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The energetic optimization of water-water heat pump heating systems

PhD dissertation theses

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1. Research actuality

In my thesis I deal with the technical and mechanical problems of the use of heat pump heating systems, primarily with the calculation and the description of the thermodynamic cycle. During my research, I have ascertained that there is much room for improvement in the area of scientific quality of planning and operation. There are numerous unresolved issues in these two areas. The most important aim of my thesis is to optimize operation, within this, to develop a so-called 'coefficient of performance maximizing mathematical model'.

In the event of realization of adequate coefficient of performance (COP) heat pump heating and cooling systems can substitute a significant amount of primary energy sources. An important part of the Hungarian National Energy Strategy is the reduction of the import of natural gas, and the replacement of use of natural gas with the use renewable energy sources by 2030.

Electric or gas-motor compressor-driven heat pumps set up in order to extract environmental low-temperature heat sources (geothermal, ground heat, ground-water, river water, etc.) are categorized as renewable energy sources. The electric compressor heat pumps are more common than the gas-driven compressor heat pumps. Their installation is cheaper, and they are easier to maintain and operate.

2. Presentation of topic:

In my thesis I deal with heat pumps that have a water-system heat source: well-water, ground-water, river water, from which the heat is derived directly or with the use of a transfer medium, and is passed to the heat pump's evaporator via a pump. The heat used by the consumer is taken from the condenser and the heating water is circulated by a pump.

The water heat pump systems consist of three flow circuits:

- cold water circuit - low temperature heat source
- thermodynamic cycle - evaporator, compressor, refrigerant, condenser and damper
- hot water circuit - heating water circulation

From an energetic point of view, the behavior of the heat pump heating system can be followed in a wide range through the coefficient of performance. The amount of pumping energy put into the cold and hot water circuit largely affects the value of the energetic coefficient of performance. Therefore, besides studying the compressor's energy requirement, one must also study the energy requirements for pumping in the other two circuits as well. For the satisfaction of consumers' needs, one must choose the solution that ensures the maximum energetic coefficient of performance from the available operational statuses. The system's energy efficiency principally depends on the heating circuit's and the cold and hot water circuit's flow rate.

In case of a given heat demand, the heat pump system's coefficient of performance is not only determined by the heat pump with specific heating power, but also by the compressor's power and the adjusted operational status of the cold and hot water circuits.

The goal of my thesis is to develop a simulation model of the hydraulic and thermal relationships of a heat pump heating system, in order to achieve energetic optimization. The energetic optimization means maximizing the coefficient of performance.

In my study, I have considered primarily the types and power magnitudes of heat pumps used in family houses and apartment buildings. I must note that the physics and mathematics models calculated to describe the thermodynamic cycle can be applied -in principle, with minor changes - to industrial use. There is no adequate orientation literature regarding the heat exchange sequences in the condenser's vapor cooling phase. To determine the heat transfer coefficient, I have built a heat pump, which consists of a plate heat exchanger and I have provided the system with measuring devices. I have also used the data from the measurements to describe the heat transfer processes in the refrigerant.

3. *Theses:*

1. thesis - An approximate method to optimize coefficient of performance on the basis of required consumer heat demand

When designing a new system based on scaling heat demand, determining the heat exchange surface, the evaporation and condensation temperature, designing a cycle with which the coefficient of performance is maximal.

During the research, we assume to know the following:

- We are familiar with the heat exchange surface, heat transfer and transmission factors and the calculation methods for the system in question
- We are familiar with the temperature of the low-temperature heat source (well-water, spring water, soil-probed heat source etc.)
- We are familiar with the consumer heat demand
- The compressor power
- The heated interior's temperature is specified.

Hypothesis and approach:

- The transmission of heat happens during the vapor -cooling phase is ten percent of the heat transmission during the condensation phase.

$$\dot{Q}_{f\H{u}t\acute{e}s} \cong \dot{Q}_{46} = \dot{Q}_{56} + \dot{Q}_{45} = \dot{Q}_{56} + 0.1 \cdot \dot{Q}_{56} = 1.1 \cdot \dot{Q}_{56}.$$

The following method demonstrated is applicable for thermal design as well, if we change the size and circulation parameters of the heat exchanger.

