ACTIVITIES OF RESTRICTED DURATION IN NETWORK TIME MODELS OF CONSTRUCTION PROJECTS

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Abstract
Activities of fixed-, flexible-, intermittent-, distributable-, and unknown duration in general network time models are discussed in the paper on basis of preset lower and upper bounds on their time extent. After a short historical review, we discuss the theoretical background and we introduce a modified Floyd-Warshall algorithm to calculate a generalized network time model (GTM) highlighting differences of well-known techniques of PERT, CPM, MPM and PDM. Finally, we publish a small construction problem to demonstrate practical application and modelling activities of these kinds.

1. Introduction

Computer aided management, dynamic time models, anticipating consequences of any changes in planned performance, being informed on actual progression are unceasing expectations and eternal challenges of modern project management. CPM, PERT, MPM, PDM are well-known acronyms of techniques inseparable of evolution of computers and developed to ease the work of experts responsible for time management of projects on various fields of human life and of economy. Principles of all before-mentioned techniques can be derived from a primal-dual pair of problems in applied mathematics, namely, the problem of finding the Longest Path(s) between two distinguished nodes of a weighted directed graph and the problem of constructing a Minimum Potentials’ system having limitations (lower and/or upper bounds) on pair-wise differences of its elements. Algorithms to solve these problems are usually based on some kind of labelling techniques (such as Dijkstra’s [1] for finding the shortest path(s)), to construct the potentials’ system first, that provides basis for identifying the longest path(s) [4][6][8][9].

Construction management frequently faces situations that time extent of some efforts (activities or sub-projects) are unknown or can be estimated with significant uncertainty or it is not expedient to set them in advance but let them be adjusted to more certain components of the project.

Subject of our research is the ways and tools of integrating these kinds of efforts (activities of flexible duration, activities broken by idle times, activities performed by parallel teams, and activities of unknown duration) in dynamic time model of construction projects.

2. Historical review

At first steps of developing modern computer aided project management techniques Kelley and Walker [6] developed a time model on a particular structure of data that can be demonstrated by a directed weighted graph with one originating node, with one terminal node, with no circular references, and with non-negative weights assigned to the edges. The structure was referred as “network”. By evolution of
computer techniques all before-mentioned restriction on data structure became omissible but the only one that is essential for a valid mathematical solution: no positive circular references allowed. They used AOA (Activity-On-Arrow) correspondence of project time elements and their in-graph representatives. Nodes are establishing direct links between preceding activities (represented by arrows entering the nodes) and succeeding activities (represented by arrows leaving the nodes). Weights of arrows are input values for calculations. The aim is to determine minimum overall timespan of the project, that is, to calculate the length of the longest by-arrow sequence(s) of directed edges (referred as Critical Path) between the originating node and terminating node of the graph (CPM_time model).

Based on fast computer-aided calculations and involving time-cost trade-off of project components focus of time planning could be set on finding an optimal time policy for shortening the overall timespan of the project in a cost-effective way. Method of optimization was derived back to recursive calculations of the network (network time analysis) and changing the input values (weights of activities) step-by-step in a purposeful way for each succeeding recalculation (CPM_cost model). Acceptable ranges of durations were set by upper bounds (normal times) and lower bounds (crash times) while the trade-off functions (cost slopes) between them was assumed to be linear.

Weights (estimates on activity durations) were also in focus of examinations in technique developed for US Navy at the end of fifties of the last Century referred as Program Evaluation and Review Technique (PERT) [7]. Uncertainty of estimating time extent of activities (sub-projects) was integrated in the model on a probabilistic basis (assuming Beta distribution, based on triplets of time estimates for each activity: optimistic, realistic, and pessimistic ones) but the CPM_time typed network time analysis was derived back to a single calculation with deterministic values (expected durations).

AOA typed correspondence of project elements and their graph representatives proved to be insufficient to model mutual dependencies of activities that can be overlapped in time. Reversed correspondence, that is, nodes representing activities of fixed duration and weighted arrows representing technological dependencies, provided tools for modelling any relative time positions of related activities. (Activity-On-Node (AON) correspondence) Assuming linear progression of each activity Fondahl [3] and Roy [9][10] proposed four basic types of relations (Finish-to-Finish, Finish-to-Start, Start-to-Finish, Start-to-Start) to guide relative time positions of succeeding activities. But special combination of these relations together with fixed durations of activities can result in a paradox of durations [5][12][15] and imply negative weights and circular references in the graph model [13].

