Budapest University of Technology and Economics
Department of Applied Mechanics

Booklet of PhD Thesis

Stability of human-driven nonholonomic systems
– Skateboarding and Driving –

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Introduction

Nowadays, human drivers are still integral parts of our transportation systems, thus, to enhance safety on the roads (more than 1.25 million people died in traffic accidents in 2013) the understanding of the human-vehicle interaction is inevitable. Although the relevance of the topic seems to be less important than the development of other technological parts of vehicles, like for example the implementation of driver assistance systems, but intensive research on human-vehicle interaction can also lead to the better understanding of the human control. The results on the field can be useful to design better algorithms even for autonomous driving beyond the medical applications which can improve the human balancing models, for example. Moreover, a proper human driver model provides an opportunity to test prototype vehicles without the presence of humans in dangerous situations. It may also help to reduce the number of vehicle recall due to inadequate tests, in which usually professional test drivers are involved, on the contrary most of the road vehicles are driven by nonprofessionals.

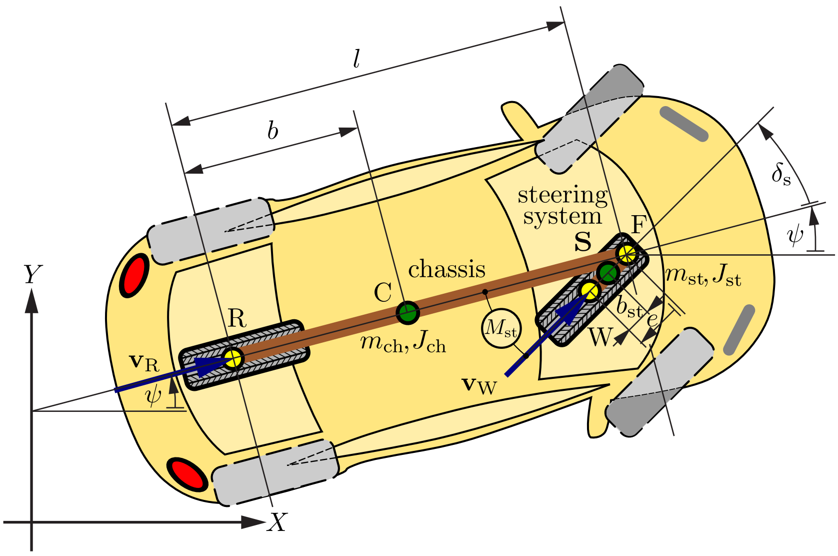


Figure 1. The single track mechanical model of the vehicle.

According to this, the aim of this thesis is to understand the human-vehicle interaction through simple nonholonomic (containing kinematic constraints) mechanical models. To pointing out the effect of some system parameters on the dynamical properties, analytical investigations are favorable, so as simple as possible models are welcomed. Hence, to model a passenger car, a single track or so-called bicycle model is used in our study with the assumption of perfectly rolling wheels that also eliminates the uncertainty parameters of the tires. To deeply understand the effect of the reflex delay in the human-vehicle interaction a simple skateboard model is also investigated in the thesis.

