EXPECTED DOWNSIDE RISK-BASED ASSET PRICING

by

DUSÁN TIMOTITY

A DISSERTATION
submitted in partial fulfilment of the requirements for the degree of
DOCTOR OF PHILOSOPHY

BUDAPEST UNIVERSITY OF TECHNOLOGY AND ECONOMICS

Supervised by Mihály Ormos, PhD

Budapest, Hungary, 2016
Contents
I. Introduction ........................................................................................................................................... 2
II. Generalized asset pricing: Expected Downside Risk-Based Equilibrium Modelling ........ 7
   II/1. Introduction ................................................................................................................................. 8
   II/2. The cause of risk-seeking behaviour ....................................................................................... 9
   II/3. Expected downside risk (EDR) as a risk measure ................................................................. 21
   II/4. Risk-seeking behaviour in EDR Environment ..................................................................... 29
   II/5. The effect of limited borrowings ............................................................................................ 33
   II/6. Expected return ....................................................................................................................... 38
   II/7. Conclusion ............................................................................................................................. 41
III. The case of “Less is more”: Modelling risk-preference with Expected Downside Risk 43
   III/1. Introduction ............................................................................................................................. 43
   III/2. The model ............................................................................................................................... 45
   III/3. Empirical results .................................................................................................................... 53
   III/4. Concluding remarks ............................................................................................................... 60
IV. Expected Downside Risk and Asset Prices: Characteristics of Emerging and Developed European Markets ........................................................................................................... 62
   IV/1. Introduction ............................................................................................................................. 62
   IV/2. Literature review ...................................................................................................................... 63
   IV/3. Data and methodology ........................................................................................................... 65
   IV/4. Separate analysis .................................................................................................................... 69
   IV/5. Aggregate analysis .................................................................................................................. 71
   IV/6. Model comparison ................................................................................................................... 73
   IV/7. Conclusion ............................................................................................................................. 77
V. Summary .............................................................................................................................................. 79
I. Introduction

Since the milestone paper of Harry Markowitz (1952) on Modern Portfolio Theory, literature on the pricing of capital assets has motivated a large number of research papers and is still one of the main research direction. However, the initial framework of Markowitz’s paper has gone through an extensive evolution since. Various branches originating from the mean-variance setting have been proposed in the subsequent decades, all of which aiming to better capture the expected return of capital assets. Researchers in the topic followed one of two, fundamentally separate directions. In particular, one subgroup has devoted its research potential to better understand the fundamentals behind the prices of assets, while the other has made an effort to discover robust factors relevant in the dynamics of prices. The former is called equilibrium asset pricing that derives the value of goods in normative economics, whereas the latter is commonly referred to as empirical finance constituting a branch of positive economics.

Although, the mean-variance model of Markowitz provided a coherent framework describing the choice of individuals under risk, it had no practical use as a tool for pricing risky assets. Nevertheless, its findings boosted a great amount of research in the topic, and in a decade, economists of the first aforementioned subgroup have made success in creating a coherent pricing setting that was finally able to provide more or less reasonable estimates for asset prices, which could yield a model that can be used in everyday practice. These findings together have been indexed in literature under the name of Capital Asset Pricing Model or CAPM (Lintner, 1965; Mossin, 1966; Sharpe, 1964 and Treynor, 1961).

This latter model was based on the idea of diversification, the fact that investors hold portfolios in order to decrease the riskiness of their portfolio. Moreover, the authors have concluded that, if investors are rational, the market is perfect and there is an infinite limit for borrowing at the risk-free rate, each market participant holds the same combination of risky assets; only the share of the risk-free asset in their portfolio differs according to their preferences. This finding also yields that the one risky portfolio held by investors is the capital market itself; therefore, individual asset prices behave according to their relationship to this market portfolio. The authors called this latter relationship as the sensitivity to the market returns, or the CAPM Beta.
However, over the subsequent years, a great amount of evidence has accumulated on the poor performance of predictions and estimations of expected return based on the CAPM. The detection of anomalistic patterns, such as seasonalities of the January effect (Rozeff and Kinney, 1976), the weekend (Cross, 1973) or the holiday effect (Ariel, 1985), and the findings on novel factors playing a significant role in driving asset prices, such as the price-to-earnings ratio (Basu, 1977) or the dividend yield (Keim, 1985) have all lead to the emergence of a new approach to asset pricing, the empirical finance. Since the first detections of such anomalies, a great amount of empirical research has focused on finding and documenting such patterns. The whole efficient market hypothesis (EMH) (Fama, 1970) is formed around these anomalistic findings, and tries to implement them in asset pricing. In particular, the weak form of efficiency, which states that prices include all the information derived from time series of financial and economic variables, denies the existence of such patterns. This market efficiency, however, is a dual hypothesis; it measures the abnormal returns or anomalies with respect to expected return derived from an equilibrium asset pricing model. Therefore, the existence of such patterns might be simply the cause of a false choice of the applied asset pricing model.

This question was first addressed in the theoretical discussion of Stephen Ross (1976) on the use of multifactor models. Since then, a vast number of papers have been published with the aim of obtaining better predictions of expected returns through the inclusion of novel risk measures, without deriving the principles of investors’ preferences. One of the best-known models implementing the predictive ability of such factors was presented by Eugene Fama and Kenneth French (1993), in which the authors included cross-sectional return differences based on the market capitalization and price-to-book value ratio of equities in addition to the market premium proposed by the CAPM. The relevance of this approach is well reflected by the fact that studies aiming to discover new, relevant factors are still getting published on the topic. For example, some control for autocorrelation between returns in the momentum factor (Carhart, 1997) and some include the effect of the liquidity on asset prices (Amihud and Mendelson, 1986; Pastor and Stambaugh, 2003).

Nonetheless, during the last three decades, researchers aiming to provide fundamental models based on the preferences of investors have also contributed to the literature on asset pricing. One subgroup of these latter models has been focusing on relating the lifetime utility of investors through capturing the negative relationship
between expected return of a capital asset and its sensitivity to changes in consumption (Merton, 1973). The other subgroup emerged from the novel findings related to individual preferences; these models either included additional elements to the standard expected utility theory introduced by Neumann and Morgenstern (2007), such as the Epstein and Zin recursive utility (1989) in Campbell et al. (2003), or implemented findings from behavioural finance such as prospect theory (Kahneman and Tversky, 1979) in asset pricing models (Barberis et al., 2001).

In this dissertation, I follow this latter approach by allowing for investors who behave according to prospect theory and are subject to heuristic biases in decision making and mental framing. Apart from the extensive amount of experimental evidence, the importance of implementing these findings from behavioural finance has been highlighted by results of recently emerged interdisciplinary studies as well. In particular, the findings of a very recent field between economics, psychology and neuro-science, the neuroeconomics, as well as results in non-human physiology (Chen et al, 2006; Lakshminarayanan et al., 2011), have confirmed the experimental evidence accumulated in the past decades on heuristics (Kuhnen and Knutson, 2005), reference dependence (de Martino et al., 2009) and prospect theory altogether (Glimcher and Fehr, 2013). Therefore, I aim to cover and implement as much of these biases as possible in the asset pricing framework presented in this dissertation.

In the current set of studies, however, I start from the origins of modern asset pricing, the Modern Portfolio Theory. As mentioned above, recent literature on the fundamental principles behind asset prices either implements behavioural findings in the mean-variance framework and under the standard equilibrium assumptions, such as infinite borrowing and a unique risk-free rate, or approach the modelling from the empirical side by searching for factors relevant in price dynamics. The key point is that practically all of these models are still based on the same assumptions defined by the CAPM fifty years ago.

Most of these standard assumptions, however, are unrealistic and have been shown to contradict the actual characteristics of financial markets. Nevertheless, in this thesis I present that relaxing the most contradictory assumptions, one can still apply the main conclusions of standard asset pricing models. In particular, by switching from the mean-variance framework to modelling based on Expected Downside Risk (EDR), the finite borrowing, non-unique risk-free interest rate, risk-seeking and price-maker investors, and non-normal distributions can be handled in one coherent model derived
from the principles of utility maximisation. This latter risk measure is defined as the expectation of the returns below the expected return, that is, the expected bad outcome. As it captures the risk in terms of outcomes, instead of the usual positive relationship between risk and required return, we find a negative relationship between EDR and expected return in general.

The thesis is structured as follows. Part II is based on the papers of Ormos and Timotity (2013a, 2013b, 2013c, 2013d, 2014a, 2016a) and presents the main modelling framework in the EDR - expected return system. In this section, I present the first two theses of the dissertation.

First, **the inclusion of Expected Downside Risk in asset pricing allows for relaxing one of the most unrealistic, yet crucial restrictions of commonly used equilibrium models, the assumption risk-averse investors.** I show that by extending the definition of market participants to investors who behave in line with prospect theory and become risk-seeking in the domain of losses, this behaviour can be implemented in the standard asset pricing framework as well.

Second, in addition to the generalized investor behaviour, **with the shift in risk measure to Expected Downside Risk, the assumptions of price-taker investors and unlimited leverage opportunity for a unique interest rate can also be relaxed.** The proposed model derives that neglecting the effect of leverage margins and liquidation or not taking into account the cross-sectional dynamics of the borrowing rate may lead to biased results in standard asset pricing models. However, with the use of EDR, one considers the tail risks as well, which latter is the driving factor of the effect of leverage margin on risk and expected return. Furthermore, EDR can also be used as a proxy for leverage limits and investor-specific interest rate; therefore, it describes individual choice subject to realistic constraints. In Part II an aggregation of these individual choices is provided as well, in which each participant behaves as a price-maker investor contributing to the dynamics of the unique, aggregate expected return. Hence, the price is not anymore taken as exogenous but individual investors themselves play an important role in its formation, that is, the assumption of price-taker investors can be relaxed.

Part III discusses the implications of the EDR risk measure in asset pricing and its relationship to anomalistic findings in experimental economics. The third thesis is related to the studies of Ormos and Timotity (2016a, 2016c), which discusses the rare, yet possible case of a negative relationship between risk and expected return.
mentioned in Part II. According to this point, by using the Expected Downside Risk as risk measure, both a positive and a negative relationship between expected return and risk can be derived under standard conditions of equilibrium asset pricing models, such as the expected utility theory and positive risk-aversion. Using this framework no alternative psychological explanation or additional boundary condition on utility theory is required to explain the phenomenon often measured in experimental economics (Brooks et al, 2014; Kahneman and Tversky, 1979; 1992; Linville and Fischer, 1991; Post and Levy, 2005).

In Part IV that is based on the paper of Ormos and Timotity (2016b), empirical evidence for the superior performance of the proposed asset pricing model is presented in developed and emerging European markets. The fourth and fifth theses based on this latter study are as follows.

First, I find that the Expected Downside Risk-based asset pricing model captures the relationship between risk and expected return with superior performance in comparison with the volatility, variance, semi-variance, CAPM Beta, and Downside CAPM Beta on Central and Eastern European and Developed Western European markets. Based on the empirical analysis of capital markets of the United States of America, the United Kingdom, Germany, France, Hungary, Poland, and the Czech Republic daily, weekly, monthly and yearly analysis provides evidence in favour of this latter point.

Second, dollar-denominated returns often perform better than regressions in the local currency both in regressions based on EDR and alternative risk measures, which indicates that international capital inflow does play an important role in asset prices. Moreover, this finding is particularly significant on Developed European capital markets, which is in contradiction with the belief of international investors having a greater influence on emerging markets compared to developed ones.

Finally, in Part V, a short summary is provided that covers the main results and theses of this dissertation.
II. Generalized asset pricing: Expected Downside Risk-Based Equilibrium Modelling

Abstract

This part of the dissertation introduces an equilibrium asset pricing model, which we build on the relationship between a novel risk measure, the Expected Downside Risk (EDR) and the expected return. On the one hand, our proposed risk measure uses a nonparametric approach that allows us to get rid of any assumption on the distribution of returns. On the other hand, our asset pricing model includes loss-averse investors of Prospect Theory, through which we implement the risk-seeking behaviour of investors in a dynamic setting. By including EDR in our proposed model unrealistic assumptions of commonly used equilibrium models – such as the exclusion of risk-seeking or price-maker investors and the assumption of unlimited leverage opportunity for a unique interest rate – can be relaxed. Therefore, we argue that based on more realistic assumptions our model is able to describe equilibrium expected returns with higher accuracy, which we support by empirical evidence as well.

Acknowledgments: We are grateful to the seminar and conference participants at the Workshop on Behavioural Economics and Industrial Organization at Corvinus University, 2014, 10th EBES Conference in Istanbul, the 10th International Scientific Conference on European Financial Systems at Masaryk University and 5th International Conference “Economic Challenges in Enlarged Europe” in Tallinn. We would like to gratefully acknowledge the valuable comments and suggestions of the Editor, Prof. Sushanta Mallick and three anonymous referees that have contributed to a substantially improved study. Mihály Ormos acknowledges the support by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences. Dusán Timotity acknowledges the support by the Foundation of Pallas Athéné Domus Scientiae.
II/1. Introduction

The most commonly applied asset pricing model, the Capital Asset Pricing Model (CAPM) (Lintner, 1965; Mossin, 1966; Sharpe, 1964), defines three sets or areas of constraints or boundary conditions: (1) there is a perfect market, which consists of four assumptions: (a) investors have constant preferences, (b) are price-takers, (c) are perfectly informed and (d) there is no transaction cost; (2) conditions of investors’ behaviour: (a) they are rationally risk-averse, therefore they hold efficient portfolios defined by Markowitz and (b) due to their rationality and perfect information have homogenous expectations; (3) assumptions of investment possibilities: (a) in addition to risky portfolios, investors may invest in risk-free assets as well and (b) they can borrow infinite amounts at this risk-free rate. Furthermore, the standard regressions behind the theoretical model assume that the returns are normally distributed. Using our proposed equilibrium asset pricing model that includes loss-averse investors described by Prospect Theory (Kahneman and Tversky, 1979) rather than Expected Utility Theory (EUT) (Von Neuman and Morgenstern, 2007) the assumptions of risk-averse and price-taker investors, normal distribution of returns (thus the required linear regression between risk and expected return (Erdős et al., 2011) and unlimited borrowing at a unique risk-free interest rate can be omitted.

In this latter setting, we apply the assumption that investors may define a reference point different from zero on their utility curve, resulting in anchoring (Ariely et al., 2003). We show that in some cases they do not refuse risk; moreover, they start to follow risk-seeking behaviour to an extent, since their expected utility can be maximized with this behaviour. Thus, the assumption of risk-averse investors cannot be valid if such investors participate in market processes. However, we show that this type of behaviour can be implemented in standard asset pricing without any problems, which yields that the inclusion of risk-seeking investors does not change the main implications of standard equilibrium models.

We approximate the perceived risk by a novel measure, the Expected Downside Risk (EDR), which is based on Value-at-Risk (Campbell et al., 2001; Jorion, 2007) and Conditional Value-at-Risk (Rockafellar and Uryasev, 2000). Through this method, we define the expected loss below the expected return weighted by its probability. The results we present in this study are significantly different from those of well-known models that approximate expected return by standard deviation, such as Modern
Portfolio Theory (Markowitz, 1952). Although the mean-variance optimization is valid under elliptical distributions as well (Chamberlain, 1983) and the normality condition has already been omitted in some regressions (such as the Markowitz 2.0 model based on Conditional Value-at-Risk (Kaplan, 2012) or Iglesias (2015)), these models still require unrealistic assumptions and miss a coherent and general approach to modern asset pricing, which we aim to introduce with our approach. In the followings, we implement the risk-seeking behaviour in an EDR - expected return environment by combining the risk-seeking behaviour and our proposed risk measure. Furthermore, we add to our model the effect of limited borrowings, where different leverage constraints and interest rates are applied for every single investor. Hence, a more realistic and precise way to explain the individual optimization method is defined, which yields the unnecessity of the assumption of unlimited leverage opportunity for a unique risk-free rate.

Subsequent to describing the individual optimization the aggregation method is discussed (i.e. how the market sets the expected return of a given asset). Here we include price-maker investors as well, who may have a significant effect on price formation by block transactions or in illiquid market segments. Finally, we provide a pricing equation that approximates the expected return by EDR and compare it against alternative risk measures.

This part is structured as the following: section 2 discusses risk-seeking behaviour and its implications for asset pricing models, then our proposed risk measure is defined in section 3. In section 4 these two are combined together and risk-seeking behaviour is shown in an EDR setting. Section 5 is related to restricted borrowing limits and their effect on portfolio choice. Section 6 implements all the previous findings in one model and describes the formulation of the expected return including the pricing equation. Finally, Part II ends with a brief conclusion. All sections are divided into theoretical and empirical subsections where empirical evidence is provided to facilitate understanding the main ideas and to support the theory.

II/2. The cause of risk-seeking behaviour

II/2.1 Risk aversion or loss aversion

According to Expected Utility Theory (EUT) (Von Neuman and Morgenstern, 2007), perceived utility is a concave function of total wealth. This leads to the law of diminishing marginal utility, which means that the marginal utility that a person derives
from consuming each additional unit of a product declines. According to the theory, this utility perception causes risk aversion, meaning that a mathematically fair investment with equal expected gain and loss would have a negative effect on expected utility.

There are two main risk aversion factors applied in economic theory: constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA) (Pratt, 1964). The first one describes the risk premium necessary to invest in a mathematically fair investment, which does not depend on the reference wealth. The second one states that risk aversion changes with the change of wealth in a constant way, meaning that reference wealth multiplied by CARA is constant over time for each investor. Both measures have advantages and disadvantages over the other; however, in the case of small changes of wealth, we can use both of them for any investor. According to Markowitz (1952), either examining the utility curve for CARA or for CRRA, the approximation of the utility of investment \( F(U(F)) \) using a Taylor series is defined as

\[
U(F) \approx E(F) - 0.5\alpha \sigma^2,
\]

where \( U(F) \) is the expected utility of an investment \( F \), \( E(F) \) its expected value, \( \alpha \) the Arrow-Pratt measure of constant absolute risk aversion and \( \sigma^2 \) the variance that measures the risk of investment \( F \). This is a key step in our model since there is a debate going on about which utility function to use in Prospect Theory. Although Kahneman and Tversky (1979) propose applying a power function similar to the one used in the CRRA approach, numerous studies show that an exponential function performs better (Smidts, 1997) or at least equally well (Beetsma and Schotman, 2001). Since CARA is independent of wealth, it makes it easier to describe the risk attitude of a given investor; therefore, we apply its corresponding exponential utility function in our model.

By accepting the assumptions that \( \sigma^2 \cdot E(r) \) variance-expected return efficient combinations can be described with a concave curve with positive slope – or in the case of unlimited borrowings with constant slope – and that investors are risk-averse (hence their CARA is positive, thus their utility curve is convex and monotonically increasing), optimal choice can be defined for any investor.

However, according to the Prospect Theory of Kahneman and Tversky (1979), investors do not behave this way, and their actions cannot be explained by Expected Utility Theory (EUT). The authors underline that investors' decision-making process is based not solely on economic rationality but subjective elements as well, which motivates them to behave in a way that would be completely irrational in standard
economic theories. According to Shefrin (2002), these behavioural patterns are heuristic-driven biases and frame dependencies. There are numerous heuristics that play important roles in decision-making, such as representativeness (that is, people overestimate the frequency of events surrounding them and they apply stereotypes in their decisions), availability bias (that is, people rely too heavily on their own experience and easily available information) or – most importantly in this study – anchoring (that is, people make reference points and adjust new information according to them). These subjective elements cause biases in investors’ expected probabilities, therefore they overestimate (and overreact to) rare events and underestimate (underreact to) frequent events.

Based on the heuristics mentioned above, Kahneman and Tversky develop a model that – in spite of EUT, which defines utility as the function of total wealth (the absolute way) – measures utility through the change of wealth (in a relative way), which prove to be a more precise method to describe investors behaviour. According to their theory, positive changes in wealth can be described with a function similar to the EUT utility function (which is concave and has monotonic growth), hence, the law of diminishing marginal utility (Gossen, 1854) stays intact. However, in the case of negative changes in wealth (losses), although investors keep being risk-averse in normal cases (since their reference point returns to zero and the slope of the curve on the loss side is on average 2.25 times the slope on the gain side, thus their utility decreases 2.25 times more for x loss than it increases for x gain), this utility function becomes convex. Therefore, they define an S-shaped value function (utility curve) (Figure II/1). This value function, however, could make current asset pricing models useless by introducing the anchoring heuristic (i.e. including previous outcomes in utility perception) as risk-seeking behaviour emerges this way and optimization with positive CARA is no longer available (i.e. no optimal portfolio with finite risk is present).
II/2.2 The emergence of risk-seeking

The Kahneman and Tversky (1979) model enables analysis of single period choices. According to their theory, the investor makes its choice from the zero reference point every time; however, in reality, this is not always true. As Shefrin (2002) argues, the heuristic-driven biases, for different reasons – such as the anchoring effect (adjusting to a reference situation), the inclusion of sunk costs or the disposition effect (where investors give up properly managing their portfolios after a massive loss and do not sell) – may motivate investors to fix a reference point other than zero on their value function and hold that at their next decision. This dynamic approach, similarly to the loss aversion of the static approach in Prospect Theory, shows that mathematically fair investments with greater volatility may provide expected utility growth; therefore, investors maximizing the variance of the chosen fair investment – up to an optimal, risk-neutral state – may increase their utility gain for a fixed expected return. Hence, this is by definition risk-seeking behaviour, which can be seen in Figure II/2 where we use the exponential utility function for CARA \((1 - e^{-ax})\). Here, we would like to add that later in Section 3.1, the results of our detailed empirical analysis on the VIX confirms that this risk-seeking behaviour is valid not only for tail risk but in the whole domain of losses, and therefore, it drives asset prices generally including tail risk.
calculations as well as the expected return. The calculation of the latter utility function is based on the equation

\[ 1 - e^{-5x} + 2.25\left(1 - e^{5(-0.05)}\right) = -2.25\left(1 - e^{5(-0.05)}\right) + 2.25\left(1 - e^{5(-0.05-0.05-x)}\right) \] (II/1)

where \(a = 5\) is an average level of constant absolute risk aversion and \(x = -5\%\) is the negative starting return indicating the reference point after a 5\% loss. Based on the Monte Carlo approximation results for \(x\), investors are risk-seeking in this case, until they are able to reach the \(x=7.18\%\) return on the positive side. Here, we have to mention that the modified exponential function has a finite slope at zero, hence, not every \((\alpha;x)\) combination has a real solution for eq. (II/1). Less risk-averse investors (with low \(\alpha\)) tend to become risk-seeking after huge declines in wealth as well but the maximal amount of loss (-\(x\)) inducing this phenomenon decreases as risk aversion increases.

