Punching shear resistance of reinforced concrete slabs without punching shear reinforcement
Summary and Theses of PhD Dissertation

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1. Introduction

Reinforced concrete slabs on columns were developed by Turner and Maillart at the beginning of the 20th century. In the early years large mushroom-shaped column capitals were used for the slab-column connections to facilitate the concentrated effects of the column reactions. In the 1950s flat slabs without capitals started to become dominant. Because of their simplicity, both for construction and for use (simple formwork and reinforcement), they have become very common for medium height residential and office buildings as well as for parking garages. The design of flat slabs is mostly governed by serviceability conditions on the one side and by the ultimate limit state of punching shear on the other side.

2. The aim of research

In this dissertation the punching shear capacity of flat slabs was investigated. The design codes typically give empirical expressions for the punching shear capacity of flat slabs, which are based on experimental investigations. The expressions of design codes can be simply and easily investigate. In these expressions two empirical parameters appear; the punching shear strength and the control perimeter.

These parameters were examined through the expressions of the current version of MSZ EN 1992-1-1 Eurocode 2.

The shear strength of reinforced concrete elements was investigated based on strength of materials. Whereupon the calculated results of shear resistance were compared with the tests results.

After the statistical analysis of punching tests, the control perimeter was determined using a shell-theoretical assumption. The upper limit value for the punching shear resistance can be verified through a simple mechanical model.

3. Punching shear resistance according to design codes

Most of design codes give the punching shear resistance of flat slabs in the form of 
\[ V_R = V_{Rc} + V_{Rs}, \]
where \( V_R \) is the punching shear strength, \( V_{Rc} \) is the estimate of concrete contribution and \( V_{Rs} \) is the estimate of shear reinforcement contribution.

Actually, the summary of resistances is theoretically incorrect, because the deformations cannot be compatible.

Nevertheless, by the detailed examination of value for \( V_{Rc} \) (the estimate of concrete contribution) the possibilities and the limits of this summary can be clarified.

For the examinations, the punching shear resistance of slabs without punching shear reinforcement was written in the following simple form

\[ V_{Rc} = v_{Rc} \cdot u_{cont} \cdot d, \]

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where \( d \) is the effective depth of the slab, \( u_{cont} \) is the control perimeter and \( v_{Rc} \) is the shear strength of the reinforced concrete element. In this expression the value of \( u_{cont} \) and \( v_{Rc} \) are both based on experimental investigations (empirical parameters).

In this dissertation the theoretical background of these empirical parameters was investigated.

Based on analogy of the shear failure of beams and the punching shear in flat slabs, in the above formula \( b_w \) can be substituted instead of \( u_{cont} \), where \( b_w \) is the smallest width of the cross-section of a beam. Thus the shear resistance of a beam without shear reinforcement can be obtained. Since the obtained expression contained merely the shear strength, as parameter, initially simple rectangular reinforced concrete beams were investigated.

### 4. Shear strength of reinforced concrete elements

During the investigation was assumed that the shear strength of a reinforced concrete member without shear reinforcement can be well characterized by the shear resistance of the concrete compression zone and both the aggregate interlock and the dowel action can be neglected.

At first, the stress distribution in cracked cross-section was investigated and based on the result of this investigation, the shear resistance of a beam without shear reinforcement was determined. The calculated values of shear capacity were compared with the test results and the value of shear resistance according to Eurocode 2.

#### 4.1. Stress distributions in concrete compression zone

The initial investigation of cracked cross-section of reinforced concrete beam finite element was carried out using analysis software. The numerical solution showed that in case of a separated crack, the singularity of the crack tip is dominant in the shear stress distribution. In case of coherent crack pattern, which is typical for reinforced concrete elements, the second-degree parabola of shear stress distribution will became dominant.

The next step was finding analytical solutions for subproblems. The concrete compression zone, which is a typical case of the plane stress condition, was investigated in details.

