ADVANCED NONLINEAR CONTROL OF VEHICLES AND THEIR FORMATIONS

JÁRMŰVEK ÉS FORMÁCIÓIK KORSZERŰ NEMLINEÁRIS IRÁNYÍTÁSA

Ph.D. Thesis Summary

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Research objectives and motivation

The current trend in automotive industry is automation in driving on a wide range of scale, from basic cruise control (i.e. tempomat) to fully automated pilot systems. Almost every recently manufactured vehicle contains certain type of driving assistance system, and may also have a certain level of autonomy, i.e. automated parking functions, pedestrian collision avoidance systems, adaptive lane keeping control. Recent high-end products of the automotive industry are capable of executing more complex maneuvers, i.e. high-speed cruising on highway by employing autopilot control system. Tremendous effort is put by automotive companies and governments into having fully automated ground vehicles in the past few decades. However, automation of ground vehicles requires precise and reliable models of the dynamics of the phenomena arising in the different maneuvers, along with accurate controllers, actuators and sensor instrumentations.

The demand for autonomous ground vehicles is essentially based on human needs. Combining the development of intelligent autonomous vehicles with the modernization of automated highway systems, a more secure and comfortable future transportation can be created. The motivation of this work is to facilitate intelligent transportation by developing advanced vehicle control algorithms in order to perform various tasks, such as optimal path planning, obstacle avoidance and high-speed and coordinated formation control. By the fact that autonomous vehicle control algorithms are able to filter out external disturbance and can be used effectively against dynamic uncertainties, automated systems are able to react to situations that cannot even be detected by humans, moreover they are lacking the possibility of human failures. Therefore, these integrated algorithms provide higher degree of efficiency to the individual users, and can also positively impact on the macro environment.

In general case, the motion of a vehicle can be described by nonlinear models which imposes additional difficulties in designing autonomous vehicle control systems. For instance, nonlinearities can be resulted from tire/ground wheel forces, aerodynamic effects, suspension systems and dynamic load changes. Thus, advanced nonlinear control systems are needed that are able to handle such complexity of vehicle dynamics. Another problem is that controlling an individual vehicle may not be sufficient in some cases. Once the automation of single vehicles is elaborated, the next step would be the formation control of these vehicles, i.e. the control of multiple autonomous (i.e. unmanned) vehicles in order to achieve a desired goal. These may include scenarios such as avoiding accidents in general transportation situations, minimizing the travel times and as well as the risk of traffic jams, employing more efficient energy usage, and decreasing human interaction. Logistics is a typical industrial sector in which a greater degree of cost-effectiveness can be guaranteed by introducing intelligent transportation.
It can be seen that high degree of innovation can be achieved at both household and industrial level by the application of automated vehicles. My objective is to contribute to the development of advanced nonlinear control of autonomous ground vehicles as far as possible, for safer and more efficient transportation.

II. Summary of New Scientific Results

1. Optimal Control of Four In-Wheel Driven Vehicles

I developed a hierarchical vehicle control system for the optimal path planning and control of four-in-wheel driven (4WD) vehicles under state and input constraints taking into account initial state perturbations. The control system is separated into three levels and its main architecture is shown in Figure 1.

The high-level controller (HOCP) is responsible for generating the optimal reference trajectories and control inputs for the two-wheel driven (2WD) vehicle model. The wheel traction and braking forces are considered as inputs of the vehicle. I elaborated two different operational modes as part of the HOCP system. The first mode can handle fixed, predefined paths in which a global time optimal control problem is solved by regulating the steering rate \( (\omega_\delta) \) in the following form:

\[
\min_{x(t), u(t), t_f} t_f + \int_0^{t_f} \omega_\delta^2(t) dt
\]

The second operational mode can accept unspecified, arbitrary tracks, i.e. when only a part of the entire track is known. For each section, a separate local optimum control problem (LOCP) is solved in a receding horizon predictive control fashion (RHPC) with appropriate continuity conditions. The objective is to maximize the
total distance covered by the vehicle during motion as follows:

\[
\min_{x(\cdot), u(\cdot)} k_1 \int_{t_i}^{t_i+\Delta t} \omega_\delta^2(t) dt - k_2 (\Gamma^\Delta_l + \Gamma^\Delta_r)
\]

This strategy generates time-suboptimal reference signals for the vehicle.

