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Booklet of PhD Thesis

Friction Effects in Mechanical System Dynamics and Control

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Introduction

Modeling of friction is one of the most difficult tasks in mechanics. This dissipation effect can have strong influence on the system dynamics and can cause complex dynamic behavior. The effect of friction is especially important in positioning automation problems. The applications subjected to position control often demand high accuracy and fast operation at the same time. The necessary performance fulfilling the above mentioned requirements is limited by not only the effect of friction but also by the digital nature of the applied motion controllers.

This research was motivated by the recognition of the interplay of the effect of these phenomena on the system dynamics. The later presented experimental results show that the response of these systems are often non-intuitive and unexpected. The presented research focuses on the understanding of the special characteristics of the dynamics induced by friction and sampling.

Thesis outline

The present thesis discusses the effects of friction on the dynamics of controlled mechanical systems. The corresponding results are presented in four chapters. In order to analyze the interplay between the friction effects and the controlled dynamics of mechanical systems, an experimental setup was designed and assembled. The modeling of this experimental setup is presented in Chapter 2 which includes detailed information on the mechanical, electric and control assumptions. During the measurements, a special concave shape vibration envelope was observed in the time history. Chapter 2 also focuses on the understanding of this special characteristic of the damped motion, and it aims to identify the simplest representative dynamic model. Later, this model is called as the sampled-data sliding-oscillator model. The system of equations describing the dynamics of the controlled mechanical system is solved in a series of analytical steps.

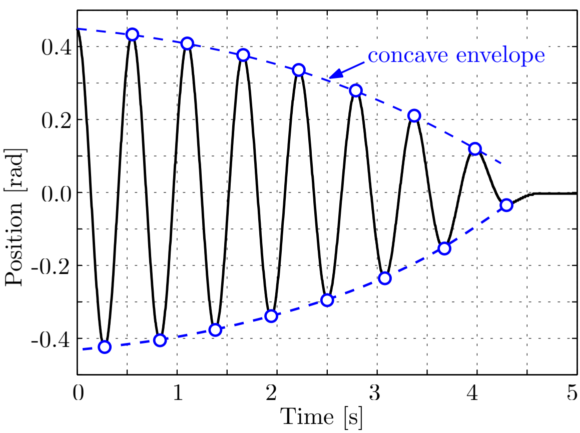
Chapter 3 deals with the examination of the dynamics of the sampled-data sliding-oscillator described above. In order to understand the effect of dry friction, first, a reference model is considered and analyzed without friction. It is followed by the analysis of the effect of dry friction on the dynamic behavior of the sampled-data sliding-oscillator model. First, some results of the corresponding literature are reproduced using passivity analysis. Then, energetic considerations are combined with classical describing function analysis to obtain an improved stability condition. Later, the stability properties of the sampled-data sliding oscillator model are analyzed by determining the consecutive vibration peaks. At the end of this chapter, the stability properties of the sampled-data sliding-oscillator model are further analyzed. The key element of this analysis is the determination of the dominant vibration frequency of the controlled motion. This is used to show how the dry friction can stabilize the otherwise unstable, digitally controlled motion.

In Chapter 4, the analysis of an effective continuous-time system model is presented. This continuous-time system model is also used for the analysis of the stability properties of the sampled-data sliding-oscillator model. In this analysis the important element is the introduction of an unstable system model with negative viscous damping.

Finally, in Chapter 5, the sampled-data sliding-oscillator model is extended by considering viscous damping and dry friction. This makes it possible to analyze the combined stabilization effect of both of the two main physical dissipation mechanisms.

Thesis 1

A special type of vibrations with concave envelope was identified during positioning experiments, which has very different decay characteristics compared to that of the classical continuous-time systems damped by viscous and/or dry friction. This concave vibration envelope is illustrated in the figure below. Based on detailed modeling considerations and analysis, the minimal number of representative system parameters were identified which are necessary to reproduce the experimentally observed phenomenon by simulation.



In sampled-data linear systems, the instability due to sampling results in vibrations with concave envelope when Coulomb friction stabilizes the motion. The observed concave envelope vibrations are reproduced by using the following model of a sampled-data sliding oscillator

$$m\ddot{x}(t) + f_{\mathrm{C}}\,\mathrm{sgn}\,(\dot{x}(t)) = -k_{\mathrm{p}}x_j\,,\;\; t \in [t_j, t_j + t_{\mathrm{s}})\,,\; t_j = jt_{\mathrm{s}}\,,\; j = 0, 1, 2\ldots,$$

where x represents the generalized coordinate as a function of time t corresponding to the modeled degree-of-freedom, m is the corresponding generalized mass, and f_{C} denotes the magnitude of the generalized dry friction force. In addition, k_{p} is the proportional control gain, t_j denotes the jth sampling instant, t_{s} is the sampling time, and x_j denotes the sampled position at the beginning of the jth sampling interval.

