Mast Vibration Reduction of Single-mast Stacker Cranes via Modern Control Methods

Overview of Ph.D. Thesis by

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1 Introduction

Since the seventies of last century the warehousing technology and machinery have had significant improvement. Beside the manually powered material handling solutions and manually controlled equipments the completely mechanized and highly automated material handling systems also have appeared. One of the typical developmental areas was the appearing of high bay warehouses with automated storage/retrieval system. These kinds of warehouses have spread rapidly due to the high price of building plots and growing amount of materials to store since they are capable of storing a large amount of goods in a small area. The essential elements in automated storage/retrieval systems (AS/RS) of high bay warehouses are the stacker cranes (a.k.a. storage and retrieval machines) which perform directly the storage/retrieval operation into/from rack position. A detailed summary of high bay warehousing system constructions, its technological subsystems and material handling machinery can be found in the books [1] and [2]. The classification of stacker cranes, its typical configurations and structural elements are introduced by the introductory sections of theses [3] and [4]. The basic definitions and terminology of these machines are summarized in the standards [5], [6]. Further technical details about the construction of advanced stacker cranes, its components and technological parameters can be found in the following brochures: [7], [8], [9], [10].

In our days the height of advanced, self-supporting high bay warehouses can reach even the $40 - 50\,\mathrm{m}$. In these high bay warehouses completely automated stacker cranes operate which enables the fast handling of a large amount of materials. The reaching of short material handling cycle-time requires high material handling capacity. Therefore the main features of applied stacker cranes are the growing structural heights and in the same time the higher operation velocities and accelerations. Another strict requirement of stacker cranes is the reliable, economical operation which forces the designers of these machines to reduce the dead-weight and to improve the payload/gross weight ratio.

An ideal stacker crane therefore has high hoisting height and capacity, fast operating velocities and low dead-weight. However, the reducing of dead-weight may cause the decreasing of cross-sectional dimensions of structural members, therefore the stiffness of frame structure is reduced. In the same time the inertial forces acting on the frame structure of stacker cranes have grown due to the increased accelerations. The high and less stiff structures are more responsive to dynamical loads. During operation undesirable vibrations, low frequency and high amplitude mast sways may occur because of the different inertial forces. The high amplitude mast vibrations may reduce the stability and positioning accuracy of the stacker crane and in an extreme case they may damage the structure. Another harmful consequence of mast vibrations is the increasing cycle-time of material handling since before the beginning of pick-up and deposit cycles with the load handling unit the settling of mast vibrations must be waited.

The aim of this thesis is the investigation of above mentioned vibrations and the analysis of its reduction methods. Practically the mast structure has two fundamental configurations: the so-called single-mast and twin-mast structures. In this work the single-mast structure (see in Figure 1) is analyzed since this configuration is more responsive to dynamic excitations and considerable mast vibrations may arise in the frame structure of these machines. In this thesis the vibrations in direction of traveling motion (i.e. the motion towards the aisle of warehouse) are investigated since they are substantially greater than the vibrations of other directions.