The fundamental principle of the approximate method is to use the fact that 90% of the useful heat is transmitted by the refrigerant in the condenser, during the condensing phase.

1. thesis

The determination of the operating point of a hot water sub-system with the approximate method, under fixed heat demand.

Because the heat demand $\dot{Q}_{fűtés}$ the compressor power P_{34} and the heated interior's temperature t_{bt} is known, t_{mk6} is determined.

$$t_{mk6} = \frac{1 - N \cdot a_1 - \sqrt{(N \cdot a_1 - 1)^2 - 4 \cdot N \cdot a_2 \cdot (N \cdot a_0 + t_{mk3})}}{2 \cdot N \cdot a_2}. \qquad (1)$$

The constant:

$$N = 1.1 \cdot \frac{P_{34} - 0.1 \cdot \dot{Q}_{fűtés}}{c_{Pmk} \cdot \dot{Q}_{fűtés}}. \qquad (2)$$

The refrigerant temperature on the inlet of the condenser.

$$t_{mk4} = \frac{P_{34}}{c_{Pmk} \cdot \frac{\dot{Q}_{fűtés}}{1.1 \cdot \Delta h_{mk56}}} + t_{mk3}. \qquad (3)$$

The hot water circuit inlet, intermediate and the outlet temperature:

$$t_{mv4} = \frac{\dot{Q}_{f\H{u}t\acute{e}s}}{(k \cdot A)_{f\H{u}t\acute{e}s}} - \frac{\dot{Q}_{f\H{u}t\acute{e}s}}{2 \cdot c_{Pmv} \cdot \dot{m}_{mv}} + t_{bt}, \quad (4)$$

$$t_{mv5} = \frac{\dot{Q}_{f\H{u}t\acute{e}s}}{(k \cdot A)_{f\H{u}t\acute{e}s}} - \frac{0.9 \cdot \dot{Q}_{f\H{u}t\acute{e}s}}{2.2 \cdot c_{Pmv} \cdot \dot{m}_{mv}} + t_{bt}, \quad (5)$$

$$t_{mv6} = \frac{\dot{Q}_{f\H{u}t\acute{e}s}}{(k \cdot A)_{f\H{u}t\acute{e}s}} + \frac{\dot{Q}_{f\H{u}t\acute{e}s}}{2 \cdot c_{Pmv} \cdot \dot{m}_{mv}} + t_{bt}. \quad (6)$$

The surface of the condensation:

$$A_{56} = \frac{1}{1.1 \cdot k_{56} \cdot \left(\frac{t_{mk6} - t_{bt}}{\dot{Q}_{f\H{u}t\acute{e}s}} - \frac{0.1}{2.2 \cdot \dot{m}_{mv} \cdot c_{Pmv}} - \frac{1}{(k \cdot A)_{f\H{u}t\acute{e}s}}\right)}. \quad (7)$$

The coefficient of performance at an integrated, cold and hot water system with the approximate method.

$$\mathrm{COP} = \frac{\dot{Q}_{f\H{u}t\acute{e}s}}{\dot{m}_{mk} \cdot \left(\mathrm{h}_{mk4}(t_{mk6}) - \mathrm{h}_{mk3}(t_{mk1})\right)} \quad (8)$$

The mass flow of the circulated cold water.

$$\dot{m}_{hv} = \frac{1}{\frac{(t_{hv1} - t_{mk1})}{\dot{m}_{mk} \cdot (\mathrm{h}_{mk3}(t_{mk1}) - \mathrm{h}_{mk1}(t_{mk1}))} - \frac{1}{(k \cdot A)_{13}}} \cdot \frac{1}{2 \cdot c_{Phv}} \cdot \quad (9)$$

Related publications: (1)

2.thesis - determining the heat transfer coefficient of vapor based on measurements

The formulas for determining the heat transfer coefficient in the vapor cooling phase found in different literatures are obscure, not precise, their validity, credibility and the margin of error are not reported. Not many articles in scientific journals deal with the heat transfer coefficient of vapor of the refrigerant flowing through plate heat exchangers. I felt this gap needed to be filled with results from a series of measurements. This is justified because in my simulation model, the vapor cooling phase's heat transfer coefficient is an input data.