Vattai and Mályusz [11] approached the network time scheduling problem in a reversed way. Find the longest path(s) and assign time potentials for the individual project components along them. They adapted the well-known Floy-Warshall algorithm [2][14] to determine all-pairs longest path(s) in the weighted directed graph and discarded all restrictions on the graph structure. They recalled the early AOA correspondence of project components and their graph representatives in a generalized way. Arrows are representing time extents (regardless of their technical meaning – activity duration or lead or lag time) and nodes are representing time positions (regardless of their technical meaning – deadline, milestone, or direct link). Applying various bounding schemes on acceptable ranges of activity durations, appropriate values of them can also be a partial output of (and not input for) network time analysis.

This paper aims at discussing practice and proper use of bounding schemes on activity durations to promote modelling some scheduling challenges typical in Construction Management.

### 3. Introducing lower and upper bounds on duration of activities

Activity durations are “bounded” in all network techniques some-how. CPM, PERT, MPM, PDM imply activities of fixed duration only and acceptable increments or delay(s) of their timing is handled by series of float- or slack values (Total Float, Free Float, Independent Float for activities, Slack for events). The novelty of proposed General Time Model (GTM) is that actual values of durations are defined by ranges bounded from down, or from up, or both, or neither – instead of fixed discrete values – while the model keeps calculable. Below we review basic types of activities of limited duration and their application in context of construction projects. Figure 1 shows proposed shields for activities indicating their bounding

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**Figure 1**

Proposed shields for activities indicating their bounding

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schemes and arrangement of time data (Early Start, Early Finish, Duration minimum, Duration, Duration maximum, Late Start, Late Finish) within them. We also introduce a denotation “CR” for a special kind of duplex relation, namely, for Finish-to-Finish (FF) and Start-to-Start (SS) relation together, with the same bounding value for defining technological breaks (succession times) between succeeding activities overlapped in time – regardless of their actual duration. (Graphics below have been created by computer application “GRProject” developed for educational purposes at BUTE DCT&M 2022-2023 to demonstrate different types of network techniques, such as CPM, MPM and GTM.)

3.1. Milestone
An “activity” duration of which is fixed to value “zero” \( (D_{\text{min}}=D=D_{\text{max}}=0) \) and is frequently referred as event or milestone. It has no time extent but time position. Mostly used to indicate special states or fulfilment of some expectations, achieving an intermediate- or ultimate target, or simply indicating a common deadline for some parallely ongoing efforts. Transition of events and activities is an inherent question in all network techniques since duration of activities can be approached as elapsing time between two events (between start and finish – see CPM and PERT activities) and events can be handled as zero-duration activities (see CPM dummy activities and MPM milestones).

3.2. Activity of unknown duration
Also referred as hammock activities [4]. Most of scheduling software applications available on the market (such as Microsoft Project, Gantt Project etc.) are – as considered – provided with tools to integrate activities of this kind in network time models (usually as summary activities) but the comparison is incorrect. In before mentioned applications time positions (starting and ending times) of these activities are not integrated in the network model but are projected from results of network time analysis. The need for integrating activities of this kind in the network time model emerges when availability of some resources assigned to them is essential consideration of the project. Such as: tower cranes, dewatering pumps, guarding-, occupying (renting) public areas etc. We can define the first time we need them during execution of the project, and we can define the last time too. Actual period of need for them is determined by other activities in the time model. Their improper use can lead to a strange situation – as an outcome of network time analysis – that the activity should be finished before it has been started (?!). It can be a true blunder – or can be an indication that progression of the activity (sub-project) would be better.
scheduled in opposing direction than it had been assumed in advance. (This later is easier to interpret at linear structures such as bridges, tunnels, retaining walls, transmission lines, highways etc.)

3.3. Intermittent activity

Activity performance of which can be paused and restarted any times as needed in the execution period. Its minimum time extent can be estimated accurately but idle times are allowable during its completion. The range of acceptable duration is set by a lower bound only ($D_{\text{min}} \leq D$) based on the estimated time needed to perform the whole activity without any break. Appropriate range of idle times and the whole timespan to complete the activity can be derived from output of network time analysis. Losses due to idle times in periods of on-site works are essential problems. Assigning alternative jobs for the teams performing these activities can be an effective way of reducing them. Such as: formwork in, formwork out or temporary support in, temporary support out at concrete works of structures like retaining walls, bridges, tunnels, high-rise buildings, etc. Workers are performing more jobs during the execution period in an alternating way ("alternating team").