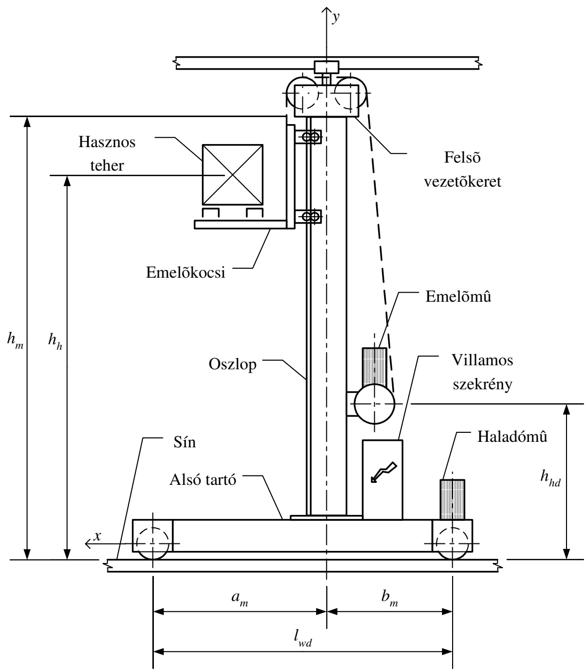


Figure 1: Single-mast stacker crane

The reduction of these undesirable vibrations as well as improving the positioning accuracy of stacker cranes is achieved via applying several methods of control theory. Motion control of stacker cranes as well as estimation of structural vibrations during design period of stacker cranes or dynamic investigation of an existing structure requires a dynamic model of the flexible structure. This model must be accurate enough to represent the dynamic behavior of an actual structure and at the same time quite simple to fulfill the requirements of controller synthesis techniques. These requirements can be satisfied by e.g. the distributed parameter continuum models (Euler-Bernoulli beam models, see e.g. [11, 12]), the various finite element models [13, 14] as well as the so-called multi-body models [15, 16] which consist of rigid bodies, elastic- and damping elements. With the comparison of these kinds of models the selection of most appropriate dynamic model can be performed. However, the dynamical properties, e.g. resonance frequencies, mode-shapes etc. of stacker cranes depend on the magnitude and position of lifted load which must taken into account during generating dynamic models. A simple linear time invariant model probably can not handle this parameter dependency in the whole load position and magnitude range. The applied dynamic modeling method must take also the above mentioned property of structure into consideration.

For applying the methods of control theory and designing several controllers the control oriented modeling of single-mast stacker cranes is also required. This kind of modeling approach needs completely different considerations than the modeling techniques mentioned in the previous paragraph (which are more suitable for dynamic analyses). The advanced methods for control design can be applied only with relatively low order dynamic models because of the complex numerical methods used in control design. During the control oriented dynamic modeling therefore a sort of model order reduction procedure may be used. The so-called performances of control system design methods must be defined also in this modeling step. The performance criteria are defined to evaluate the designed control systems. According to the objectives of this thesis two main performance criteria should be taken into account, they are the good reference signal tracking and the mast vibration attenuation. Further performance criteria may also be defined for the control design, for example they are the sensor noise attenuation or the limitation of control input in order to avoid large control signals and undesired hysteresis effects in the actuator system. In the control oriented modeling and control design the above mentioned lifted load-parameter dependance of dynamical properties also must be taken into account. In order to do this two kinds of modeling approaches are exist. In the first case the effects of varying lifted load magnitudes and positions are taken into consideration as an unstructured uncertainty model, see [17]. The second modeling approach is based on that the actual magnitude and position of lifted load can be measured online, therefore, a so-called LPV model can be defined [18, 19]. With this approach the dynamic models can be parametrized by the varying lifted load magnitude or position. Using this LPV model the dynamic properties of single-mast stacker cranes can be described in a more accurate way.

The main objective of this thesis is to apply and analyze the recent results of modern control researches in order to obtain the appointed performance criteria. One of the possible investigation directions is applying robust control systems [17, 20]. In the robust control design methods the performance criteria is defined by the help of special weighting functions. For the control design purposes the dynamic model of system that we would like to control is extended with the above mentioned weighting functions, therefore the so-called augmented model (or generalized plant) is defined. According to the performance criteria special input- and output signals (the so-called disturbance input w and performance or controlled output z) can be defined for the generalized plant. The aim of robust $\mathcal{H}_\infty$ control design (see e.g. [21]) is to minimize the $\mathcal{H}_\infty$ norm (i.e. gain) of the $w \to z$ transfer function of closed-loop system. With the varying of parameters in the weighting functions a good trade-off can be achieved between conflicting performance criteria, as well as the so-called robust stability of closed-loop system and the performance robustness can be guaranteed.

Another possible investigation opportunity is analyzing the so-called gain scheduled controllers, the main objective of these controllers is to control nonlinear systems. The design of classical gain scheduled controllers is based on the local linearization of a nonlinear model in a set of equilibrium points, thus the control design methods for linear systems further remain applicable. The nonlinear controller from the set of locally designed linear controllers can be formulated by either interpolating the linear controllers or by switching them. In the case of the investigated stacker crane model the above-mentioned linearization is unnecessary, since the local models that are valid for "frozen" load conditions can be generated by means of the presented linear modeling methods. Unfortunately, with the classical gain scheduling method the performances and robustness of closed-loop system is guaranteed only in the local points of parameter space. However, using the LPV modeling based gain scheduling approach (i.e. LPV controllers, see [22, 23, 24]) gives performance and robustness guarantee to the closed-loop system for the whole admissible parameter trajectories. In order to reduce the mast vibrations of stacker crane in this thesis a polytopic LPV modeling based gain scheduled controller is applied [22], which minimizes the induced $\mathcal{L}_2$ norm of closed-loop system. This method is an extension of the $\mathcal{H}_\infty$ controller synthesis method of linear systems to the LPV systems. Applying the presented control methods the problems related to the varying lifted load magnitude or position may be solved.

