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SUMMARY OF PHD THESIS

A numerical calculation method for aiding construction of changing ratio gears

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Budapest, 2017

I. Description of the topic, objectives

The typical manufacturing technologies of regular cylindrical gears are based on the gear generation process. These technologies enable the production of the gears complex geometry with the help of just a few geometric parameters. In the last few decades the evolution of the various CNC manufacturing technologies enabled irregular unique manufacturing of these machine parts. Examples are the CNC wire spark cutting, CNC milling, laser cutting or the additive technologies (3D printing). In order to apply these techniques however, in contrary to the classical methods the exact geometry of the gears are needed. In the early phase of this research we created a parametric 3D CAD model of a cylindrical gear that has an exact geometry, thus enables the CNC manufacturing. We learned that generalizing the geometrical calculations we would be able to create not just cylindrical but non cylindrical gears as well.

In the field of noncircular gears geometrical calculation the most common principles are based on the gear generating manufacturing process. This can be simulated using a curve of cutting tool geometry. This mathematically described curve has to be rolled down on the noncircular pitch curve using various coordinate transformations. The envelope curve of the resulting geometry is the noncircular gears geometry. In order to handle the more complicated general cases, which are difficult to analytically solved, we applied numerical methods.

The noncircular gears occur very rarely in the practice. Since manufacturing in most cases is not an issue, the rareness can be reasoned by difficulties of the complex calculation. One of the major objectives of this research is to reduce the calculation and construction time demand in order to make these special parts spread broader. Our main goal was to create a calculation method and an algorithm that generates noncircular gears geometry easily and fast. In order to easily process the complicated general cases a numerical method was applied. In order to prove the correctness and test the applicability prototyping and testing had a very important role in our research.

The second objective of our research was to determine the possible usage of the noncircular gears. We tried to locate the fields of engineering where the application of these spatial machine parts have benefits compared to the classical engineering solutions. The correctness of the applications has to be proved by working prototypes.

It is also very important to refine – or if necessary modify – our algorithm on the base of our experiences.

By constructing and applying noncircular gears not just the geometry but also the loads, the strength and the interlocking behaviors have to be determined. This work however does not contain the discussion of these issues just the geometrical calculations. These issues can be further objectives in the future.

II. Antecedents and research methods

In the past fifty years two books with emphasized importance were published on the topic of noncircular gears. These books are the Olsons *Noncircular cylindrical gears* [1] and Litvins *Theory of gearing* [2]. The latter one created the basis of the currently most common calculations.

In the last decade the most significant research and development of this field can probably be connected to the People's Republic of China. The huge government support of these researches resulted many demanding engineering solutions and scientific articles [3], [4], [5], [6]. In these articles we can read about calculation methods, geometrical error evaluations, manufacturing technologies and many interesting engineering solutions.

Litvins method of geometrical calculation is based on the gear generation process where the tool motions are simulated by various coordinate transformations. The tool curve represents the shape of the actual tool profile. At the gear generating motion if a point of the tool curve has a tangential relative velocity vector the point would be currently in connection and is also on the envelope. This condition can be inspected with the help of the orthogonality condition. This condition claims that at a given point the scalar multiplication of the relative velocity vector and the normal vector is zero if those are perpendicular.

The pitch curves can be determined by using the rigid body velocity equation. This equation enables to determine the velocity vector of a rigid body's arbitrary point if the motion of this body is known. The relative motions of the gears can be determined by the required axis distance and ratio functions. In the coordinate systems that are fixed to the moving gears, we can determine all the points that have zero relative velocity. The curves defined by these points are the pole curves that are identical to the pitch curves.

The curve of the tool profile has a periodic nature thus it can be determined by using Fourier transformations. This method provides a continuous and differentiable curve that is necessary for determining the normal vectors.

At more sophisticated cases the Boolean operations are applied. At this method the coordinate transformations representing the tool motion are carried out discretely in finite steps. At each step the Boolean cut operation is carried out between a virtual gear and tool modelling the actual gear cutting process. These operations can be automatically carried out in some CAD systems such as AutoCAD.

