Budapest University of Technology and Economics
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Booklet of Thesis Statements

for the PhD dissertation

Dynamical Systems with Varying Time Delay in Engineering Applications

submitted by

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Overview of the dissertation

This work is devoted to the analysis of engineering systems where time delays play an important role. Time delays are often considered to be a source of unstable behavior and undesired oscillations. Thus, investigating the dynamics of time-delay systems is a highly important task in engineering. Here, engineering applications are shown where different types of delays occur: point (discrete) and distributed delays, which may be constant, time-dependent, or state-dependent. The examples include linear and nonlinear, autonomous and time-periodic dynamical systems. Although the physics of these systems is different, the underlying mathematical description and the tools of analysis are similar.

The first engineering application is the stabilization of unstable systems via control loops that include feedback delays. In order to compensate the destabilizing effect of time delays, a predictor feedback control strategy called Finite Spectrum Assignment (FSA) is investigated. The stabilization of an inverted pendulum is discussed in detail, which is important in understanding human balancing and motor control, where the human reaction delay plays an important role. This problem is described by a linear autonomous system where constant point and distributed delays occur. Via stability analysis, it is shown that the FSA controller is able to stabilize unstable systems with relatively large feedback delays, provided that an accurate estimation of the system's parameters is available (Thesis 1).

The second topic is the dynamics of vehicular traffic and the control of autonomous vehicles. The concept called Connected Cruise Control is investigated, where the participants of vehicular traffic send information about their motion to each other via wireless vehicle-to-vehicle communication. This enables the vehicles to control their motion such that the safety and mobility of traffic is improved. Since the communication is intermittent and requires a certain amount time, time delays occur in the control loops, which are time-periodic when digital controllers are used.

The negative effect of time delays on the safety and mobility is analyzed, and a predictor feedback control strategy is proposed to compensate the time delays (Thesis 2).

The rest of this work is devoted to the analysis of machine tool vibrations (chatter) in metal cutting. First, turning operations are analyzed with special attention to their nonlinear dynamics. Namely, there exists a bistable region in the space of technological parameters, where stable stationary cutting and large-amplitude machine tool chatter coexist, and machine tool vibrations occur to large enough perturbations during cutting. From practical point of view, this region is unsafe and must be avoided. Via the analysis of the governing nonlinear autonomous system with constant point delay, a simple closed-form formula is derived by which the bistable region can be computed (Thesis 3).

Then, turning processes are considered assuming low cutting speeds, where more sophisticated cutting force models are required for an accurate prediction of machine tool vibrations. Here, a distributed cutting force model is investigated, and the region of bistability is determined by analyzing the associated nonlinear system with constant and state-dependent distributed delays (Thesis 4).

Finally, low-speed milling models are investigated. It is a widely accepted experimental observation for milling processes that the maximum stable (chatter-free) axial depth of cut typically increases when the cutting speed is decreased. This so-called low-speed stability improvement phenomenon is investigated in detail via two different concepts. First, a velocity-dependent cutting force model is considered, which takes into account the vibration-induced variation of the cutting direction. Via the analysis of the corresponding time-periodic time-delay system, it is shown that this model is not able to explain the low-speed stability improvement for all milling conditions (Thesis 5). Then, a distributed cutting force model is extended to milling. This model is associated with time-periodic distributed delays, and is able to explain the low-speed stability improvement (Thesis 6).

Thesis 1

I have investigated the Finite Spectrum Assignment predictor feedback control technique, with special attention to the effect of parameter uncertainties in the internal model, and to the effect of inaccuracies in the implementation of the control law. I have analyzed the stabilization of an inverted pendulum with feedback delay. I have computed the stability charts of the governing differential equation with distributed delay, which show the ideally, theoretically, and robust stable domains in the space of the control gains for different levels of parameter uncertainties in the internal model. Based on this, I have arrived at the following conclusions.

The following statements hold regarding the stabilization of a second-order system with input delay of the form

$$\ddot{\varphi}(t) - a\varphi(t) = u(t-\tau)$$

by means of the Finite Spectrum Assignment (FSA) predictor feedback control technique using a digital controller with sufficiently small sampling time.