2 Contributions of the Thesis

Thesis 1.

I performed the high fidelity simulation oriented dynamic modeling of single-mast stacker crane. Within this I generated three kinds of dynamic models: distributed parameter (continuous) beam model, finite element model, linear multi-body model. By means of the comparison of frequency domain properties (i.e. frequency response functions) of models the validated dynamic models for the further analysis and synthesis problems are available.

Related publications: [Haj08], [HT09], [HK10], [HK11], [HG13a], [HG14a], [HG14b], [HG16a], [KH04a], [KH04b], [KH05], [KH09], [KH10], [KH11], [K+11]

- During the investigation of continuous beam model I applied the method of transfer matrices to the branching frame structure of single-mast stacker crane. By means of this method I determined the natural frequencies of structure, the parameters of mode shape functions as well as the frequency response function $G_c(\mathrm{i}\omega)$ between the tractive force input signal and the mast-tip movement output signal.

- For the investigation of mast vibrations in the direction of traveling motion I recommended a planar linear multi-body model. This model consists of rigid elements with lumped masses and elastic links between elements, the elasticity of links is provided by spiral springs. Within the multi-body modeling I derived the relationship between the parameters of multi-body model (element lengths, lumped masses and spring stiffness values) and the material- and cross-sectional properties of continuous beam.

- The selection and validation of dynamic model applied in the further analysis and synthesis problems is carried out by means of the comparison of frequency domain properties of the continuous beam model, a simple planar finite element model with line elements as well as the presented multi-body model. In order to this first I generated the frequency response functions for the finite dimensional (finite element and multi-body) models. They are denoted by $G_f(\mathrm{i}\omega)$ and $G_m(\mathrm{i}\omega)$ respectively. After that I generated the following absolute- and relative error functions of dynamic models:

$$\Delta_{af}(\omega) = \left|G_c(\mathrm{i}\omega) - G_f(\mathrm{i}\omega)\right|, \quad \Delta_{am}(\omega) = \left|G_c(\mathrm{i}\omega) - G_m(\mathrm{i}\omega)\right|,$$
$$\Delta_{rf}(\omega) = \frac{\left|G_c(\mathrm{i}\omega) - G_f(\mathrm{i}\omega)\right|}{\left|G_c(\mathrm{i}\omega)\right|}, \quad \Delta_{rm}(\omega) = \frac{\left|G_c(\mathrm{i}\omega) - G_m(\mathrm{i}\omega)\right|}{\left|G_c(\mathrm{i}\omega)\right|}.$$

Analyzing the above-mentioned functions it can be concluded that the investigated models are sufficiently accurate, the absolute and relative errors are quite low in the relevant frequency range. Either of the investigated models can be applied for the solution of further problems, however, due to its simplicity and relative low degrees of freedom the multi-body model is recommended.

Thesis 2.

I developed the control oriented modeling method of single-mast stacker crane, within this modeling approach I extended the dynamic model by performances and uncertainty models. I also recommended methods for taking the effects of varying lifted load magnitude and position into consideration in the dynamic modeling. The resulting control oriented models are suitable for the realization and investigation of several control methods.

Related publications: [HG13b], [HG14c], [HG16a]

- In the control oriented modeling the main performance criteria that I recommended are the good reference signal tracking and the proper mast vibration attenuation. Moreover by the attenuation of several sensor noises and the limitation of control input signals the properties of closed-loop system can be further improved.

- The properties of dynamic model depend on the magnitude and position of lifted load, in order to take this effect into account I recommended two kinds of methods. In the first method I took the effects of varying lifted load magnitude and position into consideration as an unstructured uncertainty model. For this I applied an output-multiplicative uncertainty model, I determined the weighting function W_m of this model by means of a model set generated with varying loading conditions.