Figure 3 shows an example of applying intermittent activity and explains principles of turning primary results of time analysis (e.g.: early starts, early finishes) to detailed schedule of its performance. (Idle times are spread in time evenly.)

Relations 1-4 below can be used to prepare a detailed schedule of intermittent activities.

\[ D = EF - ES \] (1)

\[ n = \min \{ D_{\text{min}} \cdot D - D_{\text{min}} \} \] (2)

\[ S = \frac{D}{n} \] (3)

\[ D_s = \frac{D_{\text{min}}}{n} \] (4)

Where: $D$ = scheduled timespan (Duration) to complete the activity  
$ES, EF$ = Early Start and Early Finish of the activity, output from network time analysis  
$n$ = number of periods of breakless performances  
$S$ = delay (Step) time for starting succeeding period of breakless performance  
$D_s$ = Duration of performing one breakless period
3.4. Distributable activity

A strange and to be handled carefully type of activities – but not far from construction reality. Significant expensive and unacceptably slow at construction site, and usually “accelerated” by mobilizing more parallel teams and/or machines to perform them. The number of mobilized teams/machines are usually set in advance, but – in an accepted range – can be derived from results of time analysis of the network time model. Range of acceptable duration is set by an upper bound \((D \leq D_{max})\), based on the estimated time needed to perform the whole activity by one team/machine. Since the minimum side is not set in advance, in case of uncarefully constructed time model it may occur that the activity should be finished before it has been started(?!)) – that is an evident mistake. So, both the start and the finish of these activities should be guided properly by the related activities in the time model.

Figure 4 shows an example of applying parallel teams and explains principles of turning primary results of time analysis (e.g.: early starts, early finishes) to detailed schedule of their performance.

Relations 5-9 below can be used to prepare a detailed schedule for parallel teams.

\[
D = EF - ES \\
C = \left[ \frac{D_{max}}{D} \right] \\
n = \frac{C \cdot (D_{max} - D)}{C \cdot D - D_{max}} + 1 \\
S = \frac{D}{n + C - 1} \\
D_s = \frac{D_{max}}{n}
\]
Where:  
\( D \) = scheduled time frame (Duration) to perform the activity  
\( ES, EF \) = Early Start and Early Finish of the activity, output from network time analysis  
\( C \) = number of parallel teams (Capacity) to be mobilized to perform the activity (rounded up)  
\( n \) = number of sections the activity is to be distributed (segmented) among  
\( S \) = delay (Step) time for succeeding parallel teams to start their sections  
\( D_s \) = Duration of performing the activity by one team in one section  

Remark: relations 5 - 9 are applicable if \( n > 2 \) and \( D_{max} \neq k \cdot D \) where \( k \) is an integer

### 3.5. Flexible activity

Also referred as stretchable activity. Scheduled or expected duration of the activity is restricted to an accepted range set by a lower and an upper bound \((D_{min} \leq D \leq D_{max})\). Though the rest of the activities in construction practice is of this type, most of the network techniques do not provide a tool to involve them in the time model. Assigning ranges of acceptable duration values for the individual activities makes possible to determine the best fitting durations by the network time analysis itself [8]. That is: actual durations of activities are expected outputs of network time analysis.

### 3.6. Activity of fixed duration

Scheduled or expected duration of the activity is well known and its technical content is specified exactly. Both the lower- and the upper bound of the time extent equals to the expected duration \((D_{min}=D=D_{max})\). In MPM and PDM techniques it is the basic type of activities, and their durations are pre-set input values of network time analysis. Due to the manifold relations allowed in MPM/PDM techniques consequences of any changes of duration of critical activities of this kind can be forecasted via thorough analyses of their dominant relations [5][12][15].