For the analysis of concrete compression zone, initially the linear normal stress distribution was assumed. In this case, the solution showed that the shear stresses are distributed through the compression zone as a second-degree parabola.
To extend the solution further investigation was performed with linear and non-linear assumption. The final result showed that in the plasticising concrete compression zone the variation of shear stress distribution cannot be derived from the variation of normal stress distribution.

The stress distributions in cracked cross-section in case of linear and non-linear normal stress distribution are shown in Fig. 1.

### 4.2. Shear resistance of concrete compression zone

The concrete compressive zone in a reinforced concrete beam subjected to bending is a typical case of plane stress conditions, whose behaviour can be adequately studied by Mohr’s criterion. For Mohr envelope a simple straight line – the so called Coulomb’s line – was used. Knowing the stress distributions of the concrete compression zone and the $\tau(\sigma)$ failure criteria, the shear resistance of the cracked cross-section became calculable.

The calculated value for shear resistance, which is a function of the curvature, is shown in Fig. 2. For the purpose of comparison, the figure also includes the test results of Leonhardt and Walther, for the values of shear slenderness $\lambda \geq 2.5$, where $\lambda = ald$ is the shear span-to-depth ratio. The figure also shows the value of shear resistance according to Eurocode 2.

For the value of shear resistance simple approximate expressions were given and the calculated values of shear capacity were compared with the test results. The comparisons showed that the shear resistance of a member without shear reinforcement can be well characterized by the shear resistance of the concrete compression zone. The good agreements showed that the initial assumption was correct.

Thus, it can be concluded that the calculation of shear resistance in the elastic state and in the state of bending failure must be treated separately. Moreover, the expression of shear resistance must contain a size effect factor, which is related only to the concrete compression zone and belongs to the shear strength of concrete.
The investigations showed that the shear resistance of the cracked cross-section emerges in the expressions of Eurocode 2. Thus the design value for the shear resistance $V_{Rd,c}$ is no other than the shear resistance of the cracked cross-section in elastic state. Moreover, the value of $v_{min}$ characterizes the shear failure in case of minimum area of longitudinal tension reinforcement.

The final results provided explanations for the semi-empirical expression of Eurocode 2. Thus, neither the shear resistance of an uncracked member nor the shear resistance of a plastic hinge is included in the expressions of design code. Therefore, the shear resistance around a plastic hinge may be less than the shear resistance according to Eurocode 2.

The function of $v_{RC,MOHR}$ with the ratio of longitudinal tension reinforcement is shown in Fig. 3. In these calculations, Leonhardt and Walther’s test parameters were used.

The function of $v_{RC,MOHR}$ can be well approximated by the function of $\rho l^{1/3}$. Based on the test parameters $v_{RC,MOHR} \approx 1.44 (\rho l f_{ck})^{1/3}$ approximate function can be given. Using this expression the value for the shear resistance is $V_{RC} \approx 0.17 k (100 \rho l f_{ck})^{1/3} b d$. The obtained expression is practically the same as the shear resistance of a rectangular beam according to Eurocode 2.
Further investigations showed that the expression according to Eurocode 2 overestimates the minimum value for shear resistance. Thus instead of formula $v_{\text{min}} = 0.035 k^{3/2} f_{ck}^{1/2}$, expression $v_{\text{min}} = \kappa_1/\gamma_c k^{3/2} f_{ck}^{1/2}$ is proposed, where the recommended value for $\kappa_1$ is 0.035.

5. Control perimeter

The other empirical parameter used in the expression for the punching shear resistance of slabs without punching shear reinforcement is the control perimeter. The control perimeter was defined with a distance from the edge of the column (the loaded area). The design procedure for punching shear was based on a check at the control section, which section is defined by the control perimeter.
Based on the background of shear strength of reinforced concrete elements, the control perimeter can be calculated from the test results. The calculations were based on the test data of appendix I of fib bulletin 12, which contains 200 punching tests of slabs without shear reinforcement from the year 1956 to 2000. The relative frequency histogram of the location of control perimeters is shown in Fig. 4.