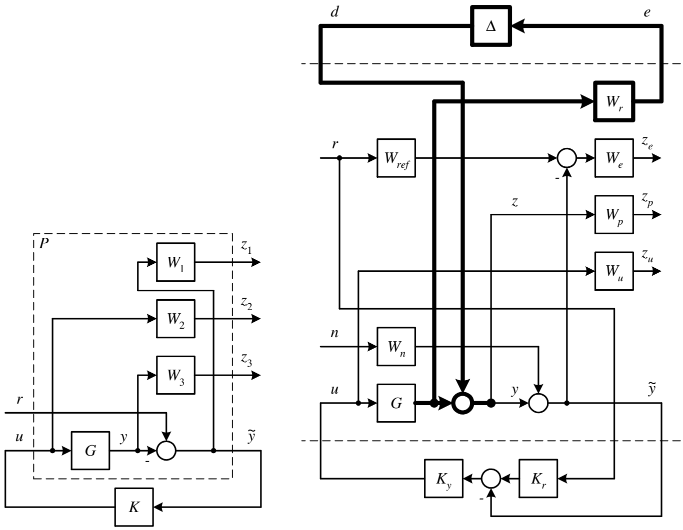
- In the second method I constructed a polytopic LPV model which is based on the measurement of lifted load position. The vertices of polytope defined in parameter space are the endpoints of allowed lifted load position range.

- For the successful controller synthesis I also performed the model order reduction of dynamic models in both cases which is based on the modal truncation method of LTI systems. In the case of LPV model a further linear transformation is required in order to recover the consistency of state variables between local dynamic models.

Thesis 3.

I developed a robust $\mathcal{H}_\infty$ control method for the linear time invariant model of single-mast stacker crane which aim is to ensure the good reference signal tracking and the proper mast vibration attenuation while the model uncertainties are taken into consideration. Within this I recommended the selection of system and uncertainty structures according to the $\mathcal{H}_\infty$ design paradigm, as well as the selection of uncertainty and performance weighting function parameters.

Related publications: [HG13b], [HG14c]



(a) Loop shaping (b) Robust control

Figure 2: Generalized plants

- In the first step of control design I applied a standard $\mathcal{H}_\infty$ control method (i.e. the mixed-sensitivity loop shaping) which is based on the nominal model of stacker crane, see Figure 2 (a). The aim of this method is to adjust the control bandwidth via weighting the sensitivity and complementary sensitivity functions of closed-loop system by weighting functions W_1 and W_3 respectively. With this the performances of closed-loop system can be analyzed. From this analysis it can be concluded that the magnitude of mast vibrations is inversely proportional to the control errors.

- The second step of control design is the actual robust $\mathcal{H}_\infty$ control synthesis. In this method the model uncertainties are taken into account by means of the weighting function W_r, see Figure 2 (b). In the generalized plant applied for control design the function W_{ref} represents the ideal behavior of closed-loop system. The error between ideal and actual closed-loop transfer function is weighted by the weighting function W_e. The mast vibrations are penalized by the W_p weighting function. The control input is limited by using the performance weighting function W_u, as well as the purpose of the weighting function W_n is to reflect the sensor noises.

- The trade-off between the cycle time of stacker crane movement (i.e. signal tracking) and the mast vibration attenuation can be achieved by means of variation the parameters of weighting functions W_e and W_p. In this analysis I adjusted the bandwidth and gain of these functions so that the weighting strategy has changed from the cycle time focusing cases to the vibration attenuation focusing cases. The results are summarized in Figure 3.

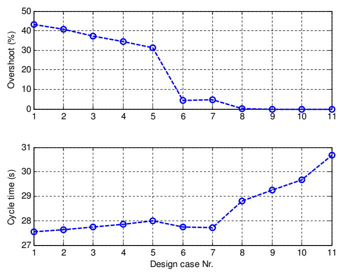
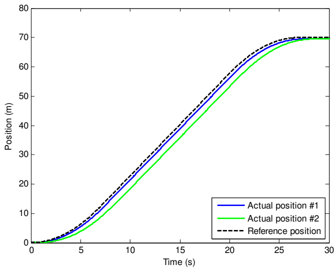


Figure 3: Overshoot of mast vibration signal and cycle times

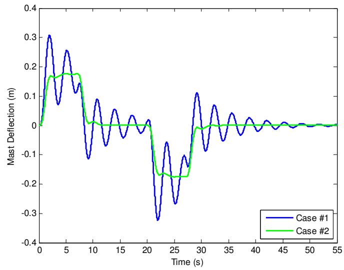
- I verified the results of robust control design via time domain analyses, dynamic simulations, see Figure 4.

