Instability phenomena of pressure relief valves

Booklet of the PhD Dissertation

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1 Introduction

This work focuses on the dynamics of pressure relief valves (PRVs) to improve the design techniques and rules by exploring and understanding their dynamic behaviour. Pressure relief valves are safety elements of pressurised systems, such as natural gas systems, chemical plants (see Figure 1 left configuration), or hydraulic power transmission circuits (see Figure 1 right configuration). They prevent the pressure level from increasing above a prescribed set pressure by releasing an excess of fluid out of the system.

The simplest and most robust type of PRVs are direct spring-loaded valves, which, due to their simplicity and reliability, are often employed in safety applications. In these pieces of equipment a single helical spring is used in order to press the poppet to the seat, thus providing a prescribed set pressure.

As Figure 1 demonstrates, PRVs are mounted to the protected system either directly or more commonly due to the practical installation or operational considerations via an upstream pipeline. Being a spring-mass system, the valve itself has its own dynamics, however, the presence of the reservoir and the connecting pipeline might lead to a risky combination of the valve body dynamics and the pipeline dynamics. Due to the complex fluid-structure interaction, such systems have a tendency to 'self-oscillate' and such dangerous oscillations are challenging to predict in the design phase.

**Figure 1:** Pressure relief valve installation examples. Left: storage unit of process technology. Right: hydraulic power transmission circuit.
2 Summary of the results

2.1 Force balance of the valve body

Investigations that aimed to deeply understand the valve body dynamics highlighted that the governing equations of its motion derived by former researchers (e.g., MacLeod (1985); Hayashi (1995); Licskó et al. (2009); Hős and Champneys (2011)) do not serve adequate accuracy when describing the poppet motion. This fact motivated a more proper derivation of the force balance on the poppet even for unsteady case. It was shown that the steady and unsteady fluid force acting on the valve body can be described by $Re_{s,s}$ Reynolds number where the characteristic velocity is defined with the difference of the fluid velocity in the upstream-side of the valve and the valve body velocity. The unsteady fluid force, moreover, is proportional to a drag coefficient. I provided formulae for the steady and unsteady drag coefficient. Considering the obtained unsteady fluid force in the governing equation of the valve body dynamics I could reproduce the experimentally observed dynamics.

Thesis # 1

Based on experiments I determined the unsteady fluid force $F_{fl}$ acting on the oscillating poppet valve body of a pressure relief valve in incompressible working medium. The fluid force was found in the following form

$$F_{fl} = A_s(p_v - p_0) + \rho Q^2 \left( \frac{1}{A_s} - \frac{\cos \phi}{A_g(x_v)} \right) + C_{dr,s,s}(Re_{s,s}) \frac{\rho}{2} A_s |v_{rel}| v_{rel}$$

where

$$A_s = D_s^2 \pi / 4, \quad A_g = D_s \pi \sin \phi x_v,$$

while

$$Re_{s,s} = \frac{v_{rel} D_s}{\nu}, \quad \text{and} \quad v_{rel} = v_{fl,s} - v_v.$$

$D_s$ [m] and $A_s$ [m$^2$] are the diameter and cross-section of the upstream-side passage while $A_g$ [m$^2$] is the flow-through area at the valve. $p_v$ [Pa] and $p_0$ [Pa] are the pressure in the upstream- and downstream-side of the valve, respectively. $\rho$ [kg/m$^3$] and $\nu$ [m$^2$/s] are the density and the kinematic viscosity of the working medium. $Q$ [m$^3$/s] is the flow rate through the valve, $\phi$ [-] is the
half-cone angle, $x_v [m]$ is the valve displacement, $C_{dr,s,s} [-]$ is the unsteady drag coefficient, $Re_{s,s} [-]$ is the instantaneous Reynolds number, $v_{rel} [m/s]$ is the relative velocity of the fluid in the upstream passage and the valve body, $v_{fl,s} [m/s]$ is the fluid velocity in the upstream passage, and $v_v [m/s]$ is valve body velocity.

