following the practice in automobile industry [165, 168], the car market segments are classified into the following groups:

- low class,
- lower-mid class,
- upper-mid class,
- high class.
Using these, the classification of some of the most popular automobiles is given in the Table 1. On the other hand, upon the manufacturing company and the number of options, the unit TOC of the cars from the same market segment might vary up to 50% [165]. Seeing that via the approach selected this might influence the model results, the choice of the representative cars is a sensitive part of the model. Even so, as the market shares are associated to the entire part of the segments (covering from the less to the most expensive), a representative automobile should be taken from the vicinity of the unit TOCs' upper limits (see chapter 5.2.). Due to the four segments considered here, this technique allows representing the unit TOC in function of the market shares [165], or in other words the travelling budget constraints and the market shares associated (see the Figure 45.). According to the location and the non-linear aspect of the points (unit TOC-cumulative market shares), I assumed that a fitting curve has the following power growth form:

\[ M_i = a_i \cdot TOC_i^b_i \]  \hspace{1cm} (5.1)

where at the simulation time \( t \), \( M_i \) represents the market share, \( TOC_i \) stands for the unit total operating cost (or the travelling budget constraint) for the market segment \( i \), and \( a_i, b_i \) are the unknown coefficients of the equation.

As the final model contains the elements of the equation (5.1), I shortly describe the simplified conventional estimation of the unknown parameters \( a_i \) and \( b_i \).

To estimate \( a_i \) and \( b_i \), the Least Squares Method is applied [169] due to its advantage in analyzing the precision of the adjusted data. Since this technique assumes a linear relationship between the dependent and the independent variables, the equation (5.1) needs to be converted [170]. By introducing \( A_i = \ln(a_i) \), \( B_i = b_i \), and \( Y_i = \ln(M_i) \), \( X_i = \ln(TOC_i) \) for all \( t \), the equation (5.1) is linearized such as the followings [170]:

\[ Y_i = A_i + B_i \cdot X_i \]  \hspace{1cm} (5.2)

To approximate the given set of data \((X_1,Y_1), (X_2,Y_2), \ldots , (X_n,Y_n)\), where \( n \geq 2 \), the best fitting curve \( f(X) \) at each \( t \) is estimated by [170]:

\[ \Pi_t = \sum_{i=1}^{n} [Y_i - f(X_i)]^2 = \sum_{i=1}^{n} [Y_i - (A_i + B_i \cdot X_i)]^2 \Rightarrow \min \]  \hspace{1cm} (5.3)

\begin{tabular}{|c|c|c|c|c|}
\hline
\textbf{Make} & \textbf{low-class} & \textbf{lower-mid class} & \textbf{upper-mid class} & \textbf{high class} \\
\hline
Audi & A2 & & A4, TT & A8, A8 \\
Citroen & C1, C2, C3 & Xsara, C4 & & \\
Fiat & Seicento, Punto, Panda & Stilo, Multipla, Barcetta & & Ulysse \\
Ford & Ka, Fiesta, Fusion & Focus & Mondeo & Galaxy \\
Mazda & 2 & 3, MX5 & 6, Premacy & MPV, RX8 \\
Opel & Astra, Corsa & Omega & Vectra & Signum \\
Peugeot & 107, 107, 206 & 307 & 406, 407 & 607, 807 \\
Renault & Clio, Modus & Megane & Laguna & Espace Vel Satis \\
Volkswagen & Fox, Lupo, Polo & Golf, Bora, Touran & Passat & Sharan, Phaeton \\
\hline
\end{tabular}

\textbf{Table 1.} Examples for the car market segments considered in this investigation [165].

[165]
Therefore, the least squares error is obtained where \( A_t \) and \( B_t \) makes the first partial derivates equal to zero [170]:

\[
\frac{\partial \Pi}{\partial A_t} = 2 \sum_{i=1}^{n} [Y_{i_t} - (A_t + B_t X_{i_t})] = 0 \tag{5.4}
\]

\[
\frac{\partial \Pi}{\partial B_t} = 2 \sum_{i=1}^{n} [Y_{i_t} - (A_t + B_t X_{i_t})] = 0 \tag{5.5}
\]

Expanding the equations (5.4), (5.5), the unknown \( A_t \) and \( B_t \) coefficients can therefore be obtained [170] with:

\[
A_t = \frac{\left( \sum_{i=1}^{n} Y_{i_t} \left( \sum_{i=1}^{n} X_{i_t} \right)^2 \right) - \left( \sum_{i=1}^{n} X_{i_t} \right)^2 \left( \sum_{i=1}^{n} Y_{i_t} \right)}{n \sum_{i=1}^{n} X_{i_t}^2 - \left( \sum_{i=1}^{n} X_{i_t} \right)^2} \tag{5.6}
\]

\[
B_t = \frac{n \sum_{i=1}^{n} X_{i_t} Y_{i_t} - \left( \sum_{i=1}^{n} X_{i_t} \right) \left( \sum_{i=1}^{n} Y_{i_t} \right)}{n \sum_{i=1}^{n} X_{i_t}^2 - \left( \sum_{i=1}^{n} X_{i_t} \right)^2} \tag{5.7}
\]

As the Figure 45 indicates, using the solutions for (5.6), (5.7) and the equations \( a_t = \exp(A_t) \), \( b_t = B_t \) the market share is finally expressed with a non-linear function of the travelling budget constraints (see chapter 5.2.). This, compared to the approach proposed in chapter 4.4., offers higher flexibility and precision in the prediction modelling, since it enables to determinate the market shares at any budget constraint. Furthermore, as expected, the curve also reflects the non-linear attribute of the population’s price sensivenes, which is in line with economic theories [26, 163, 164].

![Fig. 45. The non-linear market share estimation using the unit TOC of cars.](image)

While using the added value of the non-linear market share estimation, the proposed approach results in the Figure 46. As it shows (similarly to the chapter 4.4.) that the calculation initiates with a unit TOC evaluation for both small aircraft and cars from different market segments. Once again, following the economic theories [166] and the practices in transportation domains [167], the total operating cost is the sum of all indirect and direct costs (see chapter 5.2.). Therefore, in case of small aircraft, this could be given as:
where \( TOC_{i,t}^{SA} \) is the unit total operating cost, and \( u_{n_i} \) are the independent variables at the simulation time \( t \).

On the other hand, due to several market segments, the unit total operating cost of the personal cars is rather characterized with the following:

\[
TOC_{i,t}^{car} = v_{i,t}^1 + v_{i,t}^2 + v_{i,t}^3 + \ldots + v_{i,t}^i
\]  

(5.9)

where \( TOC_{i,t}^{car} \) is the unit total operating cost, and \( v_{we}^i \) are the independent variables for the market segment \( i \), at the simulation time \( t \).

Using the equations (5.6), (5.7), (5.9), and knowing the car market shares [165], the model estimates the parameters \((a_i, b_i)\) of the power growth function to express the budget constraints in function of the market shares (see Figure 45.). To convert this curve to a function that associates the budget constraints to the population, rather than the overall car market, the model estimates the total car ownership per population ratio. As discussed in the chapter 4.4., after experimenting with a number of different models for this task, finally an S-curve is used, based on the model proposed by Dargay [151]. The primary justification of using this work lies in the benefit of country specific prediction (see Figure 44.), which enables to take into consideration the specific characteristics of the European context (see Figure 43.). For that reason, the total car ownership is given by:

\[
V_t = \gamma \ast \exp\{\alpha \ast \exp(\beta \ast GDP_t)\}
\]  

(5.10)

where \( V_t \) represents the vehicle per population ratio, \( GDP_t \) stands for the GDP per capita, and \( \alpha, \beta, \gamma \) are the Gompertz curve coefficients, similarly to the equation (4.15).

Finally, as presented in the Figure 46., the small aircraft accessibility – or affordability in terms of costs – is determined as [26, 27, 29]:
where \( X_i \) and \( Y_i \) are respectively the natural logarithms of the unit total operating costs and the market shares of the personal cars from the segment \( i \), at the simulation time \( t \). Using \( a_i \) and \( b_i \), the equation (5.11) could be simplified to:

\[
SA_{i,t} = [a_i \ast TOC_{i,t}^{SA} \ast b_i] \ast [\gamma \ast \exp(\alpha \ast \exp(\beta \ast GDP_t))]
\]

which at each \( t \) represents the small aircraft accessibility (\( SA_t \)) in function of the parameters of the power growth function \( (a_i, b_i) \), the coefficients of the Gompertz curve \( (\alpha, \beta, \gamma) \), the unit total operating cost of SA \( (TOC_{i,t}^{SA}) \), and the GDP per capita \( (GDP_t) \).