The FSA controller is sensitive to mismatches between the parameters of the internal model and the actual system parameters. Let ε_τ and ε_a denote the error of the estimations of the feedback delay τ and the system parameter $a > 0$, respectively. Let a_{crit} denote the critical value of the system parameter a in the sense that if $a > a_{\mathrm{crit}}$, then the system cannot be stabilized for a given feedback delay. If the internal model perfectly matches the real system ($\varepsilon_\tau = 0$, $\varepsilon_a = 0$), then stabilization by FSA is possible for arbitrarily large system parameter. In case of small parameter mismatches, the following properties hold based on the numerical analysis of the special case $\varepsilon_a = 0$. The FSA controller is superior to the proportional-derivative (PD) and the proportional-derivative-acceleration (PDA) controllers with respect

to the critical system parameter a_{crit}. If the parameter mismatches are less than 25% ($\varepsilon_\tau < 0.25$), then a_{crit} for the FSA controller is larger than for the PD controller. If the parameter mismatches are less than 8% ($\varepsilon_\tau < 0.08$), then a_{crit} for the FSA controller is larger than for the PDA controller.

Related publications: [1, 2].

Thesis 2

I have investigated a vehicular string traveling on a single lane where the vehicles use connected cruise control to regulate their longitudinal motion based on the data received from other vehicles via wireless vehicle-to-vehicle communication. I have analyzed the effect of time-periodic time delays in the control loops that are caused by digital controllers and are increased by the eventual loss of data packets during communication. I have proposed two control strategies that use predictors to compensate the destabilizing effect of time delay. By means of calculating the domains of plant and string stability in the plane of the control gains and by determining the minimum achievable time gap below which stability cannot be achieved, I have obtained the following results.

Digital controllers in connected cruise control introduce time-varying time delays into the control loops of the vehicles. For the case of a leader-follower vehicle pair, predictors using the approach of Finite Spectrum Assignment are able to compensate the destabilizing effect of time delay by means of predicting the follower's velocity and position one sampling period ahead. The prediction increases the size of the plant and string stable domains in the plane of the control gains and it increases the critical sampling period (where the string stable domain vanishes) by a factor of 1.5. Packet losses

in vehicle-to-vehicle communication increase the time delay, and decrease the size of the stable domain and the critical sampling period. Prediction of the headway based on the distances that the leader and the follower travel during packet losses is able to restore the plant stable domain, whereas a finite impulse response filter on the leader's velocity data is able to improve string stability.

Related publications: [3, 4].

Thesis 3

I have investigated the occurrence of machine tool vibrations and the phenomenon of bistability for the single-degree-of-freedom mechanical model of turning processes. By means analyzing Hopf and double Hopf bifurcations associated with the nonlinear delay-differential equation that describes the dynamics of orthogonal turning, I have obtained the following results.

Consider the single-degree-of-freedom model of orthogonal turning processes, where the cutting force is a concentrated force that is a monotonically increasing function of the chip thickness, and the cutting force is the only source of nonlinearity in the machining system. Then, there exists a bistable region in the space of technological parameters, where stable stationary cutting and large-amplitude machine tool chatter coexist, and machine tool vibrations occur to large enough perturbations during cutting. The bistable technological parameter region can be estimated by the following for-

mula independently of the spindle speed:

$$\frac{a_{\mathrm{H}}-a_{\mathrm{BB}}}{a_{\mathrm{H}}} = 1-\left(\sum_{k=1}^{\infty}\frac{1}{4^{k-1}}\binom{2k-1}{k}\eta_{2k-1}\right)^{-1}$$
$$= 1-\left(1+\frac{3}{4}\eta_3+\frac{10}{16}\eta_5+\frac{35}{64}\eta_7+\frac{126}{256}\eta_9+\dots\right)^{-1},$$

where a_{H} denotes the maximum value of the chip width a associated with linearly stable stationary cutting, whereas a_{BB} denotes the minimum chip width where the phenomenon of bistability occurs (that is, the cutting process is bistable for $a_{\mathrm{BB}} < a < a_{\mathrm{H}}$). Parameters η_m $(m \in \mathbb{Z}^+)$ are dimensionless cutting force coefficients that can be obtained from the feed h_0 per revolution and the derivatives of the specific cutting force f_q that is a function of the chip thickness: $\eta_m = \big(h_0^{m-1} f_q^{(m)}(h_0)\big)/\big(m! f_q'(h_0)\big)$.