### 4. Network time analysis

In a General Time Model relative time position of each project component (activity or milestone) is defined by a pair of time potentials – one for its start \((s)\) and one for its finish \((f)\). Relations 10-11 are for calculating indices of time potentials of a project component \(m\).

\[
s = 2 \cdot m - 1 \quad \text{(start potential)} \quad (10)
\]

\[
f = 2 \cdot m \quad \text{(finish potential)} \quad (11)
\]
All restrictions on differences of time potentials are transformed to a single or to a duplex of lower bound typed limitation(s) (See relations 12-14).

\[ \pi_j - \pi_i \geq \tau_{ij} \quad \text{(lower bound)} \quad (12) \]

\[ \pi_j - \pi_i \leq \tau_{ij} \equiv \pi_i - \pi_j \geq -\tau_{ij} \quad \text{(upper bound)} \quad (13) \]

\[ \pi_j - \pi_i = \tau_{ij} \equiv (\pi_j - \pi_i \geq \tau_{ij}) \cup (\pi_i - \pi_j \geq -\tau_{ij}) \quad \text{(fixed value)} \quad (14) \]

Where: \( \pi_i \) and \( \pi_j \) are related time potentials in potentials’ system
\( \tau_{ij} \) is the bounding value set on difference of time potentials \( \pi_i \) and \( \pi_j \)

Potentials are assigned to nodes of the graph, and restriction(s) on pair-wise differences of potentials are represented by weighted directed edges – where weights are the bounding values themselves.

A modified Floyd-Warshall algorithm can be used to prepare transitive closure (\( A^n \)) of the structure matrix (\( A^0 \)) of the weighted directed graph. Cells of the former contains length of the longest path(s) (if any) between all pairs of nodes. Figure 5 shows a Delphi routine for the modified Floyd-Warshall algorithm.

<table>
<thead>
<tr>
<th>Preparing (( A^0 )) matrix</th>
<th>Preparing transitive closure (( A^n )) of structure matrix (( A^0 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>for ( i := 1 ) to ( n ) do</td>
<td></td>
</tr>
<tr>
<td>for ( j := 1 ) to ( n ) do begin</td>
<td></td>
</tr>
<tr>
<td>( a[i,j] := w[i,j] );</td>
<td></td>
</tr>
<tr>
<td>{ … }</td>
<td></td>
</tr>
<tr>
<td>end;</td>
<td></td>
</tr>
</tbody>
</table>

\* \( a[i,j] := w[i,j], \) if \((i,j) \in E; a[i,j] := M \) otherwise

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For more details see Vattai 2016 [11].

5. Sample project

To demonstrate application of above-mentioned activities here we cite some details of a small project to be elaborated by students of civil engineering studies at Budapest University of Technology and Economics. The job is to prepare time and resource estimates of building a RC retaining wall at a fictive highway correctional project. In our example height of the wall is 4 m and length of it is 400 m.

According to the assignment: time and resource estimates are to be based on calculations including quantities, standards/outputs of performances and available capacities. Quantity of formwork can cover...
maximum 100 m long a section of the structure. Progressions of works are to be synchronized in time as much as can be.

Figure 6 shows a schematic cross-section of the structure. GTM network time model of the project can be seen in Figure 7. Detailed schedule of works is presented in form of a linear schedule in Figure 8.
Sheet-wall piling is represented by a distributable activity in the model (ID3 - red lines). Formworks are modelled as alternating jobs for teams performing them (ID9 and ID11 - green lines; ID15 and ID17 - blue lines). Trench refill of small quantity along the foundation is inserted in the model as activity of unknown duration (ID12 – dotted line in ochre). Progression of removing sheet-wall is represented by a flexible activity scheduled duration of which is also an output of GTM time analysis (ID20 - red line). Other jobs are built in the time model as activities of fixed duration (grey progression lines). (Summary activities and milestones are not indicated in the linear schedule.)

6. Conclusions

After a short historical review of network techniques, we introduced a General Time Model applicable to construct network time models including activities ranges of acceptable durations of which are set by lower and/or upper bound values. Via technical examples and via a small theoretical road correctional project we demonstrated their use in context of Construction Industry. We highlighted main differences of traditional network techniques used to develop dynamic time models of different types of projects and we also pointed at the common mathematical background of them. The proposed GTM model is based on calculations of all-pairs longest paths by a modified Floyd-Warshall algorithm. Research is going on to develop even more efficient methods and to overcome disadvantages of admittedly time and memory consuming Floyd-Warshall algorithm, with special consideration of sparse graphs, that is a characteristic feature of network time models typical in construction practice.

6. References