**Figure II/2: Fair investment after previous loss**

Notes: Figure II/2 indicates the reference point change in dynamic utility perception subsequent to a loss. The horizontal and vertical axes stand for the wealth and value function changes respectively, where \(U\) indicates the utility function. The previous loss equals to -5\%, whereas the mathematically fair investment with which investors become risk-neutral gives either 12.2\% or -12.2\% in addition to the –5\% loss.

However, those who become risk-seeking maximize their risk (here the variance) for a given expected return until the utility growth due to the positive wealth change is
greater than the utility loss due to negative wealth change. At this point, they become indifferent to risk (i.e. the utility growth and loss of a further change in risk are equal). Depending on the initial portfolio, a risk-neutral curve can be defined using the following method. We create a mathematically fair investment consisting of \((x + y)\) gain and \((-x - y)\) loss both with 50% probability. Let us assume that the reference point is \((-x)\), that is, the investor has anchored to a previous loss with a notional amount of \(x\). This investor keeps following risk-seeking behaviour until the utility growth and loss become equal, therefore:

\[
U(-x) - U(-2x - y) = U(y) - U(-x)
\] (II/2)

By defining the utility on the negative side as a function of its positive equivalent, we can replace the utility for \(x < 0\) with \(U(x) = -2.25 \cdot U(-x)\), and we get:

\[
-2.25U(x) + 2.25U(2x + y) = U(y) + 2.25U(x)
\] (II/3)

therefore

\[
U(y) = -4.5U(x) + 2.25U(2x + y).
\] (II/4)

We can define an alternative version of eq. (II/4) by:

\[
\frac{2.25[2U(x) + U(y) - U(2x + y)]}{1.25} = U(y)
\] (II/5)

One can see from this equation that due to the law of diminishing marginal utility \([2U(x) + U(y) - U(2x + y)] > 0\), therefore, \(U(y)\) and \(y\) have to be positive in order to find a solution for the equation. The exact shape of the utility function can define the precise solution.

As for the variance of this investment, starting from the \(-x\) reference point choosing the mathematically fair investment with \((x+y)\) amplitude, we can define:

\[
\sigma^2 = \frac{[y - (-x)]^2 + [-2x - y - (-x)]^2}{2} = \frac{(y + x)^2 + (-x - y)^2}{2} = \frac{2(x + y)^2}{2} = (x + y)^2
\] (II/6)
Furthermore, by assuming an alternative version of this investment with a positive expected return, we get similar results. Here, we keep the -x reference point but instead of 0, an investment with an expected return of 0<x<1 is analysed. In the followings, we show that for an investment with the c1 expected return after -x loss (that is, having the (-x+c1) reference point), the variance has to decrease in order to have the same utility change – in absolute terms – for both negative and positive wealth changes. The slope of this decrease can be defined by analysing the precise utility function defined in eq. (II/5).

In order to find a solution for the equation between utility gains and losses, we define the following situation. Given a mathematically fair investment with c1 expected return combined with the original reference point of -x the new reference point becomes (-x+c1). As mentioned above, the amplitude, having been (x+y) before, has to decrease by -c2 in order to keep the equality between utility differences caused by gains and by losses, therefore

$$U(y - c_2) - U(-x + c_1) = U(-x + c_1) - U(-2x + 2c_1 - y + c_2) \quad (II/7)$$

Converting the negative side to its positive equivalent again we get

$$U(y - c_2) = 2.25U(2x - 2c_1 + y - c_2) - 4.5U(x - c_1) \quad (II/8)$$

Through eq. (II/8), it is clear that in the case of c1=0 we get the same result as eq. (II/4) (thus c2=0) and if c1=x, then

$$U(y - c_2) = 2.25U(y - c_2) \quad (II/9)$$

Due to the multiplier 2.25, eq. (II/9) can be solved if and only if both sides are 0, therefore, y=c2. According to eq. (II/4):

$$0 = 2.25[U(2(x - c_1) + y - c_2) - 2U(x - c_1) - U(y - c_2)] + 1.25U(y - c_2) \quad (II/10)$$

We define the relation between c1 and c2 by using eq. (II/10) as a function. Assuming that the utility curve is a standard exponential function for constant absolute risk.
aversion that is \((1 - e^{-ax})\), we substitute the following: \(x=c\), \(y=d\) will be constant, while \(c_1=x\) and \(c_2=y\) will be the main variables. Eq. (II/10) is now rewritten:

\[
2.25(1 - e^{-a(2c+d-2x-y)}) - 4.5(1 - e^{-a(c-x)}) - 1 + e^{-a(d-y)} \quad \text{(II/11)}
\]

The total derivative of this function by variable \(x\) is:

\[
\frac{d}{dx} \left(2.25(1 - e^{-a(2c+d-2x-y)}) - 4.5(1 - e^{-a(c-x)}) - 1 + e^{-a(d-y)}\right) = -2.25e^{-a(2c+d-2x-y)} \left(\frac{da}{dx}(-2c + d - 2x - y) - a \left(2 \frac{dc}{dx} + \frac{dd}{dx} - \frac{dy}{dx} - 2\right)\right) + 4.5e^{-a(c-x)} \left(-(c - x) \frac{da}{dx} - a \left(\frac{dc}{dx} - 1\right)\right) + e^{-a(d-y)} \left(-(d - y) \frac{da}{dx} - a \left(\frac{dd}{dx} - \frac{dy}{dx}\right)\right) \quad \text{(II/12)}
\]

Since we know that according to eq. (II/10) this function has to be zero for all changes of \(x\), its derivative also has to be zero. \(df/dx\) can be written as

\[
\frac{df}{dx} = -2.25e^{-a(2c+d-2x-y)} \left(a \frac{dy}{dx} + 2a\right) + 4.5ae^{-a(c-x)} + a \frac{dy}{dx} e^{-a(d-y)} = 0 \quad \text{(II/13)}
\]

Thus, we can describe the relation between \(x\) and \(y\) as \(dy/dx\):

\[
\frac{dy}{dx} = \frac{4.5e^{-a(2c+d-2x-y)} - 4.5e^{-a(c-x)}}{e^{-a(d-y)} - 2.25e^{-a(2c+d-2x-y)}} \quad \text{(II/14)}
\]

According to our Monte Carlo approximation for \((d - y)\) using \((c - x) = [0.01; 1]\) and \(a = [0.5; 10]\) \(\frac{d(d-y)}{d(c-x)}, \frac{d(d-y)}{d(c-x)} > 0\) for each solution, hence if the expected return \(x\) increases, \(y\) has to increase according to eq. (II/14) and thus the volatility of \((d - y) + (c - x)\) declines. In the case of small changes the same results apply for the power function used for CRRA as well.

Turning back to our original variable system \((x,y,c_1,c_2)\), we find that in the case of \(0<c_1<x\) this risk-seeking behaviour exists and the interval for variance-expected return combinations is known. Furthermore, the shape of the transition function is described by eq. (II/14).
Summarizing the above-mentioned derivation, we show that starting from the variance-expected return combination \((0; x)\) (the minimum of the interval) and ending at the combination \(((x+y)^2; 0)\), investors maximize the variance until they reach the “optimal” choice, the risk-neutral curve, which can be seen in Figure II/3. We underline that this risk-neutral function is monotonically decreasing; therefore, according to eq. (II/14), as the expected return of the investment \((c_1)\) increases, the variance \((c_2)\) has to decrease. Hence, instead of investment \(A\), an investor would choose investment \(A'\) with the same expected return and higher variance, which is completely irrational in standard equilibrium models. The milestone paper of Thaler and Johnson (1990) also confirms these results where we see a good example of investors becoming risk-seeking subsequent to losses if breaking even is possible, which latter is practically always valid in the case of capital markets.

**Figure II/3: Risk-seeking until risk neutrality**

Notes: Figure II/3 represents the optimization of risk-seeking investors. They maximize utility by increasing risk (thus decreasing EDR) until they reach the risk neutral point \(A'\). Depending on the initial portfolio choice alternatives of \(A'\) constitute the risk-neutral curve.

As our result is derived in a variance-expected return system, it can be implemented in standard equilibrium models as we show in the next section. However, standard asset pricing models require symmetrically distributed returns, which is clearly not the case in the real markets. Therefore, in Section 3 we propose a model applying Expected Downside Risk (EDR) as risk measure, which allows us to describe the same
situation with an alternative approach that is able to manage any type of return distribution.

**II/2.3 Implementation in standard models**

In order to place the above-mentioned behaviour in asset pricing theories, we combine the results of Expected Utility Theory (EUT) and Prospect Theory (PT). These two theories – although based on different assumptions – have many similar properties. Although EUT examines the utility of total wealth and PT explains the change of utility for gains and losses, both utility functions are concave and strictly monotonically increase on the positive side. We assume that the utility function described by PT is actually the same as the EUT function from the reference wealth $W = 1$. Therefore, the behaviour of completely rational investors, who are not influenced by different heuristics (such as the anchoring effect) and are perfectly informed, can be described by both EUT and PT. These market participants have risk-averse behaviour; therefore, the base model of CAPM, Modern Portfolio Theory (MPT) (Markowitz, 1952), can sufficiently explain their preferences.

However, risk-seeking behaviour can also be implemented in MPT using the following method. We have already shown that risk-seeking always has a limit where it reaches risk neutrality and this converges to zero variance with the growth of expected return (since it decreases monotonically in a variance-expected return system). This risk-neutral curve (RNC) consists of points where investors are risk-neutral, where their utility depends only on the expected return of investments and it is not influenced by risk (here variance). Therefore, investors choose the portfolio with the highest possible return on their RNC. As this curve crosses the MPT-defined curve of efficient portfolios, the efficiency frontier (EF) – by definition the portfolio with the highest expected return at a given risk level – also the optimal choice – is exactly the intersection of these two curves illustrated in Figure II/4. Thus, risk-seeking behaviour has no effect on the efficient portfolios of standard equilibrium models; therefore, it can be implemented in asset pricing regressions. However, we have to take into consideration that it may change the overall volatility of markets if the efficiency frontier is an endogenous variable as well (which we show in Section 5).
II/2.4 Evidence of risk-seeking behaviour in empirical data

The existence of risk-seeking behaviour in everyday life is shown in examples of Shefrin (2002) and Thaler and Johnson (1990). Furthermore, we provide some evidence as well in the following.

We analyse daily returns of the Standard and Poor’s 500 Index from 1950 to 2013 and find strong evidence that supports our results of risk-seeking behaviour described above. Regardless of the time, the negative return in period $t$ causes higher volatility in $t+1$. We divide our experiment into monthly and weekly analysis. We assume that two distinct effects generate the volatility change in period $t+1$. First, a massive gain or loss in $t$ affects next period volatility due to the behaviour shown above (Black, 1976; Bollerslev, 1986; Engle, 1982; Christensen et al., 2015; Zhou and Nicholson, 2015); second, the gains or losses in interval $t+1$ also influence volatility due to equilibrium pricing. With this method, we can rule out the possibility that the volatility change is only due to the price movements in the same period or vice versa. We find that extreme previous period return has a greater effect on current volatility than the return in the same period.
In our analysis, massive losses are defined as returns below the 10% percentile and massive gains as returns above the 90% percentile of the density function. We find that in the case of a massive weekly loss, the increasing effect on next period volatility is higher than in the case of monthly loss; however, both are significant. We argue that this effect is due to the availability heuristic – which drives investors to rely more on recent information – which causes previous weekly returns to have a stronger influence than previous monthly returns.

Table II/1 shows that after a monthly price fall, the results are only significant if the next month has a negative return (thus the S&P 500 is in a downtrend); however, the aggregate monthly results are also significant. In the case of a weekly price fall, regardless of the trend over the next week, volatility increases significantly.

**Table II/1: Effect of price jumps and price falls on volatility**

<table>
<thead>
<tr>
<th>Effect of Price Jump or Fall</th>
<th>Average</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility change after price jump in monthly uptrend</td>
<td>-0.65%</td>
<td>0.4316</td>
</tr>
<tr>
<td>Volatility change after price jump in monthly downtrend</td>
<td>3.36%</td>
<td>0.3294</td>
</tr>
<tr>
<td>Volatility change after price fall in monthly uptrend</td>
<td>-6.99%</td>
<td>0.0663</td>
</tr>
<tr>
<td>Volatility change after price fall in monthly downtrend</td>
<td>33.91%</td>
<td>0.0001***</td>
</tr>
<tr>
<td>Volatility change after price jump in weekly uptrend</td>
<td>-2.61%</td>
<td>0.2158</td>
</tr>
<tr>
<td>Volatility change after price jump in weekly downtrend</td>
<td>9.44%</td>
<td>0.0502</td>
</tr>
<tr>
<td>Volatility change after price fall in weekly uptrend</td>
<td>26.76%</td>
<td>0.0000***</td>
</tr>
<tr>
<td>Volatility change after price fall in weekly downtrend</td>
<td>37.03%</td>
<td>0.0000***</td>
</tr>
<tr>
<td>Avg vol change after price jump monthly</td>
<td>0.61%</td>
<td>0.4309</td>
</tr>
<tr>
<td>Avg vol change after price jump weekly</td>
<td>1.98%</td>
<td>0.2550</td>
</tr>
<tr>
<td>Avg vol change after price fall monthly</td>
<td>12.10%</td>
<td>0.0078**</td>
</tr>
<tr>
<td>Avg vol change after price fall weekly</td>
<td>31.38%</td>
<td>0.0000***</td>
</tr>
</tbody>
</table>

Notes: We used the daily historical data from the S&P 500 Index for the period of January 2, 1950 to May 6, 2013. We define price falls as a monthly (or weekly) return below the 10th percentile of the monthly (or weekly) return distribution and price jumps as monthly (or weekly) returns above the 90th percentile of the monthly (or weekly) return distribution. Uptrend means that the next period has a positive return, while downtrend stands for a negative return. Monthly (or weekly) volatilities are based on daily returns and their change is calculated using logarithmic return. The averages presented are the mean weekly and monthly volatility changes over the previous period in the indicated sub-cases and the p-values stand for the respective Student’s t-test for them with the null of the average being zero.

According to our model, this phenomenon can be explained by the risk-seeking effect after losses. Since the Standard and Poor’s 500 Index is a value-weighted index, it reflects the changes in asset allocation. In the case of price falls, investors suffer from losses and turn to riskier investments. They substitute their less risky portfolio elements
with riskier ones, thus, the increased demand for risky investments elevates their price and market capitalization, while the opposite happens for less risky ones. This way the weights of riskier investments in the S&P 500 increase; therefore, their price movements and volatility will contribute to the index more, increasing the volatility of the S&P 500 itself as well. These results are confirmed by literature on mutual fund activity as well, according to which a negative relationship was found between returns and subsequent money inflows (Warther, 1995; Goetzman and Massa, 1999; Edelen and Warner, 2001) and between contemporaneous inflow of equity and bond funds (Goetzmann et al., 2000).

II/3. Expected downside risk (EDR) as a risk measure

Since the introduction of the expected return-variance relationship in MPT and the beta in the CAPM, numerous attempts have been made to offer an alternative risk measure. Amongst the most important ones, we find Value-at-Risk (Holton, 2003), which gained popularity amongst quantitative financial professionals in the late ’80s (mainly due to the crisis in 1987). This measure provides an entirely new method to calculate the true risk of derivatives, options, and other nonstandard financial assets as it shows the maximal value that investors can lose with a predefined confidence interval (or probability), which is based on the true density function of these investments. Although it exhibits many advantages over volatility and beta, it misses the important characteristics of subadditivity and convexity (Rockafellar and Uryasev, 2000). Conditional Value-at-Risk (CVaR), also called as Expected Shortfall, is introduced to solve the problems of subadditivity and convexity, which makes CVaR one of the most precise risk-measuring techniques today (Krokhmal et al., 2002); moreover, it is then defined as a coherent risk measure with its properties of monotonicity, subadditivity, positive homogeneity, and translation invariance (Acerbi and Tasche, 2002; Csóka et al., 2007). Nevertheless, this latter risk measure has also raised another question recently: in particular, in contrast to VaR, CVaR is not elicitable, which latter flaw has yielded a number of criticisms against its use in practice. Still, as Acerbi and Székely (2014) show, both risk measures can be backtested; therefore, the inelicitability does not pose a real problem against the practical use of CVaR-based measures. However, the predefined significance level in CVaR exhibits some disadvantages in quantitative methods. Choosing the optimal level of \( \alpha \) is difficult and influences heavily
the investment choice in modelling. This problem with the subjective confidence level has been questioned in Huang et al. (2012) as well, however, their proposed risk-measure, the extreme downside risk comes with two possible flaws: first, it is based on extreme value theory, therefore, its fitted distribution may not represent the true behaviour of asset prices; second, its left tail index is difficult to interpret for the average investor, and therefore, it may not account for the true premium required. The performance measure Omega (Keating and Shadwick, 2002) avoids the subjectivity problem by measuring the ratio of the expected abnormal return below and above zero; however, its use in equilibrium modelling is fairly moderate since investors are rather interested in downside risk and expected return. Alternative measures similar to ours are semi-variance (Markowitz, 1968) and its standardized version, the downside beta (Estrada, 2007), which consider only the downside-risk of returns, however, these measures cannot be fully implemented in equilibrium asset pricing under the assumption of expected utility theory and again their relevance in the risk premium is questionable due to their complexity. We apply Expected Downside Risk as risk measure with which we can eliminate the above-mentioned disadvantages.

Our definition of EDR is based on VaR and CVaR, which are discussed in Appendix II/1 in detail. Similar to Omega, semi-variance and downside beta, EDR does not apply any subjectively fixed α percentile to measure tail risk but is defined as a risk measure that covers the loss function on the whole domain, although, in contrast to Omega, we examine losses relative to the expected return \( E(r) \) instead of zero. In other words, we define Expected Downside Risk as the negative Conditional Value-at-Risk for \( \alpha \) significance level, where \( VaR_\alpha(x) \) is the expected return. This is shown in eq. (II/15), where the EDR of investment \( x \) is described:

\[
EDR(x) = p(r_x \leq E(r_x))^{-1} \int_{r_x(y) \leq E(r_x)} r_x(y)p(y)dy, \quad (\text{II/15})
\]

where \( r_x(y) \) stands for the return distribution of portfolio \( x \), \( p(y) \) for the probability density function and \( E(x) \) for the expected return. We illustrate the EDR-\( E(r) \) system in Figure II/5, in which the Expected Downside Risk and expected return are captured by the horizontal and vertical axes. Since EDR measures the whole downside risk and considers return above zero as well, we may discover some investment opportunities that reflect well the positive time preference of investors. This phenomenon is due to
the fact that consumption in the present causes higher utility growth for investors than the same consumption in the future; therefore, they require compensation in the future in exchange for lending money (thus utility) in the present. Hence, investments may exist where \( EDR(x) \) exceeds 0 because their risk is so low that the average of negative returns in absolute meaning is smaller than the expected return itself. Even if the investment turns out to have a lower return than the expected one, the Expected Downside Risk is positive; that is, the investor realizes a positive expected return even if the return falls short of the expectation. This can be interpreted as the risk-free return defined in standard equilibrium asset pricing models; however, in reality, none of them are entirely risk-free. Therefore, \( EDR(x) \) is always lower than \( E(r) \). This can be illustrated by the line with \( s=1 \) slope in the \( EDR-E(r) \) coordinate system. None of the portfolios can reach the area below this line.

We illustrate the system of \( \{EDR(x),E(r)\} \) pairs in Figure II/5 to the right. The weights and the values of the pairs are calculated by Monte Carlo simulation with the following data and parameters. We use historical, annual returns from 1993 to 2014 of 22 different assets: 20 randomly sampled shares, the 3-month Treasury bill and the value-weighted equity index of the Center for Research in Security Prices (CRSP). During the simulation 10,000 randomly weighted portfolios are generated and for every portfolio, we calculate the average (expected) annual return, variance and the \( EDR \). In Figure II/5 the horizontal axis shows the Expected Downside Risk, while the vertical axis indicates the expected return. The efficiency frontier (the highest expected return for every \( EDR \) value) is very similar to the one in the MPT (to the left in Figure II/5) since it is concave, although its slope is decreasing instead of increasing. So, the optimization with indifference curve (which is convex in MPT) seems to have a unique solution for \( EDR \) equilibrium as well.
In Figure II/6 we show the difference between applying the normal or Gaussian PDF and using the historical distribution of the returns in EDR calculation to underline the importance of empirical risk measures. For investments with low expected return, historical distribution seems to have a lower risk than the estimation with normal PDF, while for those with high expected return the risk is higher using empirical data, the latter of which reflects the fat-tail distribution of returns. The former phenomenon is due to the fact that less risky investments (e.g. government bonds) have highly skewed distribution, thus their expected return is well above the median; therefore, the loss integrating interval to $E(r)$ is wider than in the case of normal distribution, where it would be exactly the median. The Student’s t-test with the null hypothesis of the difference being zero is rejected at extremely low p-values (less than $10^{-4}$ in magnitude), hence, applying the historical distribution indeed matters.
Figure II/6: The effect of true distribution relative to normal

EDR-Expected return pairs

Notes: Figure II/6 indicates EDR (horizontal axis) and E(r) (vertical axis) pairs using Gaussian and historical distribution of yearly returns of 10000 randomly weighted portfolios containing the annual returns of 22 different assets from 1993 to 2014: 20 randomly sampled individual shares, the value-weighted CRSP equity index and the T-bill with three-month maturity.