Thesis 4.

I developed a robust LPV control design method for single-mast stacker cranes. By means of this method the performances defined for the reference signal tracking and mast vibration attenuation can be guaranteed in a wide range of lifted load position. By means of time domain investigations I proved that the controlled system has accurate tracking behavior and in the same time good mast vibration suppression properties. Comparing the results of time domain investigations to the results of $\mathcal{H}_\infty$ *control I proved that the favorable performances are valid for a wide parameter domain.*



(a) Position



(b) Mast vibration

Figure 4: Simulation results of robust control

Related publications: [HG14c], [HG16b]

- In the first control design step I generated a classical gain scheduled controller by means of local LTI models that are valid in the fixed points of lifted load position range. I formulated the gain scheduled controller from the set of locally designed linear controllers by means of switching between the local controllers in the border points of its corresponding load position region. With this control design method the results can be extended to a wide range of lifted load position.

- In the next step I designed an LPV controller which is based on the polytopic LPV model. With this method the stability and performances of closed-loop system can be guaranteed for all admissible parameter variations. During the controller design I applied a simplified augmented plant (see Figure 5) which makes the calculations of design method easier as well as helps to reduce the conservatism of resulted controller.

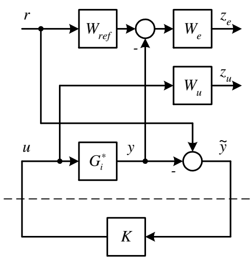
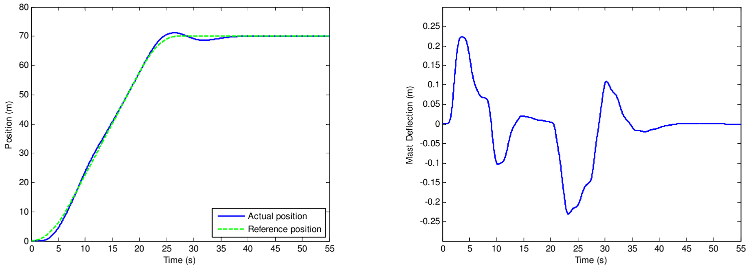


Figure 5: Generalized plant for LPV control

- I verified the results of gain scheduled control design via time domain analyses, dynamic simulations, see Figure 6. Analyzing the simulation results it can be concluded that the controlled system show an accurate tracking behavior and sufficiently suppressed mast vibrations.



(a) Position (b) Mast vibration

Figure 6: Simulation results of LPV control

3 Practical Aspects

The new scientific results of the thesis can be directly applied as the elements of control systems of stacker cranes in AS/RS systems. By means of these results the throughput capacity of automated storage/retrieval systems as well as the utilization of material handling machines can be increased. With the attenuation of harmful vibrations the additional loads of frame structures can be reduced, the operational reliability of these machines can be improved. Due to the reduced vibrations the dynamic loads of stacker cranes are also reduced, therefore, the dynamic coefficient used in strength calculations can be decreased. Eventually a stacker crane with reduced dead-weight can be designed and the operation of this machine can be more economical.

The results of dynamic modeling can also be applied for the dynamic investigations of existing structures, for exploring the possible dynamic problems as well as during the design of new stacker cranes in order to predict the dynamic behavior of designed structure. The introduced control methods can be applied again in such cases when fast and vibrationless position control of large flexible structures is required. For example the control of container cranes or tower cranes.

4 Future research

Further research directions are given in the sequel.

- The dynamic models introduced in the thesis can be further refined by means of taking the possible nonlinearity effects (e.g. nonlinear friction models, saturation, hysteresis etc.) as well as external disturbance signals into consideration. The design and analysis of nonlinear control based on this refined model can be a useful research area.

- The dynamic modeling of actuator systems and the investigation of actuator dynamics are also suggested to be considered in the further phase of research. These actuators are usually AC servo drives which provide for the movement of stacker crane machinery. Since these elements are part of the control loop they may have an important role during the controller implementation.