In this work numerical methods were applied for the calculations. These were implemented in Matlab environment that is perfectly suitable for such evaluations. In contrary to many mathematical software in Matlab environment primarily numerical approach is applied in combination with numerous control flow operations (known from the computer programming). Although the other broadly used evaluation software such as Mathematica and Maple are as well capable of carrying out numerical calculations, their strength is in the analytical calculations. The numerical methods have advantages and disadvantages compared to the purely analytical approach. In many cases the results does not come in the form of an expression but as an array of discrete values. The more complicated calculus related operations such as differential and integral can be carried out by using numerous basic multiplying and addition operations. The built in control flow statements make the automatic sequence of calculation possible that reduces the time demand of a complex evaluation. In this work the verification of the usability of the worked out application has been given a prominent role. For working out these solutions the PTC Wildfire 5 and PTC CREO2 CAD software provided assistance. Prototyping and testing had a great role in our research providing ultimate verifications to the proposed solutions. For this we applied the most diverse technologies such as CNC milling, CNC laser cutting, wire spark cutting and 3D printing.

III. Summary of the research and the thesis

In this research two different calculation algorithms had been worked out that are suitable for the generation of changing ratio gear geometries. In order to make these algorithms easier to use we provided comfortable graphic user interfaces.

The first algorithm implemented in Matlab environment was based on the modelling of the gear generation manufacturing procedure. After determining the noncircular pitch curves the algorithm generates a virtual cutting tool gear that contains all the needed profile parameters. The virtual gear generation is carried out by using various coordinate transformations on the tool gear. This provides a set of lines that consists of numerous transformed tool gear curves. The envelope of the resulting set of curves is the final shape of the noncircular gear. In order to determine the envelope curves more methods had been set up.

One of these applies a numerical approach and uses straight searching lines in order to determine the envelope curves. This method is based on finding the intersection of two straight lines by using linear equations. The discretisation of the tool gear curves results a set of small straight lines. Their intersection with the straight searching lines can easily be determined. In the first step the algorithm projects a line from the inside of the envelope toward the outside. The nearest intersection to the start point will be the first point of the envelope curve. Further points can be determined by using a loop of searching lines that is defined in the range of the last point. This loop goes from the inside to the outside of the envelope, while always crossing it (figure 1). The calculation can be accelerated by reducing the range of searching to the close environment of the last point. This method provides an envelope curve that consists of numerous straight lines. The resolution of this curve can be enhanced by reducing the size of the searching loop.

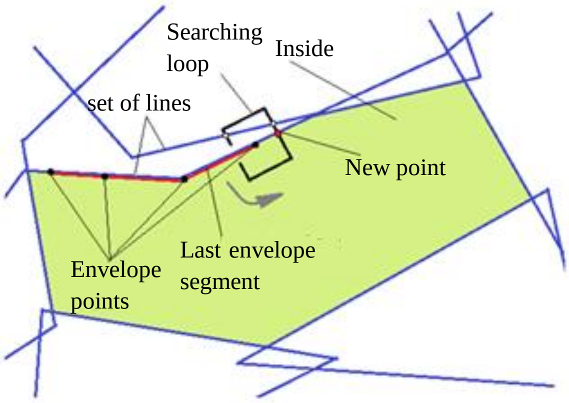


Figure 1. Sketch of the envelope calculation

One disadvantage of this process is that in specific cases it fails to determine the next envelope point. This may occur when the envelope takes a sharp bend near 180°. Another disadvantage is that this method is incapable of detecting the sharp edges. As seen in figure 2 those are usually cut off. Fortunately at a real practical gear calculation these phenomena occur rarely.

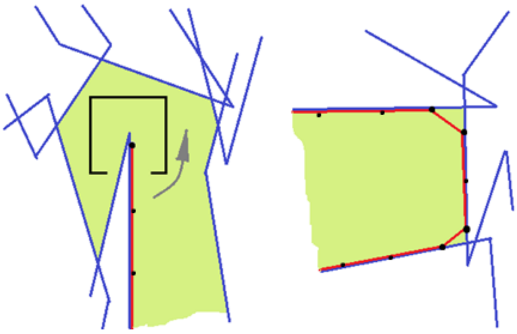


Figure 2. Problems regarding the proposed method

The shape of the searching loop has a great effect on the quality and correctness of the results. Our goal was to create a shape that can be reliably applied and causes a smaller inaccuracy at the sharp edges. It is important that this loop does not contain too many straight lines; otherwise the calculation time may increase. After experimenting with many different forms eventually an open rectangle form had been chosen that consists of three straight lines (figure 3). The flattened form enables to decrease the lengths of the envelope lines at the sharper corners minimizing the inaccuracy.