Although numerous techniques exist to solve the minimization problem for $CVaR_\alpha(x)$ (e.g. Krokhmal et al., 2002), which can easily be applied to $EDR(x)$ as well, our main goal is not to describe these methods or the calculated efficiency frontier but to combine them with the utility measure in Prospect Theory, hence, to describe the optimal choice of investors with precise quantitative analysis.

II/3.1. Testing the probability level of $EDR$

We assume that the Volatility Index of S&P 500, the VIX, also commonly known as the “fear index,” reflects well investors’ reaction to price changes. If the VIX changes much, it means that investors’ reaction to the given price movement is fairly strong, or they are influenced heavily by the given change in prices.

We examine the price changes of the S&P 500 Index from 1990 to 2013 (using the same data as in 2.4), and then compare this to the changes of the VIX Index in the same period. Our results confirm that changes in $EDR$ are a really important factor for investors; thus, the risk premium correlates strongly with our risk measure. The results show that the 5% $\alpha$ – usually chosen for Conditional Value-at-Risk – is not appropriate; however, using higher levels of $\alpha$ produces higher predictive power to describe investors’ decisions.
In our test, we examine the changes of the VIX for every $\alpha$ percentile of the distribution of S&P 500 daily returns. We apply a $t$-test to examine whether the changes of the VIX Index are significant. The p-values of the changes are presented in Figure II/7.

**Figure II/7: Student-t values for changes of VIX**

![Graph showing Student-t values for changes of VIX.](image)

Notes: The graph above indicates the p-values of Student’s t-tests with the null hypothesis that contemporaneous changes of VIX conditional on quantiles of S&P500 returns indicated on the horizontal axis are equal to zero.

According to the left side of the curve, increasing the $\alpha$ percentile initially raises the prediction power, thus returns higher than the 5% percentile of the distribution also play an important role in investors’ decision. Although we find the $p$-value reaching its minimum at the 40% percentile where it is $1.96 \times 10^{-208}$, raising $\alpha$ further to the $E(r)$ percentile (at 49%), which we also use for $EDR$, we get $8.1 \times 10^{-188}$ for the $p$-value, which is clearly not far from its minimum and is very significant. Therefore, we may state that the interval of the return distribution used for $EDR$ calculation, hence $EDR$ itself too, does have an effect on investors’ decision and seems to be a good risk measure. Here, we add that comparing $p$-values of such magnitude may seem irrelevant to the reader, however, these extremely low values are due to the high number of observations. Therefore, we argue that the trends are much more relevant than the magnitudes themselves. Optimization and collecting evidence for picking a given probability level for our risk-measure require comparing such numbers.

In addition to the aforementioned choice of the applied cutoff level for expected loss we provide in the followings a description how $EDR$ affects asset prices and expected
return and a brief empirical test whether it is able to better capture the expected return than its alternatives.

II/3.2. EDR-based asset pricing

As we mentioned before, the level of risk aversion of an investor can be defined by a unique risk aversion parameter “a” (Pratt, 1964). Since we assume that the Kahneman and Tversky utility function has the same convexity on the right side as the expected utility function in EUT, the behaviour of an individual can be described in both risk-averse and risk-seeking cases with the help of this measure. Therefore, our model is able to define the optimal choice for every investor with the constraints that the goal is utility maximization and the efficiency frontier is known.

In the case of risk aversion, the approximation of expected utility could be used in our model as well, that is

\[ U(F) \approx E(F) - 0.5a\sigma^2 \]  

(II/16)

First, we use normal distribution as approximation, which allows for tighter conditions, however, true distributions can be calculated in the same way. This way we can define \( EDR \) as the function of expected return and variance:

\[ EDR(x) = E(r_x) - 0.8\sigma \]  

(II/17)

According to eq. (II/17), we can substitute the volatility (\( \sigma \)) with the Expected Downside Risk (\( EDR \)); therefore, the approximating function II/(16) can be implemented in the \( EDR-E(r) \) system:

\[ U = E(r_x) - \frac{0.5}{0.8^2}a[E(r_x) - EDR(x)]^2. \]  

(II/18)

We can define the slope of the indifference curve in the \( EDR-E(r) \) system as the total derivative should be zero, hence

\[
\text{in the case of normal distribution } EDR = \text{CVaR}_{0.5} = \int_{E(r)}^{\infty} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(r-E(r))^2}{2\sigma^2}} dr
\]
\[
\frac{dU}{dEDR(x)} = \frac{a}{0.8^2} \left( E(r_x) - EDR(x) \right) + \frac{dE(r_x)}{dEDR(x)} \left( 1 - \frac{a}{0.8^2} \left( E(r_x) - EDR(x) \right) \right) = 0. \tag{II/19}
\]

Therefore, the sensitivity of the expected return for the change of the Expected Downside Risk is
\[
\frac{dE(r_x)}{dEDR(x)} = -\frac{a}{0.8^2} \left( E(r_x) - EDR(x) \right) \left( 1 - \frac{a}{0.8^2} \left( E(r_x) - EDR(x) \right) \right) = 1 - \frac{1}{1 - \frac{a}{0.8^2} \left( E(r_x) - EDR(x) \right)}
\]

\[
\tag{II/20}
\]

In order to determine the concavity of the indifference curve we use the second derivative, that is
\[
\frac{d^2E(r_x)}{dEDR(x)^2} = \frac{a}{0.8^2} \left( 1 - \frac{a}{0.8^2} \left( E(r_x) - EDR(x) \right) \right)^2
\]

\[
\tag{II/21}
\]

This is always positive, hence, indifference curves are convex. We distinguish three cases, which are \( EDR(x) < E(r_x) - \frac{0.9^2}{a}, \) \( EDR(x) = E(r_x) - \frac{0.9^2}{a}, \) and \( EDR(x) > E(r_x) - \frac{0.9^2}{a}. \) For the first case, we have
\[
1 - \frac{a}{0.8^2} \left( E(r_x) - EDR(x) \right) < 0, \quad \frac{dE(r_x)}{dEDR(x)} > 1.
\]

\[
\tag{II/22}
\]

For the second case, \( \frac{dE(r_x)}{dEDR(x)} \) is not differentiable, and the third case implies
\[
1 - \frac{a}{0.8^2} \left( E(r_x) - EDR(x) \right) > 0, \quad \frac{dE(r_x)}{dEDR(x)} < 0.
\]

\[
\tag{II/23}
\]

Except for extremely high risk aversion combined with very high expected return, which is highly unlikely as described in Appendix II/2, the third case applies for real situations practically always. Therefore, indifference curves have generally negative slope and are always convex, hence, optimization methods using concave efficiency frontier can determine the optimal choice in the \( EDR-E(r) \) system as well. Nevertheless, in Part III we cover the rare, yet existing case of an optimal solution with positive-slope utility as well.
II/4. Risk-seeking behaviour in EDR Environment

Standard asset pricing models based on the assumption of normally distributed returns (which overestimate the effect of diversification sometimes and do not consider the autoregression of short-term returns) cannot describe precisely risk-seeking behaviour when return distribution is asymmetric and fat-tailed. As we have mentioned before, the variance-based theory is applicable only in the case of symmetrically distributed returns. Therefore, we introduce a parameter that can describe the risk of an investment without having to consider its skewness or kurtosis, one that can define the risk on both the negative and positive side for any type of distribution. Using Expected Downside Risk we get the optimal solution for this problem.

In order to describe the EDR-based method, first, we introduce a Prospect variable [Pr(x)]. [Pr(x)] is the opposite of Expected Downside Risk; thus, it measures the expected value of the outcomes higher than the expected return. Since

$$ aEDR(x) + (1 - a)Pr(x) = E(r_x) $$ (II/24)

where $\alpha$ is the probability that return will be below the expectation, so $\alpha = p(r(x) \leq E(r(x))$; therefore, Prospect is defined by:

$$ Pr(x) = \frac{1}{(1-\alpha)} [E(r_x) - aEDR(x)] $$ (II/25)

Modifying Figure II/2 with the Prospect we get Figure II/8.
Figure II/8: Risk-seeking with Prospect

Notes: Figure II/8 represents the reference point change in dynamic utility perception subsequent to a loss. The horizontal and vertical axes stand for the wealth and value function changes respectively. $U$ indicates the utility function, $E(r)$ the expected return and $Pr$ the prospect. The previous loss equals to $-x$, whereas the mathematically fair investment gives either $+x+y$ or $-x-y$ in addition to the $-x$ loss.

This way we can omit the symmetry requirement of distributions since the $\alpha$-weighted average of the Prospect and the Expected Downside Risk always sums up the expected return. Therefore, we define every investment as fair investment by simplifying the distribution to these two outcomes. So, we omit the main assumption of modelling with variance, the requirement of symmetric distribution of returns.

Figure II/9 clearly shows the similarity of risk-seeking in the models based on variance and on Prospect. In fact, the only difference is that in the case of using Prospect the model has a solution for asymmetric discrete and continuous distributions as well, and according to eq. (II/14) the change of expected return causes an opposite change in the value of Prospect. In order to capture the relationship, we repeat eq. (II/14):

$$\frac{dy}{dx} = \frac{4.5e^{-a(2c+d-2x-y)} - 4.5e^{-a(c-x)}}{e^{-a(d-y)} - 2.25e^{-a(2c+d-2x-y)}}$$  \hspace{1cm} (II/14)

If we substitute $(d-y-(c+x))$ in eq. (II/14) (which measures the distance between the expected value of positive outcomes and the reference point) with $Pr(x)$, we are able to implement this situation in the $Pr-E(r)$ system. Based on this substitution, $y$ decreases as $c$ decreases, $Pr(x) = d + c - y - x$ has to decrease for the increase of
c and y values since both variables have a negative effect on it. Therefore, we can define a risk-neutral curve similarly to the one in the variance-expected return system, which is shown in Figure II/9. The minimum and maximum points of the interval are \( Pr=y-(x) = x+y \) for \( E(r)=0 \) and \( Pr=0 \) for \( E(r)=x \), which is the same as in the model based on variance. In fact, \( Pr \) could take on values below 0; the only restriction is that it has to be above the expected return. Although, in reality, we do not deal with investments like this, this case is included in the \( EDR-E(r) \) model as well.

**Figure II/9: Risk-seeking in \( Pr-E(r) \) environment**

![Risk-seeking in Pr-E(r) environment](image)

Notes: Figure II/9 shows the optimization of risk-seeking investors. They maximize utility by increasing risk (thus decreasing \( EDR \)) until they reach the risk neutral point \( A' \) then, as the only determinant of utility is the expected return on the risk-neutral curve

Although our model is based on *Expected Downside Risk*, not *Prospect*, we have already defined the relation between the two in eq. (II/25). Substituting this to eq. (II/14) (where \( E(r)=x \) and \( Pr(x)=d+c-y-x \)) we get

\[
EDR(x) = \frac{x-(1-a)(d+c-y-x)}{\alpha} = \alpha^{-1}((2-a)x + (1-a)y - (1-a)c - (1-a)d). \tag{II/26}
\]

Since \((1-a)\) is positive, one can see that the a growth of both \(x\) (the expected return) and \(y\) increases \( EDR(x) \); therefore, the risk-seeking phenomenon can be implemented in our \( EDR-E(r) \) model in the way illustrated in Figure II/10, where the risk-neutral curve is presented.
Figure II/10: Risk-seeking in $EDR-E(r)$ environment

Notes: Figure II/10 shows the optimization of risk-seeking investors. They maximize utility by increasing risk (thus decreasing EDR) until they reach the risk neutral point A'. Depending on the initial portfolio choice alternatives of A' constitute the risk-neutral curve.

Figure II/10 shows clearly that the efficiency frontier of the $EDR-E(r)$ model and the risk-neutral curve, similarly to the variance-based regression, produce a unique optimum again. By accepting that this risk-seeking behaviour exists, investors maximize the risk according to eq. (II/26), and thus, minimize the $EDR$ of their investment for every $E(r)$ expected return up to the frontier curve. At this point they become risk-neutral; thus, their optimal choice depends only on $E(r)$. Therefore, they choose the point with the highest expected return on their risk-neutral curve (RNC), which is an efficient portfolio, precisely the intersection of the efficiency frontier of the $EDR-E(r)$ model and the RNC as presented in Figure II/11.
Figure II/11: Risk-neutral curve and efficiency frontier

Notes: Figure II/11 shows the optimization of risk-seeking investors. They maximize utility by increasing risk (thus decreasing EDR) until they reach the risk neutral point A’ then, as the only determinant of utility is the expected return on the risk-neutral curve, they pick the highest reachable portfolio, which turns out to be exactly the cross-section of the risk-neutral curve and the efficiency frontier.

Hence, our model based on Expected Downside Risk is applicable for risk seekers as well and thus, we do not have to deny the existence of these investors. If this behaviour is limited (and is as shown above), we can implement the phenomenon, and the calculation based on the efficient frontier stays intact, the expected return for every EDR will be the same as in the case without risk-seeking. Therefore, the first thesis of this dissertation can be defined here: the inclusion of Expected Downside Risk in asset pricing allows for relaxing one of the most unrealistic, yet crucial restrictions of commonly used equilibrium models, the assumption risk-averse investors.

II/5. The effect of limited borrowings

In this section, we omit one of the main, although very unrealistic, assumptions of standard asset pricing models, unlimited borrowings. CAPM defines the capital market line (CML) as the set of efficient investment opportunities, including risk-free and risky assets, that goes to infinity. However, this assumption is fairly unrealistic (Holmes et
(al, 2015). In most of the cases, there is no opportunity to invest in such positions. Using realistic factors the expected return of portfolios is completely different from that of standard regressions. We use the finite borrowing constraint that is available at different interest rates. In order to create a leveraged model we create the following assumptions, which are more realistic than the ones used in standard models (Black, 1972; Fama and French, 2004):

- For each investor borrowing is limited (as it is either investment credit or proceeds from short sales). In the following, this limit is measured by $(1+x)$ (for example, in the case of 2:1 leverage $x=1$).
- Every time investors use leverage, the lending institution defines a margin that involves the automatic closing of the position or liquidation if the value of the portfolio reaches its $m$ percentage. Analytically this means that the return reaches a $(-1+m)$ loss; therefore, $r=(-1+m)$.

Due to these assumptions, investors gain other advantages in exchange for paying the interest rate. On the one hand, in the vast majority of contracts, they get insurance “for free” due to liquidation at the margin call. In this case, the investors cannot lose more than their own invested money; however, it would be possible through a leveraged portfolio without marginal requirements. This reduction of risk has no excess cost for them; however, they get some of the negative risk eliminated, and thus get a higher expected return, as is described in Figure II/12 and in the following analytics.

**Figure II/12: Effect of leverage with margin requirements**

Notes: The graphs represent the effect of leverage and margin call: as the leverage increases to $(1+x)$ the expected return would increase with it, however, the margin call protects both the investor and the lender from losing more than the cutoff level $(-1+f)$, therefore, further increasing the expected return.
\[
E(r)_L = E(r)_P (1 + x) - r_c x - p(r_Q < -1 + m)CVaR_{Q,p(r_Q<-1+m)} - p(r_Q < -1 + m)(-1 + m) \\
(II/27)
\]

\[
E(r)_L = E(r)_P (1 + x) - r_c x - p(r_Q < -1 + m) \cdot (CVaR_{Q,p(r_Q<-1+m)} - Var_{Q,p(r_Q<-1+m)}) \\
(II/28)
\]

where:
- \(E(r)_L\) is the expected return of L leveraged portfolio with margin requirements
- \(E(r)_P\) is the expected return of P unleveraged portfolio
- \((1 + x)\) is the leverage
- \(r_c x\) is the interest rate for borrowing multiplied by the borrowed quantity (the total cost of borrowing)
- \(p(r_Q < -1 + m)\) is the probability of Q leveraged portfolio without margin requirements generating a return below (-1+m)
- \(CVaR_{Q,p(r_Q<-1+m)}\) is the Conditional Value-at-Risk of Q portfolio at p probability
- \(Var_{Q,p(r_Q<-1+m)}\) is the Value-at-Risk of Q portfolio at p probability

Since \(CVaR \leq VaR\) is always true for fixed distribution and probability and \(r_Q\) is multiplied by \((1+x)\) for \(x\) leverage, we get increasing marginal expected return in the case of margin requirements instead of constant marginal expected return. So, the relation between the leverage of the portfolio and the expected return is not linear, and the function describing \(EDR-E(r)\) leveraged portfolios \(\frac{dE(r)}{dx}\) is not constant.

According to this deduction, one can create the following leveraged position. If \(A\) and \(B\) are unleveraged portfolios, \(EDR_A=EDR_B\) and \(E(r)_A>E(r)_B\), \(A\) is “more efficient”; therefore, it is the optimal choice. However, the reduction effect of margin requirements can have the opposite result for leveraged expected returns if \(A\) and \(B\) have different probability distributions. This causes \(E(r)_{L,A}<E(r)_{L,B}\) if the return distribution of portfolio \(B\) has fatter tails.

\[
p(r_{A(1+x)} < -1 + m)(CVaR_{A(1+x),p(r_A<-1+m)} - Var_{A(1+x),p(r_A<-1+m)}) - p(r_{B(1+x)} < -1 + m)(CVaR_{B(1+x),p(r_B<-1+m)} - Var_{B(1+x),p(r_B<-1+m)}) > [E(r)_A - E(r)_B](1 + x) \\
(II/29)
\]
where \( r_{B(1+x)} \) is the leveraged expected return of portfolio \( B \) using \((1+x)\) leverage and no margin requirements. The situation mentioned above is presented in Figure II/13 where \( A \) and \( B \) are portfolios without leverage, \( A' \) and \( B' \) are portfolios with \((1+x)\) leverage, \( D \) is the interest rate paid for borrowing and the 45° dashed line indicates the frontier that no portfolio can exist below since \( EDR \leq E(r) \).

**Figure II/13: Effect of different probability distributions on portfolios**

Notes: The graph represents two portfolios with different return distributions. Due to the limited loss (margin call), a portfolio with higher expected loss below the cutoff value could yield better leveraged opportunity while not being efficient without leverage.

According to Figure II/13, rational investors having risk-averse behaviour may hold portfolios that generate less expected return for a given risk with unleveraged conditions. Therefore, we accept the fact that investors hold positions that seem to be “inefficient” initially may be a rational choice. Furthermore, this phenomenon also contradicts the strict dominance of diversification mentioned in standard asset pricing theories. While investors can reduce the volatility to a point through diversifying their portfolios, the distribution of their returns tends to converge to normal distribution as the number of investments grows. Since normally distributed portfolios have much less probability at the tails, the positive effect of margin requirements is also decreased. Therefore, it is not enough to sacrifice everything to diversify without considering any other parameters; one has to analyse the optimal choice in regard to the effect of leverage and the liquation at the margin call.
These parameters can take on values from a wide range; however, with the necessary information, the regression is very precise. In order to describe the behaviour of investors, we use the “a” Arrow-Pratt measure of risk aversion (CARA) that can be defined through various methods such as questionnaires and observation tests (Barsky et al., 1997; Hanna and Lindamood, 2004). Information technology today allows this parameter to be measured continuously for each investor by monitoring transactions; therefore, correction can be made at any time. The optimal choice also depends on the possibilities investors may have; therefore, our model uses the calculated $EDR-E(r)$ pairs, their distributions to adjust the leveraged efficiency frontier, the borrowing constraint and the interest rate for each investor. In our regression the indifference curves (using “a” CARA) and the efficiency frontier (using x leverage limit, $r_C$ interest rate, and $EDR-E(r)$ pairs), thus the optimal choice in their tangent point, can be defined for every investor. Figure II/14 illustrates the choices where $A$, $B$ and $C$ are unleveraged, $A'$, $B'$ and $C'$ are leveraged positions, O is the optimal choice and D is the interest rate (which is risk-free, thus $EDR=E(r)=r_C$). For different leverage possibilities or interest rates, investors’ preference may change. In this case, Efficiency frontier 2 included both $A$ and $B$ portfolios but not $C$; however, for an investor with Efficiency frontier 1, $A'$ and $C'$ are efficient and $B'$ is not.

**Figure II/14: Individual optimization with leverage constraints**
Notes: The graph represents three portfolios with different return distributions. As in Figure II/13, the
efficiency frontier may change as the leverage differs. Furthermore, combining the frontier with the
highest reachable indifference curve yields the optimal portfolio choice.

II/6. Expected return

II/6.1. Formation of the expected return

Until this point we assumed that investors are price-takers, thus, one can invest in
portfolios with constant, exogenous $EDR-E(r)$ parameters. However, in the previous
section, we showed that available portfolios (and thus the efficiency frontier) can be
fairly different for various investors, especially if we accept that some of them have the
possibility of investing in highly leveraged positions. Therefore, hereafter we define
individuals as price-maker participants of the market, although their effect on prices
may differ significantly.

Institutions (such as brokerage firms) that have access to investors’ trading data are
able to estimate their behaviour based on historical actions. They can define their
clients’ risk aversion, their CARA and their utility function. In fact, every single
parameter is known to make estimations on future $EDR-E(r)$ pairs. These
approximations are assigned to each investor to define their optimal choices.
Furthermore, by aggregating these choices they can estimate the aggregate expected
return of the market (which moves the prices). However, in order to have an
aggregation, these institutions have to have a model to describe the relation between
different required returns assigned to a given $EDR$. Distinct groups of investors may
have different return requirements (especially due to leverage and interest rate
differences); however, since every publicly traded asset has a unique price, these
required returns have to add up to a unique expected return based on a function.
Although the main purpose of this study is not to define this precise function, in order
to approximate a regression we use the value-weighted aggregating function. Thus,
we add up each investor’s required return with weights equal to their invested value to
get a unique expected return.

