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Faculty of Transportation Engineering and Vehicle Engineering
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Examination of the symmetry of regular and near-regular structure’s static analysis in vehicle industry

Thesis book

by

Péter HARTH
MSc mechanical engineer, automotive engineer

Supervisors:

Dr. Péter BÉDA
DSc, Professor

Dr. Pál MICHELBERGER †
DSc, Professor Emeritus, member of the Hungarian Academy of Sciences

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INTRODUCTION AND MOTIVATION

Due to conquest of high-performance and super computers, more complex and more detailed models can be studied in statics, dynamics, vibrations and other mechanical fields as well. However, in the past couple of decades, researchers and engineers have been further impressed by the idea - although enough computational capacity is available - how can existing methods be further developed to require even fewer computational work, i.e. to achieve a shorter time in computing. Thus, assuming a given computational capacity, it would be possible to perform larger tasks or even solving multiple tasks simultaneously.

It is important to mention that the development of analytical methods has been closely linked with the development of so-called reanalysis methods that facilitate the re-execution of the calculations. For static tasks, if analysis of a structure requires a unit of computational work, then after the modification of the structure, its reanalysis requires less than one unit of work with a well-chosen reanalysis method. The extreme case, where after modifying the structure, the reanalysis method requires a unit of computational work, it does not have significant benefits. Such a method, which takes into account some of the regularity of the structure and uses it, has obvious advantages, because in the case of several modifications of the structure several times, fewer computational work will be done or the calculating engineer will have to reach the end result in a shorter time.

Regarding the number of methodologies for static analysis of structures, many methods have been developed and are under development in terms of their procedural mechanism. Initially, the scalar (now classical), and later the matrix algorithm, and in the present time, the approximation methods have come to the fore. Among the approaching methods, today one of the most popular calculation algorithms are using the finite element method. Due to their designation and their calculation methodology, exact results cannot be obtained in calculations, only an approximate result within a predefined convergence error limit. This end result depends on the human factor - as well as the experience of the calculating engineer - much more than anything else. The accuracy of the calculation is influenced by the structure of the mesh, its quantitative and qualitative parameters, the geometry of finite elements, the solution used, and so on. Contrary to the approximation methods, an exact method is available to achieve an exact result. In general speaking, the applicability of one or other method, the load itself, the size and the complexity of the structure being analysed, is strongly determined. It should be noted that some of the methods have been added over the years with techniques for reducing computational work, such as block-diagonalization.

After reviewing the literature detailed in the dissertation, it can be stated that no exact calculation method has been developed so far that makes the analysis and reanalysis of regular and near-regular structures possible in the presence of 2 or more symmetry planes applying the force method complemented by the block-diagonalization technique. Therefore, the purpose of the dissertation is to replace the aforementioned deficiency, by presenting a calculation method that allows the exact static analysis of regular and near-regular structures, further developing the relationships discussed in the group representation theory and the principle of general coupling. The further aim of the dissertation is to retrieve the static analysis of a near-regular structure that does not meet the requirements of the symmetry group C_{nv} ($n = 2, 3, 4, \dots m$) to a regular structure complying with the requirements of the symmetry group C_{nv} to allow for the re-calculation of the entire calculation task after the subsequent modification of these regular structures should not be done, but only part of it. Thus, if the structure is repeatedly modified, the need for analysis is reduced because only a part of the calculation has to be repeated again.

THESES

The Thesis 1 and 2 state the structure-dependent properties of the planar frame, frame-row, lattice-like structure and stiffened shell structures that meet the requirements of the symmetry group C_{2v}. Thesis 3 discusses the local structural modification problem of the structures applying symmetry groups C_{2v} and C_{3v}. Finally, generalizing structural local modification is discussed that fulfils the requirements of the symmetry group C_{nv}.

Thesis 1

The researchers have also examined the irreducible representations of the symmetry components (redundants) in the structure that fulfils the requirements of the given symmetry group. This symmetry-based method is used in the force method description and the symmetry properties of the regular structures (frame, frame-row, lattice-like structure) corresponding to the symmetry group C_{2v} are summarized in 3 sub theses.