### 5.2. Model variables

To provide the small aircraft prediction, the proposed non-linear analogical model considers several model variables. Since these are also influenced by a number of factors describing the characteristics of SA (e.g. the number of seats, the applied propulsion system), first I clarified which kind of aircraft should be covered in the investigation. While, the air traffic analysis (see chapter 3.1.) showed that in Europe single engine pistons are in majority, this investigation also took into consideration the turboprops and jets, in order to support further investigations in each domain and to analyze, whether the model is consistent. As a result, the aircraft fleet ranges from one of the best-selling single engine model (the Cessna 172) to the top of the light airplane hierarchy (e.g. the Beech Baron 55 or the Cessna Citation V). Due to the limitations of the available documents [138], the model considers a total of 31 aircraft over different production periods between 1960 and 2006 (see Figure 55. for the complete list).

![Fig. 47. An example of the small aircraft models considered in this investigation.](image)
the estimations also distinguish two main categories; the direct and the indirect operating costs. While the first includes those costs which could be associated with – and depend on – the type of the aircraft being operated on a specific flight (and which therefore would change if the aircraft type was changed), the second contains all other variables that are indirectly related [167, 180]. In air transportation [167, 177, 178] the direct operating costs generally cover (i) the flight crew salaries, (ii) the fuel costs, (iii) the airport and en-route charges, (iv) the aircraft insurance, (v) the rental or lease related costs, (vi) and the maintenance charges. The indirect operating costs are (i) the station and ground expenses (e.g. ground staff or office rent), (ii) the ticketing costs and (iii) other costs for example the advertisement. Even so, due to the characteristics of small aircraft and especially the owner-pilot or fractional ownership operations, some of the above mentioned categories, and particularly the indirect operating costs (for example the ticketing, catering or office related costs) are found to be irrelevant in this investigation. Therefore, I rather adapted the generally used total operating cost components to the context of the European small aircraft, which therefore considers the following elements [26, 27, 29]:

- ownership (or lease) related cost,
- air navigation charges (including en-rout and terminal),
- airport costs (parking, landing and passenger service),
- fuel cost (defined for different flight phases),
- pilot cost (or pilot hire cost),
- maintenance cost (with parts, labour and overhaul costs),
- insurance cost (including hull and liability).

Following the practice of the TOC estimations [167, 178], the above listed elements range from variable to fixed costs [178]. While the first comprises the ones that happen only when the aircraft flies [178] (e.g. fuel cost), the last groups those costs incur regardless of whether or not the airplane is in use [178] (e.g. ownership, insurance) and therefore usually given with the EUR / year basis. On the other hand, to take into account the different aircraft characteristics (e.g. the number of seats) and therefore obtain a reasonable comparison of the SA types considered, all costs should rather be expressed in EUR / seat-km. To convert the fix costs to this basis, one should divide them by the number of SA seats, the flight distance, and the amount of operations (or blocks flown) during an average small aircraft flight hour value. With respect to the variable costs, these might be first expressed for one block (accounting for the costs between the moments when the aircraft leaves the gate of departure, till it arrives to the gate of arrival [167]), then converted to the EUR / seat-km basis using the flight distance and the number of SA seats. Otherwise, the flight distance is assumed to be 150 km, as this is found to be the most frequent value of the small aircraft traffic analysis (see chapter 3.3.). The average weekly small aircraft flight hours is taken to be 22. The major reason for that is the assumption of about two round trips a day, which is reasonable in case of air taxi business or fractional ownership operations [178]. In addition, this is also in line with other relevant works [73, 90, 183] and with the peak hours of the SA traffic analysis (see chapter 3.3.).

To calculate the ownership or the lease related costs, numerous approximation models are available in the literature. For example Roskam [174] has developed a series of models such as the following equation:
\[ AMP = \log^{-1}(0.657+1.4133 \times \log WTO) \] (5.13)

While this is found to be relevant in providing predictions on the airplane market price (AMP), its bottleneck for the SA application includes the focus on business jets and particularly on aircraft with a maximum take-off weight (WTO) between 10,000 and 60,000 lbs. Therefore this estimation is rather selected a historical data approximation, since this method enables to reflect the small aircraft characteristics. In the literature, a widely used formula for such estimation is given with the equation (5.14) [175]:

\[ C_{A/C} = \frac{E(K_{A/C} \times P_{A/C} + K_E \times P_E \times n_E)}{K_C \times W_C \times V_{cr} \times B_y} \] (5.14)

where the capital cost of aircraft ownership \((C_{A/C})\) is given by the normative coefficient characterizing the effectiveness of the capital cost \((E)\), the coefficient taking into account the non-commercial operational times of the aircraft and engines \((K_{A/C}, K_E)\), price of the aircraft and engines \((P_{A/C}, P_E)\), the number of engines \((n_E)\), the coefficient of commercial load \((K_C)\), the cruise speed of the aircraft \((V_{cr})\) and the yearly flight hours \((B_y)\).

Additionally, the aircraft and engine prices might be approximated with the following formulas [175]:

\[ P_{A/C} = K_A \times K_V \times W_e (a_1 + b_1 W_e - c_1 W_e^{d_1}) \] (5.15)
\[ P_E = K_{TE} \times K_{SE} \times PO_E (a_2 - b_2 \sqrt{PO_E}) \] (5.16)

where \(W_e\) is the empty weight of the aircraft, \(PO_E\) gives the engine power, \(K_A, K_V, K_{TE}, K_{SE}\) are respectively the coefficients depending on the number of produced aircraft, the cruising speed, the type of engines, the number of produced engines (in series) and \(a, b, c, d\) are the parameters of the approximations.

However, in the context of SA the available data cannot distinguish the aircraft and engine costs, neither the coefficient of the commercial load, which results in the weakness to exploit the generally used methods. For that reason, to calculate the ownership or the lease related costs in this investigation, the list price of the simulation origin (2006) is used. Similarly to other cost analysis in the domain, a bank loan of 10 years is assumed, with a fixed nominal interest rate of 8 % [183]. According to the statistical data available, the yearly depreciation is taken from a trendline over the price of past years (see Figure 48.). After tested on several aircraft, an exponential model is used, since – with respect to small aircraft – this gives the highest R-square value [146], 0.9 in average. Furthermore, once the bank loan is reimbursed, I assumed that SA users will purchase a new aircraft, since otherwise the raised maintenance costs of an older model might result even in higher TOC [184, 185].

As for the air navigation costs, generally [186] the en-route and terminal navigation charges are distinguished. The first remunerates the costs of providing the air traffic control services along the flight, and the second paid after each take-off, as defined by each country’s civil aviation authority. With respect to en-route charges, it is levied for each
flight performed under Instrument Flight Rules (IFR) in the Flight Information Regions (FIR) falling within the competence of the Member States [167, 186]. Following EUROCONTROL [186], this charge takes into account the distance flown ($d_i$), the aircraft maximum take-off weight ($MTOW$), and the unit rate of en-route charges ($t_i$) such as the formula (5.17) indicates:

$$R = \sum_{i=1}^{n} (t_i \times d_i \times \sqrt{\frac{MTOW}{50}})$$  (5.17)

With respect to the Terminal Navigation Charges [167], it could be evaluated by the multiplication of the correction factor ($K$), the unite rate ($T_n$) and the service unit (that is the $MTOW$ to the power of 0.9). Since this investigation aims to provide a European prediction without the focus on any city pair, each the unite rate of en-route charge, the correction factor and the unit rate of the terminal navigation charge are considered with average European values, as in March 2006 [186]. This gives 49.2, 1.24 and 17.9 EUR respectively.

To take into account the airport costs, this investigation covers the landing, parking and passenger service issues, since other elements (such as catering) are not meaningful in the context of SA. Using several Aeronautical Information Publications (AIP) [187] and contacts with small airports across Europe [188] (e.g. France, Hungary, Croatia, Poland, Switzerland, Spain) the average value of each factor is obtained, as in April 2006. All together – upon the size of the airports – this results in the following range of average values:

- 4.67-7.83 Euro Cents / seat-km with respect to the pistons,
- 9.2-14.32 Euro Cents / seat-km to represent the turboprops and
- 9.4-14.4 Euro Cents / seat-km in case of the jets.