This weighted aggregation function is based on macroeconomic demand and supply
functions, where the price assigned to a portfolio is defined by the current aggregated
supply and demand on the investment opportunity. Since databases enable analysis
of who wants to make transactions at the current price with what volume and what the
required return of the given investor is, the future price (current price multiplied by the
required return) and the future volume of each investor can be defined. This way the aggregated supply (AS) function is known. According to Fama (1991), investors insist on smoothing their consumption over time; therefore, it seems to be realistic to assume that their incoming cash flows are balanced and continuous due to diversification. Furthermore, if the future cash flows are assumed to be fixed (which seems to be true according to the efficient market hypothesis by Fama (1970), in which prices reflect all the available information), the aggregated demand (AD) grows at the same pace as AS, which is the required return of investors. This means that in the discounted cash flow pricing method the exponent of the cost of alternative choice gets smaller over time. Combining the functions mentioned above we create the AS-AD system in our model, hence, one can approximate future prices of assets and their expected returns. An illustration of this approximation is shown in Figure II/15, where the initial \((t_0)\) AS and AD functions are increased by continuous return over time (represented by the axis “\(t\)” ). The functions used are \(P_S=2Q\) and \(P_D=5-2Q\), however, applying different ones does not change the results as long as aggregate supply and demand are increasing and decreasing functions of price respectively. We fix the continuous return at 20% in order to be expressive, hence, the exponential growth over time can clearly be seen. Hence, we have shown the validity of the second thesis of the dissertation: with the shift in risk measure to Expected Downside Risk, the assumptions of price-taker investors and unlimited leverage opportunity for a unique interest rate can also be relaxed.

**Figure II/15: The formation of the expected return**

Notes: The simulated data represents the evolution of the price \(P\) as the function of aggregate demand and supply \(Q\) and time \(t\). The increasing price shows the effect of the expected return over time.
We underline that this type of approximation of the expected return is very sensitive to the input data; therefore, the wider the sample is, the more precise the regression will be. The necessary data can be extracted by analysing the actions of clients in financial institutions or brokerage services; however, using our model at national level (such as under the supervision of the Securities and Exchange Commission) or at international level (for example, with the administration of the International Monetary Fund (IMF) or the European Central Bank (ECB)) could produce fairly precise approximations of future prices of capital markets.

II/6.2. Empirical evidence for the EDR-E(r) relationship

Although, due to the lack of individual data, we cannot precisely define the aggregating function a brief empirical comparison with alternative risk measures can be made for a representative investor. Assuming that risk-free rate does not vary significantly and leverage limit is a function of risk expected return can be fitted on risk measures such as variance or EDR. The former assumption is necessary to define a unique constant in the regression, while the latter is required to be able to control for the selection of portfolios. An example of the latter would be a pattern that investors who are provided higher leverage limits choose portfolios with lower risk (hence lower variance or higher EDR) (see Appendix II/3). Therefore, less risky assets are purchased by investors with higher leverage opportunities and higher risk-aversion, hence, the price of assets is not determined by the correlation with a unique market portfolio but depends on the correlation with the reference portfolio. However, this reference is conditional on the leverage limit and risk-aversion, therefore, by assuming that capital markets include a diverse set of investors, no unique systematic risk (i.e. CAPM beta) exists as it is different for every single investor. Therefore, we argue that the risk premium should be described with the total risk and hence we apply the EDR measure.

In Table II/2 we provide the empirical results of our illustration of EDR-based asset pricing. Here the overlapping yearly returns of the 340 S&P500 members traded both in 2009 and 2014 are applied in a linear estimation of the expected return. The risk measures used are β, σ, semi-variance (σ⁻), downside beta (β⁻) and EDR. The pricing equation of the EDR is estimated as
\[ E(r) = 0.15 + 0.56 \cdot EDR. \]  

**Table II/2: Linear estimations of the expected return**

<table>
<thead>
<tr>
<th>Risk measure</th>
<th>Coefficient</th>
<th>p-value</th>
<th>R-squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility</td>
<td>-0.1110</td>
<td>0.0466</td>
<td>0.01</td>
</tr>
<tr>
<td>Semi-variance</td>
<td>-0.9198</td>
<td>0.0167</td>
<td>0.02</td>
</tr>
<tr>
<td>Beta</td>
<td>-0.0181</td>
<td>0.0000</td>
<td>0.06</td>
</tr>
<tr>
<td>Downside Beta</td>
<td>-0.0217</td>
<td>0.0006</td>
<td>0.03</td>
</tr>
<tr>
<td>EDR</td>
<td>0.5655</td>
<td>0.0000</td>
<td>0.62</td>
</tr>
</tbody>
</table>

Notes: Overlapping yearly returns on the 09/10/2009 to 09/10/2014 are applied. The coefficient indicates that of the applied risk-measure, p-value stands for its Student’s-\( t \) test with the null of the coefficient being zero and the R-squared represents the coefficient of determination of the OLS estimations including a constant as well.

As Part II is a theoretical one, we consider these results promising based on the \( R^2 \) values and significance levels, however, we discuss the detailed empirical test of the model in the subsequent parts.

**II/7. Conclusion**

Our proposed model yields novel results in four main fields of asset pricing. Firstly, we introduce a novel risk measure, Expected Downside Risk. Being based on Conditional Value-at-Risk it keeps the advantages, such as subadditivity and convexity, but omits the disadvantages, such as subjectivity and the non-general approach to return distributions. This similarity is important since CVaR has gained high popularity in financial risk management nowadays (Bank for International Settlements (BIS), 2012), and therefore, switching to EDR could easily be done as they use very similar calculation methodology. Secondly, by combining Prospect Theory with Expected Utility Theory, our model is able to describe risk-seeking and risk-averse behaviour in one regression. Thirdly, we point out that the unlimited leverage assumption of existing equilibrium models can cause significant bias in asset price prediction. Using an alternative approach, our model may lead to more precise and unbiased estimations. And fourthly, through the aggregation function of individual required returns, the price-maker activity of investors can be modelled. Although due to the lack of individual data,
this function has not been tested in detail in this study we provide a brief empirical estimation of the expected return with the use of aggregated data. There have been numerous attempts to describe this aggregating with market microstructural (Brennan and Subrahmanyam, 1996; Garman, 1976) or behavioural financial approaches (Shefrin and Statman, 1994), which, combined with our ideas, could pave the way for future research. Furthermore, we underline here that these theories mainly focus on the short-term changes of prices while, in the long term, our approximation of expected return at an aggregate level through EDR seems to work well in practice as individual effects are cancelled out.

Moreover, we argue that research on asset pricing should not be driven by searching for factors with no fundamental background (although some of them may provide significant results for given periods) but should be based on well-defined theories. Our model proposes a method for asset pricing based on investors' preferences; hence, it explains the differences between expected returns with the diversity of groups investing in the given assets. Therefore, two distinct assets with equal volatility may provide different expected returns due to the difference in investors' opportunities and preferences. Although this means that expected return is not solely based on volatility, instead of including irrelevant factors in asset pricing models, future research on the topic should focus on the questions which group of investors' trade the assets that have different characteristics and what the reason is behind their choice.

Potential ways of future research could cover the individual portfolio choice of investors either by analysing the individual choice of the participants of capital markets or by testing whether the difference between the portfolio risks of investors accessing high and low leverage is significant (as suggested by Appendix II/3). An alternative potential topic could be related to the experimental analysis of investors' portfolio choice subsequent to gains or losses. Moreover, as in-sample tests of $EDR$ indicate extremely high goodness-of-fit relative to the alternatives, further in- and out-of-sample prediction models of the proposed risk measure could be tested as well using GARCH-type, VAR or other autoregressive models. Finally, the coherence, the elicitability and the backtestability of Expected Downside Risk could be also tested in a thorough mathematical analysis.
III. The case of “Less is more”: Modelling risk-preference with Expected Downside Risk

Abstract

This part discusses an alternative explanation for the empirical findings contradicting the positive relationship between risk (variance) and reward (expected return). We show that these contradicting results might be due to the false definition of risk-perception, which we correct by introducing Expected Downside Risk (EDR). Our results indicate that, when using the EDR as risk measure, both the positive and negative relationship between expected return and risk can be derived under standard conditions (e.g. expected utility theory and positive risk-aversion). Therefore, no alternative psychological explanation or additional boundary condition on utility theory is required to explain the phenomenon. Furthermore, we measure that it is a more precise predictor of expected return than volatility, both for individual assets and portfolios.

III/1. Introduction

According to Modern Portfolio Theory (MPT) (Markowitz, 1952) the utility of stochastic investment opportunities can be described as the following, by assuming either wealth-dependent (constant relative) or wealth-independent (constant absolute) risk-aversion under Expected Utility Theory (EUT) (Gossen, 1854). The Taylor approximation of expected utility yields the following relationship between expected return and variance:

\[ U(F) \approx E(F) - 0.5\alpha \sigma^2 \]  

where \( U(F) \), \( E(F) \) and \( \sigma \) stand for the utility, expected value and standard deviation of a cash-flow, while \( \alpha \) is an Arrow-Pratt measure of constant absolute risk-aversion (CARA). This well-known equation in asset pricing (e.g. Capital Asset Pricing Model by Lintner (1965); Mossin (1966); Sharpe (1964)) suggests a positive relationship between expected return and volatility under standard circumstances (i.e. positive risk-aversion
coefficient). However, contradicting empirical results (Brooks et al., 2014; Kahneman and Tversky, 1979; 1992; Linville and Fischer, 1991; Post and Levy, 2005) indicate a negative relationship between the two parameters leading to the emergence of an alternative utility theory: the prospect theory of Kahneman and Tversky (Kahneman and Tversky, 1979). The authors find that in certain cases (e.g. decision on losses) if investors have two options yielding the same expected return they tend to choose the option involving higher risk. They propose a convex, fluctuation-dependent utility function for losses as a solution to the problem.

In contrast, we argue that this behaviour deviating from that suggested by the EUT is not necessarily due to a flaw in the theory itself but the measurement of the risk perception. In line with Alonso and Prado (2015) we keep the expected utility framework while focusing on the factors driving utility; however, instead of including other types of randomness (e.g. ambiguity) the perception of risk is analysed. Although numerous novel risk measures have emerged since the MPT, such as Value-at-Risk (VaR) (Campbell et al., 2001; Jorion, 2007), Conditional Value-at-Risk (CVaR) (Rockafellar and Uryasev, 2000) or Expected Shortfall (Acerbi and Tasche, 2002) or entropy (Ormos and Zibriczky, 2014), none of them had the initial purpose of contributing to equilibrium modelling by describing risk-preference in a more precise way. Therefore, we apply Expected Downside Risk (EDR).

Subsequent to discussing our applied risk measure and its theoretical application in asset pricing, we provide an empirical analysis of its explanatory power of the average return of individual assets as well as well diversified portfolios, test whether the theoretical findings hold in reality as well and make comparison with its peers such as volatility, variance and 5% Conditional Value-at-Risk.

The rest of Part III is organized as follows: in Section 2 we introduce and discuss our proposed method for measuring risk perception, in Section 3 the empirical investigation of our proposed explanation is presented and lastly in Section 4 we provide a brief conclusion.
III/2. The model

We argue that the flaw in the conclusion of contradicting experimental and empirical results on the relationship between risk and return comes from the fact that the tests try to explain the results in a volatility or variance-based setting. This setting, as suggested by equation (III/1), indeed yields the result that investors having dominantly positive risk-aversion coefficient (Barsky et al., 1997; Hanna and Lindamood, 2004) should choose an investment option involving a lower risk for a given level of expected return.

In contrast, in line with prospect theory, we argue that investors do not focus on (in practical terms, they do not perceive) the volatility or variance of their investment but the expected loss of it. This change of risk measure also allows for including other properties of the return distribution (e.g. skewness) that have been documented to play an important role in explaining the risk premium (Astebro et al., 2014; Post et al., 2008). However, we do not assume that investors should behave according to Prospect Theory, that is, they do not necessarily follow loss aversion instead of risk aversion. We argue that keeping the EUT setting with constant risk-aversion while changing the applied risk measure, one may explain the contradicting results of risk- and loss-aversion. Our proposed measure is the expected loss of an investment labelled as the Expected Downside Risk (EDR), which is defined in the following way: similar to the Conditional Value-at-Risk, EDR measures the expected tail risk; however, it does not apply an ad-hoc probability level and considers returns below the expected return as a loss. We repeat the definition discussed in Part II, that is

\[ EDR(x) = p(r_x \leq E(r(x)))^{-1} \int_{r_x(y) \leq E(r_x)} r_x(y)p(y)\,dy. \]

(III/2)

where \( r_x \) and \( E(r_x) \) sign the return and its expectation of a given \( x \) portfolio and \( r_x(y) \) and \( p(y) \) stand for the outcomes of the \( x \) portfolio and their respective probabilities. In other words, \( EDR \), as its name suggests, measures the expected loss (risk) of investors given that their reference point is the expected return as suggested by Easterlin (1974) or Kőszegi and Rabin (2006).

This change of risk measure is a plausible modification of standard asset pricing models assuming EUT for two reasons. On the one hand, due to bounded rationality
(Simon, 1982), measures being easier to interpret are the main factors driving the focus of investors (Gigerenzer and Selten, 2002); in particular, this is an important reason why fixed monetary payments and assigned probabilities are applied in experimental and laboratory tests instead of complicated mathematical formulas (Tversky and Kahneman, 1992), such as a variance-expected return choice set. Therefore, the expected amount of money an investor can lose might be of greater relevance than the expectation of the squared deviations from the mean. On the other hand, the variance-based approach yields the optimal portfolio choice only if each compared investment has the exact standardized distribution (i.e. excluding the variance and mean, they are identical). This problem has already been solved by Value-at-Risk and Expected Shortfall due to their non-parametric approach; however, both have a pre-defined probability-level that cannot be verified fundamentally (i.e. the reason for the choice of a given probability level does not have an economic explanation). Nonetheless, in our empirical results section, we compare EDR against these risk measures as well using the most popular probability levels.

Assuming that, apart from the expected return and variance, the distribution type of the return of a given asset does not change over time (which, in reality, is a fairly valid assumption (Singleton and Wingender, 1986; Sun and Yan, 2003)) its EDR can be described as a linear function of its expected return and volatility of past return. In order to illustrate this relationship we provide an example with normal distribution as an approximation, which allows for tighter conditions; however real distributions of returns can be calculated in the same way by changing the coefficient only. We can define EDR as the function of expected return and standard deviation:

\[
EDR(x) = E(r_x) - 0.8\sigma
\]  

(III/3)

According to this equation, we can substitute the volatility (\(\sigma\)) with EDR in equation (III/1). Therefore, the approximating function can be implemented in the EDR-E(r) setting:

\[E(r_x) = \int r \cdot \left(1 - \frac{1}{\sqrt{2\pi}2\sigma} \cdot e^{\frac{-1}{2\sigma^2}}\right) dr\]

that is equal to 0.8 assuming standard normal distribution, and therefore, adding a constant (the expected return) and a multiplication by the standard deviation yield equation (III/3)

\[in the case of normal distribution EDR=CVaR_{0.5} = \int E(r) r \cdot \left(1 - \frac{1}{\sqrt{2\pi}2\sigma} \cdot e^{\frac{-1}{2\sigma^2}}\right) dr\]
In Figure III/1 we provide a numerical simulation of the indifference curve for an average $a=4$ risk-aversion coefficient (Barsky et al., 1997), where points signed by $+$, $-$ and $x$ represent utility levels of $U=1$, 2 and 3 respectively. Although these examples use normal distribution with the $0.8$ volatility coefficient seen in equation (III/3), we show later that in general the coefficient is actually not far from this value.

**Figure III/1: Indifference curves in EDR-E(r) system given a=4**

Furthermore, the generalized form of equation (III/4)

$$0 = -c \left( E(r_x) - \frac{2cEDR(x)+1}{2c} \right)^2 + EDR(x) + \frac{1}{4c} - U,$$

where $U$ stands for the utility and $c$ is a constant for each investor

$$c = \frac{0.5}{0.8^2} a,$$

suggests a more intuitive result: since the first term is always non-positive (assuming a positive risk-aversion coefficient) equation (III/7) must be valid in order to have a real solution, hence

$$EDR(x) + \frac{1}{4c} - U \geq 0.$$
For further analysis, we can define the slope of the indifference curve in the \( EDR-E(r) \) setting as its total derivative should be zero. Here, we use the general, parametric approach, hence, the deduction is valid for any distribution.

\[
\frac{dU}{dEDR(x)} = \frac{a}{v^2} (E(r_x) - EDR(x)) + \frac{dE(r_x)}{dEDR(x)} \left( 1 - \frac{a}{v^2} (E(r_x) - EDR(x)) \right) = 0, \quad (III/8)
\]

where \( v \) stands for the coefficient of volatility in equation (III/3). Therefore, the sensitivity of the expected return for Expected Downside Risk is

\[
\frac{dE(r_x)}{dEDR(x)} = -\frac{a}{v^2} \left( E(r_x) - EDR(x) \right) = 1 - \frac{1}{1 - \frac{a}{v^2} (E(r_x) - EDR(x))}. \quad (III/9)
\]

This relationship implies that

\[
1 - \frac{a}{v^2} (E(r_x) - EDR(x)) < 0, \quad \frac{dE(r_x)}{dEDR(x)} > 1 \quad (III/10)
\]

if \( EDR(x) < E(r_x) - \frac{v^2}{a} \), infinity at \( EDR(x) = E(r_x) - \frac{v^2}{a} \) and

\[
1 - \frac{a}{v^2} (E(r_x) - EDR(x)) > 0, \quad \frac{dE(r_x)}{dEDR(x)} < 0 \quad (III/11)
\]

if \( EDR(x) > E(r_x) - \frac{v^2}{a} \). Furthermore, the expectation of returns not greater than the expected return cannot be higher than the expected return itself, hence, including the constraint of \( EDR(x) \leq E(r_x) \),

\[
E(r_x) - \frac{v^2}{a} < EDR(x) \leq E(r_x) \quad (III/12)
\]

yields the “usual” positive relationship between risk and expected return. However, in equation (III/10) we have shown that in the case of small EDRs the relationship changes and a higher risk will be rewarded with lower expected return. The aforementioned relationship is presented in Figure III/2, where the curve, the dashed
line and the dotted line stand for the indifference curve, the risk-free portfolios and the

\[ EDR(x) = E(r_x) - \frac{\nu^2}{a} \]

constraint respectively. Here we use \(a=0.5\) and \(\nu=0.8\) parameters.

**Figure III/2: Limits of indifference curves in the EDR-E(r) system**

![Figure III/2: Limits of indifference curves in the EDR-E(r) system](image)

In order to analyse the case including leverage opportunity, first, we have to define whether it provides an optimal solution as well. Since the relationship is linear between the weight of the risk-free asset in combined portfolios and either EDR or E(r), if an optimal solution exists, it is equal to the point of tangent of the leveraged portfolios of

\[ E(r_x) = r_f + kEDR(x), \quad (\text{III/13}) \]

where \(k\) is a constant, and the highest indifference curve

\[ U = E(r_x) - \frac{0.5}{\nu^2} a [E(r_x) - EDR(x)]^2. \quad (\text{III/14}) \]

Based on investors’ goal of maximizing their utility we get a simple linear constraint optimization problem from equation (III/13) and (III/14), which is

\[ \max_{E(r_x), EDR(x)} \left\{ E(r_x) - \frac{0.5}{\nu^2} a [E(r_x) - EDR(x)]^2 \right\} \quad s.t. \ E(r_x) = r_f + kEDR(x). \quad (\text{III/15}) \]

Using the Lagrangian and its derivatives we get
\[ L = E(r_x) - \frac{0.5}{v^2} a [E(r_x) - EDR(x)]^2 + \lambda \left( E(r_x) - r_f - kEDR(x) \right), \quad (\text{III/16}) \]

\[ \frac{dL}{dE(r_x)} = 1 - \frac{a}{v^2} [E(r_x) - EDR(x)] + \lambda = 0, \quad (\text{III/17}) \]

\[ \frac{dL}{dEDR(x)} = \frac{a}{v^2} [E(r_x) - EDR(x)] - \lambda k = 0, \quad (\text{III/18}) \]

\[ \frac{dL}{d\lambda} = E(r_x) - r_f - kEDR(x) = 0. \quad (\text{III/19}) \]

According to eq. (III/17)-(III/18) the optimal solution for the constrained optimization problem is

\[ EDR(x)_{opt} = \frac{kv^2}{a(k-1)^2} - \frac{r_f}{k-1}, \quad (\text{III/20}) \]

\[ E(x)_{opt} = \frac{kv^2}{a(k-1)} + \frac{kv^2}{a(k-1)^2} - \frac{r_f}{k-1}, \quad (\text{III/21}) \]

We further have the condition of \( E(r(x))_{opt} \geq EDR(x)_{opt} \), therefore, the solution is valid if and only if

\[ \frac{kv^2}{a(k-1)} \geq 0 \quad (\text{III/22}) \]

\[ k \in (1, \infty) \cup (-\infty, 0]. \quad (\text{III/23}) \]

In order to define the optimal portfolio that is used in leveraging we determine the slope coefficient in the following way. Substituting back into eq. (III/14) we get

\[ U = \frac{kv^2}{a(k-1)} + \frac{kv^2}{a(k-1)^2} - \frac{r_f}{k-1} - \frac{0.5}{v^2} a \left[ \frac{kv^2}{a(k-1)} \right]^2 = \frac{v^2k-0.5v^2k^2}{a(k-1)^2} + \frac{v^2k-ar_f}{a(k-1)}. \quad (\text{III/24}) \]

Then the derivative of (III/22) is
\[
\frac{dU}{dk} = \frac{ar_f(k-1) - v^2 k}{a(k-1)^3} = \frac{(ar_f - v^2)k - ar_f}{a(k-1)^3} = 0,
\]  

(III/25)

we find that

\[
\frac{d^2U}{dk^2} = \frac{2ar_f(1-k) + v^2(2k+1)}{a(k-1)^4} = \frac{-2(ar_f(k-1) - v^2k + v^2)}{a(k-1)^4},
\]

(III/26)

which is always positive under the first order condition (III/25), therefore

\[
\frac{d^2U}{dk_0^2} = \frac{v^2}{a(k_{0} - 1)^4} > 0
\]

(III/27)

if \( \frac{dU}{dk_0} = 0 \) where \( k_0 = \frac{ar_f}{ar_f - v^2} \) stands for the extremum of the slope coefficient. These together yield that the utility function does not have a local maximum but a minimum. Furthermore, as the derivative of the leveraged utility function with respect to the slope coefficient depends on both the slope and the given risk-aversion, risk-free return and volatility combination, we define the following cases:

If \( ar_f > v^2 \)

\[
\frac{dU}{dk} \begin{cases} 
> 0 & \text{if } k < 0 \\
< 0 & \text{if } 1 < k < \frac{ar_f}{ar_f - v^2} \\
> 0 & \text{if } k > \frac{ar_f}{ar_f - v^2},
\end{cases}
\]

(III/26)

and if \( ar_f < v^2 \)

\[
\frac{dU}{dk} \begin{cases} 
< 0 & \text{if } k < \frac{ar_f}{ar_f - v^2} \\
> 0 & \text{if } \frac{ar_f}{ar_f - v^2} < k < 0 \\
< 0 & \text{if } k > 1,
\end{cases}
\]

(III/27)

These relationships are the main theoretical findings of the proposed model. They indicate that depending on the exogenous parameters of \( a, r_f, v^2 \) (i.e. the risk-aversion coefficient, the risk-free rate and the coefficient of volatility in the EDR regression) the leveraged optimization could lead to both negative and positive slope choice. Investors
maximize utility either by choosing the leverage slope closest to the $EDR = E(r)$ line or by picking the leveraged portfolio line the furthest from the 45-degree line.