Thesis 1.1

It is demonstrated that 6 one-dimensional, internal symmetry components can be assigned to the 6 unknowns in the case of static analysis of regular, planar frame that meets the requirements of the symmetry group C_{2v}, with 6 times internal indeterminacy and general external load.

Due to the single sliced basic system of the frame, it does not meet the requirements of the symmetry group C_{2v} but the *SS*, *SA*, *AS* and *AA* symmetry components do.

Only the *SS*, *AS* and *SA* in planar, and only the *AS*, *SA* and *AA* internal symmetry components in perpendicular plane fulfil the symmetry requirements regardless of the location of symmetry or antimetry plane and of the location of the cut.

In planar the *AA*, in perpendicular plane the *SS* internal load cases can be statically determined.

In other words, general external load can be divided into 4+4 (planar and perpendicular) symmetry components but in the compatibility matrix the *AA* and the *SS* blocks are missing in case of planar and perpendicular tasks [HP5].

Thesis 1.2

It is stated that in the case of static analysis of a regular, planar frame-row with the symmetry group C_{2v}, 6 (planar and perpendicular 3 + 3) one-dimensional internal and 8 (4 + 4) one-dimensional external symmetry components can be applied to general external load.

It is demonstrated that the applicability of the internal symmetry components are not independent of the number of frames. The 3 + 3 internal symmetry components used in a single frame are suitable to describe the 4 + 4 external load symmetry components during frame-row examination (Table 1. and 2.).

With this, the frame compatibility matrix can be block-diagonalized, in planar and in perpendicular cases, there are 4 or 4 blocks. I have determined the size of each blocks (number of unknowns) and I have also demonstrated the duality of each case of external load, which further reduces the computational work (Table 3. and 4.) [HP8].

Symmetry of external load	Even case			Odd case		
	X_1(*AS*)	X_2(*SA*)	X_3(*SS*)	X_1(*AS*)	X_2(*SA*)	X_3(*SS*)
SS	—	●	●	—	○	●
SA	—	●	●	—	●	○
AS	●	—	—	●	—	—
AA	●	—	—	○	—	—

Table 1.: Applicability of internal symmetry components, planar case

Symmetry of external load	Even case			Odd case		
	X_4(*SA*)	X_5(*AS*)	X_6(*AA*)	X_4(*SA*)	X_5(*AS*)	X_6(*AA*)
SS	●	—	—	○	—	—
SA	●	—	—	●	—	—
AS	—	●	●	—	●	○
AA	—	●	●	—	○	●

Table 2.: Applicability of internal symmetry components, perpendicular case

Symmetry of external load	Planar		Perpendicular	
	Even case	Odd case	Even case	Odd case
SS	n	n	$n/2$	$(n-1)/2$
SA	n	n	$n/2$	$(n+1)/2$
AS	$n/2$	$(n+1)/2$	n	n
AA	$n/2$	$(n-1)/2$	n	n
Σ	$3n$	$3n$	$3n$	$3n$

Table 3.: Regular planar frame-row static indeterminacy

Planar	*SS*	*SA*	*AS*	*AA*
Perpendicular	*AA*	*AS*	*SA*	*SS*

Table 4.: Duality of the external load

Thesis 1.3

It is demonstrated that 3 one-dimensional internal symmetry components can be applied for static analysis of regular, planar, rhythmic lattice-like structure containing 4 longitudinal beams and arbitrary number of cross-members which meets the requirements of the symmetry group C_{2v}. These are the *SS*, *SA* and *AS* symmetry components that make the compatibility matrix block-diagonally [HP7].

Thesis 2

It is demonstrated that in the case of static analysis of a regular, planar stiffened shell structure, that meets the requirement of the symmetry group C_{2v}, 8 (4 + 4 due to two symmetry planes layouts) one-dimensional internal symmetry components can be formulated for general external loads, applying the force method. With these 8 components compatibility matrix can be block-diagonalized.

It is also demonstrated that in the case of a force method static analysis of a regular, planar stiffened shell structure with the symmetry group C_{4v}, 4 internal symmetry components can be formed for cyclic external load with the compatibility matrix can be block-diagonalized.