I developed a novel algorithm for the middle subsystem that is responsible for the optimal distribution of the 2WD high-level control forces for the more realistic 4WD vehicle. The method minimizes the deviations from the total occurring forces and moments between the two-wheel and the four-wheel vehicle model in a least square sense. The aim is to find an optimal control allocation of the 4WD wheel forces such that the resulting motion of the center of gravity is similar to the one of the 2WD’s. The main advantage of this approach is that the distributed longitudinal forces can be used as nominal controls for the lower control level.

![Graph](image)

Figure 2: Time optimal lane change trajectory of the 4WD vehicle with MPC integral control and initial perturbation. The complete track is known prior and the high level optimal reference signals are computed by the HOCP system.

Since the initial state of the (real) 4WD vehicle typically differs from those of the 2WD optimal solution, hence a low-level real time control is needed at the third stage to reduce the effect of differences in the initial states. For this purpose I designed a discrete time model predictive integral controller (MPC) which intends to eliminate the errors at the end of each horizon according to the following optimal
control strategy:

\[ \min_{\delta u(\cdot)} \sum_{k=1}^{N-1} \| e_k - \delta y_k \|^2_Q + \sum_{k=0}^{N-1} \| \delta u_k \|^2_R, \quad Q \geq 0, \quad R > 0 \]

\[ Q_N (e_N - \delta y_N) = 0 \]

The above optimization problem minimizes the state perturbations of the linear time-varying 4WD model in closed loop. I provided the solution of the MPC control problem in analytical form by using Lagrange multiplier technique which gives possibility for real-time application.

Figure 3: Sub-optimal trajectory of the 4WD vehicle with MPC integral control and initial perturbation. The arbitrary track is known in piecewise sections and the high level optimal reference signals are computed by the RHPC system.

I showed by simulation that the proposed three-level control system is suitable for the optimal control of four in-wheel drive vehicles (Fig. 2 and Fig. 3). The results of the following Thesis Group are discussed in Chapter 3 of the thesis.

**Thesis Group 1** I developed a three-level hierarchical control framework and designed algorithms for the optimal control and trajectory planning of four independently actuated (4WD) ground vehicles taking into account state and input constraints. The high-level controller computes the global and local optimal control of the vehicle for fixed and arbitrary paths. The middle level controller performs optimal 4WD force distribution. The low-level controller minimizes the deviations from the trajectories under initial state perturbations in closed loop.

The results of the thesis group are published in conference papers [G14, G13, G11] and journals [G5, G3, G2].
Thesis 1.1  I proposed a high-level control system (HOCP) that provides the global time optimal and local time-suboptimal solution of two-in-wheel driven vehicles (2WD). The method generates optimal command signals by solving the dynamic nonlinear optimal control problem based on direct discretization and multiple shooting method either on fixed paths or unknown paths with receding horizon control scheme (RHPC). I elaborated a discrete low-level model predictive (MPC) integral controller of the linearized 4WD vehicle under initial state perturbations using the high-level optimal reference signals. The algorithm optimizes the perturbations with analytically solvable end constraints and ensures optimal trajectory tracking of the nonlinear 4WD vehicle.

Thesis 1.2  I provided a method for converting the high-level 2WD optimal solution to the optimal control of the realistic four in-wheel-driven vehicles. The algorithm employs linearly constrained quadratic optimization (LCQP) and assures similar motion of the center of mass points of both models (2WD/4WD) by producing optimal allocation of the longitudinal wheel forces.

2. Tracking Control of Autonomous Vehicles using Robotic Formalism

The scope of my work is to develop a tracking control system for unmanned ground vehicles which is capable of decreasing the position and orientation errors during high-speed maneuvers by preserving the dominant prescribed velocity using state estimation. I modeled the vehicle as a multibody mechanical system with moving base, and derived its dynamic model by using robotic formalism with Appell’s method and the Gibbs function. I extended the model to take into account dynamic load distribution, nonlinear longitudinal and lateral tire forces, aerodynamic resistance and load changes by altering the center of mass point of the vehicle.

The total 16 degree of freedom (DoF) model of the vehicle is constructed by four, pair-wise identical branches and consists of 21 rigid bodies as illustrated in Figure 4. The vehicle has 4 driving wheels, 2 steering columns, 4 suspensions and a 6 DoF chassis which is considered as the base of the moving platform.