- Related publications: [1], [2], [3], [4]

Thesis 2

A closed form discrete simulator was proposed for the solution of the initial value problem of the sampled-data sliding oscillator. The exact solution obtained by this simulator was used to perform semi-analytical stability analysis without the need of the numerical integration of the non-smooth dynamic equation.

Consider the discrete-time model of the sampled-data sliding oscillator in the form $\mathbf{z}_{j+1} = \mathbf{W}(t_{\mathrm{s}})\mathbf{z}_j - \mathbf{w}(t_{\mathrm{s}})\,\mathrm{sgn}\,(\dot{x}(t))$ with

$$\mathbf{W}(t_{\mathrm{s}}) = \begin{bmatrix} 1-p/2 & t_{\mathrm{s}} \\ -p/t_{\mathrm{s}} & 1 \end{bmatrix},\; \mathbf{z}_j = \begin{bmatrix} x_j \\ v_j \end{bmatrix},\; \mathbf{w}(t_{\mathrm{s}}) = \begin{bmatrix} \psi/2 \\ \psi/t_{\mathrm{s}} \end{bmatrix},\; \text{and } p = \frac{k_{\mathrm{p}}t_{\mathrm{s}}^2}{m},\; \psi = \frac{f_{\mathrm{C}}t_{\mathrm{s}}^2}{m},$$

where x_j and v_j denote the sampled position and velocity, respectively.

The steps of this stability analysis method are summarized below by considering

1. **Take n_1 steps with $\mathbf{z}_{j+1} = \mathbf{W}(t_{\mathrm{s}})\mathbf{z}_j - \mathbf{w}(t_{\mathrm{s}})\,\mathrm{sgn}\,(v_j)$ until reaching the critical state variables $\mathbf{z}_k$ before a vibration peak occurs. In the special case of $v_j = 0$, when $\mathrm{sgn}\,(v_j)$ is not applicable, take the first step with $\mathbf{z}_{j+1} = \mathbf{W}(t_{\mathrm{s}})\mathbf{z}_j + \mathbf{w}(t_{\mathrm{s}})\,\mathrm{sgn}\,(x_j)$.**
2. **Derive the switching parameter δ_k that is determined in closed form by using the kth discrete states as**

$$\delta_k = \frac{mv_k}{t_{\mathrm{s}}(k_{\mathrm{p}}x_k + f_{\mathrm{C}}\,\mathrm{sgn}\,(v_k))}\,.$$

3. **Create the modified system matrix $\mathbf{W}(\delta_k t_s)$ and the modified shifting vector $\mathbf{w}(\delta_k t_s)$ with the fraction of sampling time $\delta_k t_s$.**
4. **Evaluate an internal step to find the vibration peak as follows $\mathbf{z}_{peak} = \mathbf{W}(\delta_k t_s)\mathbf{z}_k - \mathbf{w}(\delta_k t_s)\,\mathrm{sgn}\,(v_k)$.**
5. **Create the modified system matrix $\mathbf{W}((1-\delta_k)t_s)$ and the modified shifting vector $\mathbf{w}((1-\delta_k)t_s)$ with the fraction of sampling time $(1-\delta_k)t_s$ in order to synchronize the solution with the applied uniform sampling.**
6. **Evaluate the correction step from the vibration peak as follows $\mathbf{z}_{k+1} = \mathbf{W}((1-\delta_k)t_s)\mathbf{z}_{peak} + \mathbf{w}((1-\delta_k)t_s)\,\mathrm{sgn}\,(v_k)$.**
7. **Take n_2 steps with $\mathbf{z}_{j+1} = \mathbf{W}(t_s)\mathbf{z}_j + \mathbf{w}(t_s)\,\mathrm{sgn}\,(v_j)$ starting with $j = k+1$ until reaching the next critical state variables before the next vibration peak occurs. In the special case of $v_j = 0$, when $\mathrm{sgn}\,(v_j)$ is not applicable, take the first step with $\mathbf{z}_{j+1} = \mathbf{W}(t_s)\mathbf{z}_j - \mathbf{w}(t_s)\,\mathrm{sgn}\,(x_j)$.**
8. **Repeat the 2nd and 3rd steps to reach a new $\mathbf{z}_{peak}$.**
9. **Check whether the magnitudes, $x_{peak} = \mathrm{abs}(\mathbf{z}_{peak})$, of two consecutive vibration peaks are decreasing or increasing to identify stability or instability, respectively.**

