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Computational study of collective breakdown phenomena

PhD Thesis booklet

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Background

The study of collective breakdown phenomena by the methods of statistical physics is getting a central role in understanding how crisis situations suddenly emerge. Breakdowns are observed in various fields of life. Examples include the development of a global economic or financial crisis situation, large-scale blackouts and the unforeseen failures in critical communication or transportation infrastructures. The study and development of the models describing these phenomena is of high priority.

Econophysics has been modeling economic and financial processes for two decades using the methods of statistical physics. Due to the availability of high-resolution stock exchange trading data, it has initially been focusing on interpreting equity stylized facts and equity derivatives. The past decade, however, has shown a tremendous rise in the trading volume of credit derivatives, i.e., products depending on events like bankruptcy, default or changes in the credit rating of a company or government. Such products are meant as instruments of risk reduction but they open ground for speculation, too. They also make the financial sector more and more connected, which increases systemic risk. The subprime mortgage crisis of 2007–2008 has shown that the rising volume of derivative products can lead to unforeseen instabilities in the stock exchange.

The scheme of systemic risk is not limited to economics but occurs in various infrastructures as well. As more and more systems get interconnected forming a global infrastructure, the consequences of breakdowns get more and more substantial and one has little experience in surveying the potential dangers resulting from interactions. This is well illustrated by nation-wide blackouts like the Northeast blackout of Canada in 2003 and the 2003 Italy blackout or the 2012 India blackouts. These service disruptions were triggered by engineering failures or natural disasters but the Italy blackout got special attention because those failures in the power grid caused a failure of the Internet communication network, which in turn caused a further breakdown of power stations, that is, interconnectedness increased the risks too.

Network science was created to describe the topology and functioning of complex systems established by connecting many interacting entities of the same kind on large scales. The interconnection of different kind of networks constitutes networks of networks which are in the center of interest for research. Networks that simultaneously supply and depend on each other, as pointed out in the examples, are referred to as interdependent networks. Such systems comprise several interdependent network layers can give rise to emergent phenomena that are not exhibited on the level of individual layers. These networks are known to be more vulnerable than standalone ones. Reasons include the avalanches of failures propagating back and forth between networks leading to an abrupt transition from one operating domain to another that is corresponding to malfunction. The importance of the model describing these phenomena is that this transition turns out to be a hybrid phase transition, which is both characterized by a jump in the order parameter and by critical scaling.

In the study of interdependent networks the lattices models, critical phenomena and percolation provide a good starting ground.

Objectives

The objective of the thesis was to investigate the breakdown phenomena occurring in complex systems, to improve predictions of systemic risk and to help decreasing it by using the methods of statistical physics and computer simulation.

One of the most important structured credit derivatives in modern finance is the Collateralized Debt Obligation (CDO) which is basically an insurance based on the tranches a reference portfolio of about hundred elements. It pays the excess loss on the portfolio above a threshold (attachment point of the tranche), up to a maximum value (detachment point). Due to the nonlinear payoff function, its pricing is usually only feasible by computer simulation. One of my objectives is to implement a Monte Carlo reweighting technique that makes the valuation of such contracts more precise and/or faster with a focus on events that rare but carry huge risk and make the standard techniques computationally intensive.

The loss can be approximated by a compound Poisson process where the event times are generated by a Poisson process and the event sizes come from an independent distribution. This model is analytically tractable therefore I was going to use it for benchmarking the computer simulation.

My other objective is to study the phenomena related to interdependent networks, a model used mainly to describe mutually dependent infrastructures but that can be further generalized to financial systems, too. The literature previously used a static approach in the sense that it described the effect of the random removal of a fraction of one of the layers. It seemed natural to make the model dynamic, i.e., remove the nodes one by one. After that, the objective was to study the breakdown phenomena and the hybrid phase transition accompanying them. Since the 2D lattice topology captures the layout of real infrastructures better than random (Erdős-Rényi) graphs, I gave the former special attention. My aim on one hand was to measure the critical parameters (breakdown point and exponents) related to the order parameter. On the other hand, it was important to find out whether the statistics of avalanches shows critical properties and if yes, which are the quantities of interest and how they can be related to properties of the order parameter. Another important question to face is whether the scaling relations derived in statistical physics also hold for the hybrid phase transition and whether there are scaling relations specific to this type of transition.