I assumed planar motion of the vehicle, therefore I introduced second order kinematic constraints by Lagrange multiplier method that assure zero vertical accelerations at the four tire/ground contact points. Consequently, I prescribed the dynamic model of the robot vehicle in the moving base (K1) by the following form:

\[
\begin{pmatrix}
\dot{z} \\
\lambda
\end{pmatrix} = \begin{bmatrix}
M & J^T \\
J & 0
\end{bmatrix}^{-1} \begin{bmatrix}
\tau + \tau_{\text{load}} + \tau_{\text{ext}} - h \\
0
\end{bmatrix}, \quad \ddot{z} = \begin{bmatrix}
a^T_1 & e^T_1 & \dot{q}^T
\end{bmatrix}^T
\]

However, in practical situations, the center of gravity (CoG) usually differs from the center of symmetry (origin of K1) of the vehicle, and omitting this effect can cause serious errors in path tracking by causing a drift during acceleration or...
cornering. The reason for this is that additional moments appear about axes of the frame attached to the CoG point. The most significant body of the above dynamic model in terms of mass and inertia is the chassis. Therefore, the center of mass point of the chassis (expressed in $K_1$) greatly affects the load distribution and the motion of the vehicle as well.

In order to solve the tracking problem, I proposed a two-level hierarchical control framework (Fig. 5). The low-level controller (LLC) computes the generalized torques of the vehicle and can be commanded to ensure a desired velocity and steering profile using PD-type active velocity, steering and PID-type active suspension control. The latter subsystem is employed to balance the four suspensions of the vehicle at an average steady-state level, thus minimizing the rolling and pitching effect of the chassis during high-speed maneuvers. Derivatives of the signals needed in the PID algorithms are provided by differentiating and smoothing filters based on fictitious PI control subsystems and linear process models. Furthermore, I showed that the LLC system cannot solely compensate the position and orientation tracking errors occurring due to the load variations caused by the unknown CoG point.

The high-level controller (HLC) uses the kinematic model of the vehicle to generate the reference position and orientation as well as the nominal speed and steering profiles. I formulated the path tracking task as a finite horizon linear quadratic (LQR) optimal control problem as follows:

$$
\min_{\delta u} \frac{1}{2} \delta \eta_N^T \tilde{Q}_N \delta \eta_N + \frac{1}{2} \sum_{i=0}^{N-1} \delta \eta_i^T Q_i \delta \eta_i + \delta u_i^T R_i \delta u_i
$$

The solution of the above LQR problem is a time-varying state feedback that minimizes the deviations from the reference signals and ensures that locally stable
trajectory tracking is achieved for the linearized (LTV) vehicle model about the reference trajectories.

Since the HLC employs full state feedback, the positions and orientation needed to be estimated accurately. For state estimation, I applied a non-steady state two-stage Kalman Filter based on kinematic equations with inertial (IMU) and two-antenna GPS sensors. The former measures the biased longitudinal and lateral acceleration as well as the yaw rate. The latter is used in the velocity and orientation measurements updates to further decrease the estimation error. Since the GPS and IMU systems have different refresh rates and measurement inaccuracies, both of them are used as auxiliary measures to estimate the vehicle’s state.

I verified the efficiency and robustness of the proposed control system on the nonlinear robotic model of the vehicle by a series of simulations (Fig. 6). I showed that during aggressive maneuvers (v > 30 m/s, ay = 8 m/s²) and under ±12 cm longitudinal and ±18 cm lateral change of the center of mass point, the maximum lateral path error remains less than 0.5 m. The results of the following Thesis Group are discussed in Chapter 4 of the thesis.

**Thesis Group 2** I developed a two-level autonomous vehicle control system for the high-speed (> 30 m/s) tracking control of the unmanned ground vehicles taking into account load changes caused by the unknown center of mass point based on the realistic 16 degree of freedom (DoF) multibody vehicle model. The proposed vehicle control system is suitable for online high-speed position and orientation tracking with desired speed profile with low errors (< 50 cm) if load variations are present, as well as minimizing the effect of rolling and pitching during the maneuver.

The results of the thesis group are published in conference papers [G12, G10, G9,
Figure 6: Vehicle states and control inputs provided by the two-level control system during a high-speed, overlapping longitudinal and lateral accelerating maneuver.

**Thesis 2.1** I proposed an algorithm for obtaining the dynamic model of a multibody tree-structured mechanical system with moving base using Appell’s method and the Gibbs-function. The algorithm is applied to efficiently compute the dynamic model of a 16 DoF model of the vehicle taking into account nonlinear external wheel forces, dynamic load distribution and suspension system.