Related publications: [5–9].

Thesis 4

I have investigated the single-degree-of-freedom mechanical model of turning processes by considering the cutting force as the resultant of a force system distributed along the rake face of the tool. By analyzing the corresponding equation of motion, which is a nonlinear differential equation with either constant or state-dependent distributed delay, I have obtained the following results.

Consider the single-degree-of-freedom model of orthogonal turning processes, where the cutting force is a monotonically increasing function of the chip thickness, and the cutting force is the only source of nonlinearity in the machining system. Then, the distribution of the cutting force along the rake face of the tool has the following effects on the occurrence of machine tool vibrations compared to the case of a concentrated cutting

force.

If the size of the chip-tool interface, where the cutting force is distributed, is constant in time, then the following statements hold. The boundary of the linearly stable technological parameter region is associated with subcritical Hopf bifurcation independently of the spindle speed. Thus, there exists a bistable technological parameter region, where stable stationary cutting and large-amplitude machine tool chatter coexist, and machine tool vibrations occur to large enough perturbations during cutting. The size of the bistable technological parameter region is independent of the shape of the cutting the force distribution: it occupies approximately the same portion of the linearly stable technological parameter region as in the case of concentrated cutting force.

If the size of the chip-tool interface is proportional to the uncut chip thickness, then the following statements hold. The equation of motion involves a state-dependent distributed delay. The linearly stable technological parameter region is the same as in the case of a chip-tool interface of constant size. The Hopf bifurcation associated with the boundary of the linearly stable technological parameter region is either subcritical or supercritical depending on the spindle speed. Thus, the bistable technological parameter region disappears in certain spindle speed ranges.

Related publications: [10–12].

Thesis 5

I have investigated a velocity-dependent cutting force model for milling operations, where it is taken into account that the cutting direction is affected by the vibrations between the tool and the workpiece. By deriving the time-periodic delay-differential equation that governs the tool's motion and analyzing the linear stability of its stationary solution, I have drawn the following conclusions.

In milling operations, the cutting direction at each tooth of the mill is affected by the vibrations between the tool and the workpiece. Taking this effect into account, the chip thickness and the decomposition of the cutting force into tangential and radial components depends on the vibration velocity. This leads to an additional damping term in the single-degree-of-freedom linear mechanical model of milling, where the damping is time-periodic and is inversely proportional to the spindle speed. The time-periodic damping depends on the radial immersion of milling and affects the occurrence of machine tool chatter. For high radial immersion, the additional damping is nonnegative, thus the maximum stable (chatter-free) axial depth of cut increases as the spindle speed is decreased. For low radial immersion, the additional damping is nonpositive, and the maximum stable axial depth of cut decreases as the spindle speed is decreased.

Related publication: [13].

Thesis 6

I have investigated the single-degree-of-freedom mechanical model of milling processes by considering the cutting force on each tooth of the mill as the resultant of a force system distributed along the rake face. I have derived the equation governing the tool's motion, which is a differential equation with time-periodic coefficients and time-periodic distributed delay. By means of analyzing the linear stability of the stationary solution, I have obtained the following results.

In milling operations, the distribution of the cutting force along the rake face of the mill teeth has qualitative effect on the occurrence of linear machine tool vibrations compared to the case of a concentrated cutting force. Due to the cutting force distribution, the maximum stable (chatter-free) axial depth of cut increases as the spindle speed is decreased. This phenomenon occurs both for up-milling and down-milling and for any radial immersion. The effect is more pronounced for down-milling than for up-milling.

Related publications: [14, 15].

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