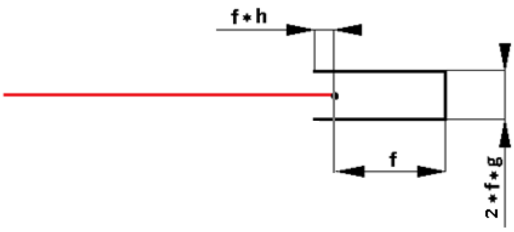


Figure 3. Sketch of the searching loop

The value *f* seen in figure 3 is the resolution parameter that can be defined arbitrarily. This parameter approximately defines the maximal length of the straight lines in the resulting envelope curve. The *g* and *h* values regulate the rate of flattening and backward shifting. Experiences showed that applying these parameters with *g=0,2 and h=0,1* values provided decent results. The resolution parameter *f* is to be chosen considering the size, the proposed accuracy and the manufacturing technology. **(Thesis 1)**

Generally the resulting curves are exported into CAD systems to work out the details of the construction. In order to make the work easier the algorithm performs post processing on the results. This is necessary on account of the numerical nature of the curves. Curves in form of polylines make 3D modelling cumbersome. A single continuous spline curve cannot be laid on the gears contour because of the sharp edges.

During the post processing the algorithm divides the curve into continuous curve segments on which individual spline curves can be laid. In order to perform this automatically the numerically determined curvature function was used (figure 4).

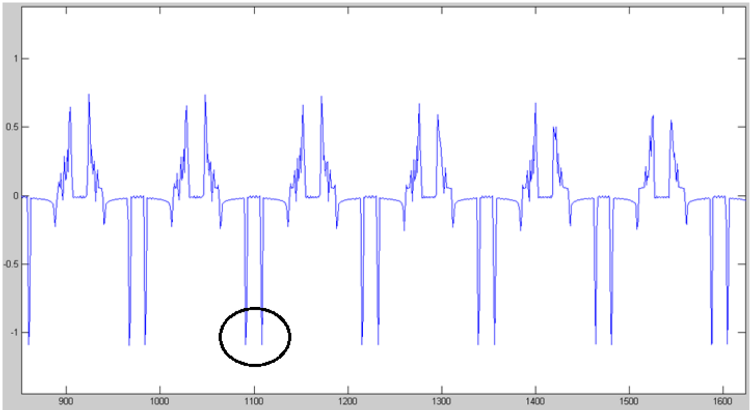


Figure 4. The nature of the numerically determined curvature function.

The characteristic nature of the evolvent profiled gear curves enables the automatic detection and separation of the flank curves, the undercut and bottom land curves and the top land curves. **(Thesis 4)**

The developed algorithm was tested on numerous worked out and implemented examples. This part of the research had the purpose to study applicability of the noncircular gears. One of the most significant developed applications of these machine parts is the special vehicle steering mechanism that uses changing ratio rack and pinion connections. This is described below.

At steering, the front wheels have a special movement. In an *idealized* case, where the vehicle's weight and the tire deformation are negligible, all axes of wheels should determine a section point that defines the vertical cornering axis. This is achieved by the Ackermann geometry. The conventional rack and pinion mechanisms that are often used at this problem always have an error, which correlates with the magnitude of steering. In practice one of the wheels is turned a bit more or less compared to the ideal Ackermann geometry that results a different driving stability. This is called pro-Ackermann or anti-Ackermann.

The major problem with conventional steering mechanisms is that in most cases a previously defined characteristic is not flawlessly implementable. The conventional steering systems consist of several four bar mechanisms. The movements of such mechanisms can be optimized by rearranging the geometry, but does not have an exact solution for most problems. Cam-follower mechanisms or changing ratio gears however make exact solution possible for most cases. Our researches enabled the application of gears or rack-pinion connections with changing ratio. These rarely applied machine parts make it possible to create a steering mechanism with arbitrary characteristics. Not like the conventional solutions, where just one rack is applied, we use separate racks at each wheel (figure 5). These racks are connected to two noncircular gears that are fixed on the steering wheel's axis. These separate changing ratio connections enable arbitrary positioning of each wheel at a specific steering wheel angle, so arbitrary steering characteristic can be implemented.