I have set up an algorithm to determine the heat transfer coefficient of R -134a vapor, based on measurements. The enthalpy of the refrigerant was calculated with the help of the temperatures recorded when exiting the compressor and at the beginning of the condensation, and with the help of the R -134a refrigerant's equation of state.

2. thesis

To calculate the heat transfer coefficient of the vapor cooling in the condenser, I suggest the following correlation:

$$\boldsymbol{\alpha_{mk45} = \frac{\lambda_{mk45}}{d_{ekv}} \cdot C \cdot Re_{mk45}{}^{n} \cdot Pr_{mk45}{}^{m}.} \qquad (10)$$

The values of constants in the specified scope:

C =0.004, n =1,046, m =1/3.

Correlation coefficient of the Nusselt number

r= 0.8632

Scope of the formula:

In regard of the flow, the scope is contained in the value of the Raynolds-number:

$$\mathbf{5000<Re<7000.}$$

In regard of thermodynamics, the scope is contained in the value of the Prandtl-number:

$$\mathbf{0{,}95<Pr<1{,}1.}$$

Related publications: (1)

3. thesis The presentation of the numeric procedure for calculating the hot water subsystem's operating point

The combined solutions of the balance equations and auxiliary equations goal is to determine the output power's values based on specified input powers. There are two different quality heat transfers happening in the condenser. The heat convection and the condensational heat dissipation of the refrigerant are sharply separated from each other, and the heat transfer coefficients for the two phases are different of magnitudes. This is why it is important to know, where the phase boundary is, and where the condensing phase begins.

During the application of the method, the cold water circuit's operational point is specified, as are the heat exchange surfaces. The numeric method shows the change of the coefficient of performance in a wide numeric range, and diagrammatizes it, depending on the different external-independent system parameters.

3. thesis

The numeric procedure for determining the system parameters for the hot water cycle, with the recording of the cold water cycle's parameters. The bases of the numeric procedure are the balance equations' linearization and the use of the Gauss-Newton method. The determination of the phase boundary is an important outcome of the use of the procedure. The procedure is invariable for the different combinations of the input data. The following block-scheme is an example for an input data combination.

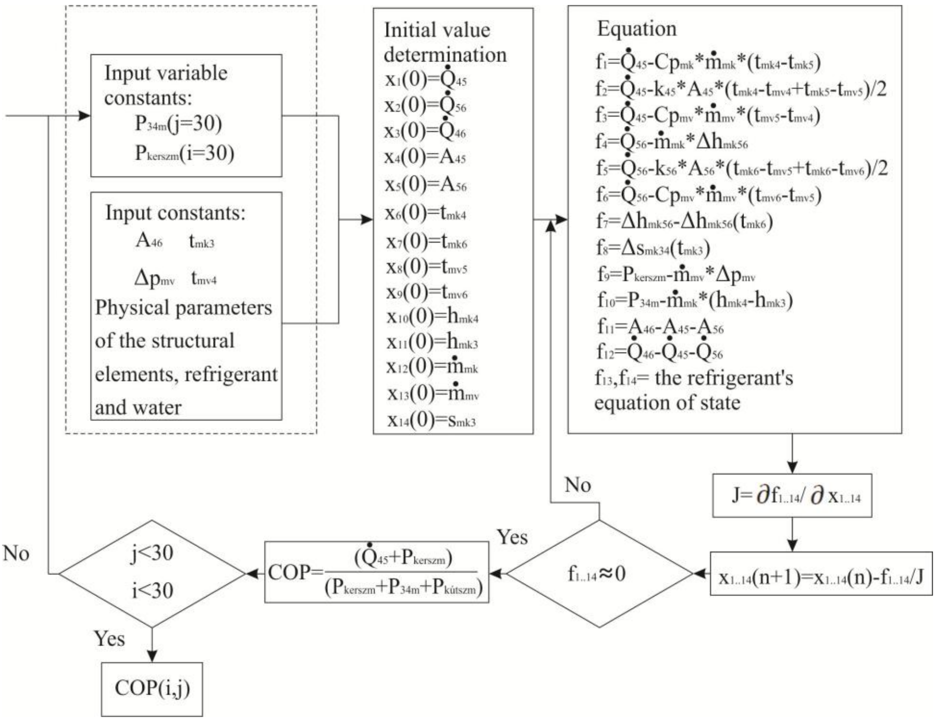

fig.1. A simplified scheme of the numeric procedure

Where indexes:

A-surface, c_p -specific heat, COP-coefficient of performance, h-specific enthalpy, k-heat transfer coefficient, $\dot{m}$ -mass flow, p-pressure, P-power, $\dot{Q}$-heat, s-specific entropy, t-temperature.

Lower indexes

1, 2, 3, 4, 5, 6-a specific points of a heat pump cycle, m-electric power consumption, mk-refrigerant, mv-hot water ,kersz- hot water circulator pump, ,kútsz-cold water circulator pump

Related publications: (2),(4),(5),(6),(10)

4.thesis - The presentation of the numeric procedure for calculating the hot water subsystem's operating point and for maximizing coefficient of performance

Numeric calculations for determining the operating point and choosing the maximal coefficient of performance with the use of equations describing the hot water sub-system and the compressor of a heating system, with recorded cold water sub-system parameters. The recorded external variables are the temperature of the refrigerant entering the compressor, the temperature of the hot water entering the condenser, the power of the compressor, the power of the hot water circulator pump, and the floor heating's, radiator's hydraulic resistance. Changing three external variables results in the following diagrams

The algorithm of the mathematical model must contain all the required numeric data, the initial values, the values of the physical parameters, and the input-independent variables and output-dependent variables must be defined. The output-dependent variables, that is, the solutions give a numeric data set. I have organized the data set into matrixes. The matrixes contain all the possible coefficients of performance for the operating point, and all the properties of the operating points. The maximal coefficient of performance and the associated properties can be chosen from all the operating points' coefficients of performance.

On fig. 2. the values of the coefficient of performance are presented in the function of the circulator pump's output powers and the compressor's output powers. A calculation has been made for several condenser surfaces and from the resulted matrices rows was chosen for a specific compressor performance. The figures contain projections, the circulator pump's powers are shown on the horizontal axis. In the figures, the curves for particular recorded compressor power values are nearly horizontal, and there occurs a maximum. Optimal circulator pump powers can be linked to the maximal coefficient of performances.

4.thesis

There exists a circulator pump power, at which the coefficient of performance reaches maximum, in case of given cold water input parameters and given compressor power. I prove my statement with the following example diagram.

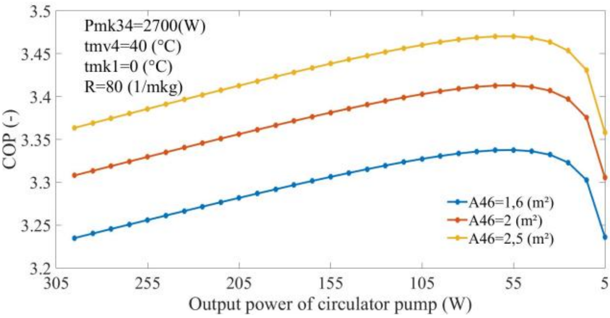

fig. 2. Values of coefficient of performance, depending on the circulator pump's output power for several condenser surfaces. The temperature of the inlet water is 30°C

Related publications: (3),(8),(9),(11)

<u>5.thesis - analytical-numerical optimization of the coefficient of performance</u>

I have developed an analytical model for maximizing the heat pump system's coefficient of performance in a hot water subsystem, depending on the quantity of the heating water circulated in the heating system and the circulator pump's power. The analytical-numerical optimization method is more complex than the numerical method. First, we must define the objective function, in this case, the examined system's coefficient of performance. The goal is the maximal value of the coefficient of performance. When searching for the maximal value, we turn out the objective function's first differential, according to the independent variable, which we equalize to zero, mathematically. The coefficient of performance can be, in theory, maximized according to both the condenser's heat transfer surface and the compressor's power demand. In this case, I do not deal with these.