The theoretical maximum of utility would be reached at the leveraged line with slope closest to unity since the limit from the right (getting closer to the $EDR = E(r)$ line on the positive side)

$$\lim_{k \to 1^+} U = \lim_{k \to 1^+} \left[ \frac{v^2 k - 0.5v^2 k^2}{a(k-1)^2} + \frac{v^2 k - ar_f}{a(k-1)} \right] = \infty \quad (\text{III}/28)$$

is infinite, while the furthest leveraged portfolio line yield only a finite utility since

$$\lim_{k \to 0} U = \lim_{k \to 0} \left[ \frac{v^2 k - 0.5v^2 k^2}{a(k-1)^2} + \frac{v^2 k - ar_f}{a(k-1)} \right] = \frac{v^2}{2a} \quad (\text{III}/29)$$

Therefore, investors would theoretically prefer the leveraged line with a slope of unity over the one with the slope of zero, however, in the real world, these portfolios are not always attainable for investors. In these cases, the leveraged portfolio line with the highest negative slope coefficient might be the optimal choice. Therefore, the third thesis of the dissertation is proved: by using the Expected Downside Risk as risk measure, both a positive and a negative relationship between expected return and risk can be derived under standard conditions of equilibrium asset pricing models, such as the expected utility theory and positive risk-aversion.

Figure III/3 summarizes the aforementioned equations where Line 1 and line 2 stand for the $EDR \leq E(r_x)$ constraint and the $k = \frac{ar_f}{ar_f - v^2}$ condition (i.e. the slope where utility is minimal) respectively.
III/3. Empirical results

III/3.1 Data
In our empirical investigations we test the realization of the $E_{DR}$-$E(r)$ equilibrium for equity portfolios, moreover, we analyse how the equilibrium is parameterized; first, for unleveraged portfolios, and second, for leveraged ones. We apply daily and annual data and statistics to investigate unleveraged and leveraged portfolios. The data used for these portfolio calculations consist of daily returns from July 31, 1993 to July 31, 2014 of the 340 constituents of S&P 500 index that have been listed both at the beginning and at the end of the period. Thus, the dataset we use is not free of survivorship bias. In order to model leveraged portfolios, we apply the mean annualized 3-month T-bill log return for the same 21 years. This dataset is used in sections 3.2 and 3.4.

In section 3.3, where no individual asset is considered, we apply longer historical time series: we use the annual returns of the S&P 500 index and the 3-month Treasury bills between 1928 and 2013.

III/3.2 Optimization for unleveraged and leveraged portfolios
In the case of unleveraged portfolios, we simulate the performance of 10,000 randomly weighted portfolios consisting of the 340 different stocks. The $E_{DR}$ calculation is based on either daily or yearly returns. The plot of these simulated portfolios using yearly statistics is presented in Figure III/4. One can see that the concave efficient frontier
representing portfolios with the highest $EDR/E(r)$ ratio always has an optimum at the point of the tangent with the convex utility functions (as shown in Figure III/1).

**Figure III/4: Unleveraged portfolios in EDR-E(r) system**

Our utility maximization results indicate that for $a=0.5$ (extremely low) and $a=4$ (average) risk-aversion the $\{EDR, E(r)\} = \{0.15, 0.27\}$ portfolio is optimal, however, for $a=10$ (extremely high) the $\{0.15, 0.24\}$ is optimal. Here, we find evidence of the surprising utility preference we have derived in the previous section: for a given level of risk (measured by the $EDR$), a portfolio with a lower expected return may provide higher utility; in particular, the utilities generated by equation (III/4) are shown in Table III/1. The results show two important patterns: on the one hand, for fixed expected return the increasing risk-aversion, as expected, decreases the utility of the stochastic payoff; on the other hand, for fixed risk-aversion, one may clearly see that for $a=0.5$ and $a=4$ a decrease in the expected return yields a loss of utility, which is in line with standard asset pricing theories, however, this relationship is the opposite for $a=10$.

**Table III/1: Utilities at different risk-aversion for fixed $EDR=0.15$**

<table>
<thead>
<tr>
<th></th>
<th>$a=0.5$</th>
<th>$a=4$</th>
<th>$a=10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(U</td>
<td>E(r)=0.27)$</td>
<td>0.2641</td>
<td>0.2224</td>
</tr>
<tr>
<td>$E(U</td>
<td>E(r)=0.24)$</td>
<td>0.2367</td>
<td>0.2132</td>
</tr>
</tbody>
</table>
We also run the simulation using daily statistics in Figure III/5. Using a similar dataset (as above) at a daily level, we find that the optimal portfolios are much further away from each other than in the annual analysis. This is mainly due to the fact that the point of tangent between the efficient frontier and the indifference curve is on the decreasing part of the latter.

Figure III/5: Unleveraged portfolios in daily analysis

In the case of leveraged portfolios, we first apply the slope optimization described in Section 2. In order to illustrate this optimization let us include a risk-free rate in the aforementioned portfolio simulation. Using the mean annualized 3-month T-bill log return for the same 21 years we get a risk free rate ($r_f$) of 2.73%. In addition, applying the OLS estimation of

$$E(r_x) - EDR(x) = v\sigma$$

(III/28)

to the yearly portfolio simulation yields a fitted volatility coefficient of $v=0.78$.

These parameters further lead to the definition of $k_0$ and yield the sign of the EDR-E(r) relationship in the following way: according to the simulated portfolios, the leveraged portfolio lines have the slope of $\frac{dE(r_x)}{dEDR(x)} = 1.64$ and $\frac{dE(r_x)}{dEDR(x)} = 0$ at the boundaries (the closest to and the furthest from the EDR=E(r) line). Utility at the boundaries is obtained by substituting back into eq. (III/24). Analysis of the utility difference of $U_{k=1.64} - U_{k=0}$ yields that for $a \in (0, 0.2855)$ $U_{k=1.64} > U_{k=0}$, hence, a positive relationship between
$EDR$ and $E(r)$ is preferred. Figure III/6 shows the leveraged portfolio optimization conditional on yearly parameters, where the red “+” sign stands for the risk-free asset while the horizontal and $\frac{dE(r_x)}{dEDR(x)} = 1.64$ sloped lines represent the optimal leverage line given $a \geq 28.55$ and $a < 28.55$ respectively. According to this optimization investors having $a < 28.55$ (the majority according to Barsky et al. (1997) and Hanna and Lindamood (2004)) choose from the leveraged portfolios on the $EDR=r+1.64E(r)$ line, while those having $a \geq 28.55$ invest into portfolios providing $EDR=r$f.

**Figure III/6: Leveraged portfolios in EDR-E(r) system**

The daily analysis yields somewhat different results. Here, we measure an average $r_f = 0.011\%$, $\nu=0.68$, and the boundary slopes range from 0 to -0.08. This case implies that for $a \in (0,155.69)$ $U_{r=0.08} > U_{r=0}$. This simulation is shown in Figure III/7. This mean that practically every investor picks portfolios from the $EDR=r-0.08E(r)$ line, and therefore, we see a negative relationship between $EDR$ and $E(r)$ at the daily frequency.
III/3.3 Ambiguous risk-preferences

Here we illustrate the aforementioned optimization with the following simple portfolio choice problem. We measure the following parameters of a well-diversified portfolio (we assume the S&P500 index behaving as the market portfolio): the average log return $E(r_m) = 9.12\%$, the annual return volatility $\sigma_M = 19.5\%$, $EDR_m = -13.07\%$, and $r_f = 3.47\%$. The $v$ parameter introduced in equation (III/8) hence takes on $v = 1.14$. Based on Barsky et al. (1997) and Hanna and Lindamood (2004) we assume an average investor having the risk-aversion coefficient of $a = 4$. Now let us look at the boundary portfolio providing minimal $EDR$ where equation (III/9) yields an infinite indifference curve slope. We know that equation (III/3) still holds, therefore

$$EDR_P = E(r)_P - \frac{v^2}{a} = E(r)_P - v\sigma_P. \quad (\text{III/29})$$

Then, equation (III/30) follows as

$$\sigma_P = \frac{v}{a} \quad (\text{III/30})$$

We further know that leveraged positions combined with the risk-free interest rate yield the following equation
The solution of the equation system yields \( E(r)_p = 11.72\%, \sigma_p = 0.2846, EDR_p = -20.67\% \) and \( U_p = -0.0447 \). Now let us measure the indifference curve of the same investor by testing the \( EDR-E(r) \) pairs with “decreased” risk. Here we mean decreased risk by increasing EDR for a given utility level, that is, let us consider a portfolio \( X \) where \( EDR_X = -10\% \). By the definition of the indifference curve \( U_p = U_X \), which gives two possible solutions of \( E(r)_X = \{-3.89\% \text{ or } 48.68\% \} \) with \( \sigma_X = \{0.0472 \text{ or } 0.4529\} \). In other words, given the type of the return distribution constant and the expected loss fixed at -10\%, investors having a risk-aversion coefficient are indifferent to the choice between a portfolio providing an expected return of -4\% or 49\%. This phenomenon might be surprising if the variance is not taken into consideration (as in most of the experimental studies) and it underlines the importance of measuring risk-preference.

### III/3.4 Robustness test and the time component

We provide a robustness test by running a linear regression model for annual, monthly and daily parameters as well. In Table III/2 we present the results of the simple linear model’s estimating expected return.

#### Table III/2: Linear estimations of the expected return

<table>
<thead>
<tr>
<th></th>
<th>Individual shares</th>
<th>Portfolios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \sigma )</td>
<td>( \text{VaR}(5%) )</td>
</tr>
<tr>
<td><strong>Yearly</strong></td>
<td>Coeff</td>
<td>2.7E-02</td>
</tr>
<tr>
<td></td>
<td>p-value</td>
<td>1.4E-01</td>
</tr>
<tr>
<td><strong>Monthly</strong></td>
<td>Coeff</td>
<td>7.2E-03</td>
</tr>
<tr>
<td></td>
<td>p-value</td>
<td>2.9E-01</td>
</tr>
<tr>
<td><strong>Daily</strong></td>
<td>Coeff</td>
<td>3.0E-03</td>
</tr>
<tr>
<td></td>
<td>p-value</td>
<td>4.9E-02</td>
</tr>
</tbody>
</table>

In contrast to the estimated positive relationship between return and risk of the volatility-based model in any terms, changing the risk measure to \( EDR \) reveals an interesting pattern: expected return and risk are only positively correlated in the short run. Regression on annual and monthly statistics indicate a positive relationship between \( EDR \) and \( E(r) \) both for individual assets and portfolios. This means that as the
expectation of the negative outcomes increases (the risk decreases), the expected return increases as well, which is against the findings of standard asset pricing theories.

Furthermore, analysing the p-values reveals that pricing regressions indicate that investors indeed seem to focus more on the expected loss instead of the simple risk measures as volatility and variance. Moreover, as the p-value is lower in $EDR$ regressions than in $VaR$ and $CVaR$ models, it seems that one considers the losses on the whole domain instead of taking into account the tail risk only (e.g. at the 5% probability level analysed). $EDR$ surpasses in goodness-of-fit the widely used CAPM and its $Beta$ measure as well in five out of the six tests, however, the $Beta$ yields much worse, insignificant estimations for individual assets. Altogether, we conclude that $EDR$ seems to be a more precise estimator of $E(r)$ than its competitors.

In order to analyse the effect of the period length on the leveraged portfolio line, we run linear regressions for distinct intervals and test whether the coefficients between risk and return show a robust pattern. Figure III/8 represents our results indicating a robust positive relationship between the coefficient of $EDR$ and the length of the analysed period. Again, this phenomenon implies that focusing on the very short term, investors seem to be risk-averse and at around 11-12 days they are insensitive to risk; for periods over 12 days a negative relationship between risk and return applies to them. Here, we highlight again that this risk-seeking behaviour exists in the sense that portfolios with different expected return may provide the same utility for given expected downside risk; however, it does not reject the Expected Utility Theory or require negative risk-aversion coefficients.
III/4. Concluding remarks

Numerous studies found evidence of investors’ behaviour contradicting the well-known positive risk-reward relationship. However, asset pricing theories based on the expected utility theory have not yet given any explanation of a negative relationship under standard circumstances. Experimental evidence shows that the risk-aversion coefficient is positive for a dominant portion of investors (and hence they are risk-averse), although, in some cases they systematically behave in a seemingly risk-seeking way.

In this study, we argue that this deviation from the expected behaviour might be due to the false definition of risk-perception instead of a flaw in the definition of the perceived utility (as previous studies suggest). Therefore, in our model we propose the Expected Downside Risk as an alternative risk measure that describes and better fits investors’ sensitivity to risk as measured by pricing regressions. Being a simple measure of the average loss relative to the expected return $EDR$ seems to yield a more perceived value of risk than the standard deviation from the historical mean, especially in the presence of a highly discrete number of choices (e.g. experimental tests). This latter statement is confirmed by our regression results as well, where expected return is more significantly driven by $EDR$ than volatility in all three period lengths for both individual assets and portfolios.
Having confirmed that *EDR* measures risk-perception better than volatility; asset pricing models based on expected utility theory can be modified by replacing the latter with the former risk measure. The solution of the optimization problem reveals that both a positive and negative relationship between expected return and *EDR* may exist under standard circumstances.

Finally, our model is supported by regression results indicating a negative relationship between risk and expected return for periods over 12 days and the “usual” positive relationship for less than 12 days.

For further research one may include the test of Expected Downside Risk in experimental settings, the detailed analysis of the effect of time on the risk-return relationship measured by *EDR* or by examining portfolio optimization where both negative and positive relationship can be found depending on the exogenous parameters, the latter of which could yield an alternative method for measuring risk-aversion.
IV. Expected Downside Risk and Asset Prices: Characteristics of Emerging and Developed European Markets

Abstract

This part discusses an empirical analysis of the Expected Downside Risk (EDR) based asset-pricing model on Central and Eastern European and Developed Western European markets. The investigated EDR risk measure, as presented below, captures the relationship between risk and expected return with superior performance. Furthermore, we also show that dollar-denominated returns often indicate a better fit than regressions in local currency suggesting that international capital inflow does play an important role in asset prices. These results are in line with the findings of Bóta and Ormos (2015), who find that efficiency on European markets (in particular, in the Central and Eastern European markets) is greater if measured in dollar-denominated returns. Moreover, we also show that the overperformance of dollar-denominated models is particularly significant on Developed European capital markets, which is in contradiction with the belief of international investors having a greater influence on emerging markets compared to developed ones.

IV/1. Introduction

Part IV aims to extend the comparative, empirical literature of asset pricing questions related to emerging and developed countries with focus on the European Union. Its main contribution lies in its methodology, in which we approximate the perceived risk of assets by Expected Downside Risk (EDR) (Ormos and Timotity, 2016a). We argue that this latter risk measure based on Conditional Value-at-Risk reflects a simpler function of risk than its alternatives, and therefore, may better capture investors’ risk perception. This latter statement is confirmed by our empirical results, where EDR outperforms its alternatives in describing the formation of the expected return using both dollar- and local currency-denomination. We also find that dollar-denominated returns often indicate a better fit than regressions in local currency when analysing separate analysis for specific countries and length of investment horizon. This latter finding is particularly significant in Developed European
capital markets, which, we argue, is in contradiction with the phenomenon of international investors having a greater influence on emerging markets compared to developed ones.

Subsequent to discussing the related literature, data and methodology, this study turns to fill the gap in the empirical literature by testing the goodness-of-fit of the EDR asset pricing model on distinct capital markets. The results are then compared with common and more sophisticated models that approximate expected return by standard deviation, variance, semi-variance (Markowitz, 1991), CAPM Beta (Lintner, 1965; Mossin, 1966; Sharpe, 1964; and Treynor, 1962) and Downside Beta (Estrada, 2002). According to the CAPM, idiosyncratic risk is supposed to have no effect on asset prices; however, previous results indicate that it may be relevant both in developed (Blitz and Van Vliet, 2007) and in emerging markets (Blitz et al., 2013). Therefore, this comparison includes both an aggregated and a separate analysis of emerging and developed markets. Moreover, in order to take into account the effect of international capital inflows, we run an analysis based on US dollar-denominated returns as well.

In Section 2 we discuss the related literature and the detailed motivation of our risk measure, in Section 3 the data and methodology used in this study are described, in Section 4 we present the detailed empirical results and in Section 5 we provide a brief conclusion on our main contributions of Part IV.

IV/2. Literature review

Our proposed risk measure, EDR is based on Value-at-Risk (Campbell et al., 2001; Jorion, 2007) and Conditional Value-at-Risk (Rockafellar and Uryasev, 2000); however, it does not measure the tail risk only but the expected loss relative to the expected return. Using the aforementioned definition its probability level becomes objective instead of an ad-hoc level, whereas it may reflect investors’ perception better as it is much more precise (in the sense of the distribution of returns) and easier to interpret than common measures such as standard deviation, variance or beta.

The second argument that supports the use of EDR is related to a behavioural approach to finance. According to Kahneman and Tversky’s (1979) Prospect Theory, investors do not refuse risk in some cases, namely when they suffer from losses in their financial position compared to previous position; in particular, they start to follow
risk-seeking behaviour to an extent, since their expected utility can be maximized with this behaviour assuming that their utility curve is concave in the domain of losses. In contrast to standard asset pricing models derived from expected utility theory, which suggest that risk-seeking (i.e. negative risk-aversion coefficient) cannot be implemented, we have shown in Part II that by using the Expected Downside Risk an optimal portfolio choice can be defined in the standard asset pricing framework as well, even if risk-seeking emerges. However, our model may predict either negative or positive relationship between risk and expected return, which could be interpreted as risk-seeking or risk-averse behaviour respectively. We have shown that changing the risk measure to EDR is a plausible modification of standard asset pricing models assuming EUT for two reasons. Due to bounded rationality (Simon, 1982), measures that are easier to interpret are the main factors driving the focus of investors (Gigerenzer and Selten, 2002). Therefore, the expected amount of money an investor can lose might be of greater relevance than the expectation of the squared deviations from the mean. Moreover, it seems obvious that investors require premium only for negative deviations from the mean, therefore, measuring only the risk (or probability) of loss may provide better results (Estrada, 2002). Furthermore, the variance-based approach yields the optimal portfolio choice only if each compared investment has the same standardized distribution (i.e. excluding the variance and mean, they are identical). However, capital assets and even market indices tend to have different characteristics both in developed and emerging markets as well (Ahmad et al., 2014). Although this problem could be solved by Value-at-Risk and Expected Shortfall due to their non-parametric nature, both have a pre-defined probability-level that cannot be verified fundamentally. These measures, however, may serve as a good base for new proxies of risk.

As discussed in Part II, Value-at-Risk exhibits many advantages over volatility and beta; however, it still misses the important characteristics of subadditivity and convexity; thus, VaR does not fulfil the requirements of a coherent risk measure of Artzner et al. (1999). Their introduction of Conditional Value-at-Risk (CVaR) solved the problems of the former; however, the predefined significance level in CVaR also exhibits some disadvantages in quantitative methods. Choosing the optimal level of \( \alpha \) is difficult and influences heavily the investment choice in modelling. This problem with the subjective confidence level has been questioned in Huang et al. (2012) as well; however, their proposed risk-measure, the extreme downside risk comes with two
possible flaws: first, it is based on extreme value theory, therefore, its fitted distribution may not represent the true behaviour of asset prices; second, its left tail index is difficult to interpret for the average investor, and therefore, it may not account for the true premium required. The performance measure Omega (Keating and Shadwick, 2002) also avoids the subjectivity problem by measuring the ratio of the expected abnormal return below and above zero; however, its use in equilibrium modelling is fairly moderate since, according to loss aversion (Kahneman and Tversky, 1979), investors are rather interested in downside risk and expected return. These flaws in existing parameters contributed to the introduction of the subject of the current analysis, the Expected Downside Risk (EDR).