In addition, it is revealed that in the case of static analysis of a regular, planar stiffened shell structure with symmetry of the symmetry group C_{4v}, 6 one-dimensional internal symmetry components can be formed for general external loads with the compatibility matrix can be block-diagonalized, applying the force method. That is generated from 4 components of the symmetry group C_{4v} and 2 selected components from the symmetry group C_{2v}. (It is determined the size of each block that are contained in Tables 5 - 8.) [HP6]

m; *n*	*SS*--	*SA*--	*AS*--	*AA*--
even	$\frac{mn}{4}$	$\frac{(n-2)m}{4}$	$\frac{(m-2)n}{4}$	$\frac{(m-2)(n-2)}{4}$
odd	$\frac{(m-1)(n-1)}{4}$	$\frac{(m-1)(n-1)}{4}$	$\frac{(m-1)(n-1)}{4}$	$\frac{(m-1)(n-1)}{4}$

Table 5.: Indeterminacy numbers to the symmetry components, if $m \neq n$, *m* and *n* are identically even or odd

m; *n*	*SS--*	*SA--*	*AS--*	*AA--*
even-odd	$\frac{m(n-1)}{4}$	$\frac{m(n-1)}{4}$	$\frac{(m-2)(n-1)}{4}$	$\frac{(m-2)(n-1)}{4}$
odd-even	$\frac{(m-1)n}{4}$	$\frac{(m-1)(n-2)}{4}$	$\frac{(m-1)n}{4}$	$\frac{(m-1)(n-2)}{4}$

Table 6.: Indeterminacy numbers to the symmetry components, if *m*≠*n*, *m* and *n* are differently even or odd

m; *n*	*--SS*	*--SA*	*--AS*	*--AA*
even	$\frac{m^2}{4}$	$\frac{m^2}{4}-\frac{m}{2}$	$\frac{m^2}{4}-\frac{m}{2}$	$\left(\frac{m}{2}-1\right)^2$
odd	$\frac{m^2-1}{4}$	$\left(\frac{m-1}{2}\right)^2$	$\left(\frac{m-1}{2}\right)^2$	$\frac{(m-3)(m-1)}{4}$

Table 7.: Indeterminacy numbers to the symmetry components, if *m* = *n*, *m* and *n* are identically even or odd

m; *n*	*SSSS*	*SSAA*	*AASS*	*AAAA*
even	$\frac{m^2}{4}-\sum_{i=0}^{\frac{m-2}{2}} i$	$\frac{m(m-2)}{4}-\sum_{i=1}^{\frac{m-2}{2}} i$	$\frac{m(m-2)}{4}-\sum_{i=1}^{\frac{m-2}{2}} i$	$\frac{m(m-4)}{4}-\sum_{i=2}^{\frac{m-2}{2}} i$
odd	$\frac{(m-1)^2}{4}-\sum_{i=0}^{\frac{m-3}{2}} i$	$\frac{(m-3)(m-1)}{4}-\sum_{i=1}^{\frac{m-3}{2}} i$	$\frac{(m-1)^2}{4}-\sum_{i=0}^{\frac{m-3}{2}} i$	$\frac{(m-3)(m-1)}{4}-\sum_{i=1}^{\frac{m-3}{2}} i$

Table 8.: Indeterminacy numbers to the symmetry components, if *m* = *n*, *m* and *n* are identically even or odd

Thesis 3

Beside the one-dimensional irreducible matrix-representations examination and structures that meet the requirement of the symmetry group C_{2v}:

Thesis 3.1

It is demonstrated that the force method static analysis of a near-regular structure that does not fulfil the requirements of the C_{2v} symmetry group can be performed by static analysis of a regular structure fulfiling the requirement of the symmetry group C_{2v}. The final load in the near-regular structure can be determined by the following relationship:

$$\mathbf{L}_{\mathrm{M}} = \mathbf{L} + \mathbf{L}_{\mathrm{C}}$$

where, $\mathbf{L}_{\mathrm{M}}$ is the final load in the near-regular structure, $\mathbf{L}$ is the final load in the regular structure, $\mathbf{L}_{\mathrm{C}}$ is the effect of the local structural modification. The $\mathbf{L}_{\mathrm{C}}$ can be calculated as:

$$\mathbf{L}_{\mathrm{C}} = -\mathbf{B}\mathbf{D}^{-1}\mathbf{B}_{\mathrm{V}}^{\mathrm{T}}\left(\mathbf{B}_{\mathrm{V}}\mathbf{D}^{-1}\mathbf{B}_{\mathrm{V}}^{\mathrm{T}} + \Delta\mathbf{R}^{-1}\right)^{-1}\mathbf{L}_{\mathrm{V}} =$$

$$\begin{bmatrix} \mathbf{L}_{\mathrm{C}}^{\mathrm{I}} \\ \mathbf{L}_{\mathbf{C}}^{\mathbf{II}} \\ \mathbf{L}_{\mathrm{C}}^{\mathrm{III}} \\ \mathbf{L}_{\mathrm{C}}^{\mathrm{IV}} \end{bmatrix} = -\begin{bmatrix} \mathbf{B}_{\mathrm{SS}}^{\mathrm{I}} & \mathbf{B}_{\mathrm{SA}}^{\mathrm{I}} & \mathbf{B}_{\mathrm{AS}}^{\mathrm{I}} & \mathbf{B}_{\mathrm{AA}}^{\mathrm{I}} \\ \mathbf{B}_{\mathrm{SS}}^{\mathrm{I}} & \mathbf{B}_{\mathrm{SA}}^{\mathrm{I}} & -\mathbf{B}_{\mathrm{AS}}^{\mathrm{I}} & -\mathbf{B}_{\mathrm{AA}}^{\mathrm{I}} \\ \mathbf{B}_{\mathrm{SS}}^{\mathrm{I}} & -\mathbf{B}_{\mathrm{SA}}^{\mathrm{I}} & -\mathbf{B}_{\mathrm{AS}}^{\mathrm{I}} & \mathbf{B}_{\mathrm{AA}}^{\mathrm{I}} \\ \mathbf{B}_{\mathrm{SS}}^{\mathrm{I}} & -\mathbf{B}_{\mathrm{SA}}^{\mathrm{I}} & \mathbf{B}_{\mathrm{AS}}^{\mathrm{I}} & -\mathbf{B}_{\mathrm{AA}}^{\mathrm{I}} \end{bmatrix} \begin{bmatrix} \mathbf{D}_{\mathrm{SS}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{\mathrm{SA}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{D}_{\mathrm{AS}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{D}_{\mathrm{AA}} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{B}_{\mathrm{SSV}}^{\mathrm{I}} & \mathbf{B}_{\mathrm{SAV}}^{\mathrm{I}} & \mathbf{B}_{\mathrm{ASV}}^{\mathrm{I}} & \mathbf{B}_{\mathrm{AAV}}^{\mathrm{I}} \\ \mathbf{B}_{\mathrm{SSV}}^{\mathrm{II}} & \mathbf{B}_{\mathrm{SAV}}^{\mathrm{II}} & \mathbf{B}_{\mathrm{ASV}}^{\mathrm{II}} & \mathbf{B}_{\mathrm{AAV}}^{\mathrm{II}} \\ \mathbf{B}_{\mathrm{SSV}}^{\mathrm{III}} & \mathbf{B}_{\mathrm{SAV}}^{\mathrm{III}} & \mathbf{B}_{\mathrm{ASV}}^{\mathrm{III}} & \mathbf{B}_{\mathrm{AAV}}^{\mathrm{III}} \\ \mathbf{B}_{\mathrm{SSV}}^{\mathrm{IV}} & \mathbf{B}_{\mathrm{SAV}}^{\mathrm{IV}} & \mathbf{B}_{\mathrm{ASV}}^{\mathrm{IV}} & \mathbf{B}_{\mathrm{AAV}}^{\mathrm{IV}} \end{bmatrix}^{\mathrm{T}} \times$$

