BUDAPEST UNIVERSITY OF TECHNOLOGY AND ECONOMICS
FACULTY OF MECHANICAL ENGINEERING

Booklet of thesis statements

related to the doctoral thesis

Dynamics of Dual-Point Rolling Bodies

submitted by

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Overview of the thesis

When a rigid body is in normal contact with two rigid surfaces, rolling or slipping motion can occur at both contact points. This situation leads to four kinematic cases of the body: dual-point rolling, dual-point slipping and two mixed rolling-slipping cases. In case of dual-point rolling, the rolling constraints at the two contact points are not independent; therefore, the contact forces are undetermined in the scope of rigid body dynamics. Hence, the dynamic condition of slipping from the Coulomb model cannot be determined.

This indeterminacy is avoided by analysing the discontinuous vector field of the dynamics of the system. In two-dimensional contact problems of rigid bodies, the Coulomb friction model leads to nonsmooth Filippov type dynamical systems. However, the three-dimensional contact with Coulomb friction is out of the scope of Filippov systems, because it leads to an isolated codimension-2 discontinuity in the phase space. For that purpose, the definition of *extended Filippov systems* is introduced, which is a natural generalisation of Filippov systems. The definition of sliding and crossing regions are defined analogously to usual Filippov systems and the construction of sliding dynamics is presented, as well (Thesis Statement 2).

The concept of extended Filippov systems is applied to the problem of the dual-point rolling body, and thus, the indeterminacy of the contact forces is avoided. This mechanical system contains two codimension-2 discontinuity sets in the phase space, and the dual-point rolling dynamics is located in the intersection of these sets. By analysing the vector field of the slipping dynamics in the vicinity of this subset, conditions are determined to decide whether there is a possibility of slipping at one or both contact points (Thesis Statement 3).

The developed analytical methods are demonstrated on two mechanical applications. One of them is the dynamics of a railway wheelset running with a constant speed on a straight track. Instead of the commonly used nonlinear creep model, the contact forces are approximated by Coulomb friction. From the discontinuous dynamical system, the conditions of slipping of the wheelset are determined. The maximum amplitude of the oscillations without slipping is derived, as well (Thesis Statement 5). In case of dual-point rolling, the effect of the amplitude of the kinematic oscillations is determined on the angular frequency of the oscillations (Thesis Statement 1).

The other application is the concept of a special flowmeter, where the flow rate is measured from the motion of a ball driven by the flow. During usual operation, the ball is in a dual-point rolling contact with the bottom and the wall of a cylindrical vessel. The analysis of the discontinuous system shows that slipping of the ball occur at one or both contact points as the flow rate increases. The variation of the parameters results in several bifurcations of the system (Thesis Statement 4).

Thesis Statement 1

Consider the model of a railway vehicle running with a constant speed v along a straight track, where the dynamics of one of its wheelsets is described by the lateral displacement y and the yaw angle ψ. Rolling is assumed between the wheelset and each rail.

i) The nonlinear dynamics of the kinematic oscillations of the wheelset is described by differential equations in the form

$$\dot{y} = a_{01}\psi + a_{03}\psi^3 + a_{21}y^2\psi + O^5(y,\psi),$$
$$\dot{\psi} = b_{10}y + b_{30}y^3 + b_{12}y\psi^2 + O^5(y,\psi),$$

where $O^n()$ denotes the n^{th} or higher order terms, and the coefficients a_{ij} and b_{ij} are determined by the geometry of the surfaces and the speed v of the vehicle.

ii) For a finite amplitude $\bar{y}$ of the kinematic oscillation, the angular frequency is given by

$$\omega_N(\bar{y}) = \omega_L \cdot \left(1 + \beta\bar{y}^2 + O^4(\bar{y})\right),$$

where $\omega_L = \sqrt{-a_{01}b_{10}}$ is the angular frequency of small-amplitude oscillations and β is the nonlinearity factor determined by

$$\beta = \frac{1}{8}\left(\frac{3b_{30}}{b_{10}} + \frac{a_{21}}{a_{01}} - \frac{b_{12}}{a_{01}} - \frac{3a_{03}b_{10}}{a_{01}^2}\right).$$

The distance between the contact points is b, the nominal rolling radius of the wheelset is r, the conicity of the wheelset is h. The radii of curvatures of the wheel and rail profiles are denoted by R_w and R_r, respectively, and their variations along the axis of the wheelset are described by the derivatives R_w',R_w'', ... and R_r',R_r'', ...

iii) The angular frequency of the small-amplitude kinematic oscillations is given by

$$\omega_L = v\sqrt{\frac{h}{br} \cdot \frac{R_w}{R_w - R_r} \cdot \left(1 + \frac{R_r h}{b\sqrt{1+h^2}}\right)}.$$

Therefore, the formula of Meijaard is valid also for profiles with varying curvature.

iv) The nonlinearity factor β is determined by the wheel and rail profiles up to the second derivatives of the radii of curvatures. For small conicity h of the wheelset, the approximate value of the nonlinearity factor is

$$\beta \approx \frac{R_w'(R_w - R_r) + 3R_r(R_w' - R_r')}{16h(R_w - R_r)^3}.$$

The small value h in the denominator of the formula shows that the variation of the curvature of the profiles has a significant effect on the change of the angular frequency for increasing amplitudes.

Related publications: [1], [2], [3], [4].