**Thesis 2.2** I elaborated a hierarchical framework for minimizing the position, orientation and velocity tracking error of the multibody vehicle in presence of load changes incorporating state estimation techniques. The control system comprises of two levels: a high-level (HLC) and a low-level (LLC) control system and the states are estimated with a two-stage Kalman Filter (KF) using realistic GPS and inertial (IMU) sensory information. I developed active suspension, speed and steering controller as part of the LLC system which ensures that the vehicle maintains the desired velocity and steering profile by employing PID-type controllers based on selectable error differential equations. I proposed a finite horizon linear time-varying quadratic optimal controller for the HLC based on the kinematic model of the vehicle that locally stabilizes the closed loop system about the perturbations from
the reference signals arising from the unknown center of mass point. The simulation results show small maximum lateral path deviation (< 0.5 m) for aggressive motion maneuvers and significant load changes near the physical limits of a compact vehicle.

3. Nonlinear Formation Control of Unmanned Ground Vehicles

I proposed two different nonlinear formation control algorithms for the coordinated control of unmanned ground vehicles (UGVs). The first method employs adaptive control with leader-follower approach as shown in Figure 7. The method is based on the kinematic (two-dimensional) model of the vehicle, that is, the input signals are considered to be the linear and angular velocities, and the outputs are the position and orientation. I assumed that the lateral velocity of the vehicle (in the moving coordinate system) can not be influenced directly, thus taking into account the typical underactuation of ground vehicles. However, indirect modification is possible by changing the angular velocity inputs, which can be related to the steering system of the vehicle.

I established the formation by prescribing the distance and the relative bearing angle between the leader and the followers. The advantage of the adaptive system is that unknown, time-varying nonlinear functions can be estimated in a decentralized setting. I assumed that the velocity information of the leader are unknown to the followers. I developed a nonlinear adaptive neural network (NN) based controller whose parameters are tuned by using online adaptation laws. The unknown nonlinear functions depending on the leader’s velocity are estimated by feedforward NN network. As a result, the followers are capable of tracking a specified position and orientation relative to the leader with increased robustness.

Figure 7: Leader-follower configuration on the $x$–$y$ plane
I proved that control law

\[ \ddot{c}_i = B_i^{-1} \left[ f_{id} + k_{iN} e_i - \dot{W}_i^T \sigma (Y_i^T \ddot{x}) \right] \]

and adaptive tuning laws

\[ \dot{V}_i = -G_i \left( \ddot{x} e_i^T \dot{W}_i^T \ddot{\sigma}' + k_{Vi} \dot{V}_i \right) \]
\[ \dot{W}_i = -F_i \left[ (\dot{\sigma} - \dot{\sigma}' \dot{V}_i^T \ddot{x}) e_i^T + k_{Wi} \dot{W}_i \right] \]

where \( F_i > 0, G_i > 0, k_{Wi} > 0, k_{Vi} > 0 \) with parameter choice of

\[ k_{iN} = k_{1i} + 2i \| \ddot{\ddot{x}} \|_F^2 + 3i \| \dot{\sigma}' \dot{V}_i^T \ddot{x} \|_F^2, \quad k_{1i}, k_{2i}, k_{3i} > 0 \]

render all closed loop error signals SGUUB (semi-globally ultimately uniformly bounded).

The above formation control system is based on kinematic equations only, however in the case of high-speed maneuvers the inertial properties of the vehicle cannot be ignored. Therefore, I developed another control system for the linear time-varying (LPV) model of UGVs based on constrained multibody interpretation.

In analytical mechanics, the motion of a body can be influenced due to acting constraint forces. The connection between analytical mechanics and formation
control is that the formation itself can be regarded as a multibody system established by constraint forces. Conversely, these constraint forces can be utilized to maintain and control the formation. However, the constraint functions need to be stabilized by appropriate feedback law. For this purpose, I introduced holonomic functions that describe the vehicle’s behavior with respect to other group members and treat the mechanical constraints in analytical setting. These functions can be easily configured, and consequently, a more flexible form of application is resulted in cases where the formation specifications are dynamically changing.

I developed a decentralized control algorithm which computes the constraint forces separately for each vehicle based on the following control law:

\[
\tau_{ci} = -\sum_{k \in A_i} \sum_{j \in B_k} W_{ki}^T \left( W_k Q_i W_k^T \right)^+ \left[ W_k P_i z_{ij} + K_{d,ki} \dot{C}_{ki} + K_{p,ki} C_{ki} \right]
\]

Using the above forces as actuator signals, the closed loop error signals can be rendered globally exponentially stable.

I verified the effectiveness of the two vehicle formation control systems during similar high-speed maneuvers as presented in Thesis Group 2, see (Fig. 8 and Fig. 9). Since these methods are not able to handle actuator signals limitations, I developed a guidance subsystem which provides slowly-varying, smooth reference signals that lead the vehicles from the initial states onto the desired formation positions, therefore minimizing the path errors together with the control efforts. The results of the following Thesis Group are discussed in Chapter 5 of the thesis.