$$\left(\begin{bmatrix} \mathbf{B}_{\mathrm{SSV}}^{\mathrm{I}} & \mathbf{B}_{\mathrm{SAV}}^{\mathrm{I}} & \mathbf{B}_{\mathrm{ASV}}^{\mathrm{I}} & \mathbf{B}_{\mathrm{AAV}}^{\mathrm{I}} \\ \mathbf{B}_{\mathrm{SSV}}^{\mathrm{II}} & \mathbf{B}_{\mathrm{SAV}}^{\mathrm{II}} & \mathbf{B}_{\mathrm{ASV}}^{\mathrm{II}} & \mathbf{B}_{\mathrm{AAV}}^{\mathrm{II}} \\ \mathbf{B}_{\mathrm{SSV}}^{\mathrm{III}} & \mathbf{B}_{\mathrm{SAV}}^{\mathrm{III}} & \mathbf{B}_{\mathrm{ASV}}^{\mathrm{III}} & \mathbf{B}_{\mathrm{AAV}}^{\mathrm{III}} \\ \mathbf{B}_{\mathrm{SSV}}^{\mathrm{IV}} & \mathbf{B}_{\mathrm{SAV}}^{\mathrm{IV}} & \mathbf{B}_{\mathrm{ASV}}^{\mathrm{IV}} & \mathbf{B}_{\mathrm{AAV}}^{\mathrm{IV}} \end{bmatrix} \begin{bmatrix} \mathbf{D}_{\mathrm{SS}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{\mathrm{SA}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{D}_{\mathrm{AS}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{D}_{\mathrm{AA}} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{B}_{\mathrm{SSV}}^{\mathrm{I}} & \mathbf{B}_{\mathrm{SAV}}^{\mathrm{I}} & \mathbf{B}_{\mathrm{ASV}}^{\mathrm{I}} & \mathbf{B}_{\mathrm{AAV}}^{\mathrm{I}} \\ \mathbf{B}_{\mathrm{SSV}}^{\mathrm{II}} & \mathbf{B}_{\mathrm{SAV}}^{\mathrm{II}} & \mathbf{B}_{\mathrm{ASV}}^{\mathrm{II}} & \mathbf{B}_{\mathrm{AAV}}^{\mathrm{II}} \\ \mathbf{B}_{\mathrm{SSV}}^{\mathrm{III}} & \mathbf{B}_{\mathrm{SAV}}^{\mathrm{III}} & \mathbf{B}_{\mathrm{ASV}}^{\mathrm{III}} & \mathbf{B}_{\mathrm{AAV}}^{\mathrm{III}} \\ \mathbf{B}_{\mathrm{SSV}}^{\mathrm{IV}} & \mathbf{B}_{\mathrm{SAV}}^{\mathrm{IV}} & \mathbf{B}_{\mathrm{ASV}}^{\mathrm{IV}} & \mathbf{B}_{\mathrm{AAV}}^{\mathrm{IV}} \end{bmatrix}^{\mathrm{T}} + \Delta\mathbf{R}^{-1} \right)^{-1} \times \begin{bmatrix} \mathbf{L}_{\mathrm{V}}^{\mathrm{I}} \\ \mathbf{L}_{\mathrm{V}}^{\mathrm{II}} \\ \mathbf{L}_{\mathrm{V}}^{\mathrm{III}} \\ \mathbf{L}_{\mathrm{V}}^{\mathrm{IV}} \end{bmatrix}$$

and

$$\Delta\mathbf{R}^{-1} = -\left[\left\langle \mathbf{R}_{\mathrm{V}}^{\mathrm{I}} \quad \mathbf{R}_{\mathrm{V}}^{\mathrm{II}} \quad \mathbf{R}_{\mathrm{V}}^{\mathrm{III}} \quad \mathbf{R}_{\mathrm{V}}^{\mathrm{IV}} \right\rangle\right]^{-1} \left(\left[\left\langle \mathbf{R}_{\mathrm{V}}^{\mathrm{I}} \quad \mathbf{R}_{\mathrm{V}}^{\mathrm{II}} \quad \mathbf{R}_{\mathrm{V}}^{\mathrm{III}} \quad \mathbf{R}_{\mathrm{V}}^{\mathrm{IV}} \right\rangle\right] + \mathbf{R}_{\mathrm{M}}\right) \left[\left\langle \mathbf{R}_{\mathrm{V}}^{\mathrm{I}} \quad \mathbf{R}_{\mathrm{V}}^{\mathrm{II}} \quad \mathbf{R}_{\mathrm{V}}^{\mathrm{III}} \quad \mathbf{R}_{\mathrm{V}}^{\mathrm{IV}} \right\rangle\right]^{-1}$$

The given relationships make the *local* structural modification of the regular structure possible performing the requirements of the symmetry group C_{2v}, including the special case of the *cutout* that represents the *geometric* modification of the structure using only the one-dimensional representation of the symmetry group [HP9].