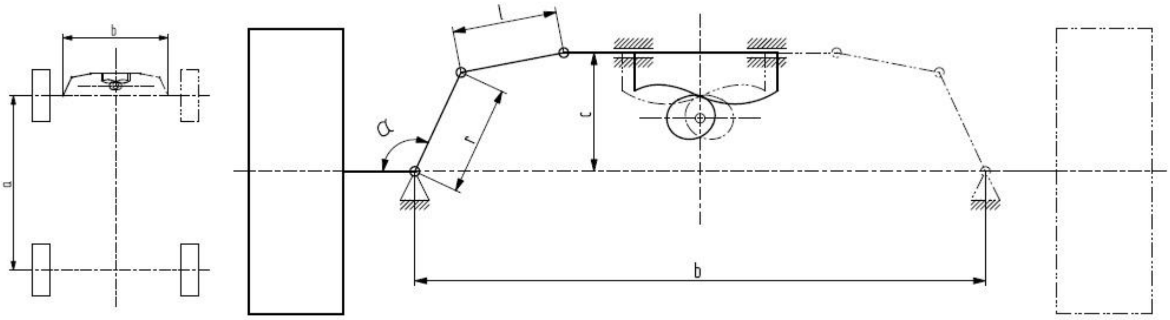


Figure 5. The kinematic sketch of the mechanism

After a special calculation method was developed, we created a simple 2D model of a steering mechanism (figure 6). For *experimenting purposes* later a fully functional model was constructed for the electric racing vehicle of BME Motion team at Formula Student (figure 7). The steering mechanism prototype implements a theoretically error-free Ackermann for a given suspension geometry. In this *experimental* project we did not take the tire deformation and slip angle into account, because the weight of the car is relatively small (~250kg). A bigger lateral force on the vehicle causes slipping at the tires at small slip angles. **(Thesis 2)**

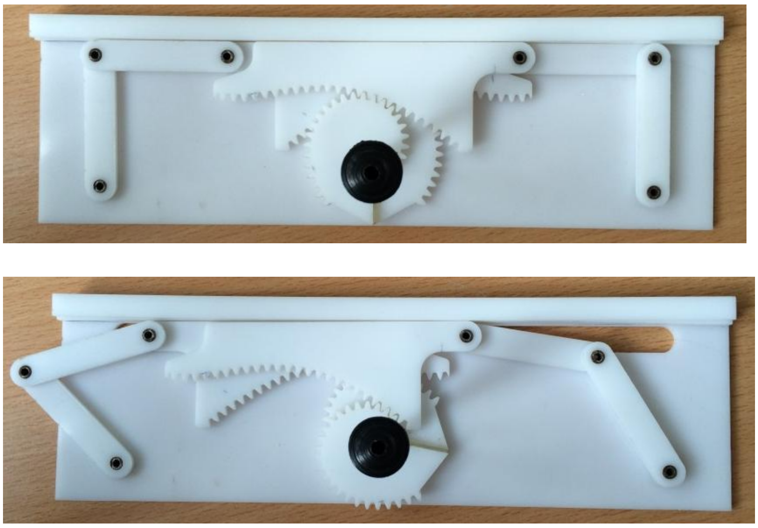


Figure 6. The 2D mockup in initial and deflected position.

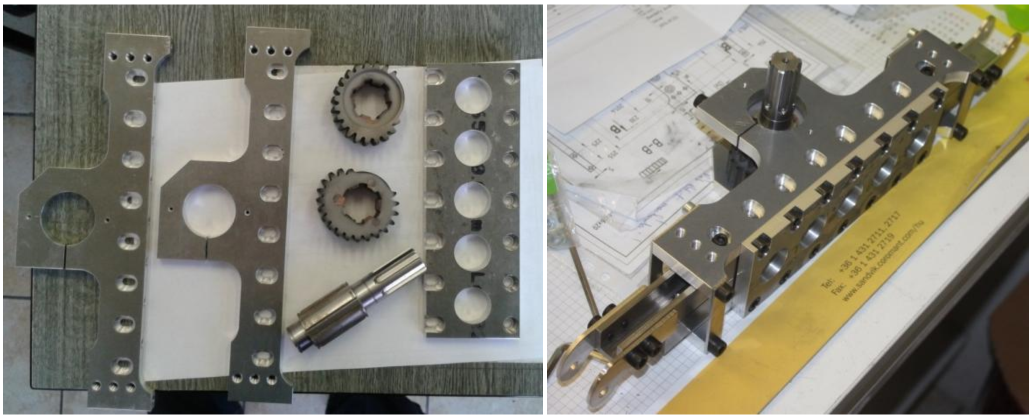


Figure 7. The fully functional prototype of the steering mechanism.

