Dynamics of Digital Force Control of Robots

PhD dissertation

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Thesis 1

By means of modeling the basic characteristics of the interaction between robots and their environment, the stability properties and dynamic behavior of a one degree-of-freedom robot subjected to proportional-derivative (PD) digital force control was determined analytically with respect to the essential design parameters.

The exact stability chart was constructed in the space of the control gains P and D, the sampling frequency $f_s = 1/\Delta t$, and the natural frequency $f_n = \omega_\mathrm{n}/(2\pi)$ of the uncontrolled and undamped mechanical system. It was proved that the stable domains become disjoint and some regions can be lost by increasing the differential gain. The possible bifurcations were identified at the stability limits, and the frequencies of the arising self-excited vibrations were also calculated.

Some characteristic points of the stability chart were given in closed form that orient the designer at the preliminary design stage of the system. In the presence of the Coulomb friction force C, the stationary force error $\Delta F = C/P$ can be minimized by increasing the proportional gain till

$$P_{max} = \begin{cases} \dfrac{1-2\cos\Delta T}{1-\cos\Delta T} \; ; & \dfrac{2\pi}{3} < \Delta T < \dfrac{4\pi}{3} \\ \dfrac{(2\cos\Delta T - 3)^2}{8(1-\cos\Delta T)} & \text{otherwise} \end{cases} \quad , \quad \Delta T = 2\pi f_s/f_n \ ,$$

where the system loses stability with self-excited vibrations. The frequency f of these vibrations relative to the sampling frequency f_s reads

$$\frac{f}{f_s} = \begin{cases} \dfrac{1}{2\pi}\tan^{-1}\left(\dfrac{\sqrt{3-4\cos\Delta T\,(1+\cos\Delta T)}}{1+2\cos\Delta T}\right) + \dfrac{1}{2} \; ; & \dfrac{2\pi}{3} < \Delta T < \dfrac{4\pi}{3} \\ \dfrac{1}{2\pi}\tan^{-1}\left(\dfrac{\sqrt{15-4\cos\Delta T\,(1+\cos\Delta T)}}{1+2\cos\Delta T}\right) & \text{otherwise} \end{cases} .$$

It was also shown, that the fastest decaying transient force error $F_e(t) \sim \exp(f_s t \ln \rho_{min})$ is provided by the control gains

$$P_{opt} = \frac{-8\cos^3\!\Delta T + 36\cos^2\!\Delta T - 54\cos\Delta T + 27}{54(1-\cos\Delta T)} \ ,$$

$$D_{opt} = \frac{8\cos^3\!\Delta T + 36\cos^2\!\Delta T - 27}{54\omega_\mathrm{n}\sin\Delta T}$$

with the minimum spectral radius (or decay ratio)

$$\rho_{min} = \frac{2}{3}\cos\Delta T \, .$$

Corresponding sections of the dissertation: 2.2.3 Stability charts, 2.2.4 The minimal force error, 2.3.2 Vibration frequencies, 2.3.3 Exponential decay.

Related publications: [4], [6], [7], [8], [20].

Thesis 2

The results presented in Thesis 1 were generalized for the case of multi degree-of-freedom (DoF) robotic systems subjected to digital force control. The applied two subsequent decompositions of the equation of motion simplified the solution of the problem to the dynamic analysis of abstract delayed oscillators. The results and also the applicability of the method were confirmed by simulation and experiments.

Kövecses's method (Kövecses, 2006) was used to present the general dynamic equations of the constrained motion of multi-DoF mechanical systems in terms of generalized coordinates. This provided the first step for the system decomposition of a common 2 DoF parallel robot with interaction force control. The minimum set of generalized coordinates associated with the actuated joints was selected. The corresponding transformation matrices were determined in closed form for the planar five-bar linkage model in question.

For the stability analysis of the applied PD digital force control, a second (modal) decomposition was applied to transform the resulted equations into a set of abstract delayed oscillators, which have the same stability and dynamic properties as those of the model analyzed in Section 2 of the dissertation.

Corresponding sections of the dissertation: 2.2.3 Stability charts, 3.2 Formulation of the equations of motion, 3.3 PD digital force control, 3.5 Simulation.

Related publications: [1], [4], [5], [16].