As for the fuel costs, first of all the total fuel quantity is defined following the official regulations of the Joint Aviation Requirements [189]. Consequently, trip, route reserve, and alternate fuel categories are considered. With respect to the trip, since the consumption also varies upon the different flight phases, it is further divided into climb, cruise, descend and taxi periods. To define the percentages of these phases form the total block time, this investigation uses the results of the Flight Safety Foundation [136], since this provides the requested data for a flight with a similar block. By adapting the values to the specific characteristics of SA (e.g. performance, flight distance), this research found the most optimal flight phases to be 22, 50, 26 and 2 percentage respectively. Then for each, the fuel consumption is estimated by the EUROCONTROL BADA [138] documents, because of its complex aircraft specific database. Finally, once the total amount of fuel is estimated, the fuel cost is given by the multiplication of the quantity with the price. Due to the relatively short block distance considered here, the fuel price is assumed to be in the category of taxable domestic operations. Using average European values in April 2006 [190], this results in 1.7 EUR / litre in case of the aviation gasoline (100LL) and 1.6 EUR / litre for the JET A fuel.

To evaluate the pilot costs, this research assumes that in 2006, SA is operated by professionals. This is reasonable in case of business, or other flights with passengers without a PPL (Private Pilot License). Therefore, upon a European survey in 2006, the
pilot hire cost is considered as 50.1 EUR per flight hour, which translates to 8.05, 2.3 and 2 Euro Cents / seat-km respectively for the pistons, turboprops and the jets.

As for the maintenance cost, generally it is approximated with the following or similar formulas with respect to the airframe \((C_{MAC})\) and engine \((C_{ME})\) categories [175]:

\[
C_{MA/C} = K_M * K_{TA/C} * W_e (a_3 - b_3 W_e) \tag{5.18}
\]

\[
C_{ME} = a_4 * K_E * K_{M,TE} * n_E * \sqrt{PO_E} \over 1 + b_4 * T_E \tag{5.19}
\]

where \(W_e\) stands for the empty weight of the aircraft, \(P_{OE}\) represents the engine power, \(K_M, K_{M,TE}, K_{TA/C}\) are respectively defining the coefficients of the applied maintenance method, the characteristics of the engine maintenance, the type of the aircraft, and similarly to the equations (5.15) and (5.16) \(a, b\) are the parameters of the approximations.

While the estimation of the parameters of the equations (5.18) and (5.19) call for maintenance planning and inspection lists, these in case of small aircraft are hardly publicly available. Therefore, this investigation rather used the Conklin and de Decker Aircraft Cost Evaluator (ACE) [184] and PlaneQuest [185] databases, because both include several SA and enable to consider labour, part, engine restoration and propeller overhaul costs. Since the maintenance cost varies upon the number of engines and the type of the applied propulsion systems, this evaluation distinguished the single and multi engine small aircraft, for each piston, turboprop and jet propulsions. Otherwise, even if generally the maintenance cost is rising with the flight hours [184, 185], the applied database is unfortunately statistically insufficient to bring this effect into consideration. Therefore, the average maintenance cost of several new and used aircraft is used in 2006, which results in the followings [184, 185]:

- single piston cost: 2.46 Euro Cents / seat-km,
- multi engine piston cost: 3.22 Euro Cents / seat-km,
- multi engine turboprop cost: 4.81 Euro Cents / seat-km,
- multi engine jet cost: 6.04 Euro Cents / seat-km.

In the literature, several costs are distinguished to take the small aircraft’s insurance into consideration. First of all, the minimum level is the third party insurance [191] which covers the owner or the operator of the aircraft against claims that may be made by other people (e.g. manoeuvring on the ground that causes a hit of an other aircraft). The next one is the hull insurance that is taken out by the owner, aiming to protect the aircraft against damage or loss. To finish, other insurances (such as war risk) are not applied in the investigation, since they are assumed to be irrelevant for small aircraft (as e.g. the war risk is only applicable for operations into countries or areas considered as dangerous). However the level of cover is also regulated by the European Parliament [192], the yearly insurance cost varies upon several factors [191]. One of them is the pilot skills and experience. Logically the more one flies the less is the insurance cost. On the other hand, popularity of the aircraft also plays a key element, since from an insurance company’s perspective the word “common” means low repair or replacement costs. Finally, the airport part could be mentioned, since characteristics such as runway length could influence the potential risk of an accident. Following these factors and the available records of the aircraft insurance
companies [193], the yearly hull and liability cost for SA is estimated to be 3 % of the aircraft value.

As for the TOC estimation of personal cars, similarly to the approach used for SA, the following elements are considered [177]:

- the ownership related cost,
- the maintenance cost,
- the fuel cost (consumption and price),
- the insurance cost,
- other costs (highway, parking).

Following the chapter 5.1., each cost element is estimated for different market segments, represented by the cars from the upper bounds of the given classes. These are assumed to be the VW Golf (base edition) for the low, the VW Passat (base edition) in case of the lower-mid, the Audi A6 (V6) to represent the upper-mid and finally the Audi A8 (V8) with respect to the high class. Otherwise, all costs are expressed in EUR / km, with – according to a European survey in 2006 – an average yearly travelling distance of 15,000 km.

To calculate the ownership costs – similarly to other investigations – this analysis takes the list price from the constructors as in 2006, assuming a 20 % down payment and four years of bank loan with an interest rate of 8 %. According to the complex statistical data on the past ownership values published by the French National Institute for Statistics and Economic Studies (INSEE) [168], the average yearly depreciation is taken as 17 % of the car value.

Contrariwise small aircraft, to take into account the maintenance cost of personal cars, numerous maintenance planning and schedules are publicly available. Here, the one of the manufactures recommended maintenance schedule [194] is used, since it provides a database for 2006, and covers the automobiles selected for each market segment. Consequently, the total cost – accounting for all parts, labour and tire – is calculated as 0.0053, 0.006, 0.0098 and 0.0138 EUR / seat-km respectively for the low, lower-mid, upper-mid and high class segments.

With respect to fuel costs, first the total consumption is estimated. Similarly to SA, since this is driven by the operation characteristics, different traffic conditions should be distinguished. However, relevant works in France and in the United Kingdom [195, 196] showed that due to the European particularities (such as the average distance between cities and the complexity of highway network), a mixed city, national road and highway circulation is reasoning to assume, with an average speed of 65 km / h. On the other hand, to take into consideration the different characteristics of the car classes (e.g. performance, weight), several fuel consumptions are considered. For this task, a detailed and yearly updated database is available from the Vehicle Certification Agency (UK) [195]. Accordingly, the following average values are assumed, all given in litres / 100 km:

- low class: 6.5 ,
- lower-mid class: 7.3 ,
- upper-mid class: 9.7 ,
- high-class: 11.9 .
As a result, the fuel cost is calculated by the multiplication of the quantity with its price, which (based on a European survey in April 2006) is assumed to be 1.3 Euros per litre for cars running with petrol, and 1.15 Euros per litre for diesels.

As for the highway cost, since different approaches in the charging systems are applied across Europe, it is based on a survey in April 2006. By weighting the distributions of the yearly fix, gate system, and those that are free of charge, the average value of 0.008 EUR / seat-km is found, which assumed to hold in this investigation.

To estimate the insurance costs, which varies from country to country, several agencies have been contacted and numerous different estimations were made [197]. The major determining factors are found to be (i) the age and the experience of the driver, (ii) the address of the owner and (iii) the value or the performance of the car. Using these, the average monthly insurance (liability and risk together) is taken as 0.011, 0.019, 0.03 and 0.05 EUR respectively, for the low, lower-mid, upper-mid and high class segments.

As for the variables of the total car ownership estimation, the model takes the Gompertz curve parameters ($\alpha$, $\beta$, $\gamma$ and GDP) as mentioned in the chapter 4.3. However the advantage of this approach lies in the fact that it enables country specific predictions, in the current state of the research, only the average European value of 0.22 is considered for the $\beta$ coefficient. Otherwise, to estimate the $a_i$ and $b_i$ parameters of the equation (5.1.), the market shares are based on the French National Institute for Statistics and Economic Studies (INSEE) [168], because their investigation covers more than 15 years of statistical data. Therefore the market shares are considered to be 12, 13, 34 and 33 percentages respectively, for the high, upper-mid, lower-mid and low class segments.

Finally, the parameters are estimated with the Matlab Software [198], due to its power in computational algorithms.