The analytical extreme presented defines the rate of the optimal hot water circulation in one step specifically. We must note, that the solution of the equation of the partial differential, which is the basis of finding the extreme, is ultimately also done numerically.

5. thesis

Analytical determination of the coefficient of performance according to the circulator pump's power demand.

The total coefficient of performance of the system:

$$\boldsymbol{COP} = \frac{\dot{\boldsymbol{Q}}_{\mathbf{46}} + \boldsymbol{P}_{\mathbf{kersz}}}{\boldsymbol{P}_{\mathbf{34}} + \boldsymbol{P}_{\mathbf{kútsz}} + \boldsymbol{P}_{\boldsymbol{kersz}}}. \qquad (\mathbf{11})$$

The system's total coefficient of performance extreme, according to the circulator pump's power demand:

$$\frac{\partial \mathbf{COP}}{\partial P_{kersz}} = \frac{\left(\frac{\partial \dot{Q}_{46}}{\partial P_{kersz}} + 1\right) \cdot (P_{34} + P_{\text{kútsz}} + P_{kersz}) - \dot{Q}_{46} - P_{kersz}}{(P_{34} + P_{\text{kútsz}} + P_{kersz})^2} = 0. \quad (12)$$

This onwards:

$$\left(\frac{\partial \dot{Q}_{46}}{\partial P_{kersz}} + 1\right) \cdot (P_{34} + P_{\text{kútsz}} + P_{kersz}) - \dot{Q}_{46} - P_{kersz} = 0. \quad (13)$$

The modified mathematical model of the condenser consists of two, connected, non-linear, concentrated parameter, implicit, algebra equation, which are expressed as $\dot{Q}_{45}$ and $\dot{Q}_{56}$.

The first equation in the modified mathematical model:

$$\dot{Q}_{45} = \left(A - \frac{\dot{Q}_{56}}{k_{56} \cdot \Delta t_{ar56}}\right) \cdot k_{45} \cdot \Delta t_{ar45}. \quad (14)$$

The differential of the upper equation according to the hot water mass flow:

$$\left[-\frac{\partial \dot{Q}_{56}}{\partial \dot{m}_{mv}} \cdot k_{56} \cdot \Delta t_{ar56} + \dot{Q}_{56} \cdot \left(\frac{\partial k_{56}}{\partial \dot{m}_{mv}} \cdot \Delta t_{ar56} + k_{56} \cdot \frac{\partial \Delta t_{ar56}}{\partial \dot{m}_{mv}}\right)\right] \cdot \frac{1}{(k_{56} \cdot \Delta t_{ar56})^2} +$$

$$+\left[-\frac{\partial \dot{Q}_{45}}{\partial \dot{m}_{mv}} \cdot k_{45} \cdot \Delta t_{ar45} + \dot{Q}_{45} \cdot \left(\frac{\partial k_{45}}{\partial \dot{m}_{mv}} \cdot \Delta t_{ar45} + k_{45} \cdot \frac{\partial \Delta t_{ar45}}{\partial \dot{m}_{mv}}\right)\right] \cdot \frac{1}{(k_{45} \cdot \Delta t_{ar45})^2} . = 0 \qquad (15)$$

The second equation of the model:

$$\dot{Q}_{56} = \dot{m}_{mk} \cdot \left[a_o + a_1 \cdot \left(-\frac{\dot{Q}_{45}}{c_{Pmk} \cdot \dot{m}_{mk}} + t_{mk4}\right) + a_2 \cdot \left(-\frac{\dot{Q}_{45}}{c_{Pmk} \cdot \dot{m}_{mk}} + t_{mk4}\right)^2\right] \qquad (16)$$

$$\frac{\partial \dot{Q}_{56}}{\partial \dot{m}_{mv}} = \frac{\partial \dot{m}_{mk}}{\partial \dot{m}_{mv}} \cdot \left[a_o + a_1 \cdot \left(\frac{-\dot{Q}_{45}}{c_{Pmk} \cdot \dot{m}_{mk}} + t_{mk4}\right) + a_2 \cdot \left(\frac{-\dot{Q}_{45}}{c_{Pmk} \cdot \dot{m}_{mk}} + t_{mk4}\right)^2\right] +$$