Another example is the application of changing profile shift. Changing ratio gears are noncircular. It means that if they are combined with a conventional cylindrical gear a changing axis distance will be given. Changing axis distance can generate a lot of difficulties and additional costs in the construction. Our goal was to create changing ratio gears that have cylindrical geometry and interlock with conventional cylindrical gears at a constant axis distance. This can be achieved by applying continuously changing profile shift. Profile shift modifies the diameter of the gears, however it does not have any effect on the ratio; regardless of that is constant or changing. The point of our calculation is that some gears with noncircular rolling curves can have circular pitch curve with the help of changing profile shift.

Applying virtual gear generating with a cutting tool gear and changing profile shift makes it possible to create cylindrical gears with noncircular rolling curves (figure 8). **(Thesis 3)**

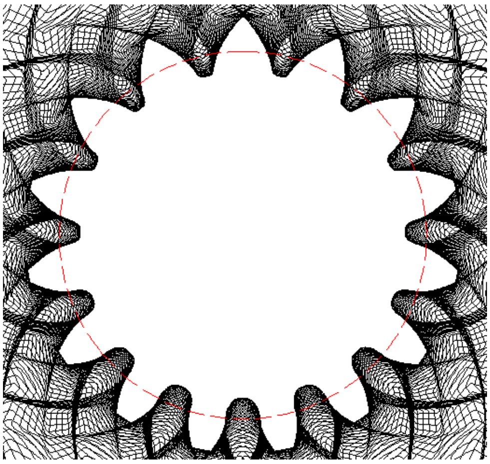


Figure 8. A cylindrical gear with changing profile shift

Besides the presented examples many more had been worked out. These helped us recognizing the weaknesses and insufficient properties of the used algorithm. It turned out that besides the changing ratio the possibility of changing axis distance would be sometimes very useful. In addition we learned that the application of changing profile parameters is crucial because if the ratio changes the loads on the teeth will also change. Changing profile parameters in axial direction would result a 3D gear that could also be useful.

After learning these problems a new approach of calculation was developed. In contrary to the first algorithm we applied the basic law of teething. Using the previously determined straight lines of action the evolvent flank curves can easily be determined. The further curves such as undercut, top land and bottom land curves are determined separately. For calculating the undercut curves two different methods had been worked out. In the first one we applied the motion function of a cylindrical tool gear. In the second, more sophisticated one we used the relative motions of each pitch curve to generate the undercut curves. The rounding of the cutting tool had been neglected in both cases. The advantage of the second method is that it generates nearly optimal undercut curves because it simulates gear generating with the interlocking gear. The result of both methods can be seen in figure 9. **(Thesis 5)**