5.3. Monte Carlo Simulation

Seeing that the aircraft considered in the simulation (see Figure 55.) result in different total operating costs, this investigation takes the mean values of a selected number of models (see chapter 6.1.) for the pistons, turboprops and jets to provide the TOCs at the simulation origin. On the other hand, as the development of the independent variables is unclear, the proposed prediction model needs to deal with several uncertain parameters to provide the future TOC estimates. While the simulation – where the uncertainty is defined by a limited number of scenarios – might provide some familiarity with the underlying mechanism, it could not support predictions for decision making and risk analysis, since it cannot quantitatively address the uncertainties present in the system (see chapter 4.1.). However, the unclear variables could also be presented by probabilistic simulations, which provide the result with a probability distribution. Due to this advantage over the other methods, the Monte Carlo Simulation (MCS) is selected that finally solves the problem by using random numbers and probability statistics [199]. As the Figure 50. indicates, its principle first consists of assigning a Probability Density Function (PDF) to the uncertain variables. The type of the distribution (e.g. normal, triangular, uniform or lognormal) should be based on (i) the conditions surrounding that variable, (ii) the best representation of the current state of knowledge and on (iii) the professional experience [200]. Once the distributions are attributed, MCS takes
randomly sampled values from them, to form one possible scenario and one result [201]. After recorded the outcome and repeated the process, this gives a range of possible scenarios with a probability distribution that finally approximates the answer to the problem.

To deal with the uncertainties, this investigation applied several distributions for the MCS [27, 28, 29]. First of all a uniform is used, when all potential values of a given independent variable happened with equal probability. Generally, these are the ones associated to small aircraft, since in Europe, no reliable data is available to estimate other probability distributions. Otherwise, the upper and lower limits of these (distributions) are obtained following the principle presented in the Table 2. Accordingly, the uncertainty in the pilot costs is driven by the characteristics of the potential SA operations. In other words this means that even if SA could be a professionally operated aircraft (or at least by pilots having a Private Pilot License), the newest technological achievements – for example the advanced cockpit environment with easy to fly flight path guidance (see chapter 2.2.) – might shift the difficulty of aircraft manoeuvrability closer to the level of personal operation, which therefore would require limited pilot skills. As mentioned in the chapter 2.1. and 2.3., this is also investigated by small aircraft relevant works; first of all by the PATS project that aims to reach this objective via aircraft automation and control theory, and secondly, by the Small Aircraft Transportation System that seeks to facilitate the pilot

<table>
<thead>
<tr>
<th>variable</th>
<th>range of potential evolution</th>
<th>based on</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pilot costs</td>
<td>personally operated SA</td>
<td>PATS, SkyCar</td>
</tr>
<tr>
<td>Air navigation charges</td>
<td>in FIRs under VFR...controlled airspace</td>
<td>EUROCONTROL</td>
</tr>
<tr>
<td>Airport charges</td>
<td>small airport...major airport</td>
<td>AIPs, survey</td>
</tr>
<tr>
<td>SA ownership</td>
<td>similar to luxury cars...to current SA</td>
<td>SATS, SkyCar</td>
</tr>
<tr>
<td>SA maintenance cost</td>
<td>advanced, more reliable materials...current SA</td>
<td>PATS, SkyCar</td>
</tr>
</tbody>
</table>

Table 2. The upper and the lower bounds of the uniform distributions assigned to the uncertain variables of the SA development.
tasks with enhanced cockpit instrument and advanced operational concepts such as the High Volume Operation at small airports. Therefore, with respect to the pilot costs, both professionally and personally operated (easy to fly) aircraft are considered.

At the same time, these flights could happen in controlled airspaces, or in Flight Information Regions (FIR) under Visual Flight Rules (VFR). Following the current experience of en-route charge payment defined by EUROCONTROL [186], this gives three average distributions: (i) from 0 to 6.22 (Euro Cents / seat-km), (ii) from 0 to 9.15 (Euro Cents / seat-km) and finally (iii) from 0 to 10.47 Euro Cents / seat-km, respectively for the pistons, turboprops and jets.

As for the airport charges, depending on airport congestions and SA preferences, the following operations are probable:

- between major airports: to take the benefits of better connectivity,
- between from small airports: to bring into play the lower costs,
- mixed operation.

Since for the last, the ratio of the major and small airport preference is unknown, finally this distribution covers all potential operations with equal probability. According to the cost estimation in the chapter 5.2., this results in three different range of potential values, respectively for the pistons, turboprops and jets.

Additionally, as other investigations show, SA could be defined to follow the past evolution of maintenance and ownership costs, or to be closer to the level of luxury cars.

<table>
<thead>
<tr>
<th>variable</th>
<th>potential values</th>
<th>type of PDF</th>
<th>units</th>
<th>based on</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pilot costs</td>
<td>0.805/0.23/0.2^*</td>
<td>UNIFORM</td>
<td>Euro Cents/seat-km</td>
<td>survey</td>
</tr>
<tr>
<td>Air navigation charges</td>
<td>0.622/0.915/0.1047^*</td>
<td>UNIFORM</td>
<td>Euro Cents/seat-km</td>
<td>EUROCONTROL</td>
</tr>
<tr>
<td>Airport charges</td>
<td>4.67...7.88/9.2...14.32/9.4...14.4^*</td>
<td>UNIFORM</td>
<td>Euro Cents/seat-km</td>
<td>AIPs, survey</td>
</tr>
<tr>
<td>SA ownership</td>
<td>0.96/1.59/0.44/5.54/0.4/9.51^*</td>
<td>UNIFORM</td>
<td>Euro Cents/seat-km</td>
<td>SATS, SkyCar</td>
</tr>
<tr>
<td>SA maintenance cost evolution</td>
<td>-3...18...3</td>
<td>UNIFORM</td>
<td>% / year</td>
<td>SATS, PATS</td>
</tr>
<tr>
<td>SA fuel consumption evolution</td>
<td>-11.7...-2.8...6.1</td>
<td>NORMAL</td>
<td>% / year</td>
<td>survey</td>
</tr>
<tr>
<td>Car ownership cost evolution</td>
<td>-1.8...0.8...0.2</td>
<td>NORMAL</td>
<td>% / year</td>
<td>INSEE, CCFA</td>
</tr>
<tr>
<td>Car fuel consumption evolution</td>
<td>-0.7...0.4...0.9</td>
<td>NORMAL</td>
<td>% / year</td>
<td>INSEE, CCFA</td>
</tr>
<tr>
<td>Car maintenance cost evolution</td>
<td>0.6...1.8...3</td>
<td>NORMAL</td>
<td>% / year</td>
<td>INSEE, CCFA</td>
</tr>
<tr>
<td>Other costs evolution</td>
<td>-0.4...0.0...04</td>
<td>NORMAL</td>
<td>% / year</td>
<td>Eurostat</td>
</tr>
<tr>
<td>Fuel price evolution</td>
<td>-1.3...4.1...9.5</td>
<td>TRIANGULAR</td>
<td>% / year</td>
<td>INSEE, US DOE</td>
</tr>
<tr>
<td>GDP per capita evolution</td>
<td>1.6...2.43...3.6</td>
<td>TRIANGULAR</td>
<td>% / year</td>
<td>Eurostat</td>
</tr>
</tbody>
</table>

Table 3. The applied probability density functions of uncertain variables. (* respectively, for piston, turboprop and jet small aircraft, assuming the average flight hours and block distance of the chapter 5.2.).

This last assumption is reasonable, since both the SATS and SkyCar projects (see chapter 2.1.), showed the evidence for the cost decrease possibilities with the use of more reliable materials and the newest technological achievements (e.g. MEMS, nanotechnology) in numerous domains, for example in aerodynamic or propulsion system design (see chapter 2.2.). Therefore, as the Table 2. indicates, the maintenance and ownership costs are assumed to range from their 2006 value to the level of luxury cars, estimated following the principle of the chapter 5.2.
As for the SA fuel consumption, car ownership, car fuel consumption, car maintenance and car other costs variables, a point estimate is available to project their future evolutions. Therefore, to scatter the potential values in proximity of the available data, a normal distribution is employed. To obtain the variance [202], confidence intervals are applied, computed by adding and subtracting past standard error units around the line of the point estimate. For this task, the following related works are considered:

- the results of this investigation (for the SA fuel consumption, since it covers more than 40 years of records),
- the documents of the French National Institute for Statistics and Economic Studies (INSEE) [168] to support the personal car ownership, fuel consumption and maintenance estimations with statistical databases over 15 years),
- the EUROSTAT publications [203, 204], since (due to lack of data) the car other costs are assumed to follow the inflation rate.

Finally, to deal with the fuel price and GDP per capita variables, the WORLD BANK [205, 206], INSEE [168] and U.S. DOT [207] records are used, since these provide all minimal, maximal and most likely future evolution values. According to the practice in the selection of the PDFs [208] this enabled to assign a triangular distribution [208], as it is presented in the Table 3.