Thesis 3.2

It is demonstrated that static analysis of a near-regular structure that does not fulfil the requirements of the symmetry group C_{nv} (n = 3, 4, ... m) can be performed by static analysis of a regular structure fulfiling the requirement of the symmetry groups C_{nv} (n = 3, 4, ... m) and C_{1v}, applying the force method. The present method uses only the symmetry components of the same meaning, regardless of the symmetry group used, i.e. only symmetric or only antimetric components.

The final load in the near-regular structure can be determined by the following relationship:

$$\mathbf{L}_{\mathrm{M}} = \mathbf{L} + \mathbf{L}_{\mathrm{C}}$$

where, $\mathbf{L}_{\mathrm{M}}$ is the final load in the near-regular structure, $\mathbf{L}$ is the final load in the regular structure, $\mathbf{L}_{\mathrm{C}}$ is the effect of the local structural modification. The $\mathbf{L}_{\mathrm{C}}$ can be calculated as:

$$\mathbf{L}_{\mathrm{C}} = -\mathbf{B}\mathbf{D}^{-1}\mathbf{B}_{\mathrm{V}}^{\mathrm{T}}\left(\mathbf{B}_{\mathrm{V}}\mathbf{D}^{-1}\mathbf{B}_{\mathrm{V}}^{\mathrm{T}} + \mathbf{\Delta R}^{-1}\right)^{-1}\mathbf{L}_{\mathrm{V}} =$$

$$\begin{bmatrix} \begin{bmatrix} \mathbf{L}_{\mathrm{C}}^{\mathrm{I}} \\ \mathbf{L}_{\mathrm{C}}^{\mathrm{II}} \\ \mathbf{L}_{\mathrm{C}}^{\mathrm{III}} \\ \cdot \\ \cdot \\ \mathbf{L}_{\mathrm{C}}^{2n} \end{bmatrix} & \mathbf{0} \\ \mathbf{0} & \begin{bmatrix} \mathbf{L}_{\mathrm{C}}^{\mathrm{IH}} \\ \mathbf{L}_{\mathrm{C}}^{\mathrm{IIH}} \end{bmatrix} \end{bmatrix} = -\begin{bmatrix} [\mathbf{B}_{\mathrm{Sn}} \quad \mathbf{B}_{\mathrm{An}}] & \mathbf{0} \\ \mathbf{0} & [\mathbf{B}_{\mathrm{S}} \quad \mathbf{B}_{\mathrm{A}}] \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \mathbf{D}_{\mathrm{Sn}} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{\mathrm{An}} \end{bmatrix} & \mathbf{0} \\ \mathbf{0} & \begin{bmatrix} \mathbf{D}_{\mathrm{S}} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{\mathrm{A}} \end{bmatrix} \end{bmatrix}^{-1} \begin{bmatrix} [\mathbf{B}_{\mathrm{SnV}} \quad \mathbf{B}_{\mathrm{AnV}}] & \mathbf{0} \\ \mathbf{0} & [\mathbf{B}_{\mathrm{SV}} \quad \mathbf{B}_{\mathrm{AV}}] \end{bmatrix}^{\mathrm{T}} \times$$