Thesis 3

To capture the basic dynamics of human-robot interaction, a uni-directional low degree-of-freedom mechanical model was derived for the analysis of digital teaching-in force control of robots. The dynamic analysis of the model with proportional force control clearly showed the applicability and limitations of the chosen robot programming method, where the outer-loop force controller was closed around the inner-loop position controller of the robot. It was shown, that the stability properties of the system, the steady-state force error, and the possible arising vibrations at the stability limits provide an essential characterization of the dynamics of human-robot interaction and cooperation.

Among the above-mentioned parameters, the stationary force error ΔF, i.e., the resistance of the robot against the operator's action, was determined relative to the constant excitation force F_0 applied by the operator. This has the form

$$\left|\frac{\Delta F}{F_0}\right| = \frac{3\Delta t}{2Pm + 3\Delta t} ,$$

where m represents the relevant mass of the teaching-in device, P is the proportional gain and Δt is the sampling time of the control loop.

The stability chart was presented in the space of the dimensionless control gain Pk/ω_{n} and the frequency ratio $f_n/f_s = \omega_{\mathrm{n}}\Delta t/(2\pi)$, where ω_{n} is the natural angular frequency of the uncontrolled and undamped mechanical system, and k is the overall stiffness of the teaching-in device. For the practically meaningful case, where $P > 0$ and $f_n/f_s < 1/4$, the force error can be minimized by increasing the the proportional gain till

$$P_{max} = \frac{f_n}{2\pi k \tan \Delta T} \quad , \quad \Delta T = 2\pi f_s/f_n \ .$$

At this upper stability limit, self-excited vibrations arise with frequency f. This frequency was determined relative to the sampling frequency f_s in closed form as

$$\frac{f}{f_s} = \frac{1}{2} - \frac{1}{\pi}\tan^{-1}\sqrt{\frac{1+2\cos\Delta T}{3-2\cos\Delta T}} \ .$$

For a prescribed decay ratio ρ, the maximum positive proportional gains were determined in the parametric form

$$\left.\begin{aligned} P_{max} &= \frac{(2\rho(\rho+1)-1)f_n}{2\pi k\sqrt{(2\rho+3)(1-2\rho)}} \\ \text{with} \quad f_n/f_s &= \frac{\cos^{-1}(\rho+\frac{1}{2})}{2\pi} \end{aligned}\right\} , \quad \text{where} \quad \rho \in \left[\frac{-1+\sqrt{3}}{2}, \frac{1}{2}\right] \ .$$

Corresponding sections of the dissertation: 4.1 Model of teaching-in device, 4.2 Steady state force error, 4.4.2 Vibration Frequencies, 4.4.3 Exponential decay, 4.5 Real parameter case study.

Related publications: [3], [12], [13], [14], [15], [19].

Thesis 4

In the REHAROB Therapeutic System (European project IST-1999-13109), an indirect (outer-loop) force control approach was implemented for the teaching-in operating mode of the robots. It was shown that the parameters of the selected PI-like control law can be tuned to provide stable and reliable teaching-in control with the minimum possible resistance against the operator's action even in the presence of large delays.

By introducing a kinematic constraint representing the position/velocity control of the robot, a simple uni-directional two degree-of-freedom mechanical model was constructed for the dynamic analysis of the teaching-in force control of REHAROB. The hardware and software dependent maximum feasible sampling frequency of the overall control loop and the deadtime of the ABB S4C+ robot controller were identified by a series of experiments.

Based on the identified mechanical and control parameters, the stability of the teaching-in force control of the REHAROB Therapeutic System was analyzed and the stability charts were determined in the space of the proportional and integral gain parameters. The central regions of the stability charts were identified where the optimal decay rates can be guaranteed for a wide range of realistic stiffness values of the physiotherapist's hand.

Theoretical results were compared to experiments by analyzing a typical taught-in robot trajectory that was recorded during the clinical tests of the REHAROB Therapeutic System. It was shown, that the lowest (characteristic) frequency of the transient part of the recorded motion corresponds exactly to the frequency calculated semi-analytically for the selected control parameters.

Corresponding sections of the dissertation: 5.4 Mechanical model of the teaching-in device, 5.5 Teaching-in force control strategy, 5.6 Model parameters, 5.7 Stability analysis, 5.8 Theory, simulation and experiments.

Related publications: [2], [9], [10], [11], [17], [18].