5.4. Sensitivity analysis and validation

Mathematical models are in general defined by a series of equations, input elements, parameters, and variables aimed to characterize the process being investigated. Therefore, the result of the model is usually a source to uncertainties, due to the lack of initial data and the choice of the key elements. Seeing as this imposes a limit to the confidence related to the output – especially for decision making – the analyst should demonstrate the relevance of the model, which calls for a sensitivity analysis and a validation.

Sensitivity analysis is a process that varies the model input parameters over a reasonable range and observes the relative change in response [209]. By doing so, the main purpose is to estimate the sensitivity of one model parameter relative to another. Sensitivity analysis is also beneficial in determining the direction of future data collection activities. For instance the parameter for which the model is sensitive (meaning that it drives a relatively large change in the result) might call for accurate determination or detailed characterization. On the other hand a model insensitive data requires the analyst to assess its possible reasons or even revise the model with updated parameters. For that reason, as the figure 51. indicates, the sensitivity analysis is a valuable tool for model building and evaluation.

There are several possible procedures to perform sensitivity analysis [210]. Among others (e.g. analytical [211, 212], computer algebra based [213, 214]), this investigation selected the most common, the Monte Carlo sampling-based method due to its ability to incorporate the probability density functions of the independent variables [210]. Similarly to MCS, this sensitivity analysis consists of the following steps [215]:

- specify the target function and select the input of interest,
- assign a distribution function to the selected variables,
- generate the matrix $M$ of inputs with the appropriate distributions,
- evaluate the model and estimate the distribution of the dependent variable,
- assess the influence and relative importance of the input elements,

\[
\begin{pmatrix}
Z_1^{(1)} & Z_2^{(1)} & \ldots & Z_r^{(1)} \\
Z_1^{(2)} & Z_2^{(2)} & \ldots & Z_r^{(2)} \\
\vdots & \vdots & \ddots & \vdots \\
Z_1^{(N-1)} & Z_2^{(N-1)} & \ldots & Z_r^{(N-1)} \\
Z_1^{(N)} & Z_2^{(N)} & \ldots & Z_r^{(N)}
\end{pmatrix}
\]

(5.20)

To visualize the \(N\) values of the inputs \(Z_i\) against the output \(Y\), scatterplots are used, due to their advantage if the parameter relationships are supposed to be non-linear. Once plotted on a common scale for \(Y\), these enable to visualize the sensitiveness of the parameters. For example, if the two variables are strongly related, then the data points should form a systematic shape (e.g. a straight line or a clear curve). On the other hand, figures representing rather a uniform could of points over the range of the input parameter is the evidence for the fact that the factor is less – or not – influential.

However, in a model with numerous parameters, the extensive treatment of the sensitivity analysis is considered to be complex. For this reason, only a limited example of the whole investigation is given here with those parameters that have the most and the least influence on the system behaviour. Using the input variables \(Z_i\) as the airport cost, GDP, fuel price and car ownership cost (to represent the travelling budget constraint) the model response to the sensitivity analysis of piston aircraft is given in the Figure 52. This shows that on average the relationships between the \(Y\) and \(Z_i\) are straight lines. The signs are reasonable, since as expected, the SA accessibility rises with GDP and car ownership growth, while it falls off by the increasing cost of the airport charges and fuel price. As the slope of the lines indicates, it is clear that \(Y\) is more sensitive to the GDP, and less responsive to the car ownership. Additionally, since none of the input variables form a
uniform cloud of points, I found the evidence that all variables should be retained in the model, and none of them can be safely ignored in the analysis [27].

On the other hand, using different scales to represent the results (see Figure 53.), one could also notice the evidence for the non-linear relationship between \( Y \) and \( Z \), as well as the distribution of the results, caused by the PDF of the input variables. Due to the characteristics of the mathematical model, these all together justify a reasonable method.

While (according to Sargent [216]) the sensitivity analysis is an operational validation method, its results only show evidence for sensitive parameter application, since – due to lack of data – it cannot be justified in the real system. Consequently, the results and the model itself necessitate a validation.

Generally, numerous approaches are distinguished for the validation of a model under its set of assumptions [216]. For the SA prediction model, the operational validation is selected, since this is where most of the evaluations take place [217, 218]. As the name indicates, this determines whether the output behaviour has the accuracy within the acceptable range for the intended purpose and end-use. To verify this, the historical data validation technique is used, since other methods – such as event validity or out-of-sample prediction accuracy – are powerless (due to the limited data) in the context of SA.

To perform the validation, this investigation uses the EUROCONTROL CFMU data [30], as historical accessibility values are unknown and real European SA ownership records are limited. On the other hand, since traffic records are not equal with accessibility values, the scale of the modelled and real data is different. To represent these in the same figure, it is further assumed that over a limited timescale (which due to the available data is taken between 1999 and 2004) there is a fixed ratio between them, and finally both are estimated in proportional evolutions relative to the validation origin.

As the Figure 54. indicates, although the model essentially follows the real data, the average difference is 6%. However, by analysing the results, I revealed the following potential reasons for the dissimilarities:
- the problems related to the real data selection, since average values are considered in the investigation that might not suitably represent the effect of seasonality,
- the meteorological constraints and the airspace congestions, which even with available SA limit the real traffic considered in CFMU,
- the model reaction time due to over and above estimations at the simulation horizon 2000 and 2003,
- the unrealistic assumption that a fix ratio between the real and the modelled data holds over the simulation horizon (as the divergent results enable to suppose between 2001 and 2003).

Nevertheless, the available statistical data do not permit to explore which of these has the most influencing or “polluting” consequence on the result. Considering this and the potential reasons listed above as limiting factors to the accuracy of the validation method proposed here, the difference might be acceptable in the forecast community, especially in the context of SA. Therefore, with the results of the sensitivity analysis and the validation, I found the evidence for the relevance of the model that employs reasonable and sensitive parameters.

**Fig. 54.** The model validation with historical data.
6. Results and conclusion

6.1. The total operating costs of small aircraft and the travelling budget constraints in 2006

According to the chapter 5.2., the total operating cost estimations cover numerous aircraft with different size and propulsion systems. From the most frequently used aircraft, 31 had available data and therefore these were considered, including best-selling single engine pistons and multi-engine jets.

Using the model variables and the calculation method mentioned previously, the model responses to the TOC estimations (at 2006) are given in the Figure 55., where the colours are associated to different propulsion systems; blue stands for the pistons, grey represents the turboprops and red symbolizes the jets. Otherwise, on the x axes, the small aircraft types are visualized (those considered in this investigation), classed in rising Maximum Take-Off Weight (MTOW). The TOC of the y axes is given in a EUR (at the 2006 level) per block basis, which once again accounts for all costs occurred between the moments when the aircraft leaves the gate of departure, till it arrives to the gate of arrival. According to the figure, while in general the TOCs are increasing gradually from (the minimal) 156.2 EUR to (the maximal) 1154.9 EUR, the shift in the propulsion system and the growth of the MTOW causes a radical alteration. Relative to the mean value of the pistons (311.5), this gives an additional cost of 241.6 % for the turboprops and 365 % in case of the jets (for the relatively short block distance considered in this investigation).

However, as the aircraft capacity varies with the MTOW, a reasonable comparison should rather be based on a cost per seat or cost per seat-km unit, as presented in the Figure 56. As it could be expected knowing the block distance considered, the pistons still offer the floor price with a minimal value of 0.258 EUR per seat-km. This is followed by the turboprops and the jets, reaching a mean of 0.602 and 0.799 EUR / seat-km respectively.

On the other hand, this figure reveals several irregularities. For instance, the Cessna 170, Beech Bonanza 33 and the Beech Baron 55 are relatively more expensive than one might suppose after the MTOW and the applied propulsion system. In case of the Baron 55, this difference is even 6.5 % higher, than the mean value of the turboprops. However, this TOC estimation is reasonable, since in general the Beech aircraft and especially the Baron 55 has always been near to the top of the piston airplane hierarchy [219]. As for the Cessna 170, this effect should be caused by the relatively old production period (between 1948 and 1956), which accounts for higher fuel consumption and maintenance costs [220].
Fig. 55. The total operating cost of small aircraft in 2006 (in EUR / block) (note: blue represents the pistons, red stands for the turboprops and grey symbolizes the jets).
Fig. 56. The unit total operating cost of small aircraft in 2006 (in EUR per seat-km) (note: blue represents the pistons, red stands for the turboprops and grey symbolizes the jets).
Therefore, to provide relevant results at the simulation origin (2006), only the most recent piston aircraft are considered, which gives the mean value as 0.274 EUR / seat-km.