In previous models the failures developed on a short timescale and the networks passively suffered their consequences. But under special circumstances, e.g., in financial or economical networks the agents may have time to react and to build new relationships. My aim was to create a dynamic model in which healing mechanism governed by random parameter hinders or slows down the abrupt collapse of the network. To study this generalized model in sufficient detail in the critical domain I had to adapt a very efficient simulation algorithm. The new model posed plenty of questions awaiting answer. I had to study how the probability of healing influences the resilience of the network, determine what the minimal healing is that stops the network from collapsing, and how the phase transition, its scaling exponents and the avalanche statistics change as a function of the healing parameter.

My objective has been fundamental research but the phenomena and models studied here are closely related to important technological and economic questions. The improvement of CDO pricing could potentially increase financial stability while understanding the possibility for healing may help to avoid infrastructural catastrophes.

Novel scientific findings

My research detailed in the thesis and the corresponding novel findings are summarized in the following thesis statements:

1. I implemented a Monte Carlo reweighting technique for the pricing of Collateralized Debt Obligation (CDO) contracts. I used this program for the pricing of CDO contracts modeled by a compound Poisson process with realistic jump time and jump size distributions. I showed numerically that with a careful choice of alternative parameters for the jump time and jump size distributions the reweighting can be used to reduce the measurement variance of the quantities needed for pricing. I showed that the decrease of the variance is in good agreement with the analytic predictions and the expected value of these quantities matches their original values without reweighting. Using the reweighting the required computational power can be reduced by up to a factor of 12 for some products. [I]

2. I studied the hybrid phase transition (HPT) that occurs in the cascading failure (CF) model on a pair of interdependent networks. I have shown that there are two sets of exponents related to the phase transition: One describes the behavior of the order parameter, which is the relative weight of the largest mutually connected component (MCC); the other one characterizes the statistics of the cascades. I defined scaling exponents for the size and fluctuation of the MCC ($\beta_m, \gamma_m, \bar{\nu}_m$), as well as for the avalanche sizes ($\tau_a, \sigma_a, \gamma_a, \bar{\nu}_a$). I studied numerically the HPT on a pair of fully interdependent 2D square lattices and measured the values of these exponents by computer simulations. [II]

3. I derived two relationships for the scaling exponent β_m that characterizes the size $m(p)$ of the MCC in the scaling regime near the critical point p_c of the system as $m(p) - m_0 \propto (p - p_c)^{\beta_m}$, where $1 - p$ is the ratio of the externally removed nodes in the system. First, I showed that $\beta_m = 1/2$ universally, for infinite range dependency links. Then I proved that $1 - \beta_m = \gamma_a$, where γ_a describes the scaling of the average size of finite avalanches as $\langle s \rangle \propto (p - p_c)^{-\gamma_a}$. This way I established a universal relationship between the two sets of exponents. [II]

4. I introduced healing into the CF model to allow the dynamic recovery of the network via forming random healing links that bridge over the failed nodes. I generalized an efficient simulation algorithm for this case. I showed via computer simulations that increasing the healing probability enhances the resilience of the network and there exist a critical healing probability where the macroscopic cascades are eliminated and the breakdown disappears in favor of the continuous shrinking of the network that is getting more and more connected. I showed that this critical healing probability also separates two different classes of behavior. Below the critical healing scaling is similar to that observed in the original system while above it scaling exponents take trivial values and the fluctuations follow the central limit theorem. [III, IV]

5. I studied the CF model with healing numerically in the regime of low healing probability w. I showed that the size $m(p, w)$ of the MCC can be mapped to a master curve as $1 - m(p, w) = a(w)[1 - m(1 - (1 - p)/a(w), 0)]$ in the $w \to 0$ limit where $a(w) = (1 - p_c(0) - \Delta p_c(w))/(1 - p_c(0))$ and the distance Δp_c from the no-healing critical point shows power-law behavior $\Delta p_c(w) \propto w^\gamma$ where γ is close to one. [III]

Publications related to the thesis statements

[I] M. Stippinger, É. Rácz, B. Vető, and Zs. Bihary. *Analytic results and weighted Monte Carlo simulations for CDO pricing*. European Physical Journal B **85** (2) (2012), p. 51. arXiv: `1105.5416`.

[II] D. Lee, S. Choi, M. Stippinger, J. Kertész, and B. Kahng. *Hybrid phase transition into an absorbing state: Percolation and avalanches*. Physical Review E **93** (2016), p. 042109. arXiv: `1512.08335`.

[III] M. Stippinger and J. Kertész. *Enhancing resilience of interdependent networks by healing*. Physica A: Statistical Mechanics and its Applications **416** (2014), pp. 481–487. arXiv: `1312.1993`.

[IV] M. Stippinger and J. Kertész. *Universality and scaling laws in the interdependent network model with healing*. (Submitted for review) (2017). arXiv: `1705.09829`.