$$\left(\begin{bmatrix} [\mathbf{B}_{\mathrm{SnV}} \quad \mathbf{B}_{\mathrm{AnV}}] & \mathbf{0} \\ \mathbf{0} & [\mathbf{B}_{\mathrm{SV}} \quad \mathbf{B}_{\mathrm{AV}}] \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \mathbf{D}_{\mathrm{Sn}} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{\mathrm{An}} \end{bmatrix} & \mathbf{0} \\ \mathbf{0} & \begin{bmatrix} \mathbf{D}_{\mathrm{S}} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{\mathrm{A}} \end{bmatrix} \end{bmatrix}^{-1} \begin{bmatrix} [\mathbf{B}_{\mathrm{SnV}} \quad \mathbf{B}_{\mathrm{AnV}}] & \mathbf{0} \\ \mathbf{0} & [\mathbf{B}_{\mathrm{SV}} \quad \mathbf{B}_{\mathrm{AV}}] \end{bmatrix}^{\mathrm{T}} + \mathbf{\Delta R}^{-1} \right)^{-1} \times \begin{bmatrix} \begin{bmatrix} \mathbf{L}_{\mathrm{V}}^{\mathrm{I}} \\ \mathbf{L}_{\mathrm{V}}^{\mathrm{II}} \\ \mathbf{L}_{\mathrm{V}}^{\mathrm{III}} \\ \cdot \\ \cdot \\ \mathbf{L}_{\mathrm{V}}^{2n} \end{bmatrix} & \mathbf{0} \\ \mathbf{0} & \begin{bmatrix} \mathbf{L}_{\mathrm{V}}^{\mathrm{IH}} \\ \mathbf{L}_{\mathrm{V}}^{\mathrm{IIH}} \end{bmatrix} \end{bmatrix}$$

and

$$\mathbf{\Delta R}^{-1} = -\left[\left\langle \mathbf{R}_{\mathrm{V}}^{\mathrm{I}} \quad \mathbf{R}_{\mathrm{V}}^{\mathrm{II}} \quad \mathbf{R}_{\mathrm{V}}^{\mathrm{III}} \quad . \quad . \quad \mathbf{R}_{\mathrm{V}}^{2n} \right\rangle \quad \left\langle \mathbf{R}_{\mathrm{V}}^{\mathrm{IH}} \quad \mathbf{R}_{\mathrm{V}}^{\mathrm{IIH}} \right\rangle\right]^{-1} \times \left(\left[\left\langle \mathbf{R}_{\mathrm{V}}^{\mathrm{I}} \quad \mathbf{R}_{\mathrm{V}}^{\mathrm{II}} \quad \mathbf{R}_{\mathrm{V}}^{\mathrm{III}} \quad . \quad . \quad \mathbf{R}_{\mathrm{V}}^{2n} \right\rangle \quad \left\langle \mathbf{R}_{\mathrm{V}}^{\mathrm{IH}} \quad \mathbf{R}_{\mathrm{V}}^{\mathrm{IIH}} \right\rangle\right] + \mathbf{R}_{\mathrm{M}}\right) \times$$
$$\left[\left\langle \mathbf{R}_{\mathrm{V}}^{\mathrm{I}} \quad \mathbf{R}_{\mathrm{V}}^{\mathrm{II}} \quad \mathbf{R}_{\mathrm{V}}^{\mathrm{III}} \quad . \quad . \quad \mathbf{R}_{\mathrm{V}}^{2n} \right\rangle \quad \left\langle \mathbf{R}_{\mathrm{V}}^{\mathrm{IH}} \quad \mathbf{R}_{\mathrm{V}}^{\mathrm{IIH}} \right\rangle\right]^{-1}$$

The given relationships make the *local* structural modification of the regular structure possible performing the requirements of the symmetry group C_{nv} (n = 3, 4, ... m), including the special case of the *cutout* that represents the *geometric* modification of the structure using only the one-dimensional representation of the symmetry groups [HP10].

POSSIBLE PROBLEMS IN THE FUTURE, RESEARCH OPPORTUNITIES

In the field of structural analysis, whether static or dynamic, symmetry has always had been a key question. In the case of symmetry, the best is to use this main layout concept during the analysis, which has obvious advantages. For large, complex structures, however, the presence and type of symmetry is not fully understood. Therefore, in the past few years, researchers began to employ how to fully automate and recognize the symmetry of structures and maximize the exploitation of symmetry. As a result, it is not the engineer performing the analysis to recognize the individual symmetry types in the structure, but it can be automated with the computer.

In the future, researches will be based on the development of a symmetry and basic (primary) system recognition algorithm that would maximize the degree of utilization of symmetry, taking the conditions of the basic system selection into account. When applying the force method, the basic system always and significantly influences the computational work.

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