Otherwise, the average contribution of the cost elements to the TOC is given in the Figure 57. Accordingly, with respect to piston SA, the pilot cost and air navigation charges are the most important parameters, reaching together 52 %. Then, with 20 % the airport cost is the third most influencing factor, followed by the fuel and maintenance elements with 10 and 9 % correspondingly. Finally, the role of the ownership and insurance cost is limited, covering barely 6 and 3 % respectively. As a result, the simulation deals with the three most weighting factors, and the TOC of the piston SA is expected to change significantly.

On the other hand, this might not be the case for the turboprops and jets, since the pilot, navigation and airport elements are less significant, not more than 33 % and 26 % in total. However, with 46 and 49 %, turboprops and jets are highly influenced by the fuel cost, which – knowing the block distance considered and the specific aircraft fuel consumptions – is reasonable. Otherwise, due to the higher list price (relative to piston SA), the ownerships are increased to 9 % in case of the turboprops, and 12 % to represent the jets. According to the TOC estimation method, this also influenced the insurance ratios, reaching 4 and 6 %

Fig. 57. The average contribution of the cost elements to the TOC of SA in 2006.

Fig. 58. The travelling budget constraints (TB) versus the unit total operating cost of Small Aircraft (SA) in 2006 (note: grey represents the cars, red indicates the SA).
respectively. As a result, unlike pistons, the TOC of the turboprops and jets are mainly driven by the fuel consumption and the fuel price. While for these two variables the simulation of this investigation accounts for a limited change, the TOC modification of both (turboprops and jets) is expected to be less than in case of the pistons.

After the review of the results related to small aircraft, the model outcome for the travelling budget constraints (TB) is presented in the Figure 58. Accordingly, with 0.074, 0.099 and 0.151 EUR / seat-km, the low, lower-mid and upper-mid segments cannot meet the cost of any small aircraft in 2006. However, the high class division offers even a superior budget (0.285 EUR / seat-km) than the unit TOC of piston small aircraft (0.274 EUR / seat-km). Even so, this value is only a fractional part of other SA’s costs, 47.3% of the level associated to the turboprops and 35.6% relative to the jets. Knowing these results and the market shares associated to each segment, one could suppose that over the simulation period, pistons would offer the highest accessibility. At the same time, since turboprops and jets are mainly fuel cost dependent, their unit TOC shift towards in the vicinity of even the high class segment is questionable.

6.2. The probabilistic evolution of the total operating costs of small aircraft and the travelling budget constraints

By running the MCS, the evolution of the independent variables over the simulation horizon is given with two dimensional probability distributions. The unit TOC development of the piston SA is represented in the Figure 59., while the turboprops and jets are given in the Figure 60., all expressed in constant 2006 EUR / seat-km (x axes), the probability (z axes) and the simulation horizon (y axes) [26]. As both indicate, the probability of reaching one particular result is decreasing gradually over the simulation horizon, since the uncertainties are raising that finally growth the range of the potential results. This also demonstrates the importance of a simulation that associates a probability to the outcome, otherwise the results would be less interpretative, like predictions based on scenarios or point estimates.
Additionally, the plot of the unit total operating costs for one particular simulation step gives traditional histograms, which enables further data analysis. By doing so for 2020 (which might be the most motivating to explore), this results in the Figure 61. [29]. As for the pistons, while the minimal and maximal value is 0.095 and 0.4642 respectively, the most probable unit TOC is 0.2007 EUR / seat-km. With 63.7 %, the majority of the results are in the vicinity of this value, located exactly in the range of 0.17 and 0.23 EUR / seat-km. On the other hand, the Figure 61. also indicates that less than one percent (0.8 %) of the outcomes are more than the mean value of 2006 (0.274 EUR / seat-km) and that the overall unit TOC decrease along the whole simulation horizon is 26.78 %.

As for the turboprops and jets, the most probable outcomes are found to be 0.453 and 0.5996 EUR / seat-km respectively. These are consistent to the characteristics of the simulation and the definition of the PDFs, since each means an overall unit TOC decrease of about 25 %. Otherwise, compared to the pistons, the turboprops and jets offer a wider range of potential values, meaning a minimum of 0.214, 0.269 and a maximum of 3.14, 4.32 EUR / seat-km respectively. According to the Figure 61. these variances are not in balance, as the probability associated to higher values than the most probable results are 0.65 for the turboprops and 0.63 with respect to the jets. Additionally, these aircraft indicate 43 % and 42 % of chance that the unit TOC in 2020 remains the same or even gets higher than it is in 2006.
However the three type of SA considered in this investigation shows numerous dissimilarities in their unit TOC evolution, the results are consistent to the characteristics of the aircraft. For instance, the Figure 60. reflects similarities in the unit TOC evolution of both turboprops and jets, which is reasonable since the difference in their cost elements’ contribution to the TOC is insignificant (see Figure 57.).
With respect to the probabilistic prediction of the travelling budget constraints, it is given in the Figure 62. (relative to the unit TOC of piston SA to offer a curve of reference). This shows that on average the travelling budget constraints are increasing over the simulation horizon. The spread of the potential results in 2020 are different for the four segments considered, within the high class varies the most, from 0.2357 to 0.3918 EUR / seat-km. The relative growths to 2006 are rational, since those reflect the differences in the price and income sensitiveness of the different market segments. Accordingly, the low class section modified the most, 17.4 %, followed by the lower-mid with 14.7 %, the upper-mid by 5.29 % and the high class with a limited 0.59 %. These results are also consistent with the investigations of Zahavi [162], as all segments show a positive influence to the GDP increase (that is the key determining factor of the TMB). On the other hand, the most probable values in 2020 for the low-, lower-mid-, upper-mid-, and high class segments are 0.0869, 0.1136, 0.159, 0.2867 EUR / seat-km respectively. Relative to their 2006 rate, these evolutions also modify the differences between the unit TOC of SA and the travelling budget constraints. Unlike in 2006, when piston SA is in the vicinity of the high class segment (see Figure 58.), in 2020 the difference between the two curves reach 30 %, and piston SA gets rather closer to the upper mid class (with 26.2 % of higher value). While the likelihood that the piston SA offers at least the same price than the one associated to the upper-mid class is 0.13, other possibilities (e.g. to reach the cost of lower-mid class) happen with a probability smaller than 0.01.

As for the turboprops and jets, their most probable unit TOC remains 58.07 and 109.1 % higher than the travelling budget constraint associated to the high class segment.
The chances to arrive at least to the same level (as the high class segment) occur with the limited probabilities of 0.034 and 0.015 respectively.

Seeing these modifications in the travelling budget constraints and the unit TOCs, one would expect that between the SA considered in this investigation, the accessibility of the pistons would change the most. Contrariwise, the shift in the accessibility of turboprops and jets is expected to be limited.

### 6.3. The probabilistic evolution of small aircraft accessibility

Knowing the travelling budget constraints and the unit TOC evolution of all piston, turboprop and jet SA, the question at this point is the SA accessibility.

Similarly to the previous chapter, the model response to the dependent variables is given with two dimensional probability distributions, all expressed in the simulation horizon (y axes), the probability (z axes) and the SA accessibility per population ratio (x axes). While the Figure 63 represents the results of the piston SA [26, 28], a comparative outcome of all aircraft category (considered in this work) is offered in the Figure 64. As each indicates (similarly to the unit TOC results), the shape of the accessibility curves along the simulation horizon reflects the uncertainty propagation along the MCS: (i) a widening range of the potential outcomes and a (ii) decreasing probability to reach one particular value.
Fig. 64. The probabilistic evolution of small aircraft accessibility (note: number 1: piston, number 2: turboprop, number 3: jet).
According to the Figure 63., the accessibility of piston small aircraft shows the most important modification, from 6 % (at the beginning of the simulation) to more than 11 % in 2020. While the initial accessibility is 1.72 and 1.12 % with respect to turboprops and jets, these cannot offer but a limited variation. On the other hand, seeing that the piston small aircraft has the leading change in the unit TOC, these results are consistent to the simulation, since finally (as expected) the piston SA meets the highest accessibility, before the turboprops and the jets.

However the Figure 63. and 64. give an insight to the model response, a probabilistic treatment of the results calls for histograms, which – similarly to the analyze of the unit TOC – are expressed for 2020. As the Figure 65. represents [29], the potential results of the piston SA ranges from 4.3 to 44.4 %. Its most frequent value is 11.67 %, meaning that
in 2020, for 11.67 % of the population SA becomes accessible, or available in terms of costs. The majority of the outcomes are also located in the vicinity of this rate, exactly 47.3 % between 9 and 13 % of accessibility. Otherwise, the distribution follows rather a log-normal shape, since relative to the most probable; the higher values are attributed more frequently than the lowers. In that manner, while 50 % of growth according to the most probable results showed the probability of 0.25, the 50 % lower ratios are happening in barely 1 % of the results.