After the statistical analysis it can be concluded, that the load bearing around the column can be investigated more adequately on the basis of the theory of bent shallow shells, than that of thin plates. Thus the load bearing around the column can be traced back to the load bearing of a shallow shell, which is located between the boundary planes of flat slab. Due to this assumption, the control perimeter can be determined.

5.1. Analysis of the shallow shell

For analyzing the load bearing around the column a paraboloid of revolution was investigated. The assumed bent shallow shell is shown in Fig. 5.

![Fig. 5. Geometry of shell around the column head](image)

The solution was obtained using the method of the generator function. From the obtained solution the membrane action can also be determined.

![Fig. 6. Membrane action and plate action as a function of the relative depth of the shell](image)

From the investigation can be concluded, that the membrane action depends only on the relative depth of the shell, denoted \( f/t \), where \( f \) is the depth of the shell and \( t \)
is the thickness of the shell (notations are shown in Fig. 5). The variation of the membrane action with the ratio of relative depth of the shell is shown in Fig. 6. The membrane action can be expressed by the following approximate formula:

\[ \mu \approx 1 - \frac{1}{1 + 1.753 \frac{f^2}{t^2}}. \]

Investigations showed the analogy of the bent shallow shell and the circular plate on elastic foundation (see in Fig. 7), where the intensity of the reaction of the subgrade is given by the curvature of the middle surface, and the fictitious Winkler-type foundation is \( C = \alpha^2 Et \). The case, in which the column reaction is distributed uniformly over the area corresponding to the cross-section of the column, was investigated in details.

The performed numerical calculations showed that the column reaction approximated by concentrated load causes an error in the membrane action, which is less than 2% for the maximum ratio of \( c/a = 0.195 \). Therefore, it can be verified that for the ratios used in practice, also for calculating the membrane action, the column reaction can be approximated by concentrated load.

### 5.2. Determination of the relative depth of the shell

The investigation of the bent shallow shell showed that the membrane action depends only on the relative depth of the shell. Thus the value of \( \frac{f}{t} \) was investigated in details. Using the approximate formula of membrane, the value of \( \frac{f}{t} \) can be calculated from the test results. These calculations showed that the mean value of relative depth is \( \frac{f}{t} = 1.32 \) and the lower limit value is \( \frac{f}{t} = 0.95 \ldots 1.00 \). For the calculation of the value of punching shear resistance, the lower limit value can be used.

The membrane action can be illustrated with compressed arches or vaults. Based on this analogy, the relative depth can also be calculated. In these calculations, the longitudinal reinforcement ratio was 1 percent.

Based on these calculations the lower limit value of relative depth was \( \frac{f}{t} = 1.30 \) and the upper limit value was \( \frac{f}{t} = 2.61 \). The upper limit value for relative depth was obtained using the finite-difference method.

The lower and upper limit values of the relative depth can be compared with the values of relative depth of the shell calculated from the test results. However, this comparison is possible only in order of magnitude, since the investigated shell was fictitious.
5.3. Determination of control perimeter

For determining the punching or control cross-section, all effects, increasing the punching shear resistance, were interpreted as an increase of the control perimeter. The control radius $r_{cont}$ can be determined from the calculated perimeter, where the control radius is the distance of the control cross-section from the centroid of the column. In the calculations was assumed that the shear resistance is determined by the shear resistance of the concrete compression zone; therefore, the basic control section $u_0 = 2\pi c$ is at the column face.

Using the expression of membrane action for $f/t = 0.95…1.00$

\[ r_{cont} = c + 1.40…1.54d \]

can be obtained. And for the mean value of relative depth which was $f/t = 1.32$

\[ r_{cont} = c + 2.44d \]

can be calculated.