IV/3. Data and methodology

IV/3.1 Data

The data used in this research includes time-series of daily, weekly and monthly returns of the top ten publicly traded companies based on market capitalisation and of the stock index of six European countries and the USA. In order to maintain a reasonably high number of observations, we apply the daily recalculated annual return methodology of Erdős and Ormos (2009). The markets analysed are chosen to fit developed and emerging markets: the Developed European markets are represented by the United Kingdom, Germany, and France, the Emerging Eastern European market group includes Poland, Hungary and Czech Republic, and numbers for the United States are tested as well as a proxy for non-European developed markets. Due to limited time-series data of the top ten stocks in emerging markets the analysed period covers the five years from 12/08/2010 to 12/08/2015. For longer periods there is no trading data for at least one of the top ten companies (as of 12/08/2015, based on market capitalization) in some countries. The risk-free return is provided by the investment horizon-adjusted yield of the local one-year government bonds in each country. The data is provided by the Thomson Reuters database. Emerging markets are commonly believed to be highly influenced by international investors. In our study, we define international investors by those whose investment is made in US dollars. This latter definition could cause two potential problems in the analysis: first, the distance effect (i.e. the negative relationship between the distance between two countries and their direct investment flows) is found to be relevant in empirical literature (Bevan and Estrin, 2004), therefore, the investments denominated
in US dollars, which mainly come from the US, are not as relevant as the effect of neighbouring countries. However, we argue that in the comparison of Central and Eastern European and Western European countries, the distance effect should not play an important role. Second, Central and Eastern European FDI is found to be supplied rather by Western European sources than the US (Frenkel et al., 2004), therefore, a euro denomination analysis could capture better the foreign presence in case of the former. However, this latter method would yield only a local currency analysis for most Western European countries, therefore, no comparison between the two regions would be possible. We argue that this duality in our analysis is important as empirical evidence shows that FDI/GDP ratio is generally higher in emerging markets than developed countries and particularly high in CEE region (Arbatli, 2011). Furthermore, according to Alfaro et al. (2004), the presence of foreign investors is an increasing function of the quality of financial markets, hence, it is worth analysing the mechanics of the foreign currency denominated expected return in the setting of market efficiency. With an indicative purpose, we test the latter hypotheses for our sample and present our results in Table IV/1, where the 2005-2014 FDI/GDP ratio is regressed on an emerging market dummy variable and time in the analysed countries. The significant dummy for emerging European markets indicates that GDP-weighted capital inflow is indeed higher in emerging markets, therefore, in addition to pricing calculations in local currency, a US dollar-denominated analysis is provided in which the US risk-free rate (i.e. one-year Treasury bill yield) is applied. With this additional calculation, the effect of international and local capital inflow can be separated.

### Table IV/1: The effect of market type on foreign investment

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>6.7262</td>
<td>0.0054</td>
</tr>
<tr>
<td>Dummy</td>
<td>4.2202</td>
<td>0.0412</td>
</tr>
<tr>
<td>Time</td>
<td>-0.7871</td>
<td>0.0288</td>
</tr>
</tbody>
</table>

Notes: The regression $FDI_{t,x} = \alpha + \beta_1 D_x + \beta_2 t$ is estimated by OLS method, in which $FDI_{t,x}$, $D_x$ and $t$ stand for the yearly foreign direct investment-gross domestic product ratio, the emerging market dummy (that is 1 for the Czech Republic, Hungary and Poland and zero otherwise) and the current year respectively.
In the empirical results section, we use a weighted test for model comparison, in which we aim to avoid the bias caused by highly volatile markets, and therefore, scale each return with the standard deviation of the given market. Table IV/2 indicates the necessity of this technique, where the daily sample descriptive statistics and differences between distinct countries are shown. For example, here one can see a large difference between the riskiness (standard deviation) of Hungarian and US returns, where the former is more than twice as big as the latter. Therefore, fitting errors of the regression in the Hungarian market could possibly contribute a lot more to the aggregated error without scaling, and thus our test results would be focused more on Hungarian data instead of general findings.

Table IV/2: Descriptive statistic of analysed countries

<table>
<thead>
<tr>
<th></th>
<th>UK</th>
<th>Poland</th>
<th>Hungary</th>
<th>Germany</th>
<th>Czech Rep</th>
<th>France</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.0041</td>
<td>0.0040</td>
<td>0.0001</td>
<td>0.0112</td>
<td>-0.0023</td>
<td>0.0062</td>
<td>0.0115</td>
</tr>
<tr>
<td>std</td>
<td>0.0320</td>
<td>0.0436</td>
<td>0.0645</td>
<td>0.0635</td>
<td>0.0437</td>
<td>0.0536</td>
<td>0.0314</td>
</tr>
<tr>
<td>skew</td>
<td>-0.3531</td>
<td>-0.3010</td>
<td>0.0645</td>
<td>-1.7120</td>
<td>-0.4746</td>
<td>-1.2246</td>
<td>-0.3394</td>
</tr>
<tr>
<td>JB</td>
<td>0.4556</td>
<td>0.5020</td>
<td>0.6601</td>
<td>0.0000</td>
<td>0.3280</td>
<td>0.0000</td>
<td>0.5376</td>
</tr>
</tbody>
</table>

Notes: The statistics represent the monthly means, standard deviations, skewnesses, kurtoses and Jarque-Bera test p-values for the market index returns of each country.

IV/3.2 Methodology

The risk parameter tested in this study, the Expected Downside Risk (EDR) measures the expected loss of an investment and is defined in the following way: being similar to the Conditional Value-at-Risk, EDR measures the expected loss; however, it does not apply an ad-hoc probability level but includes returns below the expected return, that is

$$EDR(x) = p(r_x \leq E(r_x))^{-1} \int_{r_x(y) \leq E(r_x)} r_x(y) p(y) dy.$$  (IV/1)

where $r_x$ and $E(r_x)$ show the return and its expectation of a given x portfolio respectively, $r_x(y)$ and $p(y)$ stand for the outcomes of x portfolio and their respective probabilities. EDR, as its name suggests, measures the expected loss (risk) of
investors given that their reference point is the expected return as suggested by Easterlin (1974) or Kőszegi and Rabin (2006).

Applying this risk measure and the aforementioned alternatives (volatility, variance, CAPM Beta, semi-variance and Downside Beta) for all stocks and indexes the OLS regression between the expected return and the given risk measure is run for each investment horizon (i.e. daily, weekly, monthly and yearly) and for each country separately, in which local risk-free rate is applied. In the comparison section F-tests are used on the squared residuals of the linear models to decide whether the country- and denomination-based pairs are significantly different in the competing models.

We run further robustness tests that are using weighted squared residuals, in which the squared residuals are adjusted and standardized to account for the difference in the total squared error of distinct markets and horizons. By this methodology, we can eliminate the problem of regional and time differences (i.e. the problem that residuals of yearly models and high volatility countries dominate the sum of squared residuals), and reach a much greater number of observations at the same time, which together allows for more robust test results. The methodology used for scaling is as follows. First, we estimate equation (IV/2) conditional to a $\theta$ fixed risk measure (i.e. volatility, variance, EDR, Beta, semi-variance and downside Beta), then we use the squared residuals for model comparison as in equations (IV/3) and (IV/4).

$$E(r_{t,x,h,i}, \theta_{x,h,i}) - r_{f,x,h} = \alpha_{x,h} + \beta_{x,h,i} \theta_{x,h,i} + \epsilon_{x,h,i},$$  \hspace{1cm} (IV/2)

$$SSR(r_{t,x,h}, \theta_{x,h}) = \sum_{h}^{H} \sum_{x}^{X} \sum_{t=1}^{T} \epsilon_{t,x,h}^2,$$  \hspace{1cm} (IV/3)

$$SSSR(r_{t}, \theta_{x,h}) = \sum_{h}^{H} \sum_{x=1}^{X} \sum_{t=1}^{T} \left( \frac{\epsilon_{t,x,h}}{sd(r_{x,h} - r_{f,x,h})} \right)^2,$$  \hspace{1cm} (IV/4)

where $E(r_{t,x,h,i}, \theta_{x,h})$ stands for the expected daily return of asset $i$, in country $x$, and with horizon $h$, while $\theta_{x,h,i}$, $SSR$, $SSSR$, $r_{f,x,h}$ and $sd(r_{x,h} - r_{f,x,h})$ are the applied risk measure with the former parameters, the sum of squared residuals, the scaled sum of squared residuals, the risk-free rate and standard deviation of the risk premium for a given horizon and market respectively. As shown above, $SSSR$ consists of the squared weighted residuals according to time horizon and country; in other words, it
standardizes the error terms by dividing them by the standard deviation of the risk premium for a given country and period length, then aggregates these latter. This method allows for an aggregate analysis irrespective of differences in country risk and period length.

These tests are run for the whole sample including all seven countries and by separating into the two subgroups of emerging Central and Eastern European and Developed European markets as well, hence conclusions can be drawn whether the expected return is better fitted on the proposed risk measure, whether developed and emerging markets are different in pricing idiosyncratic risk and whether international capital inflow (and therefore, US dollar-denomination) plays an important role in pricing capital assets in emerging markets.

IV/4. Separate analysis

In this section, we discuss the results of the Ordinary Least Squares (OLS) estimations of linear models separately for each country and time horizon. The applied variables are volatility (standard deviation), variance, EDR, CAPM Beta, semi-variance and Downside Beta and the regressions are run for returns in local currency and dollar-denomination and excluding and including the Jensen alpha (Jensen, 1968) as well. Our reason for applying both volatility and variance as well is to account for possible biases in the linear relationship between risk and expected return, which phenomenon has been well-documented in empirical literature (Erdős et al., 2011). Table IV/3 shows the R-squared values of linear models. Here, we only present the monthly regressions in order to save space; however, results for each regression can be found in Appendix IV/1. Results of the daily, weekly and yearly regressions also confirm our monthly findings, which latter are summarized in the followings.
### Table IV/3: Separate R-squared results for monthly return

#### α=0, returns in local currency

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>UK</th>
<th>Germany</th>
<th>France</th>
<th>Hungary</th>
<th>Poland</th>
<th>Czech Rep.</th>
</tr>
</thead>
<tbody>
<tr>
<td>vol</td>
<td>0.94</td>
<td>0.08</td>
<td>0.87</td>
<td>0.51</td>
<td>0.02</td>
<td>0.13</td>
<td>0.52</td>
</tr>
<tr>
<td>var</td>
<td>0.91</td>
<td>0.02</td>
<td>0.80</td>
<td>0.41</td>
<td>0.03</td>
<td>0.14</td>
<td>0.68</td>
</tr>
<tr>
<td>edr</td>
<td>0.86</td>
<td>0.01</td>
<td>0.78</td>
<td>0.32</td>
<td>0.10</td>
<td>0.22</td>
<td>0.77</td>
</tr>
<tr>
<td>beta</td>
<td>0.77</td>
<td>0.11</td>
<td>0.83</td>
<td>0.46</td>
<td>0.16</td>
<td>0.13</td>
<td>0.48</td>
</tr>
<tr>
<td>svar</td>
<td>0.77</td>
<td>0.03</td>
<td>0.68</td>
<td>0.27</td>
<td>0.10</td>
<td>0.15</td>
<td>0.78</td>
</tr>
<tr>
<td>dbeta</td>
<td>0.90</td>
<td>0.11</td>
<td>0.81</td>
<td>0.46</td>
<td>0.08</td>
<td>0.14</td>
<td>0.70</td>
</tr>
</tbody>
</table>

#### α=0, dollar-denominated returns

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>UK</th>
<th>Germany</th>
<th>France</th>
<th>Hungary</th>
<th>Poland</th>
<th>Czech Rep.</th>
</tr>
</thead>
<tbody>
<tr>
<td>vol</td>
<td>0.94</td>
<td>0.01</td>
<td>0.81</td>
<td>0.30</td>
<td>0.03</td>
<td>0.01</td>
<td>0.57</td>
</tr>
<tr>
<td>var</td>
<td>0.91</td>
<td>0.02</td>
<td>0.77</td>
<td>0.22</td>
<td>0.01</td>
<td>0.02</td>
<td>0.81</td>
</tr>
<tr>
<td>edr</td>
<td>0.86</td>
<td>0.01</td>
<td>0.73</td>
<td>0.18</td>
<td>0.01</td>
<td>0.04</td>
<td>0.78</td>
</tr>
<tr>
<td>beta</td>
<td>0.77</td>
<td>0.00</td>
<td>0.78</td>
<td>0.27</td>
<td>0.00</td>
<td>0.00</td>
<td>0.40</td>
</tr>
<tr>
<td>svar</td>
<td>0.77</td>
<td>0.01</td>
<td>0.77</td>
<td>0.22</td>
<td>0.00</td>
<td>0.01</td>
<td>0.76</td>
</tr>
<tr>
<td>dbeta</td>
<td>0.90</td>
<td>0.01</td>
<td>0.81</td>
<td>0.31</td>
<td>0.02</td>
<td>0.00</td>
<td>0.65</td>
</tr>
</tbody>
</table>

#### α≠0, returns in local currency

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>UK</th>
<th>Germany</th>
<th>France</th>
<th>Hungary</th>
<th>Poland</th>
<th>Czech Rep.</th>
</tr>
</thead>
<tbody>
<tr>
<td>vol</td>
<td>0.60</td>
<td>0.55</td>
<td>0.01</td>
<td>0.00</td>
<td>0.06</td>
<td>0.00</td>
<td>0.62</td>
</tr>
<tr>
<td>var</td>
<td>0.63</td>
<td>0.59</td>
<td>0.02</td>
<td>0.00</td>
<td>0.05</td>
<td>0.02</td>
<td>0.66</td>
</tr>
<tr>
<td>edr</td>
<td>0.09</td>
<td>0.88</td>
<td>0.09</td>
<td>0.24</td>
<td>0.58</td>
<td>0.14</td>
<td>0.87</td>
</tr>
<tr>
<td>beta</td>
<td>0.02</td>
<td>0.06</td>
<td>0.00</td>
<td>0.01</td>
<td>0.36</td>
<td>0.00</td>
<td>0.43</td>
</tr>
<tr>
<td>svar</td>
<td>0.34</td>
<td>0.37</td>
<td>0.00</td>
<td>0.02</td>
<td>0.48</td>
<td>0.05</td>
<td>0.75</td>
</tr>
<tr>
<td>dbeta</td>
<td>0.39</td>
<td>0.16</td>
<td>0.00</td>
<td>0.01</td>
<td>0.40</td>
<td>0.01</td>
<td>0.74</td>
</tr>
</tbody>
</table>

#### α≠0, dollar-denominated returns

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>UK</th>
<th>Germany</th>
<th>France</th>
<th>Hungary</th>
<th>Poland</th>
<th>Czech Rep.</th>
</tr>
</thead>
<tbody>
<tr>
<td>vol</td>
<td>0.60</td>
<td>0.66</td>
<td>0.00</td>
<td>0.06</td>
<td>0.12</td>
<td>0.00</td>
<td>0.70</td>
</tr>
<tr>
<td>var</td>
<td>0.63</td>
<td>0.66</td>
<td>0.01</td>
<td>0.06</td>
<td>0.11</td>
<td>0.01</td>
<td>0.82</td>
</tr>
<tr>
<td>edr</td>
<td>0.09</td>
<td>0.87</td>
<td>0.12</td>
<td>0.44</td>
<td>0.87</td>
<td>0.11</td>
<td>0.91</td>
</tr>
<tr>
<td>beta</td>
<td>0.02</td>
<td>0.66</td>
<td>0.01</td>
<td>0.08</td>
<td>0.27</td>
<td>0.05</td>
<td>0.32</td>
</tr>
<tr>
<td>svar</td>
<td>0.34</td>
<td>0.66</td>
<td>0.01</td>
<td>0.01</td>
<td>0.18</td>
<td>0.00</td>
<td>0.69</td>
</tr>
<tr>
<td>dbeta</td>
<td>0.39</td>
<td>0.64</td>
<td>0.00</td>
<td>0.02</td>
<td>0.18</td>
<td>0.00</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Notes: The results represent the coefficients of determination for given countries and models.
The coefficients indicate three main findings. First, EDR regressions are ranked amongst the top models based on the coefficients of determination. This finding is particularly valid for α≠0 cases, where EDR outperforms the competing risk measures in all European countries included in the sample. Second, developed markets seem to indicate a better fit of linear models in general as suggested by the higher average R-squared values. Third, dollar-denomination plays an important role in the goodness-of-fit of regressions. Again this pattern is representative mainly of the regressions including an estimated constant as well; in fact, if alpha is excluded, R-squared values are usually higher in linear models defined in local currency.

Moreover, one may find further interesting patterns in Table IV/3. First, variables that measure only the downside risk yield higher goodness-of-fit than those measuring the risk on the whole domain. This pattern is especially valid for the Downside Beta versus Beta; for the semi-variance, variance pair the results are mixed. Second, volatility and variance represent mainly the same type of risk. However, linearity between expected return and volatility confirms the existence of the capital market line, whereas linearity with respect to variance supports asset pricing based on utility. These two measures move together generally; however, we find some surprisingly large differences as well (for example in the case of Czech Republic). Third, one may also find it interesting that patterns representative of Developed Europe (e.g. EDR goodness-of-fit) are not valid for the US in most of the cases (namely the underperformance of EDR in the no constant case). These phenomena may be a consequence of the extreme alpha level of chosen companies in the US market, in which latter we have seen a huge run-up in the last couple of years.

In the followings, we analyse the abovementioned patterns as an aggregate behaviour of the developed and emerging subgroups, then test whether their difference can be attributed to randomness.

IV/5. Aggregate analysis

Although findings of the previous analysis have more or less robust patterns, in order to obtain a sufficiently high number of observations for the comparison of residuals, we provide the aggregated results in this section. The main patterns found above lead to the following classification: we present the goodness-of-fit results of the linear models
divided into four different subcases: asset pricing (i) in local currency without allowing for a Jensen alpha (Jensen, 1968), (ii) in dollar-denominated return without alpha, (iii) in local currency including alpha estimation, (iv) in dollar-denominated return with alpha. Average $R^2$ values are then shown for developed and emerging markets separately and in total as well for all four cases. Table IV/4 represents the first two cases.

Table IV/4: Regression R-squared values with no excess risk-free return

<table>
<thead>
<tr>
<th></th>
<th>In local currency</th>
<th></th>
<th>In dollars</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Developed</td>
<td>Emerging</td>
<td>Total</td>
<td>Developed</td>
</tr>
<tr>
<td>vol</td>
<td>0.46</td>
<td>0.27</td>
<td>0.36</td>
<td>0.31</td>
</tr>
<tr>
<td>var</td>
<td>0.38</td>
<td>0.34</td>
<td>0.36</td>
<td>0.27</td>
</tr>
<tr>
<td>edr</td>
<td>0.36</td>
<td>0.41</td>
<td>0.38</td>
<td>0.41</td>
</tr>
<tr>
<td>beta</td>
<td>0.43</td>
<td>0.17</td>
<td>0.30</td>
<td>0.31</td>
</tr>
<tr>
<td>svar</td>
<td>0.35</td>
<td>0.33</td>
<td>0.34</td>
<td>0.26</td>
</tr>
<tr>
<td>dbeta</td>
<td>0.44</td>
<td>0.31</td>
<td>0.38</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Notes: The results represent the average (i.e. daily, weekly, monthly and yearly aggregates for countries in the specific group) coefficients of determination (R-squared) of distinct linear models conditional to market type (i.e. developed, emerging or total) and to currency (i.e. local currency or dollars). In these regressions no constant is applied.

According to the regression results, in general, EDR has superior performance over the alternative measures. In particular, only for developed markets and local currency denominated returns we have found an inferior goodness-of-fit; however, we underline the fact that in these markets the Jensen alpha (which is not included in this analysis) may play an important role, hence yielding biased coefficient estimation. Another interesting result is that EDR, which measures idiosyncratic risk rather than market-related risk, performs much better relative to other measures in developed markets for US dollar-denominated regressions, which is a controversial result if one believes that the latter markets should be more efficient in terms of diversification than emerging markets. The dollar-denominated returns seem to provide a mixed picture altogether; however, again for developed markets, which are believed to be less influenced by foreign investments (e.g. Table IV/1), linear models provide higher R-squared than in emerging markets.
Second, we present the average results for the case when Jensen alpha is allowed in Table IV/5. Here, the findings are very similar to that of Table IV/4 except for that EDR yields a better fit than alternative risk measures everywhere, even in the case of local currency-denominated Developed European market returns. In particular, it is interesting to see that in every case the coefficient of determination is almost twice as high as that of the second best fit.

<table>
<thead>
<tr>
<th>Table IV/5: Aggregate R-squared values of regressions with constant</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>vol</td>
</tr>
<tr>
<td>var</td>
</tr>
<tr>
<td>edr</td>
</tr>
<tr>
<td>beta</td>
</tr>
<tr>
<td>svar</td>
</tr>
<tr>
<td>dbeta</td>
</tr>
</tbody>
</table>

Notes: The results represent the average (i.e. daily, weekly, monthly and yearly aggregates for countries in the specific group) coefficients of determination (R-squared) of distinct linear models conditional to market type (i.e. developed, emerging or total) and to currency (i.e. local currency or dollars). In these regressions constant estimation is included.

IV/6. Model comparison

In this section, we apply F-tests to find out whether the previously seen differences are significant or the results are estimated by chance. In order to reach a sufficiently high number of observations required by these hypothesis analyses we aggregate the data according to the SSSR methodology discussed in Section 3.2, hence, we compare the standardized residuals instead of the simple R-squared values, which would otherwise yield biased results due to time and region-related differences of the model residuals. First, in Table IV/6 we present the local currency denominated comparison, where ratios of the variance of standardized squared residuals of specific risk measures are shown with EDR residuals as a reference. The p-values are almost everywhere below 0.05 (the only exception is the variance-based model), furthermore, in the cases below the 5% level, they indicate that EDR significantly outperforms the competing risk
measures. The analysis reveals that by including the constant in the regression, the hypothesis that SSSR values are equal can be easily rejected in each case.