The unsteady drag coefficient $C_{dr,s,s}$ can be described with

$$C_{dr,s,s} = \begin{cases} 3.63 \cdot 10^{13} \times Re_{s,s}^{-3.63} & \text{if } a_r > 0 \\ 6.79 \cdot 10^{13} \times Re_{s,s}^{-3.71} & \text{if } a_r < 0 \end{cases}$$

with 92% (the maximum 130%) relative standard deviation in the case of $a_r > 0$ and 140% (the maximum 500%) relative standard deviation in the case of $a_r < 0$ where $a_r [m/s^2] = \dot{v}_{rel}$ denotes the time rate of change of the relative velocity. The half-cone angle of the test valve was $\phi = 30^\circ$ while the diameter ratio was $D_s/D_c = 15/32$ being $D_c$ the chamber diameter of the valve. The expressions are valid for $x_v/D_s = 0 \ldots 0.06$ displacement, $Re_{s,s} = 650 \ldots 2400$ Reynolds number, and $St = 1.7 \ldots 7.1$ Strouhal number range. The Strouhal number is defined as

$$St = \frac{f D_s}{v_{fl,s}}$$

where $f$ is the frequency of the valve body oscillation.

The above unsteady fluid force contributes to the estimation of the force resulting from the oscillating motion of the valve body.


2.2 Static stability of poppet valve equilibrium

In several cases where rapid opening is a requirement a deflecting flange is designed to the valve body to utilise the momentum of the high velocity jet that develops for small openings leading to the rapid opening of the valve. However, a sudden change in the flow pattern may occur not only at the very opening but upon varying the lift, giving rise to discontinuities in the static characteristics and leading to unreliable performance of the poppet valve as well.

Computational fluid dynamics simulations were performed to gain static characteristics of poppet valves. The investigation of the influence of the force coefficient $C_f$ on the valve performance revealed that the effective spring stiffness may become negative leading to static stability loss of the valve. Based
on linear stability analysis, the necessary condition for stable operation was determined. Moreover, it was shown that the condition is a function of two separate effects, the hydrodynamic spring stiffness and the total compression of the spring. It was concluded that the stability can be obtained either by providing \( S_f \leq 0 \) or holding the total compression of the valve sufficiently low and selecting a spring with higher stiffness for a given purpose.

**Thesis # 2**

I provided a mathematically proved explanation for the static instability of direct spring-loaded pressure relief valves. I demonstrated that the equilibrium of the valve is statically stable, if

\[
S_f (X_e + X_0) < 1,
\]

where

\[
S_f = \frac{\partial x_v C_f(x_{v,e})}{C_f(x_{v,e})} D_s \quad \text{and} \quad C_f(x_{v,e}) = \frac{F_{fl,e}}{(p_{v,e} - p_0) A_s},
\]

while

\[
X_e = \frac{x_{v,e}}{D_s}, \quad X_0 = \frac{x_0}{D_s}.
\]

\( x_{v,e} \) [m] and \( x_0 \) [m] are the valve body displacement at the equilibrium and the spring pre-compression while \( X_e \) [-] and \( X_0 \) [-] are their dimensionless delegates, respectively. \( S_f \) [-] indicates the dimensionless hydrodynamic spring stiffness and \( C_f \) [-] stands for the force coefficient. \( F_{fl,e} \) [N] and \( p_{v,e} \) [Pa] are the force exerted by the fluid acting on the valve body and the pressure in the upstream-side of the valve specified at the equilibrium of the valve body, respectively. \( p_0 \) [Pa] is the pressure in the downstream-side of the valve, \( D_s \) [m] and \( A_s \) [m\(^2\)] stand for the cross-section of the upstream-side passage, respectively.