**Thesis Group 3** I proposed two novel nonlinear formation control framework that can establish and coordinate a group of identical unmanned ground vehicles (UGVs) in formation along prescribed trajectories. Each method employs leader-follower structure and accurately capable of handling high-speed motion maneuvers (>30 m/s) while guaranteeing the stability of all closed loop error signals. The first method utilizes a kinematics based adaptive controller with online neural network approximation which is capable of handling uncertainties. The second approach is based on multibody interpretation and takes into account the vehicle dynamics. The coordinated motion is formed by imposing analytical constraint forces on the group of system.

The results of the thesis group are published in conference papers [G15, G6] and journals [G1].

**Thesis 3.1** I proposed an adaptive control law for formation control of under-actuated autonomous ground vehicle based on the leader-follower approach and kinematics equations of the vehicle. The follower vehicles have limited knowledge about the leader’s states. The unknown term containing the velocity information of
Figure 9: Formation evolution with dynamically changing constraint forces and vehicle dynamics (to scale)

The leader is estimated using neural networks with online adaptive weight tuning laws. I proved that in the frame of the decentralized formation control algorithm the closed loop error signals are semi-globally ultimately uniformly bounded. By appropriate choice of the control parameters, the tracking errors can be made small.

**Thesis 3.2** I developed a formation control system for the linear input-affine model of the vehicle based on analytical mechanics and multibody interpretation. I elaborated the necessary modification of the theory and illustrated its applicability for UGVs taking into account both underactuation and inertial properties. The coordinated motion is achieved by introducing holonomic constraint functions that describe the interaction between the vehicles. These functions are interpreted as constraint forces and the decentralized control law is determined by Lagrange multiplier method. The control system is highly configurable through the constraint functions and can be applied for various scenarios.

### III. Applications and Possible Improvements

The previously presented thesis groups are closely related to practical problems. The main motivation is the development of nonlinear vehicle control system for
autonomous transportation.

The results of Thesis Group 1 are more suitable to implement on electric cars equipped with in-wheel motors, however the control approach does not exclude other vehicle propulsion types such as hydrogen, hybrid-hydrogen or ordinary combustion engines. Furthermore, the time-optimal control strategy can be employed as part of a Collision Avoidance System based on the optimal trajectories that are determined offline and stored in a database. For a given situation of lane change or collision avoidance the nearest solution in the database can be selected, perhaps interpolation can be involved, and the new control trajectory, taking into account initial perturbations, can be performed online by using the MPC controller.

The results of Thesis Group 2 can be applied for high-speed autonomous vehicle control, i.e. in highway environment. The system provides robustness despite of load changes, such as when passengers are seated in the car, or if large longitudinal and lateral accelerations occur during maneuvering. It is possible to integrate the optimal path planner module of Thesis Group 1 into the HLC in order to optimize reference signals for the vehicle and operate more efficiently.

Thesis Group 3 can be considered as an extension of the previous two at a higher abstraction level in which coordinated control of vehicle formations can be realized. According to the results, a stable decentralized control system can be implemented for the high-speed tracking control of a leading vehicle such that the following vehicles are regulated in relative formation. The presented methods provide robustness against unknown dynamical effects, and possess a high degree of design flexibility. Among other things, automated transportation is an area where the new results can be used effectively, i.e. autonomous control of trucks moving in convoy.

Automated vehicles can offer increased traffic safety, since these systems are able to respond quicker in situations which the driver may not be able to detect, in addition the possibility of human errors can be reduced. Further directions of development may include the following:

- The optimal control framework presented in Thesis Group 1 can be advantageous if not only initial disturbances are present. Including state estimation at this stage would lead to increased robustness.

- The computation of the optimization algorithm could be improved in order to plan the optimal reference trajectory generation in real-time.

- The vehicle model using robotic formalism could be extended (Thesis Group 2). Doing so would naturally result in a more accurate model.

- The position and orientation estimation could undergo improvement. This would require either equipping more accurate sensors, or increasing the
number of sensors, or developing more sophisticated algorithms that can cope with large inaccuracies in the measurements.

- Combining the two formation control method described in Thesis Group 3 would be beneficial. It would require minimal communication between vehicles, the robustness against uncertainties could be improved and also it would remain flexible to designers due to the large set of applicable constraint functions.
Publication List

Journal Papers


Conference Papers