Table IV/6: F-test results of residuals against EDR in local currency

<table>
<thead>
<tr>
<th>Variable</th>
<th>Without alpha</th>
<th></th>
<th></th>
<th>With alpha</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ratio</td>
<td>p-value</td>
<td>ratio</td>
<td>p-value</td>
<td>ratio</td>
<td>p-value</td>
</tr>
<tr>
<td>total sample</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>volatility</td>
<td>1.6478</td>
<td>0.0000</td>
<td>2.6426</td>
<td>0.0000</td>
<td>1.5549</td>
<td>0.0167</td>
</tr>
<tr>
<td>variance</td>
<td>1.1781</td>
<td>0.1717</td>
<td>3.2219</td>
<td>0.0000</td>
<td>0.8829</td>
<td>0.4981</td>
</tr>
<tr>
<td>beta</td>
<td>3.9338</td>
<td>0.0000</td>
<td>3.1473</td>
<td>0.0000</td>
<td>3.8649</td>
<td>0.0000</td>
</tr>
<tr>
<td>semi-variance</td>
<td>2.3110</td>
<td>0.0000</td>
<td>2.7137</td>
<td>0.0000</td>
<td>2.2749</td>
<td>0.0000</td>
</tr>
<tr>
<td>downside beta</td>
<td>2.3945</td>
<td>0.0000</td>
<td>2.4422</td>
<td>0.0000</td>
<td>2.4442</td>
<td>0.0000</td>
</tr>
<tr>
<td>developed markets</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>volatility</td>
<td>2.8998</td>
<td>0.0000</td>
<td>3.4820</td>
<td>0.0000</td>
<td>2.7573</td>
<td>0.0000</td>
</tr>
<tr>
<td>variance</td>
<td>2.4539</td>
<td>0.0000</td>
<td>3.4262</td>
<td>0.0000</td>
<td>2.2280</td>
<td>0.0000</td>
</tr>
<tr>
<td>beta</td>
<td>6.9201</td>
<td>0.0000</td>
<td>3.4929</td>
<td>0.0000</td>
<td>7.5777</td>
<td>0.0000</td>
</tr>
<tr>
<td>semi-variance</td>
<td>5.7105</td>
<td>0.0000</td>
<td>3.4328</td>
<td>0.0000</td>
<td>6.1194</td>
<td>0.0000</td>
</tr>
<tr>
<td>downside beta</td>
<td>5.5178</td>
<td>0.0000</td>
<td>3.2356</td>
<td>0.0000</td>
<td>5.9264</td>
<td>0.0000</td>
</tr>
<tr>
<td>emerging markets</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>volatility</td>
<td>2.8998</td>
<td>0.0000</td>
<td>3.4820</td>
<td>0.0000</td>
<td>2.7573</td>
<td>0.0000</td>
</tr>
<tr>
<td>variance</td>
<td>2.4539</td>
<td>0.0000</td>
<td>3.4262</td>
<td>0.0000</td>
<td>2.2280</td>
<td>0.0000</td>
</tr>
<tr>
<td>beta</td>
<td>6.9201</td>
<td>0.0000</td>
<td>3.4929</td>
<td>0.0000</td>
<td>7.5777</td>
<td>0.0000</td>
</tr>
<tr>
<td>semi-variance</td>
<td>5.7105</td>
<td>0.0000</td>
<td>3.4328</td>
<td>0.0000</td>
<td>6.1194</td>
<td>0.0000</td>
</tr>
<tr>
<td>downside beta</td>
<td>5.5178</td>
<td>0.0000</td>
<td>3.2356</td>
<td>0.0000</td>
<td>5.9264</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Notes: The results represent the ratios between the SSSR of given risk measures compared to EDR. The p-values stand for the probability at which we can reject the hypothesis that the ratio is equal to one. Here, the returns are denominated in local currency.

Second, we compare the residuals of the dollar-denominated regressions in Table IV/7. These results show that, in contrast to models in local currency, only 2 differences out of 5 are significant at 5% for regressions without constant. However, in the cases where alpha is included p-values remain well below the 5% level again, which confirms that EDR significantly outperforms the alternative risk measures in the dollar-denominated regressions as well.
Table IV/7: F-test results of residuals against EDR in US dollars

<table>
<thead>
<tr>
<th>Variable</th>
<th>Without alpha</th>
<th>With alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ratio p-value</td>
<td>ratio p-value</td>
</tr>
<tr>
<td>volatility</td>
<td>1.2650 0.0501</td>
<td>1.5536 0.0169</td>
</tr>
<tr>
<td>variance</td>
<td>0.9055 0.4075</td>
<td>1.5746 0.0139</td>
</tr>
<tr>
<td>beta</td>
<td>3.5241 0.0000</td>
<td>1.5408 0.0191</td>
</tr>
<tr>
<td>semi-variance</td>
<td>1.1291 0.3113</td>
<td>1.5240 0.0223</td>
</tr>
<tr>
<td>downside beta</td>
<td>1.7704 0.0000</td>
<td>1.5164 0.0239</td>
</tr>
<tr>
<td>volatility</td>
<td>2.5773 0.0000</td>
<td>1.7179 0.0034</td>
</tr>
<tr>
<td>variance</td>
<td>2.2289 0.0000</td>
<td>1.6726 0.0054</td>
</tr>
<tr>
<td>beta</td>
<td>8.1801 0.0000</td>
<td>1.6577 0.0062</td>
</tr>
<tr>
<td>semi-variance</td>
<td>2.7523 0.0000</td>
<td>1.9919 0.0002</td>
</tr>
<tr>
<td>downside beta</td>
<td>3.5648 0.0000</td>
<td>2.1846 0.0000</td>
</tr>
</tbody>
</table>

Notes: The results represent the ratios between the SSSR of given risk measures compared to EDR. The p-values stand for the probability at which we can reject the hypothesis that the ratio is equal to one. Here, the returns are denominated in US dollars.

One may find some common patterns in the results of Table IV/6 and IV/7. In particular, the ratio of $EDR$ against either Beta or downside Beta is always much higher in the cases of the Emerging European markets than in developed markets. This finding is in line with the hypothesis that the inclusion of idiosyncratic risk in asset prices (as measured by $EDR$ regressions) is rather a property of the less efficient, emerging markets than the developed ones.

Another interesting pattern is that the exclusion of the Jensen alpha has a much stronger influence on residuals in the case of Emerging European markets. In developed markets, either by looking at results in local currency or in USD, $EDR$ performs always significantly better than the alternatives. This finding suggests that using a precise risk measure (such as $EDR$) mispricing of capital assets, that is excess risk-free return, in this case, may have a greater relevance in less efficient markets.

Moreover, we have run the difference analysis using a bootstrapped Levene-test as well to account for a potential sensitivity to non-normality of our results, however, at
5% significance level the results of Table IV/5 have been confirmed, that is, EDR significantly outperforms its analysed alternatives even if we assume no normality of the residuals.

Our following F-tests summarized in Table IV/8 are related to the null hypothesis that the variance of residuals of dollar-denominated regressions is equal to those yielded by regressions in local currency. Although not significant in most of the cases, in each test where the p-value is below the 0.05 level, the ratio higher than unity confirms higher residual variance for local currency regressions (thus, the superior performance of US dollar-denominated returns).

Table IV/8: F-test results of local currency residuals against US dollars

<table>
<thead>
<tr>
<th>Variable</th>
<th>total sample</th>
<th>developed markets</th>
<th>emerging markets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ratio</td>
<td>p-value</td>
<td>ratio</td>
</tr>
<tr>
<td>volatility</td>
<td>1.0962</td>
<td>0.4436</td>
<td>1.2814</td>
</tr>
<tr>
<td>variance</td>
<td>1.1029</td>
<td>0.4139</td>
<td>1.5240</td>
</tr>
<tr>
<td>Without</td>
<td>EDR</td>
<td>1.1255</td>
<td>0.3239</td>
</tr>
<tr>
<td>alpha</td>
<td>beta</td>
<td>0.9952</td>
<td>0.9681</td>
</tr>
<tr>
<td></td>
<td>semi-variance</td>
<td>1.3930</td>
<td>0.0058</td>
</tr>
<tr>
<td></td>
<td>downside beta</td>
<td>1.1643</td>
<td>0.2044</td>
</tr>
<tr>
<td>With</td>
<td>volatility</td>
<td>1.0870</td>
<td>0.4864</td>
</tr>
<tr>
<td>alpha</td>
<td>variance</td>
<td>1.0648</td>
<td>0.6002</td>
</tr>
<tr>
<td></td>
<td>EDR</td>
<td>1.0025</td>
<td>0.9837</td>
</tr>
<tr>
<td></td>
<td>beta</td>
<td>0.8180</td>
<td>0.0939</td>
</tr>
<tr>
<td></td>
<td>semi-variance</td>
<td>1.3291</td>
<td>0.0178</td>
</tr>
<tr>
<td></td>
<td>downside beta</td>
<td>1.2764</td>
<td>0.0420</td>
</tr>
</tbody>
</table>

Notes: The results represent the ratios between the SSSR of given risk measures in local currency compared to the residuals in US dollars. The p-values stand for the probability at which we can reject the hypothesis that the ratio is equal to one.

One may find an interesting pattern in the latter results: in particular, in developed markets the overperformance of US dollar-denominated regressions is significant for
variance, $EDR$, and beta; however, in emerging markets the performance comparisons yield small enough p-values nowhere.

In order to further increase the number of observations, we present the aggregate analysis as well in Table IV/9. Here, it is interesting to notice that regressions in dollar-denominated returns significantly outperform the local currency in developed markets irrespective of the inclusion of a constant in the regression. However, in emerging markets we cannot see such patterns, which indicates that market participants investing in US dollars have a greater relative influence on asset prices in developed EU markets than in emerging EU markets. This finding may be considered as a contradiction with the common belief and the findings of Table IV/1 confirming that presence of foreign investments is superior in emerging markets than in developed countries.

### Table IV/9: Aggregated F-test results of local currency residuals against US dollars

<table>
<thead>
<tr>
<th></th>
<th>total sample</th>
<th>developed markets</th>
<th>emerging markets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ratio</td>
<td>p-value</td>
<td>ratio</td>
</tr>
<tr>
<td>Without alpha</td>
<td>1.1038</td>
<td>0.0431</td>
<td>1.5252</td>
</tr>
<tr>
<td>With alpha</td>
<td>1.0178</td>
<td>0.7181</td>
<td>1.5157</td>
</tr>
</tbody>
</table>

Notes: The results represent the ratios between the SSSR of residuals including all risk measures aggregated in local currency compared to the residuals in US dollars. The p-values stand for the probability at which we can reject the hypothesis that the ratio is equal to one.

**IV/7. Conclusion**

In Part IV we have presented two main findings. First, we have shown that linear regressions based on Expected Downside Risk (EDR) – in general – perform significantly better than its alternatives in explaining the expected return (i.e. the risk premium) both in emerging and developed markets consisting of six sample countries in the European Union and the United States. Hence, the fourth thesis of this dissertation can be defined: the Expected Downside Risk-based asset pricing
model captures the relationship between risk and expected return with superior performance in comparison with the volatility, variance, semi-variance, CAPM Beta, and Downside CAPM Beta on Central and Eastern European and Developed Western European markets.

In particular, the F-test results of the developed and emerging market subgroup analysis confirm that $EDR$ always yields significantly higher goodness-of-fit in developed markets, which are believed to be more efficient. In emerging markets by including the Jensen alpha estimation, which seems to play an important role here, the same results apply, while excluding the constant the F-tests significant at 5% always support the superior performance of $EDR$.

Second, we have shown that regressions based on dollar-denominated returns outperform the ones in local currencies in most cases. Moreover, the aggregated F-tests show that these differences are very significant at a reasonably high number of observations for developed markets; however, for emerging markets we cannot reject the null hypothesis that dollar-denominated regressions and local currency regression yield equal variances of residuals. Therefore, the fifth thesis of this dissertation is valid: dollar-denominated returns often perform better than regressions in the local currency both in regressions based on EDR and alternative risk measures, which indicates that international capital inflow does play an important role in asset prices.

This finding is surprising as it goes against the common belief of foreign investments having a greater influence on emerging markets.

Further ways of research connected to this study may reveal novel findings on the effect of regressions on returns denominated in foreign currency; the analysis could also be extended to cover other emerging and developed markets as well or to find further evidence for the difference in risk-perception of foreign and local investors.
V. Summary

Capturing the expected returns of capital assets has always been an important cornerstone in financial analysis. The first model that was able to provide a coherent theoretical framework for their dynamics was introduced by Henry Markowitz in the Modern Portfolio Theory (1952). Since his milestone paper, numerous attempts have been made to discover new underlying factors behind price dynamics, such as the Capital Asset Pricing Model or the Fama-French three-factor model. However, these asset pricing models were defined under assumptions that are not always valid in reality. In this dissertation, I aimed to relax the most unrealistic assumptions of these equilibrium asset pricing models by introducing a novel risk measure, the Expected Downside Risk. My main findings related to this new asset pricing model are presented in the five, following theses.

First, the inclusion of Expected Downside Risk in asset pricing allows for relaxing one of the most unrealistic, yet crucial restrictions of commonly used equilibrium models, the assumption of risk-averse investors (Ormos and Timotity, 2013a, 2013b, 2013c, 2013d, 2014a, 2016a).

Second, in addition to the generalized investor behaviour, with the shift in risk measure to Expected Downside Risk, the assumptions of price-taker investors and unlimited leverage opportunity for a unique interest rate can also be relaxed (Ormos and Timotity, 2013a, 2013b, 2013c, 2013d, 2014a, 2016a).

Third, by using the Expected Downside Risk as risk measure, both a positive and a negative relationship between expected return and risk can be derived under standard conditions of equilibrium asset pricing models, such as the expected utility theory and positive risk-aversion (Ormos and Timotity, 2016a, 2016c). These results provide further evidence from asset pricing for the findings in experimental economics related to the existence of risk-seeking behaviour.

Fourth, I find that the Expected Downside Risk-based asset pricing model captures the relationship between risk and expected return with superior performance in comparison with the volatility, variance, semi-variance, CAPM Beta, and Downside CAPM Beta on Central and Eastern European and Developed Western European markets (Ormos and Timotity, 2016b). This finding highlights the relevance of EDR-based asset pricing in finance, suggesting that the
generalised theoretical considerations and extension of the assumptions indeed yield a model with better fit.

Fifth, dollar-denominated returns often perform better than regressions in the local currency both in regressions based on EDR and alternative risk measures, which indicates that international capital inflow does play an important role in asset prices (Ormos and Timotity, 2016b).

Altogether, these theses and findings confirm both from the theoretical and the empirical side that Expected Downside Risk captures better the dynamics of expected capital asset returns. On the one hand, keeping only the most realistic assumptions yields a generalised equilibrium model; on the other hand, the fitting results indicate that this generalisation leads to a greater pricing performance of the proposed model compared to alternative asset pricing models.
References

Related publications by the author


Non-related publications by the author


Ormos, M., Timotity, D., 2016g. Differences in Implied and Realized Volatility: Risk-seeking and Conditional Heteroscedasticity in Prospect Theory, Under review


Ormos, M., Timotity, D., 2016j. Intertemporal Choice and Dynamics of Risk Aversion, 30th European Conference on Modelling and Simulation, Regensburg, Germany

Other references

Acerbi, C., Székely, B., 2014. BACKTESTING EXPECTING SHORTFALL.


Appendix II/1

The VaR for a fixed $\alpha\%$ shows the value that with $(1-\alpha)\%$ probability the return of an investment will be above. An alternative explanation is that $VaR_\alpha$ is the $\alpha$ percentile of the density function of return. By defining $f(x,y)$ as the function of loss where $x$ is a chosen portfolio and $y$ is a random variable, the probability of $f(x,y)$ not exceeding a $\zeta$ value (loss) is $\psi$:

$$\psi(x, \zeta) = \int_{f(x,y) \leq \zeta} p(y)dy$$  \hspace{1cm} (A1.1)

Assuming a fixed $x$ portfolio, this equation is the same as the cumulative distribution function of the $f(x,y)$ loss function in $\zeta$. According to this function, the Value-at-Risk ($VaR_\alpha$) is:

$$\zeta_\alpha(x) = \min\{\zeta \in \mathbb{R}: \psi(x, \zeta) \geq \alpha\}$$  \hspace{1cm} (A1.2)

meaning the first point on the cumulative distribution function of the return of the $x$ portfolio with greater cumulative probability than $\alpha$. $CVaR_\alpha$ can be described with the same technique:

$$\varphi_\alpha(x) = (1-\alpha)^{-1} \int_{f(x,y) \geq \zeta_\alpha(x)} f(x,y)p(y)dy$$  \hspace{1cm} (A1.3)

where $f(x,y) \geq \zeta_\alpha(x)$ is $(1-\alpha)$ according to the definition of $VaR$, hence it becomes the denominator. The interpretation of eq. (A1.3) is that $VaR_\alpha(x)$ is the $\alpha$ percentile of the $f(x,y)$ loss function, while $CVaR_\alpha(x)$ is the probability-weighted average (expected value) of losses greater than $VaR$. Therefore, $CVaR_\alpha(x) \geq VaR_\alpha(x)$ is always true (Rockafellar and Uryasev, 2002).
Appendix II/2

First, in order to precisely simulate risk aversion, we have to take into consideration that the modified utility curve may have other risk aversion coefficients by measuring the changes of wealth instead of the total wealth. Hence, we follow the method of Barsky et al. (1997) but instead equating

\[ 0.5U(xW) + 0.5U(2W) = U(W) \] \hspace{1cm} (A2.1)

we measure

\[ U(-(1-x)E(r)) = U(E(r)). \] \hspace{1cm} (A2.2)

Plugging in the modified utility function we get

\[ 2.25U\left(1 - e^{-a(1-x)E(r)}\right) = 1 - e^{-aE(r)}, \] \hspace{1cm} (A2.3)

and thus we get the risk aversion coefficient as

\[ a = \ln \frac{1.25}{e^{-E(r)}(e^{1-x}-1)}. \] \hspace{1cm} (A2.4)

Table A/1 represents the results of our Monte Carlo estimation for the risk aversion coefficients where the rows and columns stand for the expected return and the x values used by Barsky et al. respectively.
<table>
<thead>
<tr>
<th>x/E(r)</th>
<th>50%</th>
<th>66.70%</th>
<th>80%</th>
<th>90%</th>
<th>92%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>0.665896</td>
<td>1.16164</td>
<td>1.740915</td>
<td>2.485312</td>
<td>2.718606</td>
<td>3.203772</td>
</tr>
<tr>
<td>2%</td>
<td>0.675896</td>
<td>1.17164</td>
<td>1.750915</td>
<td>2.495312</td>
<td>2.728606</td>
<td>3.213772</td>
</tr>
<tr>
<td>3%</td>
<td>0.685896</td>
<td>1.18164</td>
<td>1.760915</td>
<td>2.505312</td>
<td>2.738606</td>
<td>3.223772</td>
</tr>
<tr>
<td>4%</td>
<td>0.695896</td>
<td>1.19164</td>
<td>1.770915</td>
<td>2.515312</td>
<td>2.748606</td>
<td>3.233772</td>
</tr>
<tr>
<td>5%</td>
<td>0.705896</td>
<td>1.20164</td>
<td>1.780915</td>
<td>2.525312</td>
<td>2.758606</td>
<td>3.243772</td>
</tr>
<tr>
<td>6%</td>
<td>0.715896</td>
<td>1.21164</td>
<td>1.790915</td>
<td>2.535312</td>
<td>2.768606</td>
<td>3.253772</td>
</tr>
<tr>
<td>7%</td>
<td>0.725896</td>
<td>1.22164</td>
<td>1.800915</td>
<td>2.545312</td>
<td>2.778606</td>
<td>3.263772</td>
</tr>
<tr>
<td>8%</td>
<td>0.735896</td>
<td>1.23164</td>
<td>1.810915</td>
<td>2.555312</td>
<td>2.788606</td>
<td>3.273772</td>
</tr>
<tr>
<td>9%</td>
<td>0.745896</td>
<td>1.24164</td>
<td>1.820915</td>
<td>2.565312</td>
<td>2.798606</td>
<td>3.283772</td>
</tr>
<tr>
<td>10%</td>
<td>0.755896</td>
<td>1.25164</td>
<td>1.830915</td>
<td>2.575312</td>
<td>2.808606</td>
<td>3.293772</td>
</tr>
<tr>
<td>11%</td>
<td>0.765896</td>
<td>1.26164</td>
<td>1.840915</td>
<td>2.585312</td>
<td>2.818606</td>
<td>3.303772</td>
</tr>
<tr>
<td>12%</td>
<td>0.775896</td>
<td>1.27164</td>
<td>1.850915</td>
<td>2.595312</td>
<td>2.828606</td>
<td>3.313772</td>
</tr>
<tr>
<td>13%</td>
<td>0.785896</td>
<td>1.28164</td>
<td>1.860915</td>
<td>2.605312</td>
<td>2.838606</td>
<td>3.323772</td>
</tr>
<tr>
<td>14%</td>
<td>0.795896</td>
<td>1.29164</td>
<td>1.870915</td>
<td>2.615312</td>
<td>2.848606</td>
<td>3.333772</td>
</tr>
<tr>
<td>15%</td>
<td>0.805896</td>
<td>1.30164</td>
<td>1.880915</td>
<td>2.625312</td>
<td>2.858606</td>
<td>3.343772</td>
</tr>
<tr>
<td>16%</td>
<td>0.815896</td>
<td>1.31164</td>
<td>1.890915</td>
<td>2.635312</td>
<td>2.868606</td>
<td>3.353772</td>
</tr>
<tr>
<td>17%</td>
<td>0.825896</td>
<td>1.32164</td>
<td>1.900915</td>
<td>2.645312</td>
<td>2.878606</td>
<td>3.363772</td>
</tr>
<tr>
<td>18%</td>
<td>0.835896</td>
<td>1.33164</td>
<td>1.910915</td>
<td>2.655312</td>
<td>2.888606</td>
<td>3.373772</td>
</tr>
<tr>
<td>19%</td>
<td>0.845896</td>
<td>1.34164</td>
<td>1.920915</td>
<td>2.665312</td>
<td>2.898606</td>
<td>3.383772</td>
</tr>
<tr>
<td>20%</td>
<td>0.855896</td>
<td>1.35164</td>
<td>1.930915</td>
<td>2.675312</td>
<td>2.908606</td>
<td>3.393772</td>
</tr>
</tbody>
</table>

It is clear that the results, although somewhat different, are not that far from those measured by Barsky et al. The risk aversion coefficients are thus in the same order of magnitude and can be used for describing portfolio choice.