I developed a design method to prevent static instabilities, that is:

- if possible, it is recommended to ensure the \( S_f < 0 \) condition in the whole valve body displacement range, thus any spring stiffness can be selected, or

- if \( S_f > 0 \) condition occurs for any displacement range, the statically stable operation can be ensured for the whole displacement range by
selecting a spring with \( k \) [N/m] spring stiffness

\[
k > \frac{p_{\text{set}} A_s}{x_{0,\text{max}}}, \quad \text{where} \quad x_{0,\text{max}} = \left( \frac{1}{S_{f,\text{max}}} - \frac{x_{v,\text{max}}}{D_s} \right) D_s
\]

where \( p_{\text{set}} \) [Pa] denotes the set pressure, \( S_{f,\text{max}} \) [-] indicates the maximum value of the dimensionless hydrodynamic spring stiffness characteristics, while \( x_{v,\text{max}} \) [m] is the allowed maximum displacement of the valve body.


2.3 Dynamic instability of a pipe-valve system

In order to explore the dynamic behaviour of a pipeline-valve system I carried out experiments on a hydraulic system consisting of a positive displacement pump, a simple direct spring loaded valve, and a hydraulic hose connecting them (see Figure 1 right configuration). At certain flow rate and spring pre-compression range self-excited vibration of the valve body was observed. Time histories at the two end of the hose and the frequency of the valve body oscillation suggested the presence of a standing pressure wave in the pipeline. I developed a model of a distributed parameter system in order to explore the stability of the system. The mathematical model of the pipe-valve system is suitable for investigating the nonlinear dynamics of the valve system. The global dynamics showed qualitative and quantitative agreement with the experiment.

An interesting outcome was that although the results qualitatively agreed with the ‘no-pipe-model’ presented by Hős and Champneys (2011), the oscillation frequency remained constant for a wide parameter range (both in terms of flow rate and set pressure). Moreover, this frequency coincides with the ones of some pipe harmonic modes, which suggests that the initial valve stability loss (Hopf bifurcation) immediately couples with the internal dynamics of the pipe and the latter dominates the behaviour of the system.

From a more practical point of view, it was clearly seen that there is a critical spring pre-compression below which the valve is unconditionally stable. Qualitatively explaining, the higher the spring pre-compression is, the smaller the valve openings are and the more intense the acoustical feedback inside the pipe is. This critical spring pre-compression can be found by simple linear stability analysis during the design phase.
Thesis # 3

I developed a mathematical model that describes the dynamics of a pressure relief valve system consisting of a pipe and a pressure relief valve with conical poppet valve body and which is suitable for studying the nonlinear dynamic behaviour of the valve system. The model was developed in such way that, while capturing the pipeline dynamics, it also allows linear and nonlinear stability analysis. With the help of the model it was shown that the pipeline dynamics plays an essential role in the development of the instability, hence it cannot be neglected. With the help of the model I also showed that there exists a critical spring pre-compression below which the valve is stable. This critical spring pre-compression can be found by linear stability analysis during the design phase. I proved the model applicability by means of experiments.


2.4 Model reduction of a reservoir-pipe-valve system

Experiment and simulation results suggested that in such valve system where the valve is connected to a tank via a long pipe (see e.g., Figure 1 left configuration), standing quarter pressure wave establishes in the pipe. This has motivated the development of a reduced-order model of the valve system which is yet capable of capturing the dynamics of the system close to the loss of stability. The pipeline dynamics is described by two ordinary differential equations coupled to the reservoir pressure dynamics and the Newtonian equation of motion of the valve body. The model can be employed both in the case of compressible (e.g., air, natural gas) and incompressible (e.g., water, oil) working medium. With the help of the model quick and accurate parameter analysis can be performed. Moreover, applying linear stability analysis the model can be employed to determine the boundary of loss of stability of the system. The applicability of the model was verified in industrial project environment for gas pressure relief valve system.