$$+ \left[\left(\frac{-\partial \dot{Q}_{45}}{\partial \dot{m}_{mv}} \cdot \dot{m}_{mk} + \dot{Q}_{45} \cdot \frac{\partial \dot{m}_{mk}}{\partial \dot{m}_{mv}}\right) \cdot \frac{1}{c_{Pmk} \cdot \dot{m}_{mk}} + \dot{m}_{mk} \cdot \frac{\partial t_{mk4}}{\partial \dot{m}_{mv}}\right] \cdot \left[a_1 + a_2 \cdot 2 \cdot \left(\frac{-\dot{Q}_{45}}{c_{Pmk} \cdot \dot{m}_{mk}} + t_{mk4}\right)\right]. \quad (17)$$

The differentials of the inner variables in the equations above can be expressed using the chain-rule, as following:

$$\frac{\partial \Delta t_{ar45}}{\partial \dot{m}_{mv}} = -\frac{\partial \dot{Q}_{45}}{\partial \dot{m}_{mv}} \cdot \left(\frac{1}{2 \cdot c_{Pmk} \cdot \dot{m}_{mk}} + \frac{1}{2 \cdot c_{Pmv} \cdot \dot{m}_{mv}}\right) + \dot{Q}_{45} \cdot \left(\frac{\frac{\partial \dot{m}_{mk}}{\partial \dot{m}_{mv}}}{2 \cdot c_{Pmk} \cdot \dot{m}_{mk}{}^2} + \frac{1}{2 \cdot c_{Pmv} \cdot \dot{m}_{mv}{}^2}\right) + \frac{\partial t_{mk4}}{\partial \dot{m}_{mv}}. \quad (18)$$

$$\frac{\partial \Delta t_{ar56}}{\partial \dot{m}_{mv}} = \frac{\left(-\frac{\partial \dot{Q}_{56}}{\partial \dot{m}_{mv}} - 2 \cdot \frac{\partial \dot{Q}_{45}}{\partial \dot{m}_{mv}}\right) \cdot \dot{m}_{mv} + \dot{Q}_{56} + 2 \cdot \dot{Q}_{45}}{2 \cdot c_{Pmv} \cdot \dot{m}_{mv}{}^2} - \frac{\frac{\partial \dot{Q}_{45}}{\partial \dot{m}_{mv}} \cdot \dot{m}_{mk} - \dot{Q}_{45} \cdot \frac{\partial \dot{m}_{mk}}{\partial \dot{m}_{mv}}}{c_{Pmk} \cdot \dot{m}_{mk}{}^2} + \frac{\partial t_{mk4}}{\partial \dot{m}_{mv}}. \quad (19)$$

$$\frac{\partial \alpha_{mv}}{\partial \dot{m}_{mv}} = 0.2121 \cdot \frac{\lambda_{mv}}{d_{ekv}} \cdot \left(\frac{d_{ekv}}{\eta_{mv} \cdot A_{ek,v}}\right)^{0.78} \cdot 0.78 \cdot \dot{m}_{mv}{}^{-0.22} \cdot Pr_{mv}^{\frac{1}{3}}. \quad (20)$$

$$\frac{\partial \alpha_{mk56}}{\partial \dot{m}_{mv}} = 4.118 \cdot \frac{\lambda_{mk6}}{d_{ekv}} \cdot \left(\frac{d_{ekv} \cdot \left(1 - x + x \cdot \left(\frac{\rho_{mk6}}{\rho_{mk5}}\right)\right)}{A_{ek,v} \cdot \eta_{mk6}}\right)^{0.4} \cdot 0.4 \cdot \dot{m}_{mv}{}^{-0.6} \cdot Pr_{mv6}^{\frac{1}{3}} \cdot \frac{\partial \dot{m}_{mk}}{\partial \dot{m}_{mv}}. \quad (21)$$