Therefore, based on these coefficients, we are able to apply the \( EDR(x) + \frac{0.8s^2}{\alpha} < E(r_x) \) constraint of eq. (II/23) to the simulation presented in Figure A/1, where \( \alpha = 3 \) is used.
Even by considering this extremely risk-averse case of $a = 3$, portfolios can be found below the constraint, hence, marginal indifference curves at these points have negative slope and thus optimal choice can be found.
Appendix II/3

Without the loss of generality the assumption can be made that the efficiency frontier \( \sigma \)-E(r) pairs are described by a concave power function. Let us define this function as

\[
E(r) = (\sigma - \beta)^\alpha + \gamma; \beta > 0, \ 0 < \alpha < 1.
\] (A3.1)

The utility maximizing investor then has the problem of

\[
\max_{\sigma, E(r)} U \text{ s.t. } E(r) = (\sigma - \beta)^\alpha + \gamma,
\] (A3.2)

which yields the solution of

\[
\frac{dU}{d\sigma_{opt}} - \lambda \alpha (\sigma - \beta)^{\alpha-1} = 0,
\] (A3.3)

\[
\frac{dU}{dE(r)_{opt}} + \lambda = 0,
\] (A3.4)

\[
\sigma_{opt} = \frac{\alpha (\sigma_{opt} - \beta)^{\alpha-1}}{\alpha},
\] (A3.5)

where \( \sigma_{opt} \) stands for the portfolio risk at the optimum. We also know that the leveraged position can be defined as

\[
E(r)_L = r_f + L(E(r) - r_f); \sigma_L = L\sigma,
\] (A3.6)

where \( L \) stands for the leverage (e.g. \( L=1 \) with no leverage). The latter gives the solution of

\[
\sigma_{L, opt} = \frac{\alpha (\sigma_{opt} - \beta)^{\alpha-1}}{L\alpha}.
\] (A3.7)

The difference between the leveraged and unleveraged optimization is shown in Figure A/2, where equations (A3.5) and (A3.7) are represented by the cross-sections. The
solid, dashed and dotted lines stand for the left hand side of equation (A3.5) and the right hand sides of equations (A3.5) and (A3.7) respectively. The parameters are fixed at \([\alpha, \beta, a, L] = \{0.5, 0.1, 4, 2\} \).

**Figure A/2 Slope of the indifference curve**

The analytical and graphical interpretations indicate that investors accessing higher leverage limit or being more risk-averse (i.e. \(a\) is high) choose portfolios with lower risk that is \(\sigma_{L, opt} < \sigma_{opt}\).
Appendix IV/1.

R-squared values of regressions separately for countries and time horizons.

First, daily, weekly, monthly and yearly R2 values are presented for the case with $\alpha=0$ and local currency.

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>UK</th>
<th>Germany</th>
<th>France</th>
<th>Hungary</th>
<th>Poland</th>
<th>Czech Rep.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vol</td>
<td>0.93</td>
<td>0.05</td>
<td>0.91</td>
<td>0.51</td>
<td>0.00</td>
<td>0.15</td>
<td>0.53</td>
</tr>
<tr>
<td>var</td>
<td>0.94</td>
<td>0.01</td>
<td>0.89</td>
<td>0.42</td>
<td>0.00</td>
<td>0.15</td>
<td>0.81</td>
</tr>
<tr>
<td>edr</td>
<td>0.91</td>
<td>0.04</td>
<td>0.90</td>
<td>0.49</td>
<td>0.03</td>
<td>0.20</td>
<td>0.60</td>
</tr>
<tr>
<td>beta</td>
<td>0.82</td>
<td>0.21</td>
<td>0.88</td>
<td>0.45</td>
<td>0.16</td>
<td>0.11</td>
<td>0.25</td>
</tr>
<tr>
<td>svar</td>
<td>0.89</td>
<td>0.01</td>
<td>0.90</td>
<td>0.31</td>
<td>0.05</td>
<td>0.15</td>
<td>0.53</td>
</tr>
<tr>
<td>dbeta</td>
<td>0.93</td>
<td>0.04</td>
<td>0.92</td>
<td>0.44</td>
<td>0.06</td>
<td>0.18</td>
<td>0.60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>UK</th>
<th>Germany</th>
<th>France</th>
<th>Hungary</th>
<th>Poland</th>
<th>Czech Rep.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weekly</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vol</td>
<td>0.94</td>
<td>0.06</td>
<td>0.90</td>
<td>0.48</td>
<td>0.01</td>
<td>0.14</td>
<td>0.59</td>
</tr>
<tr>
<td>var</td>
<td>0.93</td>
<td>0.00</td>
<td>0.87</td>
<td>0.37</td>
<td>0.01</td>
<td>0.14</td>
<td>0.85</td>
</tr>
<tr>
<td>edr</td>
<td>0.93</td>
<td>0.03</td>
<td>0.87</td>
<td>0.44</td>
<td>0.06</td>
<td>0.18</td>
<td>0.69</td>
</tr>
<tr>
<td>beta</td>
<td>0.87</td>
<td>0.07</td>
<td>0.88</td>
<td>0.41</td>
<td>0.17</td>
<td>0.10</td>
<td>0.25</td>
</tr>
<tr>
<td>svar</td>
<td>0.87</td>
<td>0.06</td>
<td>0.85</td>
<td>0.31</td>
<td>0.10</td>
<td>0.14</td>
<td>0.80</td>
</tr>
<tr>
<td>dbeta</td>
<td>0.91</td>
<td>0.00</td>
<td>0.90</td>
<td>0.48</td>
<td>0.07</td>
<td>0.18</td>
<td>0.68</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>UK</th>
<th>Germany</th>
<th>France</th>
<th>Hungary</th>
<th>Poland</th>
<th>Czech Rep.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vol</td>
<td>0.94</td>
<td>0.08</td>
<td>0.87</td>
<td>0.51</td>
<td>0.02</td>
<td>0.13</td>
<td>0.52</td>
</tr>
<tr>
<td>var</td>
<td>0.91</td>
<td>0.02</td>
<td>0.80</td>
<td>0.41</td>
<td>0.03</td>
<td>0.14</td>
<td>0.68</td>
</tr>
<tr>
<td>edr</td>
<td>0.86</td>
<td>0.01</td>
<td>0.78</td>
<td>0.32</td>
<td>0.10</td>
<td>0.22</td>
<td>0.77</td>
</tr>
<tr>
<td>beta</td>
<td>0.77</td>
<td>0.11</td>
<td>0.83</td>
<td>0.46</td>
<td>0.16</td>
<td>0.13</td>
<td>0.48</td>
</tr>
<tr>
<td>svar</td>
<td>0.77</td>
<td>0.03</td>
<td>0.68</td>
<td>0.27</td>
<td>0.10</td>
<td>0.15</td>
<td>0.78</td>
</tr>
<tr>
<td>dbeta</td>
<td>0.90</td>
<td>0.11</td>
<td>0.81</td>
<td>0.46</td>
<td>0.08</td>
<td>0.14</td>
<td>0.70</td>
</tr>
</tbody>
</table>
### Yearly

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>UK</th>
<th>Germany</th>
<th>France</th>
<th>Hungary</th>
<th>Poland</th>
<th>Czech Rep.</th>
</tr>
</thead>
<tbody>
<tr>
<td>vol</td>
<td>0.93</td>
<td>0.05</td>
<td>0.78</td>
<td>0.29</td>
<td>0.10</td>
<td>0.29</td>
<td>0.72</td>
</tr>
<tr>
<td>var</td>
<td>0.77</td>
<td>0.01</td>
<td>0.63</td>
<td>0.13</td>
<td>0.09</td>
<td>0.23</td>
<td>0.89</td>
</tr>
<tr>
<td>edr</td>
<td>0.13</td>
<td>0.37</td>
<td>0.02</td>
<td>0.05</td>
<td>0.53</td>
<td>0.54</td>
<td>0.96</td>
</tr>
<tr>
<td>beta</td>
<td>0.32</td>
<td>0.00</td>
<td>0.69</td>
<td>0.12</td>
<td>0.07</td>
<td>0.16</td>
<td>0.04</td>
</tr>
<tr>
<td>svar</td>
<td>0.79</td>
<td>0.00</td>
<td>0.63</td>
<td>0.17</td>
<td>0.06</td>
<td>0.25</td>
<td>0.81</td>
</tr>
<tr>
<td>dbeta</td>
<td>0.91</td>
<td>0.00</td>
<td>0.79</td>
<td>0.33</td>
<td>0.08</td>
<td>0.33</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Second, the daily, weekly, monthly and yearly R2 values are presented for the case with \( \alpha=0 \) and dollar denominated returns.

### Daily

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>UK</th>
<th>Germany</th>
<th>France</th>
<th>Hungary</th>
<th>Poland</th>
<th>Czech Rep.</th>
</tr>
</thead>
<tbody>
<tr>
<td>vol</td>
<td>0.93</td>
<td>0.06</td>
<td>0.83</td>
<td>0.30</td>
<td>0.05</td>
<td>0.00</td>
<td>0.54</td>
</tr>
<tr>
<td>var</td>
<td>0.94</td>
<td>0.01</td>
<td>0.81</td>
<td>0.23</td>
<td>0.05</td>
<td>0.00</td>
<td>0.83</td>
</tr>
<tr>
<td>edr</td>
<td>0.91</td>
<td>0.04</td>
<td>0.82</td>
<td>0.28</td>
<td>0.04</td>
<td>0.01</td>
<td>0.57</td>
</tr>
<tr>
<td>beta</td>
<td>0.82</td>
<td>0.18</td>
<td>0.80</td>
<td>0.26</td>
<td>0.00</td>
<td>0.00</td>
<td>0.27</td>
</tr>
<tr>
<td>svar</td>
<td>0.89</td>
<td>0.03</td>
<td>0.82</td>
<td>0.12</td>
<td>0.01</td>
<td>0.00</td>
<td>0.53</td>
</tr>
<tr>
<td>dbeta</td>
<td>0.93</td>
<td>0.07</td>
<td>0.84</td>
<td>0.22</td>
<td>0.02</td>
<td>0.00</td>
<td>0.62</td>
</tr>
</tbody>
</table>

### Weekly

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>UK</th>
<th>Germany</th>
<th>France</th>
<th>Hungary</th>
<th>Poland</th>
<th>Czech Rep.</th>
</tr>
</thead>
<tbody>
<tr>
<td>vol</td>
<td>0.94</td>
<td>0.07</td>
<td>0.83</td>
<td>0.35</td>
<td>0.03</td>
<td>0.01</td>
<td>0.55</td>
</tr>
<tr>
<td>var</td>
<td>0.93</td>
<td>0.01</td>
<td>0.80</td>
<td>0.26</td>
<td>0.01</td>
<td>0.01</td>
<td>0.82</td>
</tr>
<tr>
<td>edr</td>
<td>0.93</td>
<td>0.04</td>
<td>0.80</td>
<td>0.31</td>
<td>0.01</td>
<td>0.02</td>
<td>0.66</td>
</tr>
<tr>
<td>beta</td>
<td>0.87</td>
<td>0.08</td>
<td>0.82</td>
<td>0.28</td>
<td>0.00</td>
<td>0.01</td>
<td>0.30</td>
</tr>
<tr>
<td>svar</td>
<td>0.87</td>
<td>0.04</td>
<td>0.74</td>
<td>0.30</td>
<td>0.00</td>
<td>0.02</td>
<td>0.83</td>
</tr>
<tr>
<td>dbeta</td>
<td>0.91</td>
<td>0.12</td>
<td>0.81</td>
<td>0.36</td>
<td>0.02</td>
<td>0.01</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>US</td>
<td>UK</td>
<td>Germany</td>
<td>France</td>
<td>Hungary</td>
<td>Poland</td>
<td>Czech Rep.</td>
</tr>
<tr>
<td>----------</td>
<td>------</td>
<td>------</td>
<td>---------</td>
<td>--------</td>
<td>---------</td>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td><strong>vol</strong></td>
<td>0.94</td>
<td>0.01</td>
<td>0.81</td>
<td>0.30</td>
<td>0.03</td>
<td>0.01</td>
<td>0.57</td>
</tr>
<tr>
<td><strong>var</strong></td>
<td>0.91</td>
<td>0.02</td>
<td>0.77</td>
<td>0.22</td>
<td>0.01</td>
<td>0.02</td>
<td>0.81</td>
</tr>
<tr>
<td><strong>edr</strong></td>
<td>0.86</td>
<td>0.01</td>
<td>0.73</td>
<td>0.18</td>
<td>0.01</td>
<td>0.04</td>
<td>0.78</td>
</tr>
<tr>
<td><strong>beta</strong></td>
<td>0.77</td>
<td>0.00</td>
<td>0.78</td>
<td>0.27</td>
<td>0.00</td>
<td>0.00</td>
<td>0.40</td>
</tr>
<tr>
<td><strong>svar</strong></td>
<td>0.77</td>
<td>0.01</td>
<td>0.77</td>
<td>0.22</td>
<td>0.00</td>
<td>0.01</td>
<td>0.76</td>
</tr>
<tr>
<td><strong>dbeta</strong></td>
<td>0.90</td>
<td>0.01</td>
<td>0.81</td>
<td>0.31</td>
<td>0.02</td>
<td>0.00</td>
<td>0.65</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>UK</th>
<th>Germany</th>
<th>France</th>
<th>Hungary</th>
<th>Poland</th>
<th>Czech Rep.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>vol</strong></td>
<td>0.93</td>
<td>0.00</td>
<td>0.49</td>
<td>0.01</td>
<td>0.01</td>
<td>0.18</td>
<td>0.79</td>
</tr>
<tr>
<td><strong>var</strong></td>
<td>0.77</td>
<td>0.05</td>
<td>0.38</td>
<td>0.00</td>
<td>0.02</td>
<td>0.14</td>
<td>0.92</td>
</tr>
<tr>
<td><strong>edr</strong></td>
<td>0.13</td>
<td>0.34</td>
<td>0.10</td>
<td>0.13</td>
<td>0.27</td>
<td>0.41</td>
<td>0.97</td>
</tr>
<tr>
<td><strong>beta</strong></td>
<td>0.32</td>
<td>0.06</td>
<td>0.44</td>
<td>0.00</td>
<td>0.00</td>
<td>0.07</td>
<td>0.14</td>
</tr>
<tr>
<td><strong>svar</strong></td>
<td>0.79</td>
<td>0.09</td>
<td>0.41</td>
<td>0.00</td>
<td>0.00</td>
<td>0.14</td>
<td>0.93</td>
</tr>
<tr>
<td><strong>dbeta</strong></td>
<td>0.91</td>
<td>0.00</td>
<td>0.49</td>
<td>0.01</td>
<td>0.01</td>
<td>0.16</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Third, daily, weekly, monthly and yearly R2 values are presented for the case with $\alpha \neq 0$ and local currency.
Fourth, the daily, weekly, monthly and yearly R2 values are presented for the case with $\alpha \neq 0$ and dollar denominated returns.
<table>
<thead>
<tr>
<th></th>
<th>Daily</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>US</td>
<td>UK</td>
<td>Germany</td>
<td>France</td>
<td>Hungary</td>
<td>Poland</td>
<td>Czech Rep.</td>
</tr>
<tr>
<td>vol</td>
<td>0.59</td>
<td>0.61</td>
<td>0.02</td>
<td>0.02</td>
<td>0.00</td>
<td>0.02</td>
<td>0.79</td>
</tr>
<tr>
<td>var</td>
<td>0.63</td>
<td>0.63</td>
<td>0.02</td>
<td>0.03</td>
<td>0.00</td>
<td>0.02</td>
<td>0.89</td>
</tr>
<tr>
<td>edr</td>
<td>0.42</td>
<td>0.78</td>
<td>0.00</td>
<td>0.09</td>
<td>0.10</td>
<td>0.00</td>
<td>0.87</td>
</tr>
<tr>
<td>beta</td>
<td>0.00</td>
<td>0.12</td>
<td>0.00</td>
<td>0.09</td>
<td>0.30</td>
<td>0.03</td>
<td>0.16</td>
</tr>
<tr>
<td>svar</td>
<td>0.58</td>
<td>0.07</td>
<td>0.06</td>
<td>0.09</td>
<td>0.02</td>
<td>0.01</td>
<td>0.44</td>
</tr>
<tr>
<td>dbeta</td>
<td>0.52</td>
<td>0.10</td>
<td>0.08</td>
<td>0.07</td>
<td>0.14</td>
<td>0.01</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>Weekly</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>US</td>
<td>UK</td>
<td>Germany</td>
<td>France</td>
<td>Hungary</td>
<td>Poland</td>
<td>Czech Rep.</td>
</tr>
<tr>
<td>vol</td>
<td>0.62</td>
<td>0.67</td>
<td>0.00</td>
<td>0.03</td>
<td>0.14</td>
<td>0.01</td>
<td>0.73</td>
</tr>
<tr>
<td>var</td>
<td>0.69</td>
<td>0.69</td>
<td>0.00</td>
<td>0.03</td>
<td>0.12</td>
<td>0.00</td>
<td>0.85</td>
</tr>
<tr>
<td>edr</td>
<td>0.53</td>
<td>0.70</td>
<td>0.10</td>
<td>0.13</td>
<td>0.58</td>
<td>0.02</td>
<td>0.88</td>
</tr>
<tr>
<td>beta</td>
<td>0.17</td>
<td>0.43</td>
<td>0.00</td>
<td>0.12</td>
<td>0.33</td>
<td>0.01</td>
<td>0.20</td>
</tr>
<tr>
<td>svar</td>
<td>0.51</td>
<td>0.24</td>
<td>0.00</td>
<td>0.00</td>
<td>0.09</td>
<td>0.01</td>
<td>0.80</td>
</tr>
<tr>
<td>dbeta</td>
<td>0.45</td>
<td>0.74</td>
<td>0.04</td>
<td>0.00</td>
<td>0.32</td>
<td>0.02</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>Monthly</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>US</td>
<td>UK</td>
<td>Germany</td>
<td>France</td>
<td>Hungary</td>
<td>Poland</td>
<td>Czech Rep.</td>
</tr>
<tr>
<td>vol</td>
<td>0.60</td>
<td>0.66</td>
<td>0.00</td>
<td>0.06</td>
<td>0.12</td>
<td>0.00</td>
<td>0.70</td>
</tr>
<tr>
<td>var</td>
<td>0.63</td>
<td>0.66</td>
<td>0.01</td>
<td>0.06</td>
<td>0.11</td>
<td>0.01</td>
<td>0.82</td>
</tr>
<tr>
<td>edr</td>
<td>0.09</td>
<td>0.87</td>
<td>0.12</td>
<td>0.44</td>
<td>0.87</td>
<td>0.11</td>
<td>0.91</td>
</tr>
<tr>
<td>beta</td>
<td>0.02</td>
<td>0.66</td>
<td>0.01</td>
<td>0.08</td>
<td>0.27</td>
<td>0.05</td>
<td>0.32</td>
</tr>
<tr>
<td>svar</td>
<td>0.34</td>
<td>0.66</td>
<td>0.01</td>
<td>0.01</td>
<td>0.18</td>
<td>0.00</td>
<td>0.69</td>
</tr>
<tr>
<td>dbeta</td>
<td>0.39</td>
<td>0.64</td>
<td>0.00</td>
<td>0.02</td>
<td>0.18</td>
<td>0.00</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>Yearly</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>US</td>
<td>UK</td>
<td>Germany</td>
<td>France</td>
<td>Hungary</td>
<td>Poland</td>
<td>Czech Rep.</td>
</tr>
<tr>
<td>vol</td>
<td>0.45</td>
<td>0.78</td>
<td>0.13</td>
<td>0.05</td>
<td>0.00</td>
<td>0.00</td>
<td>0.84</td>
</tr>
<tr>
<td>var</td>
<td>0.44</td>
<td>0.81</td>
<td>0.12</td>
<td>0.06</td>
<td>0.01</td>
<td>0.01</td>
<td>0.89</td>
</tr>
<tr>
<td>edr</td>
<td>0.06</td>
<td>0.98</td>
<td>0.86</td>
<td>0.60</td>
<td>0.69</td>
<td>0.25</td>
<td>0.98</td>
</tr>
<tr>
<td>beta</td>
<td>0.16</td>
<td>0.79</td>
<td>0.09</td>
<td>0.11</td>
<td>0.01</td>
<td>0.09</td>
<td>0.00</td>
</tr>
<tr>
<td>svar</td>
<td>0.42</td>
<td>0.77</td>
<td>0.07</td>
<td>0.04</td>
<td>0.00</td>
<td>0.01</td>
<td>0.90</td>
</tr>
<tr>
<td>dbeta</td>
<td>0.37</td>
<td>0.73</td>
<td>0.10</td>
<td>0.03</td>
<td>0.00</td>
<td>0.01</td>
<td>0.84</td>
</tr>
</tbody>
</table>