As for the turboprop and jet small aircraft, their most frequent accessibility in 2020 is at 2.35 and 1.57 % respectively. These give the overall growths to the vicinity of 37 % for each, which is reasonable, as they reflect the unit TOC evolutions and the market shares associated. Otherwise, similarly to piston SA, both turboprops and jets represent a distribution where higher values – relative to the most frequent result – are more often to happen. Accordingly, the accessibilities with at least 50 % more than the most probable outcome, still happen in 38.9 and 18.3 % of the total results. Additionally, the turboprops might even reach the 11.67 % of accessibility related to the pistons, but only with the limited probability of 0.001.

6.4. Conclusion

The importance of a specific method that replies to the context of the European small aircraft was already mentioned in the previous chapters. On the other hand, the literature review in 2006 showed that none of the investigations with small aircraft prediction in mind focused on the European context. The application or adaptation of the generally used methods in air transportation was not promising either, due to numerous drawbacks. Firstly, the lack of adequate statistical data to support the computation of the coefficients, and secondly their linear relationship between the dependent and independent variables that – according to Dargay [151] – might not fully reflect the non-linear attribute of the technology adoption.

Therefore, to reply to the specific requirements, and to cope with uncertainties, the proposed solution is based on an analogical approach, using an S-curve form the innovation diffusion theory. As a result, the developed model is based on the total operating cost evolution of both small aircraft and cars from different market segments. Using such approach over other techniques, this investigation found the evidences for the following model advantages:

- considers personal cars, which might become similar to small aircraft,
- offers a model structure does not suffer of the problem of gathering adequate statistical records,
- provides a country specific approach that enables to consider the differences in the European transportation characteristics and socioeconomic data (see Figure 43. and 44.),
- gives a non-linear model, which reflects numerous technology adoption phases and the dissimilarities in the price and income sensitiveness of different social classes (see Figure 45.),
- accounts for a saturation level where the demand reaches its maximum value regardless of the independent variables (unlike the generally used prediction methods in air transportation).
To deal with uncertainties in the estimation of the total operating costs, this investigation selected the Monte Carlo Simulation. The primary justification of this – over other methods – lies in the following benefits:

- probabilistic approach that is capable to handle the variables with a range of potential value (e.g. pilot cost of small aircraft),
- applies probability statistics and random numbers: model results are given with probabilities (unlike predictions based on scenarios or point estimates),
- can support decision making and risk analysis (e.g. analysis of the impact of small aircraft on ATM).

Seeing that the choice of the approach and the selection of the independent variables might be a source to uncertainties – that could pose a limit to the confidence related to the output – the relevance of the model was also demonstrated with sensitivity analysis and validation. Accordingly, the investigation showed the evidence for the fact that none of the model parameters can be safely ignored in the analysis, since the scatter plots represented systematic shapes, and therefore all variables were strongly related (see Figure 52.). Additionally, the signs of the relationships (between the dependent and independent variables) are found to be reasonable, and they also follow a non-linear characteristic, as expected from the equation 5.12.

However the historical validation showed a difference between the real and modelled data, the variation remained within an acceptable range in the forecast community, and especially in the context of the European small aircraft. Therefore, the sensitivity analysis and the validation altogether showed the evidence for the relevance of the model, which is valid and employs reasonable parameters.

Finally, the model response to the Monte Carlo Simulation showed that a shift in the unit TOC of all types of small aircraft (piston, turboprop and jet) was possible. Accordingly, the following most probable results are found for 2020:

- 0.2007 EUR / seat-km to represent the pistons,
- 0.453 EUR / seat-km for the turboprops,
- 0.5996 EUR / seat-km with respect to the jets.

While the distribution of the outcomes in the vicinity of the above listed values was in balance for the pistons, turboprops and jets showed a probability of 0.65 and 0.63 respectively, to receive higher unit total operating costs. Additionally, these two aircraft indicated 43 % and 42 % of chance that the unit TOC in 2020 remained the same or even got higher than it was in 2006.

On the other hand, this investigation also demonstrated that the shift in the unit TOC and other independent variables could influence the small aircraft accessibility, which in 2020 gave the following most frequent values:

- 11.67 % in case of the pistons (94.5 % of growth relative to 2006)
- 2.35 % for the turboprops (increased by 36.6 % from 2006)
- 1.57 % to characterize the jets (40 % higher than it is in 2006)

Otherwise, the distribution of the results followed rather a log-normal shape, since relative to the most probable; the higher values were attributed more frequently than the lowers. In that manner, while 50 % of growth according to the most probable results
showed the probability of 0.25, 0.389, and 0.183 respectively for the pistons, turboprops and jets, the 50% lower ratios were happening in less than 1% of the model outcomes. The analysis of these values found, that the results were reasonable, as all reflected:

- the characteristics of each small aircraft,

- the contribution of the cost elements to the total operating costs (for example as one would expect after the relatively short flight distance, turboprops and jets were more expensive than the pistons, which also demonstrated the general practice that advanced propulsion systems and higher flight speeds are more reasonable – in terms of costs – at block distances above 600 km),

- the characteristics of the simulation and the definition of the probability density functions (e.g. the evolution of the independent variables and the range of potential values),

- the non-linear attribute of the price and income sensitiveness (the proportional modification of the unit TOCs and the accessibilities are unequal).

Altogether the results of the developed prediction model showed the evidence for the use of the analogical approach to assess the growth of small aircraft in Europe.
7. Summary and theses

7.1. Summary

As in 2007 the European small aircraft investigations are limited and therefore the available data is statistically inadequate to support further investigations in decision making and risk analysis (for example the impact of SA on ATM), our problem was to provide a long-term demand / prediction model in order to establish the background related to the (European) SA.

This problem first required to gather and analyze the available information on the past development phases and to name the potential future directions that small aircraft might take. As these records were limited or not accessible, the initial investigation covered a European small aircraft air traffic analysis. Using the arguments on numerous flight characteristics (for example the most frequently applied propulsion system, flight altitude, flight distance) this analysis enabled to restrict the area that had to be focused. Further tasks included the revision of the potential prediction / demand model structures to evaluate which methods were in use and how they could be applied to project the European small aircraft activity. The next task was to develop the prediction model. In this investigation the proposed solution covered an analogical approach using an S-curve from the innovation diffusion theory, since over other methods (based for example on regressions or the log-linear models), this enabled to address the uncertainties in the small aircraft development, and to reflect the characteristics of the European context (e.g. dissimilarities in the price and income sensitiveness of the different social classes and countries). As a final task, a Monte Carlo Simulation was defined, and the model relevance was demonstrated via a sensitivity analysis and a historical data validation technique.

This PhD dissertation was divided into seven chapters.

Chapter 1, the introduction gave the problem definition, actuality, the tasks to be solved and the proposed methodology.

Chapter 2 provided a literature review of the related works. As a first issue, the worldwide small aircraft concepts were discussed to understand the vehicle class being analyzed and to name the possible operational approaches (e.g. regional, extra- and intra-urban). Then the state-of-the-art addressed the short-, mid- and long-term technological challenges to demonstrate the feasibility of answering to the bottlenecks such as the total operating cost, the required pilot skills or the difficulty of the cockpit environment. This chapter also summarized the available SA in 2006 to support the model with initial aircraft characteristics, and finally introduced the potential demand and prediction methods.

As the relevant European investigations are found to be limited, and therefore the available data was statistically insufficient to support a model, the Chapter 3 illustrated the small aircraft traffic analysis, which finally provided the European arguments to limit the focus of the investigation.

Chapter 4 presented the methodology and modelling including the analysis of the generally used prediction / demand models, the development of the initial linear model and some more advanced methods based on the analogical approach and the innovation diffusion theory.

Chapter 5 introduced the proposed approach and the estimation of the model variables. It also coped with the problem of uncertainties with a Monte Carlo Simulation,
as over other methods, this might support further decision making and risk analysis with probability distributions. Using a sensitivity analysis and a historical data validation, this chapter showed the evidence for the relevance of the model.

Chapter 6 gave the results of the simulation, including the budget constraints, the small aircraft total operating cost and accessibility values along the whole simulation horizon.

Finally, the Chapter 7 gave the summary, the theses and the recommendations for the future works.

