Charge and spin dynamics in low dimensional systems

Szabolcs Vajna

Supervisor: Dr. Balázs Dóra
Professor
Department of Physics
BME

Budapest University of Technology and Economics
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Introduction

Out-of-equilibrium physics has become a forefront of condensed matter research recently [Polkovnikov et al., 2011], initiated by the experimental advances achieved with ultracold atomic gases [Bloch et al., 2008]. Understanding the physics of systems driven away from equilibrium by various protocols is not only a subject of pure academic interest, but it also provides promising applications in quantum computation and quantum information processing, as well as it allows to engineer properties of materials [Bukov et al., 2015]. The most studied protocols are the sudden quench and the periodic driving. The former denotes a single, abrupt change in the interactions describing the system, e.g. in the magnitude of an electric or a magnetic field, while the latter describes a continuously changing, but time-periodic environment. In either cases, both the transient dynamics, which describes the time evolution right after turning on the perturbation, and the steady states, which characterize the response at late times, may exhibit peculiar features.

An important development was made for example in 2013, when the non-analytical time evolution of a quantity called dynamical free energy was reported in a quench in the transverse field Ising model [Heyl et al., 2013], which is a quantum spin model in an external magnetic field. Based on the analogy found between the time evolution of this quantum system and the temperature driven phase transitions of classical systems, they named the phenomenon dynamical phase transitions. Since its discovery, a remarkable effort has been focusing on understanding the details and the robustness of this phenomenon, see e.g. [Canovi et al., 2014; Heyl, 2014; Heyl, 2015; Huang et al., 2016].

Another interesting scenario was outlined in Ref. [Dóra and Moessner, 2010], where the creation of electron-hole pairs was analyzed in graphene following a sudden switching on of an electric field. This is a condensed matter realization of Schwinger’s mechanism [Schwinger, 1951], which was discovered originally in quantum electrodynamics. The two dimensional graphene is not the only example which hosts low energy excitations described by a relativistic dispersion relation. The edge or surface states of the recently discovered topological insulators [Qi and Zhang, 2011] and the low energy bulk excitations of the topological Dirac and Weyl semimetals [Burkov and Balents, 2011] are also described by the one, two or three dimensional Dirac equations, thus provide a further opportunity to examine fascinating QED effects, like Schwinger’s pair creation, Klein tunneling, or the chiral anomaly, in condensed matter systems.

Considering the long time dynamics, a central goal of non-equilibrium physics is to characterize stationary states in a similar way to statistical mechanics does for non-driven systems weakly attached to environments. One of the key ques-
tions is whether closed systems, which are isolated from their environment, can thermalize under their own dynamics, when they are initially in a non-equilibrium state [Rigol et al., 2008; Cassidy et al., 2011]. The same question can be asked about periodically driven systems either isolated from [Lazarides et al., 2014; D’Alessio and Rigol, 2014], or weakly attached to an environment [Dehghani et al., 2014; Iadecola et al., 2015; Shirai et al., 2016].

Objectives

The main goal of my research was to study non-equilibrium phenomena in various analytically treatable systems, to analyze the excitations generated by the driving protocol, and to study measures like the dynamical free energy, or the induced current.

The first few examples of dynamical phase transitions published in the literature supported the conjecture that their occurrence is closely related to equilibrium quantum phase transitions of the underlying model. My first objective was to understand better the relationship between these two phenomena. In this regard I investigated various systems with rich equilibrium phase diagrams. In addition, I was also interested in extending the notion of dynamical phase transitions to two spatial dimensions.

The second objective of my research was to generalize the results of [Dóra and Moessner, 2010] to Weyl semimetals, that is, to analyze Schwinger’s pair creation mechanism in 3D materials characterized by relativistic dispersion relation near the Fermi energy.

Finally, I intended to investigate the occupation of the stationary states in periodically driven systems, and its impact on transport properties. In particular, the photocurrent in the quantum spin Hall insulator had been studied previously in the average energy concept [Dóra et al., 2012], which determines the filling of steady states from a heuristic principle. We wanted to go beyond, and determine the occupation of the steady states and the photocurrent when the system is weakly attached to a thermal environment.