Thesis # 4

To ease the prediction of the dynamic instability of a pressure relief valve connected to a reservoir via pipe I derived a reduced model (quarter-wave model). The model supposes a standing quarter wave for velocity and pressure distribution inside the pipe. The pipeline dynamics is described by
two ordinary differential equations coupled to the reservoir pressure dynamics and the Newtonian equation of motion of the valve body. The model provides quantitatively accurate solution only close to the equilibrium. The applicability of the model was verified with measurements and with numerical simulations of a model that describes the fluid mechanics more proper in the system. The model can be employed both in the case of compressible (e.g., air, natural gas) and incompressible (e.g., water, oil) working medium. With the help of the model quick and accurate parameter analysis can be performed in the phase of valve design.

**Related publication:** Bazsó et al. (2014b).

### 2.5 Analysis of reservoir-pipe-valve system

A two-parameter bifurcation analysis was performed on the quarter wave model. I introduced a suitable nondimensionalisation which led to the identification of seven dimensionless parameters that completely specify the problem. The two key dimensionless parameters were the mass flow rate into the reservoir and the length of the pipe. It was found that there are two independent forms of Hopf bifurcation; a so-called capacity instability which involves the valve alone and occurs when the flow rate is too low and a coupled acoustic resonance which involves quarter waves within the pipe that occurs when the pipe is too long. Secondary bifurcations are also traced including curves of grazing bifurcations where the valve body first impacts with its seat.

Using a mixture of simulation and numerical continuation, it was found that these instabilities interact in a complex bifurcation diagram that involves Hopf-Hopf interaction, bi-stability through catastrophic grazing bifurcations, and subcritical torus bifurcations for long enough pipes. A particularly important finding is a range of parameters for which the capacity instability is subcritical so that large amplitude impacting chaotic motion coexists with the stable equilibrium operation.

**Thesis # 5**

With the help of quarter-wave model for incompressible flow I showed that as function of the mass flow rate through the valve and pipe length, two forms of dynamics instability independent of each other are present in the system. The first one is the so-called capacity instability which typical of oversized pressure relief valves (lower flow rate than the nominal capacity) and regarding to its mechanism only the valve becomes unstable which then
couples to the reservoir pressure dynamics and excites it. The other one is the so-called quarter-wave instability. In this case the valve-pipe system is the one which becomes unstable and typically occurs when the pipe is too long.

Employing a time domain simulation and numerical continuation, it was found that both instabilities may be present in the system that involves additional, much more complex dynamics, e.g., Hopf-Hopf bifurcation, bistability, or subcritical torus bifurcation. Moreover, I found that for a particular parameter range stable equilibrium and large amplitude impacting oscillation coexist. The later finding suggest the necessity of nonlinear analysis beside the linear stability analysis.

Related publication: Bázsó et al. (2014a).

3 Contribution

3.1 The static stability of poppet valve equilibrium

An example of such undesired valve jumps that is mentioned in Section 2.2 was experienced in an industrial project demonstrated in Figure 2. The figure illustrates an operation sequence of the pressure relief valve: opening,
relieving, and closing. At $t \approx 5\,\text{s}$ the pressure reached the set pressure; the valve opened suddenly. However, as the pressure was further increased a second, undesired jump occurred at $t = 9.7\,\text{s}$ and then two more at $t = 49\,\text{s}$ and $t = 61\,\text{s}$. Those displacement values at which valve jump was expected based on the theory reported in Section 2.2 are demonstrated with dashed lines in Figure 2. Good agreement with the experiment was found.

3.2 Quarter wave model

The findings regarding the quarter wave model were utilised in the course of an industrial project in which the client was interested in the stability map of a particular gas system consisting of a reservoir, pipe, and pressure relief valve similar to that one presented in Figure 1. The stability map of the existing system is shown in Figure 3. A series of measurements were run with different pipe lengths and the stability of the valve was observed for three different mass flow rates. Green circles indicate stable runs, red triangles and squares show the unstable ones. The solid line depicts the boundary of loss of stability predicted by linear stability analysis of the quarter-wave model. It can be seen that the analysis resulted in good estimation for the boundary of loss of stability.

![Figure 3: The stability map of a pressure relief valve system.](image-url)
Own publications


Bibliography