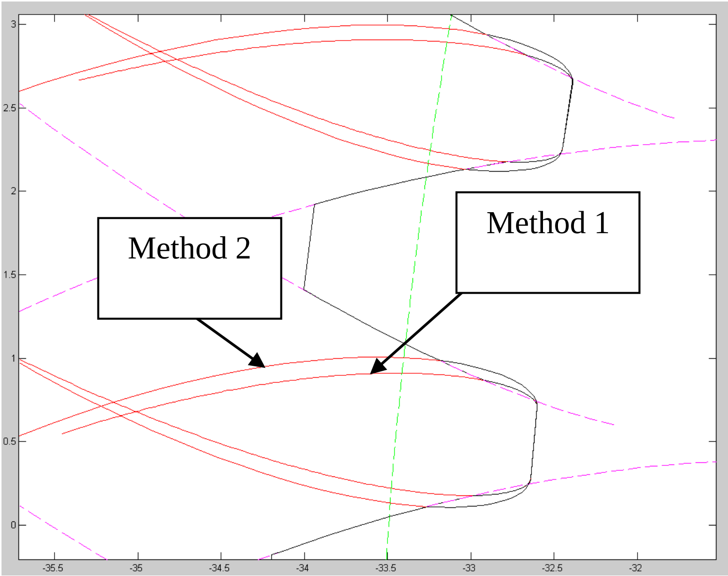


Figure 9. The resulting undercut curves

The main advantages of this newer approach are the increased calculation speed and the possibility to define the tooth parameter individually on each flank side. These options also enable the application of 3D parallel axis gears. In the algorithm a more generalized pitch curve generator module had been worked out that can handle not just changing ratio but changing axis distance as well. Like the first algorithm this also has a decent graphic user interface. For testing the method two pairs of special gears had been generated and manufactured. The first one is a pair of 2D gears with changing ratio, changing axis distance and changing profile parameters. The second example was a pair of 3D gears with changing ratio, changing axis distance and changing profile parameters in both pitch curve direction and axis direction (figure 10).

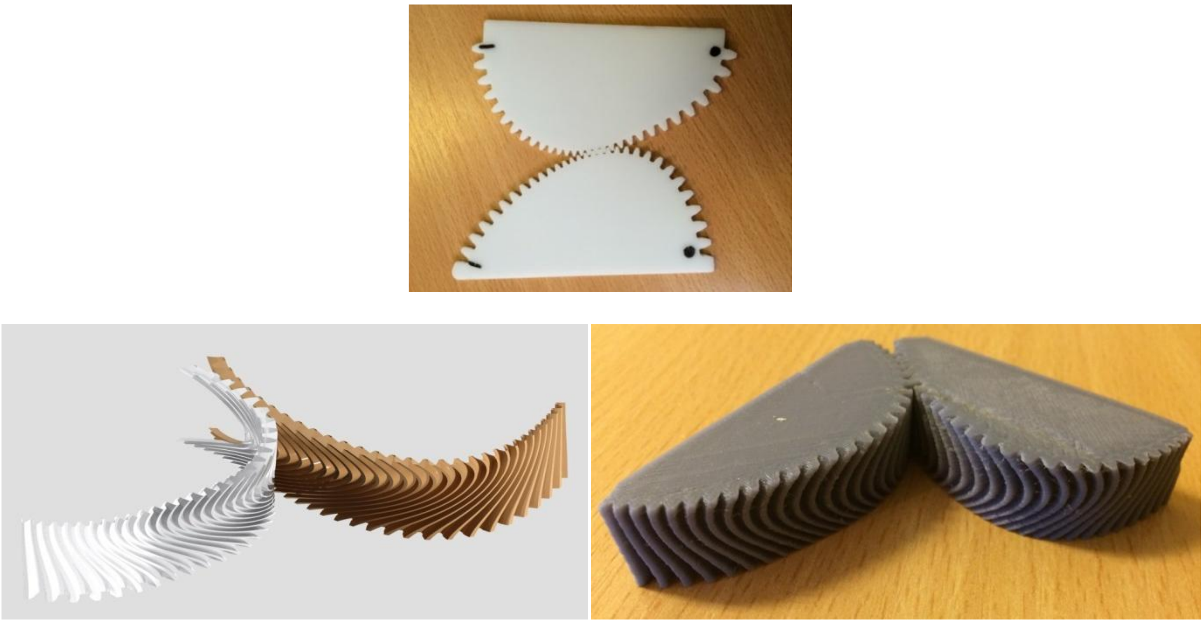
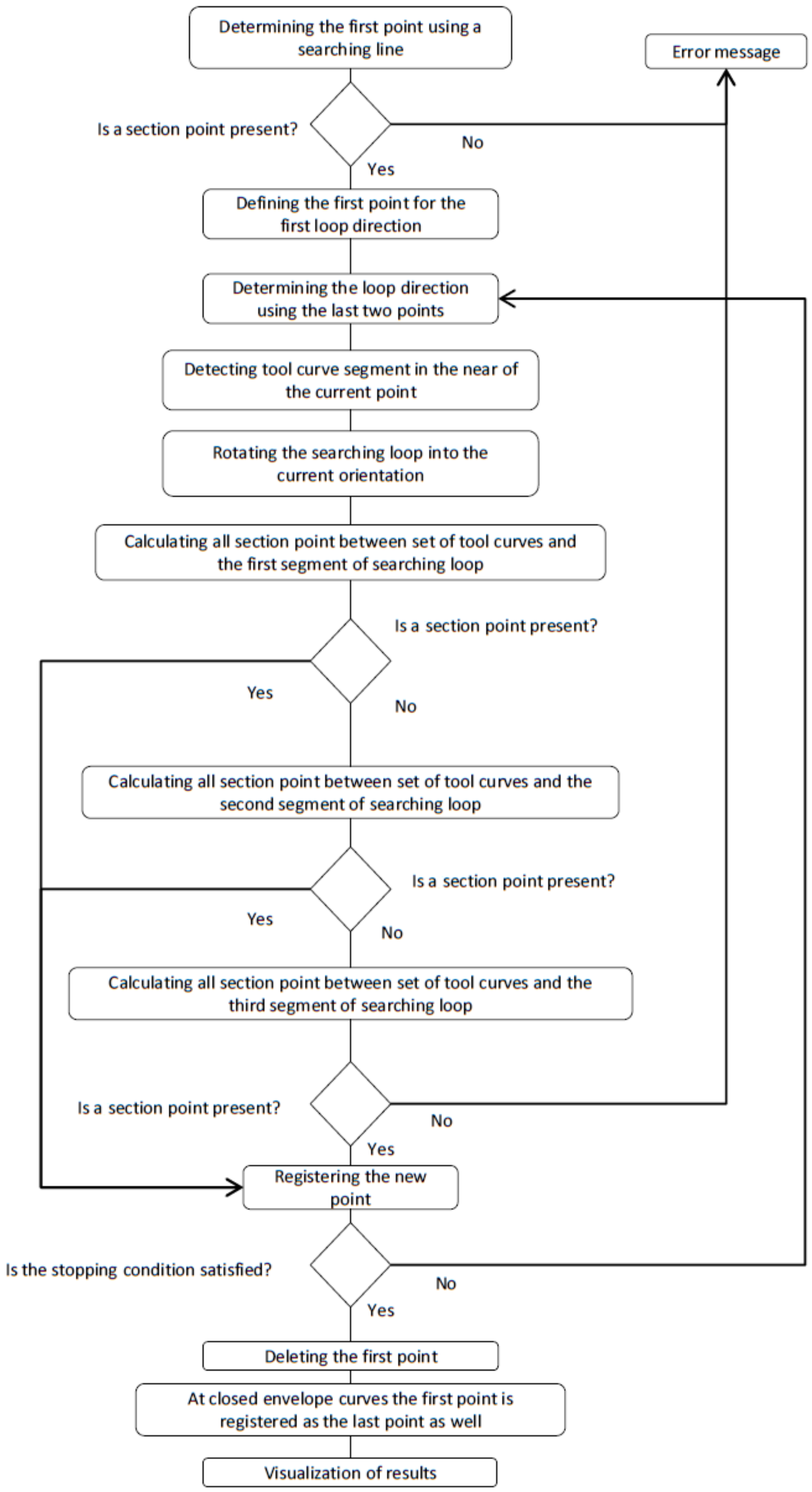