$$\frac{\partial \alpha_{mk45}}{\partial \dot{m}_{mk}} = 0.004184 \cdot \frac{\lambda_{mk45}}{d_{ekv}} \cdot \left(\frac{d_{eqv}}{A_{ek,v} \cdot \eta_{mk45}}\right)^{1.046} \cdot \dot{m}_{mk}{}^{-0.6} \cdot Pr_{mk45}{}^{\frac{1}{3}} \cdot \frac{\partial \dot{m}_{mk}}{\partial \dot{m}_{mv}}. \quad (22)$$

$$\frac{\partial \dot{m}_{f}}{\partial m_{v}} = 0.04116 - 0.0008216 \cdot \mathrm{t}_{vi} - 0.01521 \cdot 2 \cdot \dot{m}_{v} + 4.986 \cdot 10^{-6} \cdot t_{vi}^{2} + +0.0001369 \cdot \mathrm{t}_{vi} \cdot 2 \cdot \dot{m}_{v} + 0.002625 \cdot 2 \cdot \dot{m}_{v}{}^{2}. \quad (23)$$

The solution to the resulting equations must be done numerically, using the Gauss-Newton iteration. The values that ensure convergence must be determined based on preliminary calculations.

Related publications: (7)

5. *Application proposals*

During my scientific work over the years, I have come across numerous ill-measured and poorly constructed heat pump heating systems. It has always been a challenge for engineers to choose the hot water circulator pump. In some cases, the circulator pump is either over- or undersized. The oversized heat pump heating systems use unnecessary energy to circulate hot water, whilst in the case of the undersized systems, the circulator pump couldn't satisfy the hot water mass flow demand minimum. My work assists the work of practicing engineers and makes the planning easier and more precise.

6. *Publications:*

[1] Garbai L.,Nyers Á.: Hőszivattyúk kondenzátorában végbemenő gőzhűtés hőközlési tényezője

[2] Nyers Á., Garbai L.: Effect of the Condenser Surface on the Condenser Efficiency,6rd International Symposium "EXPRES 2014." Szabadka,Szerbia. 2014.

[3] Nyers Á., Garbai L.: The Coefficient of Performance of Heat Pump Condenser Depending on Hot Water Circuit Properties, Magyar Épületgépészet, 2014/1-2 (2014).

[4] Nyers, J.,Ficko, M., Nyers Á. -Analysis of theEnergy-Optimum of Heating System withHeatPumpusingGeneticAlgorithm. 4.International Symposium "EXPRES 2012." Page.17-20. Subotica, Serbia. 09-10 03. 2012.Proceedings ISBN 978-86-85409-70-7

[5] Nyers J., Garbai L., Nyers Á.: Analysis of Heat Pump's Condenser Performance by means of Mathematical Model, International J. Acta Polytechnica Hungarica 11, no. 3 (2014). IF: 0.471

[6] Nyers J., Garbai L., Nyers Á.: Hőszivattyú kondenzátorának koncentrált paraméterű stacioner matematikai modellje,Magyar Épületgépészet 7-8 (2013): 1-4.

[7] Nyers J., Garbai L., Nyers Á.: Modified mathematical model of heat pump's condenser for analytical optimization, Energy, 80 (2015). IF:4.844.

[8] Nyers, J.,Nyers, Á. - COP of Heating-CoolingSystemwithHeatPump. 3. International Symposium "EXPRES 2011.", Page 17-21, Subotica, Serbia. 2011 dec. ISBN 978-1-4577-0095-8,

[9] Nyers, J., Nyers, Á. - COP of IndividualHeatPump and HeatingSystemusingHeatPump. 5rd International Symposium "EXPRES 2013." Page 26-30, Subotica, Serbia. 21-23. 03. 2013.Proceedings ISBN 978-86-85409-82-0,

[10] Nyers J., Nyers Á.: Hydraulic analysis of heat pump's heating circuit using mathematical model, Tihany: ICCC 2013 IEEE 9th International Conference on Computational Cybernetics, 2013.

[11] Nyers J., Nyers Á.: Investigation of Heat Pump Condenser Performance in Heating Process of Buildings using a Steady-State Mathematical Model, Energy and Buildings 75 (2014). IF:2.465.