7.2. Future works and perspectives

Further investigations of the developed small aircraft prediction model could cover numerous areas. Firstly, instead of estimating the model parameters (e.g. GDP, budget constraints) with average numbers, one could apply the country specific values (as given in the Figure 43. and Figure 44.) and provide the model results to each European country that is considered in this investigation. This model enlargement would also enable to analyze the dissimilarities in the projected small aircraft accessibility, and to evaluate which countries are the most or the less concerned.

On the other hand, based on the model projections, further works could concentrate on the analysis of the impact of SA on the ATM, in order to assess how small aircraft might be integrated to the air transportation system and to study whether it would call for a change or demand enhancements (e.g. due to bottlenecks at airports surrounding: see Figure 38.). Based on the outcome of this last analysis, future investigations could also provide proposals and recommendations to further ATM or small aircraft development.

Finally, another area of interest might be to discover the potential small aircraft accessibility at other geographical regions, for example the Asia / Middle East or South America.

7.3. Contributions / theses

1. I discovered that the European small aircraft activity is driven by the country specific (socioeconomic and transportation) characteristics (e.g. the GDP, the budget spent on travelling) and the factors influencing the total operating cost of SA (see chapter 4.4.), like the technological developments in aircraft industry (e.g. the advanced aerodynamic design, novel / alternative propulsion systems, simplified cockpit environment, autonomous aircraft operation: see chapter 2.2.) [5, 29].

   1.1 The investigation demonstrates [5, 6, 135] that unlike between 1950 and 1995 when the technological developments in the small aircraft industry were limited and therefore considered as irrelevant factors to estimate the demand, in 2006 such developments are leading to a relatively inexpensive small aircraft (see chapter 2.1.) – similarly to the Small Aircraft Transportation System (see Figure 2.), the UK Jetpod (see Figure 4.), or the SkyCar (see Figure 7.) – and become a driving factor of the European SA activity.

   1.2 I found the evidences [5, 6] for the fact that using the technological achievements in automation [9], cockpit developments (e.g. weather information [16], synthetic vision systems: see Figure 13., Figure 21.) and the first operation concepts (e.g. High Volume Operations [63]), small aircraft
pilots could meet an operational environment that is closer to the level of personal automobiles, and therefore it require less skills (see chapter 2.2.).

1.3 The analysis showed that the small aircraft activity is also a factor of the budget constraint, as is fix proportion of income devoted for travelling (see chapter 4.4., chapter 5.1.) [26, 27, 28, 29].

1.4 This investigation revealed [8] that the driving factors of the European small aircraft activity are different from those in the United States, since Europe (i) has a higher population density, (ii) a high speed train network, (iii) consist of numerous countries with different transportation characteristics / socioeconomic data (see Figure 43. and Figure 44.), and (iv) at the same level of GDP its population travels about three times less than the North-Americans (see chapter 2.1., chapter 2.4. and chapter 4.1.).

2. I found that the enlargement of European small aircraft traffic could cause conflict situations with the conventional air traffic at the vicinities of the major European airports (see Figure 37. and Figure 38.) [11, 12, 133, 135].

2.1 Using the FAA weight category for small aircraft and the real flight data from CFMU (see chapter 3.1.), I estimated the characteristics – such as the number of flights a day (see Figure 29.), the flight distance (see Figure 32.), the flight altitude (see Figure 33.), the propulsion technology distribution (see Figure 34.) – and the most frequently used geographical areas of the European small aircraft traffic (see Figure 30. and Figure 31.) [11, 12, 133, 135].

2.2 As a criteria for the safety of air navigation, I defined the Flight Level, where both small aircraft and conventional air traffic has a minimal impact on the other (see chapter 3.4.) [135].

2.3 Applying the safety criteria within both small aircraft and conventional flights had a minimal influence on each other, I assessed the potential region of conflicts due to airspace use, which found to be the range of 130 km at the airport surroundings (see Figure 38.) [133, 135].

3. I discovered that the conventional demand / prediction models are powerless to be employed for projecting the European small aircraft development (see chapter 4.1.) [111].

3.1 I found [8] that the available US small aircraft demand models are irrelevant with respect to the European context, since Europe – unlike the US – (i) covers numerous countries with different socioeconomic characteristics (see Figure 44.) and transportation habits (see Figure 43.), (ii) offers a high speed train network that might become an alternative to SA, and (iii) has specific geographical conditions with smaller distances between the major cities that might result in the application of different propulsion systems (see chapter 4.1.).

3.2 I demonstrated [111] that it is not reasonable to apply the conventional demand / prediction models of the air transportation (e.g. log-linear) to the European small aircraft (see chapter 4.1.), since these methods require the past data on the activity to predict, which in case of the European SA is limited – if not non-existent – while the available information is insufficient to capture the relevant small aircraft driving factors (e.g. the required pilot skills, the value of simplified cockpit environment, or the shift in the total operating cost).
3.3 I revealed [111] that the potential application of the linear methods (or those that provide linear elasticities) are limited in the context of the European small aircraft, as these could not reflect the non-linear market adoption phases (see Figure 40.) and the dissimilarities in the price and income sensitiveness of the different social classes (see Figure 45.).

4. Following the analysis of the small aircraft stochastic adoption process, I introduced a Markov diffusion model, which – after statistical linearization – resulted in an initial prediction model (see chapter 4.2.) [11, 12, 111, 135].

4.1 Following the progress of the small aircraft activity, I determinate the independent variables of the initial SA prediction model (see Figure 39.) [111, 135].

4.2 I demonstrated [111, 135] that the initial prediction model might be based on two linear matrixes within A could define the internal dynamics of the system, and B represented the outer (control) influence (see equation 4.8.).

4.3 Using several scenarios (for the independent variables) and numerous potential matrix coefficients, the evidence for the initial prediction model applicability was found (see chapter 4.2.) [11, 12].

4.4 I found [111, 135] that the major advantage of the initial linear approach lies in the potentialities of using independent variables with a predefined role / weight, and that the model could assist to analyze the internal dynamics of the underlying mechanism (such as the effect of regulation or technological achievements on the small aircraft development) (see chapter 4.2.).

4.5 I revealed [111] that due to the limited available data on the European small aircraft activity, the major drawback of the initial prediction model is the determination of the matrix coefficients (see chapter 4.2.).

5. Using an S-curve form the innovation diffusion theory to project the personal car ownership development as an analogical approach, I established a general non-linear small aircraft prediction model for the European context (see Figure 46. and equation 5.11.) [26, 27, 28, 29].

5.1 For the applicability of the developed non-linear SA prediction model, I established a method to estimate the total operating cost of small aircraft (see chapter 5.2.) [26, 29].

5.2 The parameters of the non-linear small aircraft prediction model have been examined with a sensitivity analysis in order to prove that all elements are sensitive and reasonable to use (see Figure 52. and Figure 53.) [27].

5.3 The results of the small aircraft prediction model were validated via a historical data validation technique that gave an error rate of about 6 % in a five year period (see Figure 54.).

5.4 I demonstrated [28, 29] the applicability of the developed SA prediction model with a Monte Carlo Simulation, since this is capable to represent the unclear parameters with probability distributions and therefore quantitatively address the uncertainties in the system (see Table 2. and Table 3.).
Appendix A: The technical characteristics of the selected small aircraft (as of 2006)
List of publications

Refereed journal in foreign language (published in Hungary):


Refereed journal in Hungarian (published in Hungary):


Refereed international conference in foreign language:


Non-refereed international conference in foreign language:


Non-refereed local conference in foreign language:


Non-refereed research report:


References


https://www-local.eurocontrol.fr/research/shorttermsupport/dancefolder/index.html
(20/04/2007)


41. Peakin, W.: “Flying will be as easy as driving a car”. The Observer, February 25, 2001.


185. PlanQuest.com: “Operating Costs”


188. L’Union des Aéroports Français: “Les aéroports Français”
http://www.aeroport.fr/ (03/04/2006)


191. Mackey, B.: ”Insurability and Cost, Part Two: Pilots”. EAA Aircraft Insurance Plan (Falcon Insurance Agency)


194. Manufacturers recommended maintenance schedule
http://www.edmunds.com/maintenance/MaintenanceServlet (29/05/2006)

195. Vehicle Certification Agency, United Kingdom
http://www.vcacarfueldata.org.uk/index.asp (30/05/2006)


www.mathworks.com (23/04/2007)


200. Decisioneering: “What is Monte Carlo Simulation?”
201. Wittwer, J.W., “Monte Carlo Simulation Basics” From Vertex42.com  
http://vertex42.com/ExcelArticles/mc/MonteCarloSimulation.html (01/06/2004)


204. Statistical Office of the European Communities (Eurostat), Economy and Finance, Sustainable Development Indicators: “Inflation rate”. EUROSTAT.  