- Related publications: [5]

Thesis 3

By considering the velocity dependent effective viscous damping coefficient obtained through describing function analysis, and by providing that the energy dissipated by this damping term and the energy generated due to sampling are balanced, the critical vibration amplitudes of velocity were derived which describe the finite domain of attraction of the stable desired position.

The critical initial velocity $v_{0,cr}$ at which an unstable limit cycle exists is

$$v_{0,cr} = \frac{8 f_C}{\pi k_p} \frac{1}{t_s}$$

with $x_0 = 0$, and where f_C denotes the magnitude of the generalized dry friction force, k_p is the proportional control gain, and t_s is the sampling time of the applied proportional controller.

If the motion starts from $x_0 = 0$ and $v_0 < v_{0,\mathrm{cr}}$, the systems will reach the desired position in a stable way. When $v_0 > v_{0,\mathrm{cr}}$, the vibrations increase.
• Related publications: [5]

Thesis 4

When the motion of the sampled-data sliding oscillator starts from a non-zero initial position with zero velocity, and the higher harmonics due to sampling are neglected, the critical initial position at which the controlled system has an unstable periodic motion (limit cycle) is

$$x_{0,\mathrm{cr}} = \frac{\rho^{\pi/\vartheta}+1}{\rho^{\pi/\vartheta}-1}\,\frac{\psi}{p}, \ \textbf{with}\ \rho = \sqrt{\frac{p+2}{2}}\ \textbf{and}\ \tan(\vartheta) = \frac{\sqrt{p(16-p)}}{4-p}.$$

In these expressions, the dimensionless parameters $p = k_\mathrm{p} t_\mathrm{s}^2/m$ and $\psi = f_\mathrm{C} t_\mathrm{s}^2/m$ include the dynamic and control parameters.
• Related publications: [5], [6]

Thesis 5

The general form of the equation of motion of an uncontrolled second-order oscillator with Coulomb friction and negative viscous damping is

$$\ddot{x}(t) - 2\zeta\omega_\mathrm{n}\dot{x}(t) + \omega_\mathrm{n}^2 x(t) = \begin{cases} -f_0^+\omega_\mathrm{n}^2, & \dot{x}(t) > 0 \\ f_0^-\omega_\mathrm{n}^2, & \dot{x}(t) < 0 \end{cases}.$$

In this equation, ω_n is the undamped natural angular frequency and ζ is the damping ratio that corresponds to the considered negative damping. In addition, f_0 represents the static deformation under the fictitious load of C, thus $f_0^\pm = C^\pm/k$, where parameter k is the generalized stiffness.

In case of the second-order oscillator with Coulomb friction and negative viscous damping, the critical initial positions of the mass at which unstable limit cycle exists are

$$x_{0,\mathrm{cr}} = \frac{f_0^-\varepsilon + f_0^+}{\varepsilon - 1} \quad \textbf{or} \quad x_{1,\mathrm{cr}} = -\frac{f_0^+\varepsilon + f_0^-}{\varepsilon - 1},$$

where $\varepsilon = \exp(\zeta\pi/\sqrt{1-\zeta^2})$. When $f_0^- = f_0^+ = f_0$, the Coulomb friction model is symmetric, $x_{1,\mathrm{cr}} = -x_{0,\mathrm{cr}}$, and the critical initial position is

$$x_{0,\mathrm{cr}} = f_0 \frac{\varepsilon + 1}{\varepsilon - 1} = f_0 \coth\left(\frac{\pi}{2}\frac{\zeta}{\sqrt{1-\zeta^2}}\right).$$

- Related publications: [6]