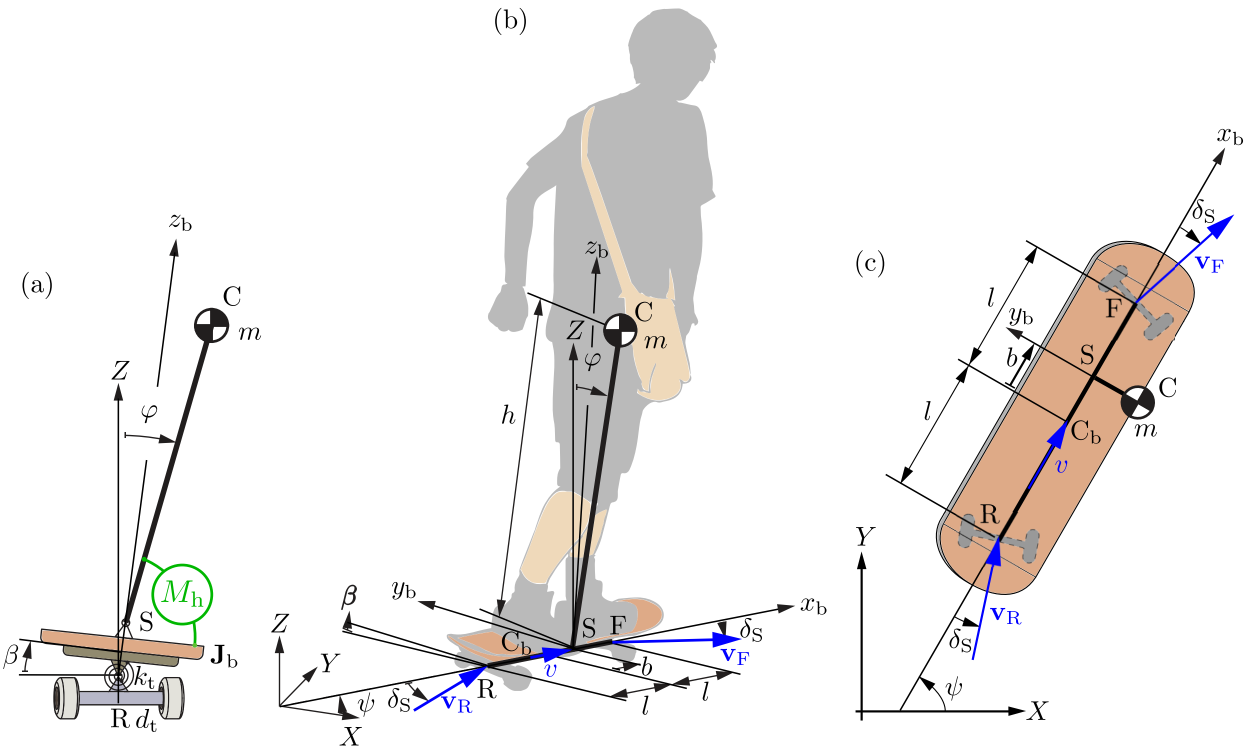


Figure 2. Mechanical model of the skateboard-skater system; panel (a) shows the rear view of the system, panel (b) shows an isometric one, while panel (c) shows the top one.

First, a nonlinear analysis of a single track vehicle model (see Fig. 1.) is performed, and the results are presented with the help of case studies on American type shopping carts, passenger cars and harvesters. The importance of the critical speed, where the rectilinear motion of the vehicle turns to be linearly unstable, is highlighted in terms of the caster length of the steered wheel. With the help of a simplified model, the positive and negative effects of acceleration and breaking are discovered. In addition to the detected critical level of deceleration, it is also shown that the arising unstable turning motion around the straight breaking maneuver makes the handling even more challenging.

Skateboard stability was also investigated with special focus on the skater's reaction delay. Based on this research, the mechanical model of skateboards (see Fig. 2.) can be simplified, and the suspension stiffness can be tuned to be optimal from stability point of view. Moreover, identified unsafe speed zones explain the seemingly unpredictable falls during an accelerating maneuver.

Main Results

Thesis Statement 1

Based on the in-plane bicycle model, where ideal rolling of the rigid wheels is considered, and the driver handling effort is modeled by a linear PD controller produced internal torque between the chassis and the steered wheel having trail, the following statements hold:

(a) **The forward motion of a front wheel steered automobile is globally stable, while, due to the presence of an unstable limit cycle, reverse motion is just locally stable up to the critical speed**

$$v_{\mathrm{cr}} := -\frac{k_{\mathrm{d}}(l-e)^2}{lJ_{\mathrm{st}}+eJ_{\mathrm{ch}}+m_{\mathrm{ch}}be(b+e)}\,,$$

where m_{ch} and J_{ch} are the mass and mass moment of inertia of the chassis, respectively, b characterizes the position of the center of mass, J_{st} stands for the mass moment of inertia of the steering mechanism with control gain (or torsional damping) k_{d}, while $e>0$ represents the caster length (trail) of the steered wheels.