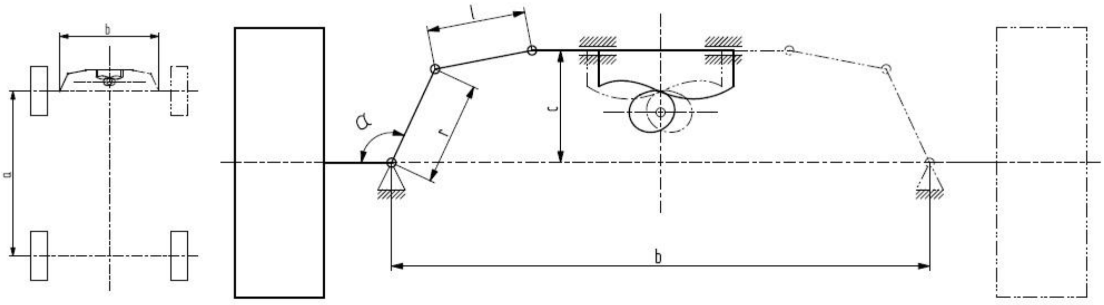
Figure 10. The generated and manufactured gears

In this research only the parallel axis gears had been studied. Changing ratio transmissions however may occur at 3D gears as well, for example bevel gears or worm gears. As optional further development the application of 3D gears is very promising because of the huge leap of recent computer sciences. The complex geometrical calculations can be performed more effectively. Further improvement of the algorithm would be the integration of load and elasticity calculations in order to reduce the time demand of engineering.