- The results of this thesis can be extended to other kinds of stacker crane structures. In practice stacker cranes with more complicated frame structures (e.g. twin-mast structures or truss structures) can be found as well. Moreover, the so-called mini-load systems for the high speed handling of unit loads (e.g. boxes, small containers) for small and light parts are also frequent. Investigation of the applicability of proposed methods for the above-mentioned machines can be a challenging research area.

Publications of the author related to the thesis

[Haj08] Sándor Hajdu. Mast vibration damping of shelf storage and retrieval machines. In *14th Building Services, Mechanical and Building Industry Days, Debrecen*, pages 235–240, 2008.

[HG13a] Sándor Hajdu and Péter Gáspár. Egyoszlopos magasraktári felrakógépek többtestmodelljei. In *Műszaki Tudomány az Észak-kelet Magyarországi Régióban Konferencia*, pages 509–518, Debrecen, 2013.

[HG13b] Sándor Hajdu and Péter Gáspár. Investigation of the influence of lifted load on dynamical behavior of stacker cranes through unstructured uncertainties. In *CINTI 2013 : Proceeding of the 14th IEEE International Symposium on Computational Intelligence and Informatics, Budapest: IEEE Hungary Section*, pages 179–184, 2013.

[HG14a] Sándor Hajdu and Péter Gáspár. Distributed parameter modeling of single-mast stacker crane structures. *PERIODICA POLYTECHNICA: TRANSPORTATION ENGINEERING*, 42(1):1–9, 2014.

[HG14b] Sándor Hajdu and Péter Gáspár. Finite element modeling of single-mast stacker cranes. *DEBRECENI MŰSZAKI KÖZLEMÉNYEK*, 13(1):55–70, 2014.

[HG14c] Sándor Hajdu and Péter Gáspár. From modeling to robust control design of single-mast stacker cranes. *Acta Polytechnica Hungarica*, 11(10):135–149, 2014.

[HG16a] Sándor Hajdu and Péter Gáspár. Multi-body modeling of single-mast stacker cranes. *International Journal of Engineering Systems Modelling and Simulation*, 8(3):218–226, 2016.

[HG16b] Sándor Hajdu and Péter Gáspár. Reducing the mast vibration of single-mast stacker cranes by gain-scheduled control. *International Journal of Applied Mathematics and Computer Science*, 2016. (in press).

[HK10] Sándor Hajdu and Béla Kulcsár. Dynamical analysis of storage and retrieval machine frames. In *16th Building Services, Mechanical and Building Industry Days, Debrecen*, pages 71–76, 2010.

[HK11] Sándor Hajdu and Béla Kulcsár. Dynamical analysis of storage and retrieval machine frames. *INTERNATIONAL REVIEW OF APPLIED SCIENCES AND ENGINEERING*, 2(1):51–55, 2011.

[HT09] Sándor Hajdu and Zsolt Tiba. Introduction to dynamic modeling of rack stackers for high-bay warehouses. *ZESZYTY NAUKOWE POLITECHNIKI POZNANSKIEJ*, 1(10):15–23, 2009.

[K+11] Béla Kulcsár et al. Egyoszlopos magasraktári felrakógépek vázszerkezetének igénybevételei. In *XVI. FMTÜ Nemzetközi Tudományos Konferencia = International Scientific Conference, Kolozsvár*, pages 169–172, 2011.

[KH04a] Béla Kulcsár and Sándor Hajdu. Dynamic modeling of transversal mast vibration of storage and retrieval machines. In *microCAD 2004, L Section: International Scientific Conference, Miskolc*, pages 71–76, 2004.

[KH04b] Béla Kulcsár and Sándor Hajdu. Magasraktái felrakógépek oszloplengéseinek dinamikai modellezése. *GÉP*, 55(12):14–20, 2004.

[KH05] Béla Kulcsár and Sándor Hajdu. Vibration damping effects and methods in the structure of shelf storage and retrieval machines. In *microCAD 2005, O Section: International Scientific Conference, Miskolc*, pages 67–72, 2005.

[KH09] Béla Kulcsár and Sándor Hajdu. Magasraktári felrakógépek oszloplengéseit csillapító hatások. In *Fiatal Műszakiak Tudományos Ülésszaka XIV.: Nemzetközi Tudományos Konferencia, Kolozsvár*, pages 111–114, 2009.