The calculated values of control perimeter were practically the same as the control perimeters calculated from the test results. Thus, the lower limit value of control perimeter is $1.5d$ and the mean value is $2.5d$. The relative frequency histogram of the location of control perimeters was shown in Fig. 4.

The good agreement of the calculated result of the control perimeter showed that the membrane effect is an important part of the load bearing around the column. Hence the load bearing around the column can be investigated more adequately on the basis of the theory of bent shallow shells, than that of thin plates.

It can be concluded, that tracing back the load bearing around the column to load bearing of a shallow shell, was successful. Due to the good agreement of the calculated result, a theoretical proof was given for the location of the control perimeter.

5.4. Maximum punching shear resistance

On the basis of the theory of bent shallow shells and the shear resistance of concrete compression zone, a simple mechanical model can be given. The model showed that the punching shear resistance must have an upper limit value. The calculation gave a maximum value of $V_{Rcs} = 1.44…1.59V_{Rc}$ for the punching shear resistance of slabs with shear reinforcement. Thus, the upper limit value of Eurocode 2 can be verified, where the upper limit value for $V_{Rcs}$ is $k_{max}V_{Rc}$, and the recommended value for $k_{max}$ is 1.5. The model also showed that the upper limit value depends on the punching crack inclination. Therefore the upper limit value depends on the punching shear reinforcement.

The constructed model and the existence of the upper limit value verified that the expression for $v_{Rd,cs}$ according to Eurocode 2 is theoretically incorrect. According to the model, the summary of resistances can only be perform on the basis of the theory of bent shallow shells.
6. New scientific results and theses

Based on the investigations the following three theses have been formulated.

**Thesis 1:** It was shown, that the shear stress distribution in concrete compression zone of cracked cross-section is given by the sum of two components: the singularity of the crack tip and the second-degree parabola stress distribution, which is independent from normal stress distribution over the cross-section.

1a It was shown, that in case of a separated crack the singularity of the crack tip is dominant in the shear stress distribution. In case of coherent crack pattern, which is typical for reinforced concrete elements, the second-degree parabola of shear stress distribution will became dominant.

1b It was verified, that in the plasticising concrete compression zone the variation of shear stress distribution cannot be derived from the variation of normal stress distribution.

**Thesis 2:** It was shown, that the shear resistance of the concrete compression zone of the cracked cross-section is a reliable estimate of the shear resistance of a rectangular reinforced concrete beam without shear reinforcement. For the calculation of the value for shear resistance of the concrete compression zone the depth of the concrete zone must be equal the value of the depth of the compression zone of the cracked cross-section and the shear strength of the concrete zone must be derived from the Mohr-Coulomb’s criterion. Merely the shear resistance of the concrete compression zone appears in the calculation formula for shear resistance of a rectangular reinforced concrete beam without shear reinforcement according to Eurocode 2.

2a It was shown, that the shear resistance of a reinforced concrete member without shear reinforcement is a function of the curvature. The shear resistance of a reinforced concrete can be calculated with simple approximate expressions; however in case of using approximate expressions different expressions must be given in the elastic state and in the state of bending failure. The expression of shear resistance, whether it is function or approximate formula, must contain size effect factor, which is related only to the concrete compression zone and belongs to the shear strength of concrete.

2b It was verified, that the design value for the shear resistance of a rectangular reinforced concrete beam without shear reinforcement, given by the expression of $V_{Rd,c} = C_{Rd,c} k (100 f_{ck})^{1/3} b d$ according to Eurocode 2, is no other than the shear resistance of the concrete compression zone of the cracked cross-section in elastic state. Moreover the minimum value for the shear resistance, given by the expression of $V_{Rd,c} = v_{min} b d$, is the shear resistance of the concrete compression zone for the minimum area of longitudinal tension reinforcement. Thus the expressions of the design code do not contain the shear resistance of the uncracked member and the shear resistance of the plastic hinge.
2lc It was verified, that the expression according to Eurocode 2 overestimates the minimum value for shear resistance, which is given by the expression of $V_{Rd,c} = \nu_{\min} b d$. Thus instead of formula $\nu_{\min} = 0.035 k^{3/2} f_{ck}^{1/2}$ expression $\nu_{\min} = \kappa_1 \sqrt[3]{k} f_{ck}^{1/2}$ is proposed, where $\kappa_1 = 0.035$.