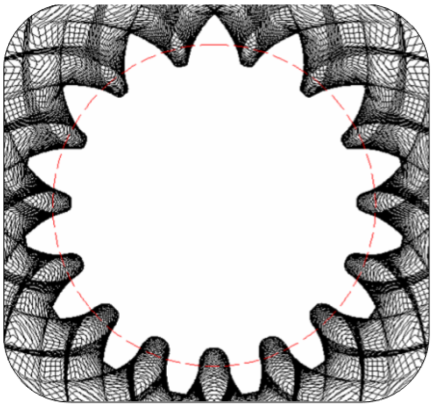
1. **Thesis:** The envelope defined by the transformed curve of the tool that simulates the gear generation process can be determined by the following proceeding. From the current point of the envelope a loop of searching lines detect the next point by using numerical calculations as described in the chart below.



2. **Thesis:** By engineering the currently used vehicle steering mechanisms the desired steering characteristics (the kinematic relation between the turning angles of the wheels) cannot be exactly implemented. It can only be approximated by using optimization. Using the method described in thesis 1 I worked out a steering mechanism that is suitable for the implementation of arbitrary steering characteristics. The mechanism contains two separate changing ratio racks and pinion connections that make arbitrary characteristic possible (as seen in the figure below). [I/2]



3. **Thesis:** Between the limits of undercut and pointed teeth changing profile shift can be applied in order to generate changing ratio cylindrical gears. This process can only work if the gear generation is implemented with cutting tool gear and the interlocking gears geometry equals to it. These special gears can operate with a conventional cylindrical gear providing alternating ratio at a constant axis distance. [I/4]



4. **Thesis:** The numerically generated gear geometry consists of numerous line segments. After importing these into 3D CAD systems the work may be cumbersome. Spline curves can only be laid on continuous line segments. I created an algorithm that can automatically detect and separate each curve segment of the teeth such as flank curves, undercut curves top land and bottom land curves. For this purpose the numerically determined curvature functions were used.

5. **Thesis:** I worked out a method that is capable of generating undercut curves that are close to the optimum as they cut less into the flank curves. For this purpose the relative motion of the pitch curves was used to model gear generation with the interlocking noncircular gear. [II/2]

IV. The applicability of the results

The utilization of this research summarized in this paper has numerous possibilities. The implemented examples that were presented in this work were not worked out just to present the calculations but to explore the application possibilities as well. Based on our experience these special gears can be effectively used to create alternating angular velocities or torques, balancing mechanisms or to implement specific kinematic motions. The created algorithms make possible to create such noncircular gears fast and easily.

V. References

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[2] F. Litvin, A fogaskerék kapcsolás elmélete, Budapest: Mőszaki Könyvkiadó, 1971.

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VI. List of own publication

I. Journal articles

[I/1] **MODELING NONCIRCULAR GEARS WITH COGAL SOFTWARE** Bendefy András, Piros Attila, DESIGN OF MACHINES AND STRUCTURES 3:(1) pp. 5-17. (2013)

[I/2] **ARBITRARY VEHICLE STEERING CHARACTERISTICS WITH CHANGING RATIO RACK AND PINION TRANSMISSION** Bendefy András, Piros Attila, Horák Péter, ADVANCES IN MECHANICAL ENGINEERING*, December 2015; vol. 7, 12: 1687814015619279,* IF.0.58

[I/3] **BURKOLÓGÖRBÉK NUMERIKUS MEGHATÁROZÁSA MÁTRIXOS MÓDSZERREL** Bendefy András, Horák Péter, GÉP 2014/6-7:pp. 9-53 paper 65. (2014)

[I/4] **CYLINDRICAL GEARS WITH CHANGING RATIO** Bendefy András, Horák Péter, Periodika Polytechnica, 1. szám 9870, 2017

II. Conference articles

[I/1] **MODELING GEARS WITH COGAL SOFTWARE** Bendefy András, Bendefy Zoltán, Bachrathy Dániel, Gépészet 2012, (2012), pp. 1-7. ISBN: 978-963-313-055-1

[I/2] **GEAR PAIR GENERATION WITH THE METHOD OF TRANSPOSED LINES OF ACTION** Bendefy András, Horák Péter, 14th International Design Conference, DESIGN 2016, pp. 129-136