[KH10] Béla Kulcsár and Sándor Hajdu. Magasraktári felrakógépek vázszerkezetének dinamikai vizsgálata. In *OGÉT 2010 - XVIII. Nemzetközi Gépészeti Találkozó, Nagybánya*, pages 265–268, 2010.

[KH11] Béla Kulcsár and Sándor Hajdu. Dynamical modelling possibilities of storage and retrieval machine structures. In *microCAD 2011, N Section: XXV. International Scientific Conference, Miskolc*, pages 87–92, 2011.

References

[1] László Felföldi. *Anyagmozgatási kézikönyv.* Műszaki Könyvkiadó, 1975.

[2] József Prezenszki, editor. *Logisztika I. (Bevezető fejezetek).* Budapesti Műszaki Egyetem Mérnöktovábbképző Intézet, 1998.

[3] Werner Bopp. Untersuchung der statischen und dynamischen Positionsgenauigkeit von Einmast-Regalbediengeräten. Dissertation, Institut für Fördertechnik Karlsruhe, 1993.

[4] Roland Aßmann. Untersuchungen zur Feinpositionierung von Regalbediengeräten. Dissertation, Institut für Fördertechnik Karlsruhe, 1997.

[5] FEM 9.001. *Terminology; Dictionary, storage and retrieval machines.*

[6] FEM 9.101. *Terminology; Storage and retrieval machines – definitions.*

[7] Mecalux. *Stacker Cranes for Pallets.*
URL: http://img.mecalux.com/external/catalogue/en-UN/pdf.

[8] MLOG Logistics GmbH. *KardexMlog ImageBrochure.*
URL: http://www.kardex-mlog.com/en/products-mlog/storage-and-retrieval-machines.html.

[9] SSI SCHÄFER. *Die Familie der Regalbediengeräte.*
URL: http://media.ssi-schaefer.de.

[10] Viastore systems. *Storage/Retrieval machines.*
URL: http://www.viastore.com/downloads.

[11] Saeid Bashash, Amin Salehi-Khojin, and Nader Jalili. Forced vibration analysis of flexible Euler-Bernoulli beams with geometrical discontinuities. In *2008 American Control Conference Westin Seattle Hotel, Seattle, WA, USA, June 11-13*, 2008.

[12] Győző Ludvig. *Gépek dinamikája.* Műszaki Könyvkiadó, 1983.

[13] David V. Hutton. *Fundamentals of finite element analysis.* McGraw-Hill, 2004.

[14] István Páczelt. *A végeselem-módszer alapjai.* Miskolci Egyetemi Kiadó, 1993.

[15] Jorge Angeles and Andrés Kecskeméthy. *Kinematics and Dynamics of Multi-body Systems.* Springer-Verlag, 1995.

[16] Javier García de Jalón and Eduardo Bayo. *Kinematic and Dynamic Simulation of Multibody Systems – The Real-Time Challenge*. Springer-Verlag, 1994.

[17] Kemin Zhou, John C. Doyle, and Keith Glover. *Robust and Optimal Control*. Prentice Hall, New Jersey, 1996.

[18] Jeff S. Shamma. Analysis and design of gain scheduled control systems. Ph.D. thesis, M.I.T. Laboratory for Information and Decision Systems, Cambridge, MA, USA, 1988.

[19] Jeff S. Shamma and Michael Athans. Analysis of gain scheduled control for nonlinear plants. *IEEE Transactions on Automatic Control*, 35(8):898–907, 1990.

[20] Kemin Zhou and John C. Doyle. *Essentials of Robust Control*. Prentice Hall, New Jersey, 1998.

[21] John C. Doyle et al. State-space solutions to standard $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control problems. *IEEE Transactions on Automatic Control*, 34(8):831–847, 1989.

[22] Pierre Apkarian, Pascal Gahinet, and Greg Becker. Self-scheduled $\mathcal{H}_\infty$ control of linear parameter-varying systems: a design example. *Automatica*, 31(9):1251–1261, 1995.

[23] A. Packard and G. Balas. Theory and application of linear parameter varying control techniques. In *American Control Conference, Workshop I, Albuquerque, New Mexico*, 1997.

[24] J. Bokor and G. Balas. Linear parameter varying systems: A geometric theory and applications. In *16th IFAC World Congress, Prague*, 2005.