**Thesis 3:** It was shown, that for calculating the punching shear resistance, the load bearing around the column can be investigated more adequately on the basis of the theory of bent shallow shells, than that of thin plates.

3la It was shown, that the load bearing around the column can be traced back to the load bearing of a shallow shell, which is located between the boundary planes of a flat slab.

3lb It was shown, that the membrane action depends only on the relative depth of the shell. For calculating the value for punching shear resistance the value of $f/t = 0.95...1.00$ can be used.

3lc Using the expression of membrane action, for $f/t = 0.95...1.00$ $r_{cont} = c + 1.40...1.54$, and for $f/t = 1.32$ $r_{cont} = c + 2.44d$ can be calculated. These values are practically the same as the control perimeters calculated from the test results. Thus for the location of control perimeter a theoretical proof was given.

3ld It was shown, that the column reaction approximated by concentrated load causes an error in the membrane action, which is less than 2% for the maximum ratio of $c/a = 0.195$. Therefore, it can be verified that for the ratios used in practice, also for calculating the membrane action, the column reaction can be approximated by concentrated load.

3le On the basis of the theory of bent shallow shells and the shear resistance of concrete compression zone, a simple mechanical model can be given. The model showed that the punching shear resistance must have an upper limit value. The calculation gave a maximum value of $V_{Rcs} = 1.44...1.59V_{Rc}$ for the punching shear resistance of slabs with shear reinforcement. Thus, the upper limit value of Eurocode 2 can be verified, where the upper limit value for $V_{Rcs}$ is $k_{max} V_{Rc}$ and the recommended value for $k_{max}$ is 1.5.

3lf The constructed model and the existence of the upper limit value verified that the expression for $V_{Rd,cs}$ according to Eurocode 2 is theoretically incorrect. In this expression the correct value is $1.00 V_{Rd,c}$ instead of $0.75 V_{Rd,c}$.

Related publications of theses 1 and 2 are [1] [4].
Related publications of thesis 3 are [2] [3] [4].
7. Related publications


8. Further possible research directions

The calculated value for shear resistance showed that the shear resistance of a cracked cross-section near a plastic hinge might be less than the shear resistance according to Eurocode 2. However this prediction could not be verified, because such experiment were not available in professional literature. Thus this problem requires further experiments.

Further, detailed investigations are required also with reference to the $\tau = f(\sigma)$ failure criteria, since it may vary with the compressive strength of concrete. In these investigations the aggregate size as parameter also must appear. In this context, there might be relation between the failure criteria of concrete compression zone and Guandalini’s empirical expression.

Since an axisymmetric case was studied, further investigations are necessary in non-axisymmetric cases, for example in case when the column is situated near an edge or a corner. The effect of the strong ring reinforcement on the punching shear resistance must also be examined, because in this case the membrane action is much more significant. This strong ring reinforcement around the column has a major practical benefit, since it can be placed between the top and bottom reinforcement layers.

Based on the results of this study, the verification model for shear resistance of beams according to Eurocode 2 must be examined in detail, since there is an obvious contradiction between this verification model and the verification model for punching shear resistance of flat slabs. While the punching shear resistance of slabs with shear reinforcement is given in the form of $V_R = V_{Rc} + V_{Rs}$ according to Eurocode 2, the estimate of concrete contribution is not included in the expression given for shear resistance of a beam with shear reinforcement. Thus it needs to be investigated, how the shear resistance of a beam with shear reinforcement can be given in the form of $V_R = V_{Rc} + V_{Rs}$, and how the reliable value for crack inclination angle can be defined.