Methods

I have applied various analytical techniques and numerical methods to study the non-equilibrium behaviour of systems under various conditions. In quench problems I applied standard tools to diagonalize the Hamiltonian of simple spin chains and BCS superconductors, such as the Jordan-Wigner transformation or the Bogoliubov transformation. I studied dynamical phase transitions through
the complex zeros of the dynamical partition function, called Fisher zeros in
the literature. I determined the excitations induced by a suddenly switched on
electric field by using analytical approximate solutions to the time dependent
Schrödinger equation, which I also compared with numerical results obtained
from an explicit Runge-Kutta method. The periodically driven systems were
studied within the Floquet-theory, and the dissipation induced by the coupling
to a thermal environment was studied within a generalized Lindbladian formal-
ism, by the Bloch-Redfield equations.

New scientific results

The main results of my thesis are summarized in the thesis statements below.

1. I have shown analytically on the example of the 1D quantum XY spin
chain in a transverse magnetic field that dynamical phase transitions can
not only show up when the non-equilibrium quench protocol connects dif-
ferent equilibrium phases, which was found in Ref. [Heyl et al., 2013] in
the transverse Ising model, but also when the initial and final Hamiltoni-
ans characterizing the quench protocol are in the same phases. Depending
on the parameters of the pre-quench Hamiltonian, I explicitly determined
the domain for the post-quench parameters on the equilibrium phase di-
agram, where dynamical phase transitions occur.

This result is published in paper [P1].

2. I have studied dynamical phase transitions in generic one-dimensional
two-band topological insulators and topological superconductors whose
topological invariants are either the winding number or the $\mathbb{Z}_2$ invariant.
I have proved for this class of models that a sudden quench protocol which
connects equilibrium phases characterized by different topological num-
bers implies the occurrence of dynamical phase transitions. Furthermore,
the number of nonequilibrium timescales, which determine when the sin-
gularities appear in the time evolution, is bounded from below by the
difference between the topological numbers characterizing the initial and
final set of parameters. I have illustrated this finding on the example of
a generalized Su-Schrieffer-Heeger model.

These results are published in paper [P2].

3. I have studied dynamical phase transitions in two-dimensional two-band
topological insulators and topological superconductors whose topological
invariant is the Chern number. I have proved for this class of models that a
sudden quench protocol which connects equilibrium phases characterized by Chern numbers of different absolute values implies the occurrence of dynamical phase transitions. I have also found a qualitative difference between dynamical phase transitions in 1D and 2D. While the former is characterized by jumps in the first time derivative of the dynamical free energy, in the latter case the jumps appear only in the second time derivative. I showed that this is a consequence of Fisher zeros filling areas in 2D rather than forming lines, which happens in 1D. I have illustrated these findings on the example of the Haldane honeycomb model.

These results are published in paper [P2].

4. I have investigated Schwinger’s pair creation mechanism and the non-linear response of Weyl semimetals. I have determined the full time evolution of the characteristic function of the total number of electron-hole pairs created by the electric field as well as the induced current. The distribution function of pairs crosses over from a Poissonian profile characterizing short time dynamics to a Gaussian one describing long times. The contribution of a Weyl node to the total current shows a peculiar non-monotonic behaviour: the quick initial increase of the polarization current is followed by a slow decay, which is taken over by the increasing conduction current at long times. I have demonstrated that the time evolution of the current can be translated to the conductivity of a disordered sample within a generalized Drude theory.

These results are published in paper [P3].

5. I have determined the occupation of the Floquet quasienergy bands and the induced photocurrent in the presence of dissipation in a quantum spin Hall insulator edge irradiated by a circularly polarized light. As such, I have generalized the results of [Dóra et al., 2012], which applied the heuristic average energy concept to determine the same quantities in the absence of dissipation. I found that their prediction, that is, a transition occurs as a function of the driving frequency from a quantized to non-quantized photocurrent, remains true also in the dissipative model attached to a zero temperature heat bath, but the value of the transition frequency is lower by a factor of two in the latter treatment. Furthermore, although the occupation profile of the quasienergy bands are qualitatively similar in the two methods, the strong dependence on the bath spectral parameter is not captured by the simple average energy concept. In addition, I have developed an analytical approximate method to study the effect of photon-absorption resonances appearing at finite system-bath couplings,
which lead to a further mixing of band occupations and to a weak violation of the quantization of the photocurrent in the low frequency regime. These results are published in paper [P4].

**Publications**

Publications related to thesis statements:


Further publications:


References