Thesis 6

The detailed dynamic analysis of the sampled-data sliding oscillator, where the viscous damping effects are also considered, results in the stability chart shown in the figure below. The corresponding dynamic model has the characteristic multipliers

$$z_{1,2} = \frac{1}{2}(\varepsilon(1-P) - P\theta + 2) \pm \frac{1}{2}\sqrt{\delta} \quad \text{with} \quad \varepsilon = \mathrm{e}^{-\theta} - 1\,,$$

where

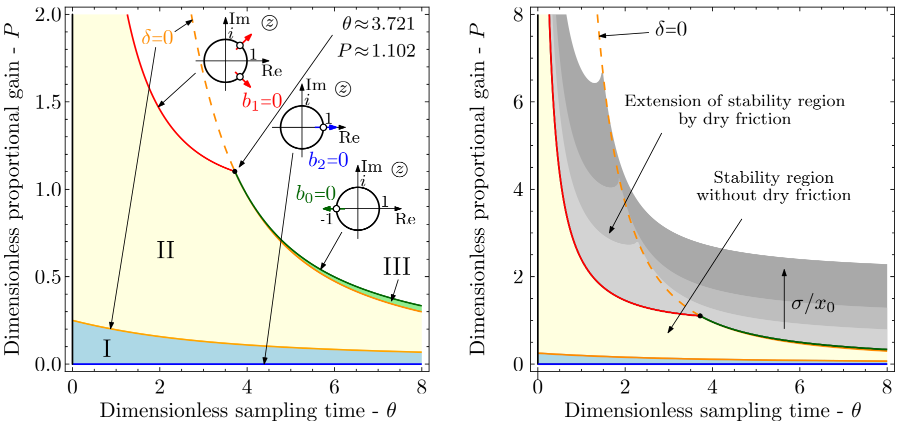
$$\delta = P^2(\mathrm{e}^{-2\theta} - 2\mathrm{e}^{-\theta} + 1 + 2\theta\varepsilon + \theta^2) + P(2\theta\varepsilon - 2(\mathrm{e}^{-2\theta} - 2\mathrm{e}^{-\theta} + 1)) + \mathrm{e}^{-2\theta} - 2\mathrm{e}^{-\theta} + 1\,.$$

In these expressions, the dimensionless proportional gain $P = mk_\mathrm{p}/b^2$ and the dimensionless sampling time $\theta = bt_\mathrm{s}/m$, where m is the generalized mass, b is the generalized viscous damping coefficient, k_p is the proportional control gain, and t_s is the sampling time.

In this case, based on the detailed algebraic analysis of the characteristic equation of the model, the corresponding stable domain of control parameters is determined, and it is illustrated in the plane of the dimensionless sampling time θ and the dimensionless proportional gain P in the left panel of the figure below. In the left panel of this figure the colored regions represent the different stable domains of control parameters, where the characteristic multipliers $z_{1,2}$ have the following properties:

- In region I, $z_{1,2} \in \mathbb{R}$, and $0 < \mathrm{Re}(z_{1,2}) < 1$. The system shows first-order dynamics; no vibrations will develop, and at the boundary of stability saddle-node bifurcation occurs.

- In region II, $z_{1,2} \in \mathbb{C}$ with $\mathrm{Im}(z_{1,2}) \neq 0$. The system has similar oscillations as an under-damped second-order system, and at the boundary of stability Neimark-Sacker bifurcation occurs.



- In region III, $z_{1,2} \in \mathbb{R}$, and $-1 < \mathrm{Re}(z_1) < 0$ or $-1 < \mathrm{Re}(z_2) < 0$. The control force alternates, and at the boundary of stability period-doubling bifurcation occurs.

In the left stability chart above, the root separation curve is $\delta = 0$, and the different type of stability boundaries are defined by

$$b_0 = 0 \iff P = \frac{2\left(e^{\theta}+1\right)}{e^{\theta}(\theta-2)+\theta+2},$$

$$b_1 = 0 \iff P = \frac{e^{\theta}-1}{-\theta+e^{\theta}-1},$$

$$b_2 = 0 \iff P = 0\,.$$

The right panel shows that Coulomb friction stabilizes the motion at two different types of stability boundaries where Neimark-Sacker and period-doubling bifurcations take place. In both cases, the stability limit of the extended stable domain is at the critical dimensionless proportional control gain

$$P_{\mathrm{cr}} = \frac{\sigma}{x_0}\coth\left(\frac{\pi}{2}\frac{\ln(\rho)}{\vartheta}\right),$$

where $\rho = \mathrm{abs}\left(z_{1,2}\right)$ and $\tan(\vartheta) = \arg(z_1)$. In these expressions, $\sigma = m f_{\mathrm{C}}/b^2$ where f_{C} is the magnitude of the generalized dry friction force, and x_0 is the initial position/perturbation with $v_0 = 0$.

- Related publications: [7], [8], [9], [10]

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