Thesis Statement 2

By introducing the definition of *extended Filippov systems*, the concept of Filippov systems is generalised to vector fields that are discontinuous in an isolated codimension-2 manifold. In these systems, the directional limit of the vector field at the discontinuity set takes continuously many values. From the resulting *limit vector field*, the sliding and crossing regions of the discontinuity manifold are defined by using the concept of *limit trajectories*. In the sliding region, the sliding dynamics is defined by taking the convex combination of the limit vector field.

Related publications: [5], [6].

Thesis Statement 3

Consider a rigid body in normal contact with two rigid surfaces by assuming Coulomb friction. At both contact points, slipping or rolling can occur, which leads to four kinematic cases of the body: dual-point slipping, dual-point rolling and two mixed slipping-rolling cases. In the case of dual-point rolling, the rolling constraints at the two contact points are not independent. Consequently, the contact forces are undetermined in the scope of rigid body dynamics, and thus, the conditions of slipping cannot be determined from the contact forces.

i) In case of dual-point slipping, the dynamics of the body leads to an extended Filippov system with two intersecting codimension-2 discontinuity manifolds. In each of the mixed slipping-rolling cases, the dynamics is given by a Filippov system, and it is restricted to one of these discontinuity sets. In case of dual-point rolling, the dynamics is restricted to the intersection of the two discontinuity manifolds.

Assume that the dynamics of the four kinematic cases of the body are compatible to each other in the sense that the dynamics of each lower dimensional system coincides with the sliding dynamics generated from the corresponding higher dimensional system.

ii) The analysis of the limit trajectories of slipping cases at the dual-point rolling submanifold shows whether there is a possibility to the transition from dual-point rolling to slipping cases. Therefore, the condition of slipping can be determined in the scope of the rigid body dynamics without the calculation of the contact forces.

Related publications: [7], [8], [9], [10], [11].

Thesis Statement 4

Consider the model of the rotating ball flowmeter containing a ball moving along the edge of a cylindrical vessel with vertical axis; the ball is driven by the swirling flow inside the device. The difference between the radii of the vessel and the ball is d, and the dimensionless mass moment of inertia is j. It is assumed that the flow velocity v_f around the vessel is constant, and its effect on the ball is modelled by a linear drag force. The resultant acceleration from gravitation and buoyancy effect is denoted by g. The contact between the ball and the vessel is modelled by Coulomb friction with a uniform coefficient μ for both the static and the dynamic cases.

i) The analysis of the resulting extended Filippov system shows that in the case $\mu < j$, the stationary state of dual-point rolling case exists if

$$v_f < \sqrt{\frac{gd}{\mu}} \cdot \sqrt{\frac{\mu^2(\mu+1)}{\mu^2(j+1)+j-\mu}}.$$

Otherwise, the ball starts slipping at both contact points. In case $\mu > j$, the stationary state of dual-point rolling case exists if

$$v_f < \sqrt{\frac{gd}{j}},$$

otherwise, the ball starts slipping at the contact point on the bottom of the vessel.

By calculating the equilibrium solutions of the slipping cases, as well, the bifurcation diagrams of the system can be created for the bifurcation parameter v_f.

ii) In the system, a fold bifurcation and a degenerate nonsmooth transcritical bifurcation occur, moreover, the variants of nonsmooth fold and persistence bifurcations occur related to the codimension-2 discontinuity manifolds.

The analyses of these bifurcations show significant limitations of the corresponding flowmeter device.

Related publications: [7], [8], [9].

Thesis Statement 5

Consider the model of a railway vehicle running with a constant speed along the straight track, where the dynamics of one of its wheelsets is described by the lateral displacement y and the yaw angle ψ. Assume that the linearised differential equations of the kinematics oscillations are written into the form $\dot{y} = \omega_L \eta b \cdot \psi$ and $\dot{\psi} = -\omega_L/(\eta b) \cdot y$, where b is the distance between the contact points on the two rails, η is a dimensionless geometric parameter, and ω_L is the linear angular frequency of the kinematic oscillations.

Assume that the forces acting on the wheelset from the vehicle are modelled by the axle load F_{load} and the elastic forces of effective stiffnesses k_y, k_ψ of the lateral and yaw suspensions, respectively. The wheelset has a mass m and a mass moment of inertia J_ψ about the vertical axis. The contact between the wheelset and the rails is modelled by Coulomb friction with a uniform coefficient μ for both the static and the dynamic cases.

i) The analysis of the resulting extended Filippov system shows that for small conicity of the wheelset, the dual-point rolling in a state (y, ψ) is realizable if

$$\left(\frac{k_y^2}{m^2} - \omega_L^2\right)^2 y^2 + \left(\frac{k_\psi^2}{J_\psi^2} - \omega_L^2\right)^2 \left(\frac{J_\psi}{m}\right)^2 \psi^2 < \left(\frac{\mu F_{\text{load}}}{m}\right)^2,$$

otherwise, the wheelset starts slipping at both contact points. Moreover, there is no possibility for the slipping of the wheelset at only one of the contact points.

ii) The maximum amplitude of the oscillations without slipping is determined by $\min(\bar{y}_1, \bar{y}_2)$**, where**

$$\bar{y}_1 = \frac{\mu F_{\text{load}}/m}{\left|k_y^2/m^2 - \omega_L^2\right|}, \qquad \bar{y}_2 = \frac{\eta \mu F_{\text{load}}\, b^2/J_\psi}{\left|k_\psi^2/J_\psi^2 - \omega_L^2\right|}.$$

Related publications: [3], [10], [11].

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