(b) **By increasing the caster length ($|e|$) of the wheels, in case of rear wheel steering ($e<0$), the subcritical Hopf bifurcation of the forward rectilinear motion ($v<0$) at the critical speed v_{cr} becomes a supercritical one, and the loss of stability is signaled by appearance of small amplitude self-excited vibrations. For further increasing of the caster length, the loss of stability results in a turning motion due to the subcritical saddle-node bifurcation.**

Related publications: (7)

Thesis Statement 2

The stability of a uniform accelerating maneuver can be investigated with the help of a simplified single track vehicle model, where the steering effort of the driver is modeled by a linear PD controller. Based on the analysis of the arising explicitly time dependent system with the conventional approach (frozen-time eigenvalues method), with investigations of the analytical solution and the Lyapunov function, and with numerical simulations, the following statement can be proved:

The accelerating maneuver is globally stable for any acceleration if the longitudinal speed v is greater than

$$v_{\mathrm{cr}} := -\frac{lk_{\mathrm{d}}}{J_{\mathrm{st}}}\,,$$

where k_{d} is the differential gain in the control system of the steering, l is the wheelbase of the vehicle and J_{st} is the mass moment of inertia of the steering mechanism. Due to the coexisting unstable turning motions, locally stable breaking can be achieved for $v>v_{\mathrm{cr}}$ if the deceleration is less than

$$a_{\mathrm{cr}} := \frac{lk_{\mathrm{p}}}{J_{\mathrm{st}}}\,,$$

where k_p is the proportional gain.
Related publications: (7)

Thesis Statement 3

A simple mechanical model of skateboarding can be composed, in which the skater is modeled via a lumped mass on a rigid bar connecting to the rigid massless skateboard by a hinge. Pure rolling of the wheel-sets can be considered, and the balancing effort of the skater can be taken into account by a linear PD controller with reflex delay. Based on this mechanical model of the skater-skateboard system, the following statements can be established:

(a) **To achieve stability, the skater reaction delay τ must be in the range**

$$\sqrt{\frac{h}{g}}\max\left(0; U-\sqrt{2-U^2}\right)<\tau<\tau_\mathrm{cr}:=\sqrt{\frac{h}{g}}\left(U+\sqrt{2-U^2}\right),$$

with the dimensionless

$$U=\frac{m\sqrt{gh}bv\tan\kappa}{k_\mathrm{t}l+mhv^2\tan\kappa},$$

where h is the height of the center of mass of the skater, m is the mass of the skater, b characterizes the skater's longitudinal position on the board, $2l$ is the length of the board, κ is the angle between the ground and the pivot axis in the suspension system, k_t is the torsional stiffness of the wheel suspension system, v is the longitudinal speed and g stands for the gravitational acceleration.

(b) **The maximal allowable reaction delay is larger for non-zero speed than for zero speed if the skater stands ahead of the center of the board ($b>0$), while the critical delay is smaller for the standing behind case ($b<0$), i.e. skateboarding is easier if the center of mass is located ahead.**

(c) **For certain system parameters, there exists such speed range even for $\tau<\tau_\mathrm{cr}$, where the rectilinear motion cannot be stabilized.**

(d) **For a given reaction delay $\tau\in(\sqrt{2g/h},\tau_\mathrm{cr})$, the rectilinear motion is unstable for both low and infinitely large speeds, although the stabilizability changes four times meanwhile the longitudinal speed increases.**

Related publications: (2; 11; 12)

Thesis Statement 4

Using an extended mechanical model of the skater-skateboard system, where ideal rolling is assumed for the wheels, the board is modeled by a body with mass and mass moment of inertia, while the skater is represented by a lumped mass only and the balancing effort is modeled via a delayed linear PD controller, the following statements can be proven:

(a) **If the damping in the suspension system is negligible, the rectilinear motion of the skateboard is stable for certain delay ranges only and the simplified model (without mass moment of inertia) can be used to give an upper estimation of the critical delay.**

(b) **With realistic skateboard parameters, all the inertial parameters of the board, i.e. the mass and mass moment of inertia, have marginal effects on the linear stability of the rectilinear motion due to the presence of the damping in the suspension system.**

Related publications: (9; 10)

Thesis Statement 5

In the mechanical model of the skater-skateboard system in which the skater is considered as a lumped mass on a rigid bar connected to the rigid massless skateboard by a hinge, the ankle stiffness of the skater can be taken into account by a torsional spring. The balancing effort of the skater can be modeled by a linear PD controller with reflex delay. Based on this mechanical model, one can establish the following statements:

(a) **A performance index**

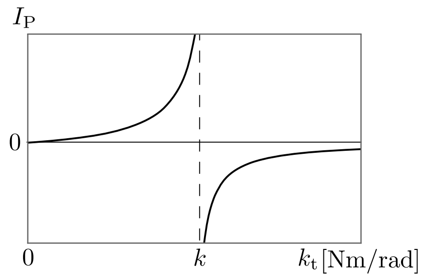
$$I_{\mathrm{P}} := \frac{k}{k-k_{\mathrm{t}}} \frac{k_{\mathrm{t}} l + hmv^2 \tan\kappa}{mghl}$$

can be introduced, where k is the ankle stiffness, m is the mass of the skater, h is the height of the center of mass, k_{t} refers to the torsional stiffness of the suspension, $2l$ is the length of the board, κ is the angle of the kingpin, v is the longitudinal speed and g stands for the gravitational acceleration. The larger the performance index I_{P}, the larger the stable domain of the control gains.

(b) **From stability point of view, the optimal and worst suspension stiffness parameters are**

$$k_{\mathrm{t}}^{\mathrm{opt}} := k - \epsilon \quad \text{and} \quad k_{\mathrm{t}}^{\mathrm{worst}} := k + \epsilon\,,$$

respectively, where ϵ is a small positive number (i.e. $0 < \epsilon << k$), see the figure below.



The dependence of the performance parameter on the torsional stiffness of the skateboard's suspension system.

Related publications: (5)

Thesis Statement 6

Uniformly accelerating maneuver (downhilling) of skateboards can be investigated in wide speed range by the mechanical model of the skater-skateboard system, which assumes the ideal rolling of the wheels, considers the skater as a lumped mass and the balancing effort via a delayed linear PD controller with passive ankle stiffness. The stability of the arising time dependent linear neutral delay-differential equation can be analyzed via the frozen-time eigenvalues method (conventional approach) and it can be also verified via numerical simulations. According to these the following statements can be established:

(a) **Considering realistic parameters, the frozen-time eigenvalues method can identify such control gains for which the downhill motion of the skateboard is practically stable.**

(b) **Acceleration makes skateboarding easier since the larger the acceleration, the larger the area of the stable control gain domain.**

(c) **There exist unsafe speed zones, where even small perturbations lead to such vibration amplitude, which can be identified in practice as unstable motion. But there exist such scenarios for which these unsafe zones can be crossed without having any practically relevant vibrations.**

Related publications: (1; 3; 5; 6)

Thesis Statement 7

Path following control of the skateboards can be studied with the help of a simpler mechanical model of the skater-skateboard system, where ideal rolling of the wheels is assumed, the skater is modeled together with the board as a massless rigid body with a lumped mass and the path tracking effort is modeled via a delayed linear PD controller utilizing the lateral position and lateral speed of the skateboard. The following statements can be derived:

(a) **For a given non-zero reaction delay of the skater, there is no such control gains for the linear feedback control, which can ensure stable path following for arbitrary chosen realistic longitudinal speeds.**

(b) **For perfectly fast reaction, i.e. for zero reaction delay, the stable path following cannot be achieved, however, the switched off control case can be stable in Lyapunov-sense.**

Related publications: (4; 8)

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